To Build or Not to Build? Capital Stocks and Climate Policy*

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Abstract

Applying climate policy in practice means considering capital stocks: some assets will produce pollution whenever they are used, and some will not. Therefore long-term abatement plans should influence current investment. Moreover, newer technologies exhibit learning-by-doing in the deployment of associated infrastructure. We investigate these ideas from both theoretical and numerical perspectives. An increasing carbon tax will reduce investments in assets that pollute, and so reduce emissions in the short term: our "irreversibility effect". As such the Green Paradox has a converse if we focus on demand side capital stock effects. We also show that the optimal subsidy increases with the deployment rate: our "acceleration effect". Considering second-best settings, we show that, although carbon taxes achieve stringent policy targets more efficiently, subsidies to the "renewables" sector deliver higher welfare when policy targets are more mild.

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1 Introduction

How soon should we ensure that all new investments are "green"? In particular, what is the optimal time to stop investment in fossil-fuel-based power plants? The world continues to make big investments into their construction, particularly coal-based plants: estimates suggest that almost 1 trillion US dollars of such investments are planned (Shearer et al. 2016). Given the long lifetimes of fossil fuel based power plants, the emissions embodied in this infrastructure potentially undermine stringent long-term climate objectives, such as the 2°C target (see Pfeiffer et al. 2016). As such, a fast coal phase-out strategy is considered as one of the necessary conditions to achieve a transformation in line with the Paris Agreement. Some countries such as the UK, Finland and France have significantly reduced their power production from coal in recent years and announced the phasing out of coal completely in the coming 10-15 years. In addition, production of electricity from renewable sources has become more competitive, expanding dramatically, primarily due to the decline in costs driven by learning-by-doing effects. These considerations prompt another key question: how much should we invest in the clean energy sector?

Economists advocate a withdrawal from polluting sectors driven by carbon pricing. And investments in the clean energy sector are driven by subsidies. So a natural third question is to ask: which of these policy instruments (carbon tax or subsidy) is more efficient in terms of maximizing social welfare in a second-best setting, where only one instrument is available?

In this paper, we study these questions both theoretically and numerically. Our analysis is in two complementary parts. First, we explore the properties of irreversible investment decisions (Arrow 1968, Arrow and Kurz 1970, Greenwood et al. 1997) in a simplified and very general model. Since irreversibility is an important feature of almost any investment decision, the model presents messages that are of a general nature and provide intuition. The model characterizes optimal irreversible investment decisions when it is known that returns on this capital are due to fall. This has important implications for investment decisions even without uncertainty.[1] And similarly, we explore investments, returns and optimal subsidies when the price of investments undergoes learning-by-doing (Wright 1936, Arrow 1962).

We then quantify the importance of irreversibility and learning-by-doing in a dynamic general equilibrium climate-economy model. This is based on DICE (Nordhaus 2014a) but deviates in two important ways. Firstly, the energy sector is modeled explicitly, incorporating both irreversibility in a "dirty" sector and learning-by-doing in a "clean" sector. And secondly, using the damage function of Nordhaus (2014a), we also consider scenarios in which global temperature changes do not exceed 2°C. This stringent target makes both the irreversibility and the learning-by-doing more important, and it is more in line with current international aspirations. Given the two externalities present in our model (global warming and learning-by-doing), we consider cases in which both a carbon tax and subsidy instruments (the first-best setting) or only one of the two instruments (the second-best) are available.

The four main findings of the paper are as follows. First, we establish a theoretical result for the relationship between climate policies and investment in dirty capital stocks, which we call the "irreversibility effect": if dirty capital cannot be converted to other capital, then it is optimal to stop investing in dirty capital earlier (compared to a case in which investment is reversible). Irreversibility in investment implies an earlier shift to investment in the clean sector, to avoid a future stranding of assets in the dirty energy sector. This shift therefore reduces emissions in the short term. We thus demonstrate that irreversibility effects on the demand side *enhance* the effects

[1] Irreversibility of investment features prominently in the modern theory of firm-level investment under uncertainty, e.g., Abel (1983), Pindyck (1991), Dixit (1992), and in the "putty-clay" framework of Atkeson and Kehoe (1999).

of a carbon tax in the short term. This is in contrast with the standard Green Paradox (GP) effect,[2] which focuses on the *suppliers* of a fossil fuel resource and shows that the knowledge of an increasing carbon tax will increase extraction of fossil fuels and will thus *counteract* the effects of the carbon tax in the short-term. Moreover, at the time at which investment in dirty fossil fuel infrastructure stops, returns on its existing assets go above those of the general economy. From the perspective of an investor, this result makes perfect sense. In the long-term, returns on this investment will fall, and thus current investments are only attractive when short-term excess returns (relative to those of the general economy) are sufficient to compensate for future losses.[3]

Second, we provide a simple expression for the optimal subsidy on technologies whose price evolves as a result of "learning-by-doing". This subsidy increases with depreciation and the learning rate, and also with the growth of capital in this sector. We call this the "*acceleration effect*" for technology policy. Thus suppose, for example, that a carbon tax on substitute sectors makes the clean technology more competitive and so enhances growth in this sector. Due to the acceleration effect, the subsidy for this sector should *also* increase (despite the fact that the market is already more favorable). So the importance of learning-by-doing is accentuated by the early withdrawal from the dirty energy sector.

Third, our quantitative results support our theoretical findings and illustrate that the net (of depreciation) rate of returns on dirty capital infrastructure with irreversible investments follows an unusual trajectory: while initially matching the returns in the general economy, it rises above the returns in the general economy when investment in dirty capital stops, and remains higher for some period of time. Within this period and for some time thereafter, investment in dirty capital will be equal to zero, although the dirty capital is not underutilized. However, net returns on dirty capital will fall eventually, reaching zero once this capital is underutilized. Quantitative results illustrate that the timing of these effects depends on the climate policy target: the irreversibility effect is present only if policy objectives are stringent enough.

We also observe the acceleration effect on the optimal subsidy when we compare mild to more stringent targets. This is because with more stringent targets, the renewable sector grows faster, and consistent with our theoretical result, the subsidies must be larger. We also find that with higher values of the learning rate, there is faster growth in the renewable sector in the current period compared with later decades (for instance, 2050 or 2100), suggesting that there is less need to stimulate renewable sector growth in later periods once the clean technology matures.

Finally, we quantitatively explore which instrument – carbon tax or subsidy – under the second-best setting yields a lower welfare loss compared with the first-best situation. We show that under a less ambitious climate policy, the economy is better off with the subsidy, while carbon pricing induces a lower welfare loss compared with the subsidy if climate policies are more ambitious.

These results further relate to the literature in the following ways. On the theory side, the irreversibility and the acceleration effects are novel, to the best of our knowledge, and are related to two branches of the literature. First, our theoretical result linking investment irreversibility and an earlier end to investment in polluting infrastructure is closest in spirit to the findings of Arrow (1968), who was the first to study investment irreversibility in a deterministic setting. He showed that optimal irreversible investment is characterized by alternating periods of positive gross investment and zero gross investment.[4] In relation to these studies, we develop a stylized model to explicitly

[2]See e.g., Sinn (2008), Jensen et al. (2015), Sinn (2015).

[3]This extra premium on irreversible investment even without uncertainty is also called the commitment premium, see for example Bernstein and Mamuneas (2007).

[4]A similar result, but within the Ramsey model of optimal capital accumulation, was obtained by Arrow and Kurz (1970), who show that an optimal solution can have any number of alternating intervals in which the non-negativity constraint (on investment) is binding or not. Such solutions, they conclude, become in essence a computational

demonstrate this effect and related patterns in the rates of return on irreversible investments and apply our results to the case of a polluting industry. Second, there is an extensive literature[5] that has explored the robustness of the Green Paradox effect by considering various extensions of its typical underlying resource model. Even though the irreversibility effect complements other mechanisms against the GP discussed in the literature, it is conceptually different as it focuses on the *demand* side of a fossil fuel resource.

On the quantitative side, our results relate to other two strands of research. On the one hand, there is an extensive literature that investigates relative merits of carbon taxes and renewable subsidies to address climate change.[6] However, these studies generally abstract from consideration of different climate policy targets under second-best settings with irreversible investment decisions.[7] On the other hand, a rich and growing literature has developed integrated assessment models to study a number of different climate change issues. Papers assessing future emissions from the energy sector include Pfeiffer et al. (2016) and Davis et al. (2010). However, these do not use the dynamically optimizing frameworks of the economics literature. Other climate-economy models generally ignore the interplay between irreversible investment decisions, inertia in energy systems, and climate policies, on which this paper focuses.[8]

Finally, our paper belongs to the literature on path dependence and climate change.[9] We contribute to this literature by analyzing the implications of path dependence embodied in carbon-intensive infrastructure for the design of optimal climate change policies.

In terms of broader implications of our results, this paper contributes to the debate on characteristics of optimal policy to combat climate change. Some advocate a "gradual slope" in policy implementation because economic growth implies that the current generation is poor relative to

problem.

[5]For instance Gerlagh (2011) focuses on the strong and weak GP and explores whether increasing fossil extraction costs counteract the (strong) GP, and whether imperfect energy substitutes may make the weak and the strong GP vanish. Michielsen (2014) investigates how the existence of a virtually non-exhaustible resource like coal can work against the GP mechanism. Smulders et al. (2012) show how an announced carbon tax can, through anticipation of higher future energy prices and thus lower production, lead to higher emissions in the short run. See also van der Ploeg (2013) and van der Ploeg and Withagen (2014).

[6]For instance the literature has argued that one of the advantages of using carbon pricing is that it can help to minimize the cost of pollution control. Fischer and Newell (2008) show that reliance on non-price policy instruments often leads to higher abatement costs. In a more recent study, Fischer et al. (2017) show that even with multiple market failures, a pricing policy remains the most cost-effective option for reducing emissions. See also Gerlagh and van der Zwaan (2006), who use a top-down energy-economy model to compare five instruments, including carbon taxes and renewable subsidies, in terms of costs, efficiency and their impacts on the composition of the energy supply systems. See also Baranzini et al. (2017) and references therein.

[7]A burgeoning theoretical literature investigates the forms of policy interventions in second-best settings. Examples include an analysis of optimal carbon taxation as part of distortionary fiscal policy (Barrage, 2014), policy intervention via carbon taxes and research subsidies as well as alternative policies to encourage the transition to a green economy (Acemoglu et al., 2016), and an analysis of how carbon taxes combined with green alternatives can increase the abandonment of fossil fuels (Rezai and van der Ploeg, 2016). None of these studies, however, has analyzed optimal policy interventions when investment decisions are irreversible.

[8]An exception is Rozenberg et al. (2017), who examine the trade-off between efficiency and political feasibility of climate change mitigation policies (in terms of avoiding stranded assets) and assess costs and dynamics of the transition from polluting to clean capital using different instruments. They compare the first-best carbon price and the second-best feebates or mandates designed to minimize costs while avoiding stranded assets. However, by focusing on a different set of questions, they do not find the effects pertaining to the irreversibility and learning-by-doing as we do in this paper; for example, their model implies that all investment in the dirty sector will cease as soon as climate policy is implemented. Their analysis is closely related to Williams (2012), who develops a stylized model of environmental regulation to argue that phased-in carbon prices reduce costly capital adjustments.

[9]See Fouquet (2016); Aghion et al. (2014), and Aghion et al. (2016). The papers relevant to our analysis are Grubb et al. (1995), Wigley et al. (1996), Grubler and Messner (1998), Goulder and Mathai (2000), Vogt-Schilb et al. (2012) and Rozenberg et al. (2017).

future generations, and so should not bear large costs of emission reductions. Moreover, doing so reduces pressure for premature retirement of the existing dirty capital stock, and it provides valuable time to develop low-cost, low-carbon-emitting technologies.[10] Others counter this line of reasoning by arguing that an effective way to reduce abatement costs is to accelerate learning-by-doing.[11] We find that early investment in the renewable sector is crucial, not only to accelerate the decline in the costs of clean energy, but also to prevent later stranding of assets that use fossil fuels. Our quantitative results within the second-best setting (with only one policy instrument available) emphasize the importance of adopting carbon pricing, an instrument that can facilitate a rapid decarbonization of the global power sector under an ambitious climate policy target like that set under the Paris Agreement. However, considering the past 10-20 years, relatively unambitious policy has manifested itself in a large part through subsidies on renewables; if that less ambitious level of emission reductions had been optimal, that single choice of instrument may well have been an excellent second best. Our quantitative results under a second-best setting where carbon pricing action is restricted by political economy issues illustrate that the subsidy needs to be higher (if the carbon tax is lower than the optimal first-best level) to compensate. In later periods once renewable technologies reach maturity, there is no need to maintain the same level of subsidies. Thus, the subsidies help to overcome political constraints in the short-run to reach the climate target, and help to develop the renewable sector to play its role in the economy in the longer term.

Finally, our paper speaks to the debate on stranded assets and climate policy.[12] The literature so far has been dominated by studies that estimate the amount of existing fossil fuel reserves to remain in the ground to limit climate change to less than two degrees of warming. For instance, according to McGlade and Ekins (2015), an estimated one-third of oil reserves, half of gas reserves and more than 80% of known coal reserves are referred to as "stranded". As we show, the implications for investment are different when one considers the stranding of assets that use the fuel.

The rest of the paper is organized as follows. In the next section we present a simple analytical model in which we characterize optimal irreversible investment decisions when the anticipated returns on those investments will fall in the future. In Section 3 we consider a simple model of investment with learning-by-doing. Section 4 describes how we set up the full dynamic general equilibrium climate-economy model to quantify the theoretical results. Section 5 sets out the results from the simulations of the climate-economy model. Section 6 provides a discussion and the final section concludes. Details on the calibration, and proofs of technical results, are provided in the Appendices.

2 A Simple Model of Irreversible Investments

First we model key features of the economy in isolation, in order to clearly present the theoretical results. The models analyzed here will be embedded in our full structure in Section 4. We consider the implications of irreversibility in investments in capital stocks whose economic productivity will

[10] W. Nordhaus was one of those in the past who recommended "gradual slope", but he recently argued that a target with a limit of 2°C "appears to be unfeasible with reasonably accessible technologies" (Nordhaus 2016). Wigley et al. (1996), among others, argue that the cost-effective emissions pathway is one that departs only gradually from the emissions baseline.

[11] Still, some authors find that learning-by-doing has an ambiguous impact on the timing of emissions abatement (Tol 1999, Goulder and Mathai 2000).

[12] Caldecott (2017) argues that the term "stranded assets" has been used to describe various situations. We follow Caldecott et al. (2013) and define stranded assets as assets that have suffered from premature write-down before the end of their technological life.

decline (cf. Arrow 1968, Arrow and Kurz 1970).[13] Our key example is investment in fossil-fuel-using power stations, but as discussed earlier, many other illustrations exist, such as the structure of transportation systems and the design of cities.

Underlying the results we derive is the very simple point that for investment in a sector to be worthwhile, the net present value returns from that sector must match those available in the wider economy. It is insufficient for these returns to match only in the period in which investment would take place.

2.1 The Household's problem

Consider a representative household, which holds (invests in) k_t of a certain capital asset and can make an additional irreversible investment of i_t in each period t. The irreversible investment asset offers a period-t return of r_t and depreciates at rate δ. There are other sources of income, written as o_t, and the household's per-period consumption is c_t, so their budget constraint is $i_t + c_t = r_t k_t + o_t$ where $i_t = k_{t+1} - (1-\delta)k_t$ and $i_t \geq 0$.

Write the standard ratio from the Euler equation as $e_{t+1} := \frac{u'(c_t)}{\beta u'(c_{t+1})} - 1$ where u is a utility function and β is the utility discount factor. Make a minor assumption that there exist $\epsilon > 0$ and $R \gg 0$ with $-\delta + \epsilon < e_t < R$ for all t, that is, e_t is bounded and bounded away from minus depreciation, $-\delta$.

We now analyze this model (proofs are provided in Appendix A). First:

Proposition 2.1. *For any $s_0, s_1 \in \mathbb{Z}_+$, investment $i_t > 0$ holds for all $t \in \{s_0, \ldots, s_1\}$ only if $r_t - \delta = e_t$ for $t \in \{s_0 + 1, \ldots, s_1\}$.*

To understand this intuitively, suppose there is an asset, "general capital", in which there is non-zero investment in every period, and whose rate of return r_t^g may be treated as exogenous. Then $r_t^g - \delta = e_t$ for all $t \geq 1$ (the Euler equation). So for non-zero investment in two assets (the "general capital" and another assets) over a time period, their net returns must match.

The obvious point of Proposition 2.1 contrasts with the following, more interesting case. Suppose the net return from the irreversible investment asset drops *below* e_t at some time t: changing economic conditions mean that this asset is no longer as productive as it was. Then, compared to the case where net returns do not drop below e_t at time t, we stop investing at an *earlier* time, and reap *excess* returns on the irreversible investment for some of the intervening period. Write $\Delta_{t,s} = \prod_{s'=1}^{s} \frac{1}{1+e_{t+s'}}$ for the compound consumption discount factor. Then

Proposition 2.2. *Suppose that $i_0 > 0$ and $r_t - \delta < e_t$ for $t \in \{s_1, \ldots, s_2\}$. Then there exists a time*

[13] Irreversibility is a critical element of the model because it makes some investments obsolete due to changes in the rates of return caused by stringent climate policy targets. This situation when the equipment, once installed, is no longer viable in economic terms can be modeled in a "putty-clay" framework. This framework assumes that dirty-electricity-producing firms can turn variable raw capital that is malleable ex ante ("putty"), into capital goods with certain technological characteristics, including energy efficiency. However, once the choices have been made, and the coal-fired plant is in operation, there is a fixed factor ratio, or frozen structure ("clay"). This property of capital structures implies that the plant will either be fully utilized or scrapped at some point in time when it loses its economic value. The relevant literature that uses putty-clay productions function are Atkeson and Kehoe (1999) and Casey (2017). The putty-clay framework should allow simultaneous use of old and new vintages and is particularly useful in explaining phenomena like entry and exit of firms, especially through the fact that equipment has become obsolete in an economic sense, or to explain drives of changes in final-use energy intensity in the US as done by Casey (2017).

$s_0 \leq s_1 - 1$ *such that* $r_{s_0} - \delta > e_{s_0}$ *and* $i_t = 0$ *for* $t \in \{s_0, \ldots, s_2 - 1\}$. *Moreover,*

$$\sum_{s=1}^{s_1-1} (1-\delta)^{s-1} \Delta_{0,s}((r_s - \delta) - e_s) \geq \sum_{s=s_1}^{s_2} (1-\delta)^{s-1} \Delta_{0,s}(e_s - (r_s - \delta)) \tag{1}$$

Thus the net returns $r_t - \delta$ from this asset follow an unusual trajectory: initially matching the consumption discount rate path e_t, or the net returns from a "general capital" asset, we see that $r_t - \delta$ rises above e_t at some point before it falls beneath. Investment is zero while returns follow this pattern. (This pattern is illustrated in Section 5, Figure 2).

It is mathematically possible that the minimal time s_0 found by Proposition 2.2 satisfies $s_0 = s_1 - 1$: investment i_t merely ends one period before the net economic return drops below the general level in the economy. However, because equation (1) must hold, such a solution would be surprising in reality if the time periods (and thus the depreciation rate) are moderately small. Recall from Proposition 2.1 that while investment is ongoing, the net return on the asset must match e_t. So the only non-zero term on the left hand side of equation (1) would be for $s = s_1 - 1$. The excess of $r_{s_1-1} - \delta$ above e_{s_1-1}, in that period alone, would have to be large enough to compensate for the entire (discounted and depreciated) sequence of periods $s \in \{s_1, \ldots, s_2\}$ in which $r_s - \delta < e_s$. If there is only a moderate change over time in both r_s and e_s, and if depreciation and discounting are also moderate, then returns will have to rise, and investment will have to cease, at some earlier point in time.

From the perspective of an investor, these short-term excess returns make perfect sense. If the investor knows that, in the long-term, returns on this infrastructure will fall, then it is not an attractive investment. However, the prospect short-term additional gains will compensate for long-term losses. These short-terms gains will indeed be realized if all other investors are similarly ending investment early.[14]

Both Propositions 2.1 and 2.2 follow straightforwardly from a technical lemma (Lemma A.1 in Appendix A) that presents the shadow price on the irreversibility constraint, $i_t \geq 0$ as the net present value of investment in this asset, relative to the opportunity cost.

Now we consider what this means for the quantity of total holdings of this irreversible asset. If there are L_0 identical households in the economy, each of size $\frac{L_t}{L_0}$, then the investment behavior of Proposition 2.2 simply scales up. We use capital letters to denote total capital K_t and total investment I_t for the irreversible asset. We assume, as is implied by standard models, that in each period, returns r_t are monotone strictly decreasing in capital stock K_t. So the pattern of investment implied by Proposition 2.2 implies *a short-term decrease in the dirty energy capital stock, relative to a world in which investments are reversible (and so underutilization is never an issue).*

To explore this, consider an otherwise identical model in which we relax the constraint $i_t \geq 0$ – allowing holdings of this capital stock to be converted back into cash for consumption or other purposes. We use tilde to refer to variables in this modified model ($\widetilde{K}_t$, $\widetilde{I}_t$, and so on). We suppose that e_t is unchanged by relaxing the constraint $i_t \geq 0$, because the sector concerning the irreversible asset is very small in relation to the rest of the economy.

Corollary 2.3. *Suppose that* $I_0 > 0$ *and there exists* $t_1 \geq 1$ *such that* $\tilde{I}_{t_1} < 0$. *Then there exists* $t_0 < t_1$ *such that* $I_{t_0} < \tilde{I}_{t_0}$ *and* $K_t < \tilde{K}_t$ *for* $t \in \{t_0 + 1, \ldots, t_1\}$.

[14]Our results can be illustrated with a historical example, for which we are grateful to Roger Fouquet. In the first half of the 19th century, the introduction of steam engines brought cheaper and more comfortable medium and longer distance travel than had previously been provided by stagecoaches (pulled by horses). Coach companies responded to this heightened competition from railways by ceasing investment into equipment and horses, driving their prices even higher, which inevitably accelerated the transition to railways (Fouquet, 2012).

In the short term, less is invested in the irreversible capital stock, relative to a world in which investments are reversible.

2.2 The Irreversibility Effect in Climate Change Economics

In this paper we apply the observations of Section 2.1 to a model of climate change economics. We are particularly concerned with capital investments in installations, such as coal fired power stations, which will burn fossil fuels. The quantity of fuel demanded, and burnt, is associated with the quantity of appropriate capital infrastructure available and in use. If, in the extreme case, this relationship is Leontief, then Corollary 2.3 implies:

Corollary 2.4. *[**The irreversibility effect**]Suppose emissions are directly proportional to the utilized fraction ζ_t of installed infrastructure that uses fossil fuel. Assume that investment in this infrastructure is non-zero in the first period, but there exists a time $t_1 \geq 1$ such that this infrastructure would be globally divested if it could be. Then, for some period leading up to t_1, emissions are below the level they would reach if divestment were possible.*

That is, capital stock effects on the *demand* side for fossil fuels *enhance* the effect of a carbon tax in the short term.

This result contrasts with those of the Green Paradox from Sinn (2008), who considers a shift from a business-as-usual regime to one with carbon pricing which increases over time, with reversible investments. Fossil fuel *suppliers* respond by accelerating extraction of their fossil fuel stocks, potentially increasing emissions in the short term. But if we take a model demonstrating the Green Paradox and alter it to accurately reflect irreversibility in investments that *use* this fossil resource, we project lower short-term emissions in the carbon-pricing scenario. Those who *demand* the fossil resource are concerned with the long-term profitability of their investments, and a reduction in investments in fossil-fuel-using assets leads to a reduction in emissions.

Thus, irreversibilities have opposing implications depending on whether we consider suppliers, or demanders, of fossil fuel. It is important to bear this distinction in mind when considering the question of stranded assets. The specification of a model will determine which effect will dominate, by showing the net of the demand-side and supply-side effects on the level of emissions.

The proof of Corollary 2.3 showed that incorporating irreversibility also means that there will be greater holdings of that asset at some date after t_1. However, it does not necessarily follow that emissions will be greater after this date, because we allow underutilization of capital stocks.

3 A Simple Model of Investing with Learning-By-Doing

Learning-by-doing is often cited as a rationale for subsidizing renewable electricity. The theory of learning-by-doing is motivated by simple observation: production performance (either in the form of productivity or cost reduction of technology) tends to improve with the accumulation of experience. We are particularly interested in the form that was specified by both Wright (1936) and Arrow (1962): each doubling of cumulative deployment reduces prices by the same factor, known as the "learning rate".[15] Empirically, the existing literature has found substantial evidence that the

[15] Wright (1936) was the first one to describe the concept of learning, after observing a uniform decrease in the number of direct labor hours required to produce an airframe for each doubling of the cumulative production of the plant under consideration.

price of renewable energy evolves in this way, although a causal relationship has not been finally established.[16]

To model this, we consider stocks of a 'learning-by-doing (LBD) asset', H_t (writing h_t for household-level holdings as before). The notation reminds readers that the LBD asset embodies human capital in the form of knowledge, as well as the infrastructure itself. The form of this knowledge is embodied in the price p_t^H of installing this infrastructure, such that $i_t^H = p_t^H(h_{t+1} - (1-\delta)h_t)$. This price depends on the total installed capacity H_t which aggregates individual holdings h_t. Thus $p_t^H = G(H_t)$. Of particular interest is Wright's Law: there exists a constant $\lambda > 0$ with

$$p_t^H = G(H_t) = p_0^H \left(\frac{H_t}{H_0}\right)^{-\lambda}. \tag{2}$$

.

Learning-by-doing gives rise to an externality. So we first explore the optimal program of investment found by a social planner. We contrast this with the behavior of households who act as price-takers, to better identify and understand the optimal subsidy.

3.1 The Social Planner's Case

The social planner optimizes total welfare $\sum_{t=0}^{\infty} \beta^t L_t u\left(\frac{C_t}{L_t}\right)$, where L_t is the population size and C_t is total consumption. This is subject to the investment equations $I_t^H = p_t^H(H_{t+1} - (1-\delta)H_t)$; the investment bounds $I_t^H \geq 0$; the price evolution given in (2); and the budget constraints $I_t^H + C_t = f_t(H_t, O_t)$, where we have written $O_t = L_t o_t$ for the economy-level aggregate of "other" incomes, so that we can write $f_t(H_t, O_t)$ for the production function. The planner will treat O_t as exogenous, which is a harmless assumption if all externalities in the remainder of the economy have been internalized.

Define the *direct return* on investments in the LBD asset to be $r_{t+1}^s := \frac{1}{p_{t+1}^H} \frac{\partial}{\partial H_{t+1}} f_{t+1}(H_{t+1}, O_{t+1})$: that is, we account for the price of investments.[17] Because this investment price changes over time, the direct return does not reflect the full value to the social planner of investments in our LBD asset. So we use the discrete time version of the definition of Jorgenson (1967) to define a *shadow return* on these investments:

$$R_{t+1} := \frac{\mu_t^H - \beta(1-\delta)\mu_{t+1}^H}{\beta u'(C_{t+1}/L_{t+1})} \tag{3}$$

Here, μ_t^H is the shadow price on the investment equation $I_t^H = p_t^H(H_{t+1} - (1-\delta)H_t)$, and so gives the total shadow value of marginal investments in the LBD asset. To find the return realized in period $t+1$, we subtract the discounted depreciated shadow value going further forward. As usual, everything is measured relative to the marginal value today of consumption tomorrow.

To show how natural the shadow return is, and relate it to the direct return, we write again $e_{t+1} := \frac{u'(C_t/L_t)}{\beta u'(C_{t+1}/L_{t+1})} - 1$ and show:

[16] Lindman and Soderholm (2012) use aggregate data and show that learning externalities are present in wind turbines and solar panel costs. Such studies based on aggregate data, however, are unable to disentangle the effect of exogenous technological change from the effect of leaning-by-doing thus masking the diverse drivers of technology costs (see also Nordhaus 2014b). Nemet (2006) for instance finds that after accounting for measures of technological change and the cost of inputs, learning has only weak explanatory power for solar panel costs. Much more recently, Lafond et al. (2017) use hindcasting techniques to assess this model, and find that it provides a very good fit. Bollinger and Gillingham (2014) provide evidence for cost reductions due to learning-by-doing across installation contractors of solar photovoltaics in California from 2002 to 2012.

[17] We write r_t^s to distinguish from the notation for the market rate of return r_t, which we will use in Section 3.2.

Proposition 3.1. *Suppose that investment into this sector will be non-zero next period, i.e.* $I^H_{t+1} > 0$*. Then* $R_{t+1} - \delta = e_{t+1}$*, and*

$$\underbrace{\frac{p^H_t}{p^H_{t+1}} R_{t+1}}_{\text{shadow return}} = \underbrace{r^s_{t+1}}_{\text{direct return}} - \underbrace{\frac{p^H_t - p^H_{t+1}}{p^H_{t+1}}(1-\delta)}_{\text{price effect}} - \underbrace{(H_{t+2} - (1-\delta)H_{t+1})\frac{G'(H_{t+1})}{p^H_{t+1}}}_{\text{learning effect}} \quad (4)$$

Thus the Euler equation holds for *shadow* returns (as distinct from direct returns, for which it does not hold). The factor p^H_t / p^H_{t+1} on R_{t+1} in (4) is needed because, as defined, R_{t+1} values returns relative to the price p^H_t of investment at the moment at which the investment is made, while direct returns r^s_{t+1} are valued relative to the price p^H_{t+1} at the time at which we receive the return.

Relative to next-period prices, then, the shadow return is composed of three terms. One is the "direct return". Next, we observe a "price effect", from the dependence of prices on time. In period $t + 1$, an additional unit of renewable capital costs p^H_{t+1}, but it would have cost p^H_t in period t. If prices are decreasing over time then this gives an incentive to delay investment, and so reduces the shadow return on investment in period t. However, the reduction is mediated by the extent to which capital will depreciate. Indeed, this effect arises because we assume that prices are constant within each year, so that there is an incentive to delay investments until the benefits of learning can be enjoyed. Thus, the size of the effect will depend on the time-periods we use. In practice, technological innovations cannot be shared instantaneously, and so fixing prices within a year may be reasonable for our practical context (Section 4).

The final term, which we call the "learning effect", arises due to our assumption of learning-by-doing. It incorporates the marginal change in price in the LBD asset due to our holdings of this asset, valued against their price at time $t + 1$. This marginal change in price is multiplied by how many units of the asset we will invest in, in period $t + 1$. One must not be confused by the negative sign: typically $G'(H) < 0$ (prices decrease with capacity), and $H_{t+1} > (1 - \delta)H_t$ (investment is positive), so that the learning effect is typically positive.

The net effect of the price and learning effects may be positive or negative, and so the total return on renewables may be greater than, or less than, their direct net return (see Corollary A.2 for examples of each case and a discussion). However, the price effect will be taken into account by small rationally optimizing firms, whereas the learning effect will not, because in our specification, learning-by-doing is a pure externality. So, as we will see next, the optimal subsidy in a decentralized model is equal to the learning effect. It follows that investing in the LBD asset becomes worthwhile from a social perspective before it is individually rational: if investment will take place in the near future, it is socially optimal to start earlier than an individual would choose to.

3.2 Learning-By-Doing and the Acceleration Effect

We assume that households act as price-takers on the LBD asset. Due to the positive externality, there will be under-investment without intervention. So we introduce a subsidy, τ_t, which is conveniently expressed as a subsidy on the rate of return. Now we may write the household's budget constraint as $i^H_t + c_t = (r_t + \tau_t)p^H_t h_t + o_t$, where o_t represents other sources of income (as in Section 2.1). These investments are characterized by $i^H_t = p^H_t (h_{t+1} - (1 - \delta)h_t)$ and $i^H_t \geq 0$. The subsidy is paid for out of lump sum taxation; as the households are price-takers, this taxation may be incorporated into o_t. Meanwhile, a final goods firm maximizes its profits $f_t(H_t, O_t) - r_t p^H_t H_t - p^o_t O_t$, where p^o_t is the price they must pay for access to other assets.

Again there are L_0 households in the economy, each of size $\frac{L_t}{L_0}$, so that the consumption of a representative individual is $\frac{L_0 c_t}{L_t}$.

Proposition 3.2. *Suppose that any externalities in o_t have been internalized. The subsidy τ_t which optimizes consumer welfare $\sum_{t=0}^{\infty} \beta^t L_t u\left(\frac{L_0}{L_t} c_t\right)$ is equal to the learning effect:*

$$\tau_t = -\left(H_{t+1} - (1-\delta)H_t\right) \frac{G'(H_t)}{p_t^H}.$$

This expression is even simpler if $G(H_t)$ follows Wright's Law (2). Write g_t^H for growth $\frac{H_{t+1}-H_t}{H_t}$. Then:

Corollary 3.3. ***[The Acceleration Effect]*** *If $G(H_t) = p_0^H \left(\frac{H_t}{H_0}\right)^{-\lambda}$, then*

$$\tau_t = \lambda \left(g_t^H + \delta\right).$$

In particular, the subsidy τ_t increases with g_t^H.

Thus, the subsidy to the LBD asset is a straightforward function of its growth rate. Contrary to models which prescribe a short-term subsidy to this sector, the specification we use implies that this subsidy is positive as long as there is any investment in this sector, even only to replace depreciating stock. Moreover, if a change in information or policy makes the LBD asset more attractive in the economy, and so it starts to accumulate faster irrespective of the subsidy, we *also* increase the subsidy to this asset, spurring growth that is faster still. We call this the *acceleration effect* for technology policy.

For an illustration, see Figure 4a in Section 4.9, where we consider the optimal subsidy under both 'mild' and 'stringent' climate policy targets (as will be precisely defined later). There we see that, in the short term, more ambitious targets to decarbonize the economy, which will incentivize faster deployment of renewable technologies, also increase the optimal subsidy to investments in these technologies. In the longer term, stringent climate policy targets mean that we have already developed a greater capacity of this capital stock, and so its growth rate drops to a lower level than under mild policy targets; the subsidy is therefore also lower.

4 The Full Model

This section outlines the full dynamic general equilibrium climate-economy model which is used for quantitative analysis. The derivations of the equations that define the solution of the model are given in Appendix D. To summarize, the model presents a climate-economy structure, where, unlike other leading climate-economy models,[18] we differentiate between three capital stocks:[19] general capital, "clean" and "dirty", with irreversibility in investments characterizing the latter two capital stocks, as in Section 2 above. We allow underutilization of dirty capital stocks, once they become uncompetitive. In addition we assume that the "clean" sector is characterized by "learning-by-doing": costs of new technologies decline as a function of cumulative installed capacity in the sector, as in Section 3. The climate module uses the representation of the carbon cycle, temperature system, and climate-economy feedbacks based on the DICE framework (Nordhaus, 2014a), but calibrated to an annual time step (Cai et al., 2015, 2016).

There are five production sectors and thus there are five types of firms: final-goods producing firms, aggregate-electricity producing firms, dirty-electricity producing firms, a single fossil-fuel

[18] See, for example Golosov et al. (2014), Barrage (2014), Acemoglu et al. (2016), Nordhaus (2008), Rezai and van der Ploeg (2017).

[19] In a similar way, but within a different context, Greenwood et al. (1997) developed the importance of investment in differentiated capital stocks for growth and technological change.

extracting firm and firms producing electricity from renewable sources. All firms operate under perfect competition. Notably, the fossil-fuel-extracting firm maximizes the present value of its profits, subject to the standard depletion equation as in Rezai and van der Ploeg (2017), internalizing the effect of depletion on future extraction costs and on present and future revenue. Producers of renewable energy maximize the present value of their profits, taking the market price of renewable energy and the stock of accumulated knowledge about using renewable energy as given.

Turning to the demand side of the economy, we are interested in the behavior of a representative household who does not internalize the learning-by-doing externality and treats all prices as given. Finally, there are three sources of carbon dioxide emissions: general output production, electricity production from dirty energy inputs, and land use. Climate change affects productivity in the final goods producing sector.

4.1 The households' problem

We are interested in the behavior of a representative household. There are L_0 households (defined as the population size of the economy at the initial period, which in our calibrated model is 2012), and the size of the family at time t is $\frac{L_t}{L_0}$, where L_t is the population size at period t.[20]

We consider all variables on a per-household basis, denoted using lowercase letters, while capital letters denote aggregate variables (over all households). For instance, we will write $k_t^g = \frac{K_t^g}{L_0}$, where K_t^g is the aggregate general capital stock and H_t is the aggregate stock of renewable energy knowledge and capital. The household seeks to maximize the sum of the welfare of individual family members, that is:

$$\sum_{t=0}^{\infty} \beta^t \frac{L_t}{L_0} u\left(\frac{C_t}{L_t}\right) = \sum_{t=0}^{\infty} \beta^t \frac{L_t}{L_0} u\left(\frac{L_0}{L_t} c_t\right) \tag{5}$$

where C_t is aggregate consumption and $c_t := \frac{C_t}{L_0}$ is per-household consumption. The household owns a representative share of all three capital assets and the five sorts of companies. We denote r_t^D, r_t^H and r_t^g as the rate of return on capital assets in fossil-fuel-using (dirty) capital, renewable (clean) capital, and general capital (used in the production of final-goods producing firms), respectively. Further, we write w_t for the wage, Π_t^g for the total profit from the sale of the final goods, Π_t^D for the total profit from the sale of fossil-fuel-based electricity, Π_t^H for the total profit from the sale of "clean" electricity, Π_t^{DE} for the total profit from the sale of fossil fuels, and Π_t^E for the total profit from the sale of aggregate electricity, so that the aggregate profit is $\Pi_t = \Pi_t^g + \Pi_t^D + \Pi_t^H + \Pi_t^{DE} + \Pi_t^E$, and the per-household profit is $\pi_t := \frac{\Pi_t}{L_0}$.

In each period, the household faces the following budget constraint:

$$\begin{aligned} i_t^g + i_t^D + i_t^H + c_t = {} & \frac{L_t}{L_0} w_t + \pi_t + r_t^g k_t^g + r_t^D p_t^D k_t^D + r_t^H p_t^H h_t \\ & + \frac{1}{L_0}\left(\tau_t^D (D_t^E + D_t^g) - \tau_t^H p_t^H H_t\right) \end{aligned} \tag{6}$$

where i_t^g is investment in general capital, i_t^D is investment into dirty capital used in the production of dirty electricity, i_t^H is investment in capital used in the clean sector, k_t^g, k_t^D, h_t are capital stocks in the general, dirty and clean sectors respectively, τ_t^D is the carbon tax, τ_t^H is the subsidy, and D_t^E and D_t^g are carbon emissions in the dirty and general sectors respectively. Since we measure fossil and renewable energy capital in gigawatts (GW), p_t^D and p_t^H are the respective prices of fossil fuels and renewable energy capital in \$/GW. The price p_t^H of renewable energy capital falls with our

[20] Table A.2 in the Appendix B provides a summary of variables' notation and definition.

embodied technological progress in the renewable energy knowledge and capital stock, and evolves as (Arrow, 1962):

$$p_t^H = G(H_t) = p_0^H \left(\frac{H_t}{H_0}\right)^{-\lambda} \tag{7}$$

However, as we treat a household as very small, we assume that their investment in renewable energy capital does not influence its price, so that the learning-by-doing externality arises. That is, the household takes p_t^H as given. The price of fossil fuel capital will be fixed, so that $p_t^D = p^D$.

Finally, we assume that the household receives rebates on the taxes and pays for the subsidies (the last two terms in the right hand side of the budget constraint), but as we assume households are small they cannot affect these levels.

The capital stocks in the general, dirty and renewable sectors are accumulated according to the following equations respectively:

$$i_t^g = k_{t+1}^g - (1-\delta^g)k_t^g \tag{8}$$

$$i_t^D = p_t^D(k_{t+1}^D - (1-\delta^D)k_t^D) \tag{9}$$

$$i_t^H = p_t^H(k_{t+1}^H - (1-\delta^H)k_t^H) \tag{10}$$

where δ^g, δ^D, and δ^H are depreciation parameters, and

$$i_t^D \geq 0 \tag{11}$$

$$i_t^H \geq 0 \tag{12}$$

are the irreversibility assumptions - a non-negativity constraint on the rate of accumulation of dirty and clean capital.

4.2 The final-goods firms' problem

The final goods are produced by identical firms, but output is damaged by climate change. Because this sector exhibits constant returns to scale, we can work with aggregate variables, and so write output:

$$Y_t = \Omega(T_t)f(Y_t^g, E_t)$$

where T_t is the temperature change from pre-industrial levels, $\Omega(T_t)$ is the damage factor ($1-\Omega(T_t)$ is the ratio of damage to output), E_t is electricity and Y_t^g is "general" (non-electricity) output.

The final-goods firms maximize

$$\sum_{t=0}^{\infty} q_t \Pi_t^g = \sum_{t=0}^{\infty} q_t \left(\Omega(T_t)f(Y_t^g, E_t) - r_t^g K_t^g - w_t L_t - p_t^e E_t - \Psi_t - p_t^{fuel} D_t^g\right) \tag{13}$$

where $q_t := \beta^t \frac{u'(c_t)}{u'(c_0)}$ is a compound discount factor for the relative price of consumption in period t, expressed in period 0 units.[21] To produce final goods, these firms rent (aggregate) capital K_t^g, hire labor L_t, purchase aggregate electricity E_t at price p_t^e, and buy fossil fuel D_t^g from fossil fuel extracting firms at price p_t^{fuel}. The firms spend money on abatement Ψ_t, which is assumed to abate a fraction η_t of emissions via the following relation:

$$\Psi_t = \frac{\phi_{1,t}\eta_t^{\phi_2}}{(1-\eta_t)^{\phi_3}} Y_t^g \tag{14}$$

[21] See Appendix D.1 for more detailed discussion on the derivation of compound interest for the firms' problems.

so that they face the emissions constraint given by:

$$D_t^g = \sigma_t(1-\eta_t)Y_t^g \tag{15}$$

where ϕ_2 and ϕ_3 are parameters and σ_t represents the ratio of carbon-equivalent emissions to output, all of which evolve exogenously along with the parameter $\phi_{1,t}$, as in Cai et al. (2016). Firms do not take into account their emissions' impact on the pollution stock and thus on productivity. In other words, firms take $\Omega(T_t)$ as a given. This, in a conjunction with the knowledge externality in the renewable sector, represents a "twin-market failure" (Jaffe et al., 2005).

For the solution of the model, we assume that the function for production before damages takes the constant elasticity of substitution (CES) form (Hassler et al., 2012):

$$f(Y_t^g, E_t) = \left[(1-\theta)(Y_t^g)^{1-1/\kappa} + \theta\,(E_t)^{1-1/\kappa}\right]^{\frac{1}{1-1/\kappa}}.$$

and

$$Y_t^g = f_t^g(K_t^g, L_t) = A_t^g(K_t^g)^{\alpha}(L_t)^{1-\alpha}.$$

Here, θ, κ, and α are parameters, A_t^g is a technology process in the general sector, K_t^g is general capital and L_t is labor. Both A_t^g and L_t evolve exogenously in the same way as in Cai et al. (2016).

4.3 The aggregate-electricity-producing firms' problem

These firms again face constant returns to scale, so we can work with aggregate variables. They produce aggregate electricity $E_t = f_t^E(H_t, \Gamma_t^{ED})$ which is a combination of fossil fuel production capacity Γ_t^{ED}, and clean production capacity H_t, with these inputs being priced at p_t^{EH} and p_t^{ED} respectively. They sell their output at price p_t^e, so that the firms maximize the present value of their profits :

$$\sum_{t=0}^{\infty} q_t \Pi_t^E = \sum_{t=0}^{\infty} q_t \left(p_t^e f_t^E(H_t, \Gamma_t^{ED}) - p_t^{EH} H_t - p_t^{ED}\Gamma_t^{ED}\right) \tag{16}$$

In modeling the electricity sector, we follow Papageorgiou et al. (2017)[22] and assume a CES production function of renewable production capacity H_t and dirty production capacity Γ_t^{ED}:

$$E_t = f_t^E(H_t, \Gamma_t^{ED}) = A_t^E\left(\omega H_t^{\xi} + (1-\omega)(\Gamma_t^{ED})^{\xi}\right)^{1/\xi}, \tag{17}$$

where A_t^E is a technology process in the electricity sector and ω and ξ are CES parameters.

4.4 The dirty-electricity producing firms' problem

The dirty electricity producing firms are fossil-fuel-based power stations, which combine existing infrastructure (such as coal-based power plants) with fossil fuels via a Leontief production function. Again, due to constant returns, we work at the aggregate scale:

$$\Gamma_t^{ED} = \min[\zeta_t K_t^D, D_t^E/\nu] \tag{18}$$

where K_t^D is total capital in dirty electricity production, $\zeta_t \in [0,1]$ is the utilization rate, and ν is the conversion rate from fossil fuel to electricity. The Leontief function implies a fixed ratio between utilized fossil fuel energy capital and dirty fuel use:

$$D_t^E = \nu\zeta_t K_t^D. \tag{19}$$

[22] We do not use their model, in which overall energy is a combination of electricity and "other dirty energy", as in their model the latter requires no capital input and so is disproportionately favored under optimization.

The firms buy fossil fuel D_t^E at price p_t^{fuel}, rent the dirty capital infrastructure at rate r_t^D, and sell their output Γ_t^{ED} to the aggregate electricity producing firms at price p_t^{ED}. So, the firms in this sector maximize the present value of their profits:

$$\sum_{t=0}^{\infty} q_t \Pi_t^D = \sum_{t=0}^{\infty} q_t \left(p_t^{ED}(\zeta_t K_t^D) - r_t^D p^D K_t^D - p_t^{fuel} D_t^E \right) \tag{20}$$

subject to emissions constraint (19), and a constraint on the utilization rate: $\zeta_t \leq 1$.

4.5 The fossil-fuel-extracting firm's problem

We treat fossil fuel extraction as being handled by a single large firm (to give rise to the Hotelling equation and the Green Paradox). This firm maximizes the present value of its profits, by taking the market price of fossil fuel, p_t^{fuel} as given and internalizing the effect of depletion on future extraction costs and resource availability:

$$\sum_{t=0}^{\infty} q_t \Pi_t^{DE} = \sum_{t=0}^{\infty} q_t [p_t^{fuel} - \tau_t^D - G^D(S_t)](D_t^E + D_t^g) \tag{21}$$

where τ_t^D is a tax on the production of fossil fuels. Fossil fuels are extracted from finite reserves; the stock remaining at time t is denoted S_t. The evolution of this stock follows from the standard depletion equation (see e.g. Rezai and van der Ploeg (2017)):

$$S_{t+1} = S_t - (D_t^E + D_t^g). \tag{22}$$

The fossil fuel extraction cost per unit is given by:

$$G^D(S_t) = \gamma_1 \left(\frac{S_0}{S_t} \right)^{\gamma_2}, \tag{23}$$

where γ_1 and γ_2 are parameters. This equation implies that when a smaller stock is left, the extraction cost will be higher.

4.6 The renewable energy firms' problem

The renewable sector is composed of small firms, who do not internalize the learning-by-doing externality (7). That is, they take the stock of accumulated knowledge about using the renewable energy H_t as given, with a rental rate r_t^H. They receive a subsidy of τ_t^H on their dollar-valued holdings of renewable energy capital H_t and sell their output to the aggregate-electricity-producing firms at price p_t^{EH}. The firms take all prices as given, so on aggregate they maximize:

$$\sum_{t=0}^{\infty} q_t \Pi_t^H = \sum_{t=0}^{\infty} q_t [p_t^{EH} - p_t^H (r_t^H - \tau_t^H)] H_t. \tag{24}$$

Note that in the "simple model" of Section 2 we did not model renewable energy firms explicitly, so in that model we wrote the subsidy as accruing to the householder, who also owns the capital.

4.7 Climate system, emissions and damages

The carbon dioxide emissions D_t have three sources: "general" output production D_t^g; electricity production from using fossil fuel D_t^E; and land use D_t^{land}.

$$D_t = D_t^E + D_t^g + D_t^{\text{land}} \tag{25}$$

Land-use emissions D_t^{land} are set exogenously as by Cai et al. (2016). We use the climate system of Cai et al. (2016), which adapts the climate system of DICE2013 (Nordhaus, 2014a) to an annual time step. As this component of our model has been described extensively in the previous literature, we omit explanation here, and simply denote the mapping from emissions to temperature by:

$$T_t = \mathcal{W}_t(D_0, \ldots, D_{t-1}) \tag{26}$$

where T_t is global atmospheric temperature change over pre-industrial levels, D_s is fossil-fuel-related pollution at time $s < t$ and the warming function $\mathcal{W}_t$ relates these two variables.

Finally, the damage factor for "DICE damages" is given by

$$\Omega_t(T_t) = \frac{1}{1 + \varsigma_1 T_t^{\varsigma_2}}, \tag{27}$$

where ς_1 and ς_2 are parameters. The damage function in climate change economics is very controversial (see, for example Weitzman 2009, 2010; Cai et al. 2015). In fact there do not exist well-founded estimates of damages for even moderate temperature changes, and so the possibility to dictate optimal climate policy based on damage estimates is limited. However, a great deal of discussion in real-world policy-making focuses on limiting global temperature changes to 2°C. We simulate this constraint by letting

$$\Omega_t(T_t) = \frac{1}{(1 + \varsigma_1 T_t^{\varsigma_2})\,(1 + \varsigma_3\,(T_t/2)^{\varsigma_4})} \tag{28}$$

with a small positive parameter $\varsigma_3 = 0.001$ and a large exponent parameter $\varsigma_4 = 50$. Thus, when atmospheric temperature increase T_t is smaller than 2°C, the new damage factor given by (28) is almost the same as (27), but when T_t is larger than 2°C, the new damage factor will imply larger damages than (27). This new damage factor (28) will be referred as the stringent damage factor.[23]

4.8 Decentralized equilibrium versus the social planner's optimal solution

To find an optimal solution of the decentralized model, we formulate it as that of a principal who must choose an allocation from among those that can be implemented as a decentralized equilibrium, bearing in mind how the other economic participants (the "agents") will respond. In the optimal taxation literature such conditions imposed on the (Ramsey) principal are known as implementability conditions. We solve it using mathematical programming with equilibrium constraints. The details are in Appendix D.

The previous sections laid out the decentralized equilibrium model. To retrieve the values of the optimal carbon tax and optimal subsidies that could replicate the first-best allocation in the decentralized equilibrium model, we also outline a social planner model where the social planner maximizes social welfare given constraints describing the carbon cycle, temperature, damages and fossil fuel depletion, and the capital accumulation equations. See Appendix C for details.

[23]Numerically, if we impose a constraint that requires that the atmospheric temperature increase should never exceed 2°C (i.e., $T_t \leq 2$ °C for all time t), then an optimization solver may provide a binding solution with $T_t = 2$ °C for many years. This binding solution is not meaningful physically as in reality we cannot maintain the temperature at a given level. In addition, from an economic view, a reason for a policy to control $T_t \leq 2$ °C is that climate damage from $T_t > 2$ °C could be much more serious, so we use the damage factor (28) to address this potential serious damage from $T_t > 2$ °C.

4.9 Subsidy and carbon tax

In the decentralized equilibrium, there are two instruments: a subsidy on renewable capital, τ_t^H, and a carbon tax, τ_t^D. There are various scenarios related to the choice of policy instruments. We differentiate between four cases: (1) a no policy scenario in which we set $\tau_t^D = 0$ and $\tau_t^H = 0$; (2) the optimal policy version, in which both instruments are freely chosen to maximize the principal's objective; (3) $\tau_t^D = 0$ and the subsidy is chosen freely to maximize the principal's objective; (4) $\tau_t^H = 0$ and the carbon tax is chosen freely to maximize the principal's objective. Clearly, the second policy yields the same outcome as the social planner's problem, and it is the first-best, which we prove in the appendices. Cases (3) and (4) are situations with second-best policies.

Next, we define:

Definition 4.1. The *social cost of carbon*, χ_t is the shadow price on carbon emissions, relative to the shadow value of output. That is, if μ_t^D is the shadow price of Equation (25) constraining total emissions, then:

$$\chi_t := \frac{\mu_t^D}{u'(C_t/L_t)}.$$

Again we will write $g_t^H = \frac{H_{t+1}-H_t}{H_t}$ and we prove (see Appendix D.8):

Proposition 4.2. *The decentralized equilibrium allocation coincides with the solution to the social planner's problem if carbon taxes are set as the social cost of carbon χ_t, which is equal to the marginal effect on future welfare of present emissions*

$$\chi_t = -u'\left(\frac{C_t}{L_t}\right)^{-1} \sum_{m=1}^{\infty} \beta^m u'\left(\frac{C_{t+m}}{L_{t+m}}\right) \frac{\partial Y_{t+m}}{\partial D_t}; \tag{29}$$

and if subsidies are set equal to the "learning effect":

$$\tau_t^H = -(H_{t+1} - (1-\delta^H)H_t)\frac{G'(H_t)}{p_t^H} = \lambda(g_t^H + \delta^H). \tag{30}$$

This verifies that the theoretical insights on learning-by-doing from the "simple model" in Section 3 all carry across to the full model. That is, Corollary 3.3 holds and we have an "acceleration effect". In particular, an increase in the carbon tax which reduces investment in and utilization of dirty energy capital and so increases deployment of the substitute renewable energy capital, *also* implies an increase in the optimal renewable energy subsidy.

Naturally, Proposition 4.2 also shows that we can examine optimal policy by using a social planner's model, which is easier computationally. However, we do not restrict attention to this simpler case; we are also very interested in worlds without optimal (first-best) policy. If only the tax, or only the subsidy, are in use, then Proposition 4.2 does not apply. We explore such scenarios with our numerical results.

5 Quantitative results from the calibrated model

This section presents the quantitative results in four parts.[24] The first investigates the links between irreversible investment decisions and climate policies. We compare optimal policies with and without a stringent climate target (using the stringent damage factor (28) or the DICE damage factor (27)

[24] The calibration of the model is described in Appendix B.

respectively). In addition, we illustrate the importance of the irreversibility in investment decisions relative to the case in which investments are reversible (the irreversibility effect). In the second part we study the acceleration effect pertaining to an early start of investment in the renewable sector. Next, we study the impact of climate policy stringency on the optimal carbon tax as well as the effect of learning-by-doing on the optimal carbon tax. Finally, we study the implications of the second-best policies for social welfare and the dynamics of the model.

The initial period in our model is 2012. The model could be run under various scenarios that can be differentiated along three different dimensions: (1) damage function (DICE damage factor (27) vs. stringent damage factor (28)); (2) irreversible vs. reversible investments; and (3) the choice of policy instruments (optimal tax and subsidy vs. second-best policies). The runs of the decentralized equilibrium under the combined optimal tax and subsidy are equivalent to the runs of the social planner model (the first-best policy).

5.1 Irreversible investment and its implications

First, we want to understand how the optimal paths of variables depend on the irreversibility assumption coupled with different climate policy targets. We notice that the effect of irreversibility (compared with when the investment is reversible) becomes quantitatively important only if the climate policy objective is ambitious enough. Figure 1 shows that the paths of investment on dirty energy are almost the same with reversible and irreversible investments under a mild climate policy objective (the DICE damage factor) in this century, but they are distinctly different from each other under a more ambitious climate policy target (the stringent damage factor).

These results emphasize the importance of setting ambitious climate policies to induce permanent fuel energy switching. The strong path dependence embodied in carbon-intensive infrastructure suggests that mild climate change policies (those based on the DICE damage factor) would not induce fast shifts away from dirty energy towards green energy, as would be required to meet the Paris Agreement objectives,[25] as we see in Figure 8 in Section 5.4.

Further, Figure 1 shows that with irreversible investments and the stringent policy target, there is no investment in dirty energy after 2020. In contrast, when investment is reversible, the decumulation rate of the dirty capital stock is unlimited, and we keep investing in this capital stock until 2027 (another seven years), when we start turning dirty capital stock into general capital, a process that continues until about 2075. However, we never entirely stop using the dirty capital stock because of the imperfect substitutability between dirty and clean energy in electricity production. So, since we decumulated the dirty capital stock sufficiently in the preceding decades, investment in the dirty capital stock resumes after 2075 under reversible investment.

These dynamic patterns of investment in dirty energy with the (ir)reversible investments and the stringent damage factor correspond to the dynamics of returns on those investments shown in Figure 2. The theoretical counterpart of this figure is Proposition 2.2 in Section 2. First, the figure shows that we end investment in the dirty capital stock when the investment is still attractive with the rate of return, $r_t^D - \delta^D$, exceeding the rate of return on the general economy, $r_t^g - \delta^g$. This is because we will only invest in infrastructure that will become obsolete if the short-term benefits from that investment compensate for future losses. Thus even without uncertainty, returns to irreversible investment require a premium.[26] Even if we end investment at around 2020, we continue to fully

[25] This finding echoes the one in Meng (2016), who estimates the strength of path dependence in the electricity sector for the U.S. Midwest and shows that a permanent decline in U.S. electricity sector emissions would require shocks of larger magnitude and longer duration than that of recent natural gas prices.

[26] This finding has empirical support from previous studies on irreversible investment in other contexts. For example, Bernstein and Mamuneas (2007) develop a simple model of production and investment with costly disinvestment to

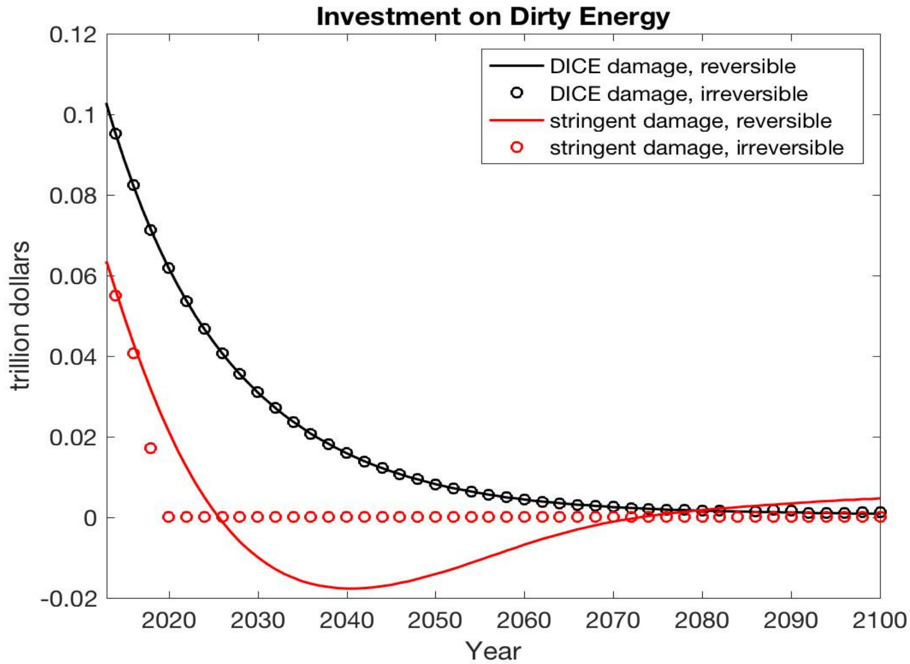


Figure 1: Investment in the Dirty Energy Capital Stock

utilize the dirty capital stock for about another 25 years, until 2045, when the return on dirty capital (r_t^D) reaches zero and we start underutilizing the dirty capital stock.

In the medium-term, as we end investment sooner than in the counter-factual (when disinvestment is a viable option), the economy continues to hold a smaller amount of dirty capital stock under irreversibility compared to the reversible case, until about 2037 (solid lines in Figure 3a). After that year, however, the economy holds larger stocks of dirty capital in the long-run if investment is irreversible. This result is due to path dependence: capital cannot be converted into other forms of capital stock. However, if we take into consideration the underutilization of the dirty capital stock in the irreversible investment case (circles in Figure 3a), then in the long-run, the same total amount of dirty capital stock will be *utilized* under irreversible and reversible investment decisions (Figure 3a).

Because emissions from the dirty energy sector are directly proportional to utilized dirty capital, the utilization curves in Figure 3a also give the pattern of emission levels. We show the level of these emissions, as well as total emissions (i.e., D_t, including those from the general economy and land-use) in Figure 3b.

5.2 The Acceleration Effect

The theoretical result in Section 3.2 gave the optimal subsidy, when the optimal carbon tax is also present (Corollary 3.3): $\tau_t^H = \lambda\left(g_t^H + \delta^H\right)$. This formula implies that (i) the subsidy continues as long as there is investment in the renewable sector, (ii) the subsidy increases with the learning

estimate the magnitude of the premium associated with irreversible investment in the telecommunications industry, assuming future telecommunications capital acquisition prices are random variables. Their findings indicate that the premium increases the user cost of capital by 70%, which implies an average hurdle rate of 14% over the period 1986-2002. Using different methods and framework, Pindyck (2005) provides similar estimates of the telecommunications hurdle rate.

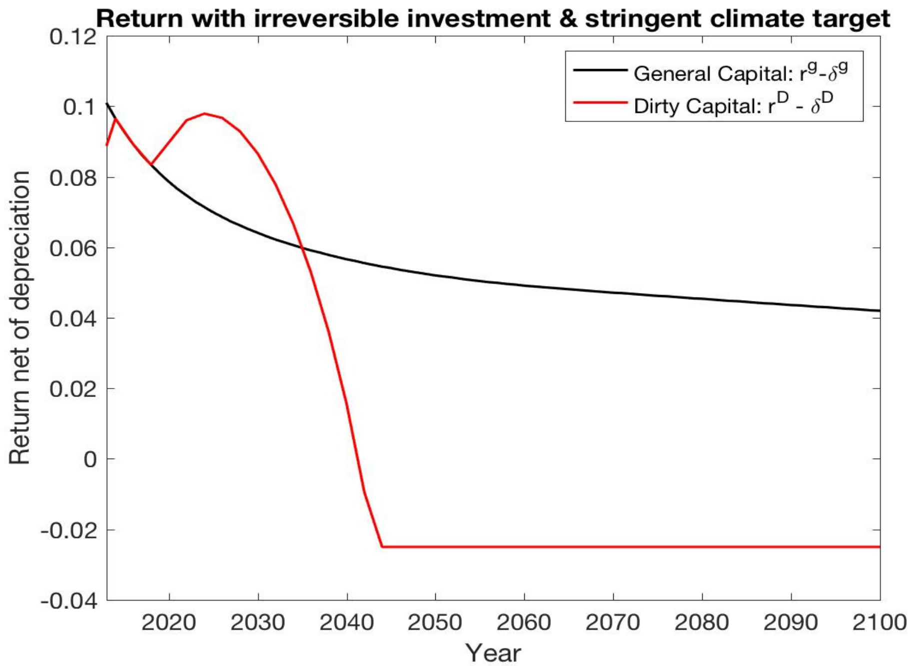
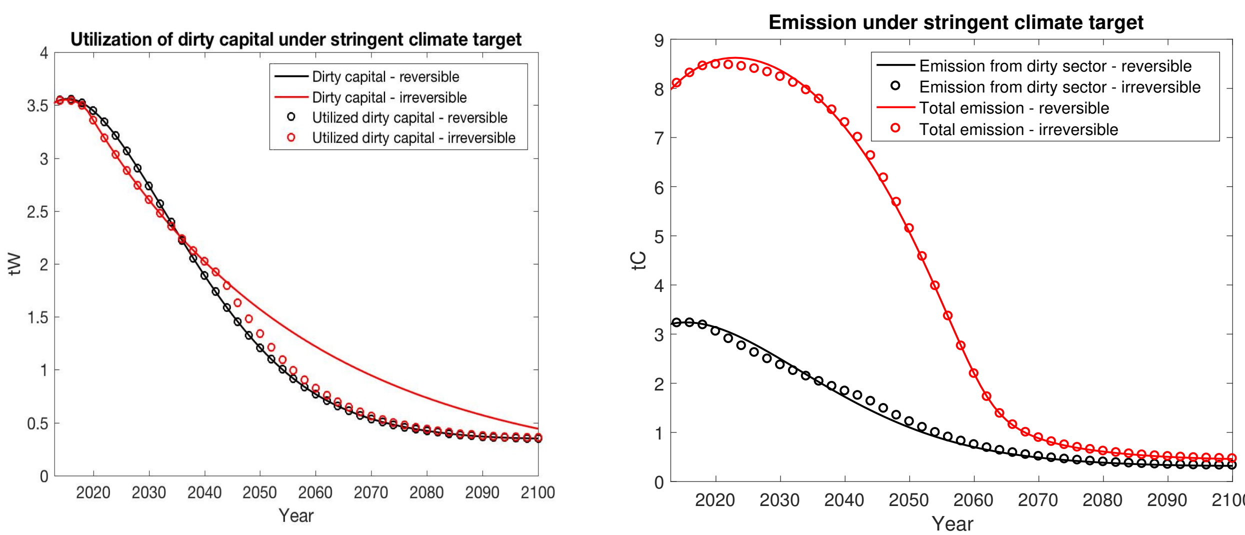


Figure 2: Return on general and dirty capital



a. Dirty energy capital stock and its utilization   b. Emissions

Figure 3: Dirty capital and emissions under the stringent climate policy target.

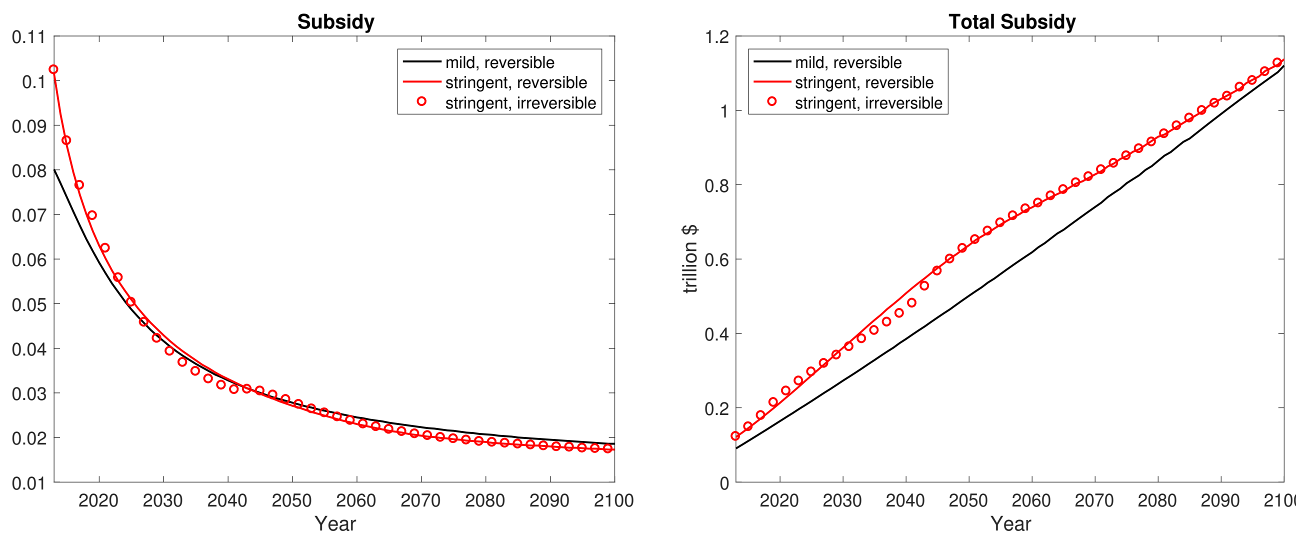
coefficient λ, and (iii) the optimal subsidy is higher when renewable capital grows faster. There are four different mechanisms that can lead to higher capital accumulation in the renewable sector and consequently to a higher level of subsidies: (1) more stringent climate policy targets, (2) the dirty sector could be shrinking faster than in the reversible case due to the irreversibility effect, (3) a higher learning rate (from higher R&D in renewables), and (4) under second-best scenario when a carbon tax is not possible, it could be optimal to grow the renewable sector faster to crowd out the dirty energy sector. We here investigate the first three of these channels. We will consider second-best policies, which encompass many important effects, in Section 5.4.

5.2.1 Channel 1: stringent climate policy

Figure 4a plots the optimal subsidy τ_t^H, and Figure 4b plots the total subsidy (i.e., total amount of dollars paid), $p_t^H \tau_t^H H_t$, under mild and stringent climate policy targets.

We observe in Figure 4a that the subsidy is higher under the stringent climate policy target in the initial decades following 2012. Because we use less fossil fuels in this scenario, we must generate more of our electricity from renewables, and so the latter sector is initially growing faster than it is in the mild policy scenario. From mid-century onward, this order reverses: if we ignore the additional effects due to irreversibility, a higher subsidy is paid under the mild policy target. This is because in the stringent policy scenario, a substantial volume of renewable energy capital stock has already been built by this point, and thereafter its growth rate is slower; thus, by the acceleration effect, so also is the optimal subsidy.

We also observe these effects when we plot the *total* subsidy paid to all holders of renewable energy capital H_t, in Figure 4b. This shows that payments are always higher under the stringent policy target, even when the subsidy (and indeed price) of the capital stock are lower, because of the size of H_t. (The decline in growth in these subsidy payments, starting at around 2040, mirrors the decline in growth in H_t already seen at this time). Still, the total subsidies of the three scenarios are converging as we approach the end of the century.



a. The subsidy τ_t^H to renewable energy

b. The total subsidy to renewable energy: $\tau_t^H p_t^H H_t$

Figure 4: Optimal Subsidies under Our Three Main Scenarios.

5.2.2 Channel 2: interaction with the irreversibility effect

In Figure 4, we plotted the optimal subsidies under the stringent climate policy with both *irreversible* and *reversible* investment decisions. There is a complex relationship between the subsidy level and the results of the irreversibility effect discussed in Section 5.1. As seen in Figures 1 and 3a, in the early periods the dirty sector shrinks faster in the irreversible case, as compared with the reversible case. However, this pattern reverses so that after around 2035 there is more dirty energy capital in use in the irreversible case. Underutilization begins around 2040 but there is still excess dirty energy capital in use until around 2070, when the trajectories converge.

So in those earlier years, when we build fewer coal-based power plants, we must be building a greater volume of renewables instead. This explains the greater subsidy for renewables in the irreversible case, visible in Figure 4a up to around 2020.

However, after this point, the subsidy to renewables drops below that for the reversible case. This is in anticipation of the greater dirty energy capacity that will remain in the economy due to irreversibility (instead of being decumulated, as in the reversible case). Thus, until 2045, the subsidy in the irreversible case remains below that of the reversible case – and even below that of the mild policy target.

But as the use of dirty capital begins to approach that of the irreversible case, it becomes necessary to again accelerate the deployment of the substitute renewable capital. Thus, from 2045, the subsidy to renewables in the irreversible case again exceeds that of the reversible case. The total subsidies paid are approximately the same in both cases at this point (Figure 4b) because less of this capital stock has been accumulated in the irreversible case, due to the prolonged reduction in investment.[27]

5.2.3 Channel 3: learning rate

Figure 5 presents the level of subsidies for three different years (2018, 2050 and 2100) under different values of the learning parameter λ. There are two important features to note. First, the level of subsidies increases with the value of learning parameter. Second, the subsidy level for 2018 follows a convex pattern with respect to the value of λ, whereas it follows a more linear pattern for the other two years.

Both of these patterns can be explained by referring to our theoretical result: $\tau_t^H = \lambda\left(g_t^H + \delta^H\right)$ (Corollary 3.3). The primary increase of subsidy with λ is clear. In addition, the degree of convexity of the subsidy level will be determined by the growth of deployment of renewables, g_t^H (our acceleration effect). Specifically, for 2018, there is higher growth of renewables with a higher learning rate, because this higher rate makes it more economically advantageous to expand the sector. Thus, at that time period, the relationship is convex. However, in 2050 and 2100, the higher growth in renewables has already taken place and the renewables are already functioning in the economy as mature technologies. As such there is no need to grow the renewable sector as fast in 2050 and 2100 as in 2018.

5.3 Optimal Carbon Taxes

The social cost of carbon is generally considered to be the most important parameter in climate change economics. If all other externalities are internalized, the optimal carbon tax should be set to this level (Proposition 4.2), which is equal to the marginal effect on future welfare of present emissions, via their effect on output. The tax is normalized according to the marginal utility of individual consumption at the time at which it applies.

Figure 6 displays the impact of climate policy stringency on this tax, for the cases of reversible investments, where stringency is measured by the parameter ς_4 in (28). We use three levels of stringency: $\varsigma_4 = 50$ is the default stringent climate policy (the "stringent" policy target used in previous sections), $\varsigma_4 = 20$ the medium stringency, and $\varsigma_4 = 10$ the low stringency. We also include results for the mild policy target (as defined in previous sections). The impact of stringency on carbon tax is large. For example, in 2050, the optimal carbon tax under the mild , low stringency,

[27] Related literature has investigated the optimal time path for innovation policy, see, for example Gerlagh et al. (2009) and Gerlagh et al. (2014). For instance, the latter show that if the patent lifetime is finite, the optimal subsidy starts at a high level, providing an incentive to accelerate R&D investments, and then falls over time.

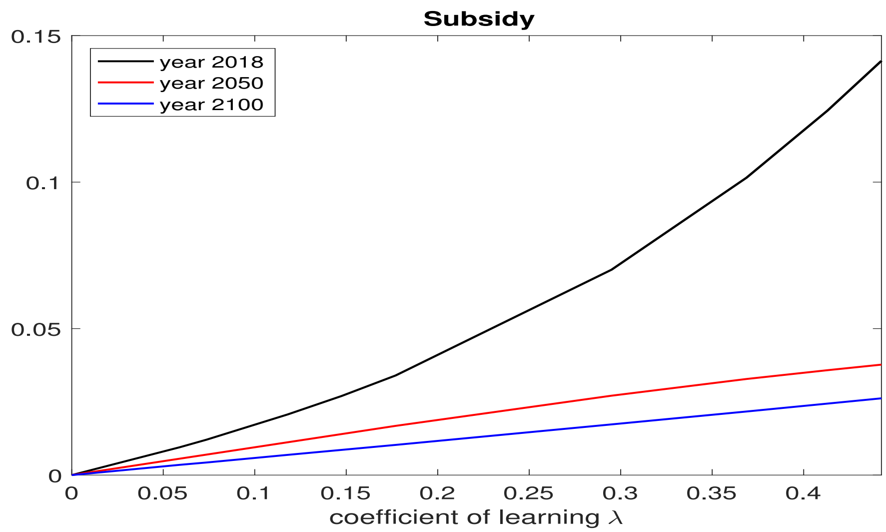


Figure 5: Optimal subsidy for different values of learning rate λ in the reversible and stringent scenario.

medium stringency, or the default stringent climate policy is \$222/tC (USD per ton of carbon), \$395/tC, \$490/tC, and \$553/tC, respectively.

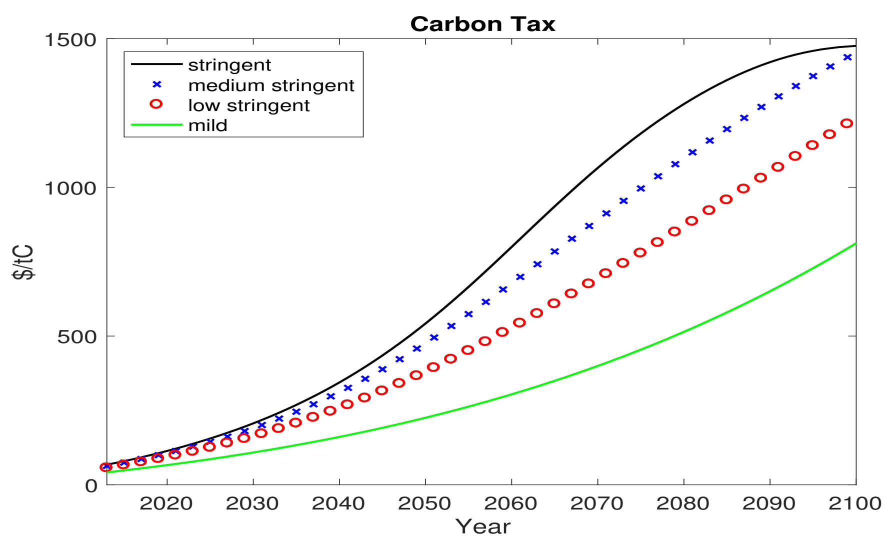


Figure 6: The Effect of Policy Stringency on the Optimal Carbon Tax

Figure 7 presents the effect of learning-by-doing on the carbon tax in the case with reversible investments: with a stringent climate policy target, the carbon tax is higher *without* learning-by-doing as it could be expected. With learning-by-doing, the associated subsidized roll-out of renewable energy technologies means that emissions are lower, both in the current period and in the future. It follows, in the stringent scenario, that the marginal effect on future welfare of current emissions is lower: a lower carbon tax is optimal. Intuitively, this is because cheap low-carbon energy means that stringent policy targets can be met without imposing a higher carbon tax.

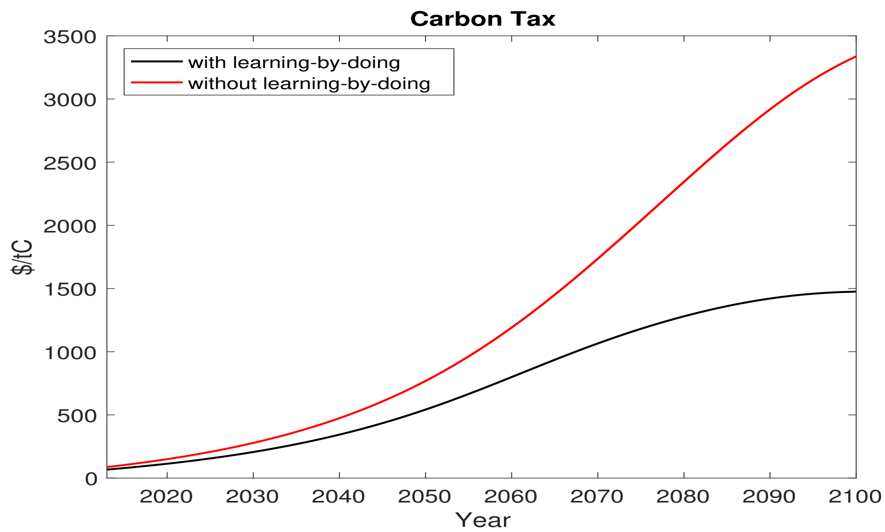


Figure 7: Optimal Carbon Tax with/without learning-by-doing in the stringent and reversible scenario.

5.4 Second-best policies

5.4.1 Subsidy Versus Tax

As Proposition 4.2 showed, the decentralized equilibrium with the optimal carbon tax on the externality created by fossil fuel use, combined with the optimal subsidy on the leaning-by-doing externality in the renewable sector, implements the optimal allocation obtained in the social planner's problem (the first best). In practice, however, one of these two policy instruments may be unavailable, and policy makers thus have to rely on second-best policies. In this section we compare the relative performance of these two policy instruments when used alone, under alternative climate policy objectives and (ir)reversible investment decisions. This is an important exercise given the landscape of debate regarding optimal climate policy. While these second-best policies considered represent two extremes, considering these extremes gives us the extent of the differences, and results for intermediate policies may be interpolated (we consider an intermediate case in Section 5.4.2 below).

Moreover, a tax-only policy is advocated in theory by those who criticize subsidies as expensive and inefficient (see e.g. Helm 2012). Conversely, if we look at actual implementations, we see that the European Union has spent a large amount on subsidies to achieve its target of 20% renewable energy by 2020, while allowing the carbon price in its trading scheme to fall to very low levels. In between, many advocate the necessity of mixed policies, while stressing the critical importance of carbon pricing.[28]

We contribute to this debate, arguing that in a second-best world, the policy instrument that should be used depends on how stringent climate policy objectives are. More specifically, under mild climate policy targets, as in the case with the 'DICE' damage factor (27), the economy is better off (in social welfare sense) with the optimal subsidy as a policy instrument. In contrast, under more stringent climate policy targets, as in case with the stringent damage factor (28), the economy is

[28]Bowen (2011) argues that "other policies are needed, too, particularly to promote innovation and appropriate infrastructure investment, but cannot be relied upon by themselves to bring about the necessary reductions to emissions. Carbon pricing is crucial."

	Optimal tax zero subsidy	Optimal subsidy zero tax
Reversible investment mild climate policy target	1.90%	1.59%
Reversible investment stringent climate policy target	2.52%	5.59%
Irreversible investment stringent climate policy target	2.49%	3.56%

Table 1: Second-best policies: welfare loss, % of initial period consumption

better off if the optimal carbon pricing policy is adopted (see Table 1).[29] As the results reported in the Table 1 further indicate, irreversibility in investment decisions does not affect the relative ranking of these policy instruments.

We now discuss the reasons why the economy does better with innovation policy (the optimal subsidy and zero tax) in the mild case, and why it is less desirable to use the same policy under the stringent case.

Figure 8 shows the temperature, emission, and tax levels under mild climate policy targets (the left panels) and stringent climate policy targets (the right panels), both assuming reversible investments. The top-left and middle-left panels show that with only carbon pricing, temperature and emissions paths closely follow those under the first-best policy. This is accomplished with a (slightly) higher level of carbon tax than under the first-best scenario. If we consider the more stringent climate policy case, we observe a similar pattern of paths for temperature and emissions: with carbon pricing only, they closely follow the paths of the first-best (the top-right and middle-right panels of Figure 8). The second-best tax level is again higher than the first-best counterpart.

The intuition behind these results is as follows. With only carbon pricing, there is a risk of being "locked into" the ways of producing electricity that are currently cheap: coal-based power plants.[30] Meanwhile, the alternative (producing electricity from renewables) is currently more expensive and may not become competitive in the future. As a result, a higher level of carbon taxes on the fossil fuel extracting firms is needed compared with the first-best scenario. But since the size of the dirty sector in the energy sector of the economy is large relative to the renewable sector, this policy of making the dirty sector "less competitive" through carbon taxes is relatively more costly (in welfare terms), than the policy of making the renewable sector competitive through direct subsidies. Contrary to carbon pricing, subsidies directly stimulate investment in renewable energy and, once clean technologies develop and become competitive, the renewable sector crowds out the dirty energy sector. Under less ambitious climate policy, this subsidy appears sufficient, as well as less costly than carbon pricing (given also the relatively smaller size of the clean sector).

On the other hand, achieving the more stringent climate policy target through innovation policy is extremely difficult as it requires decarbonization of the large dirty energy sector. Adoption of the instrument which directly targets that sector – carbon pricing – is a policy that is associated with relatively higher welfare.

Finally, the emissions and temperature paths with carbon pricing only, irrespective of the assumptions about the stringency of climate policy targets, closely follow the ones of the first-best

[29] These findings are in line with ones in Gerlagh and van der Zwaan (2006) who use a long-term top-down model with a decarbonization option through carbon capture and storage to show that carbon taxes do better for stringent targets, and subsidies do better for modest targets. However, this paper uses a different approach, analyzing the implications of the second-best instruments for climate policy in a transparent setting.

[30] See Unruh (2002) and Jaffe et al. (2005) for further discussion.

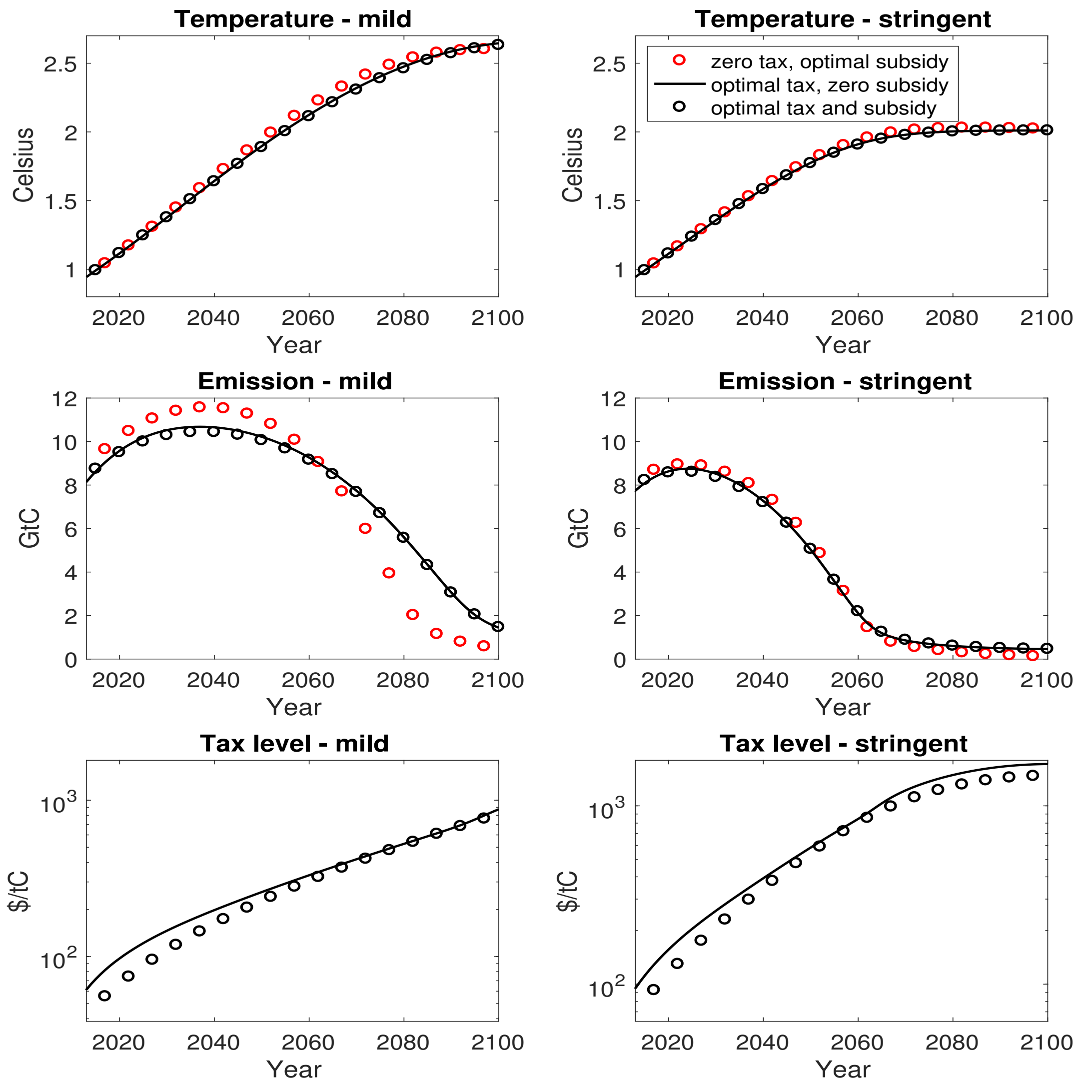
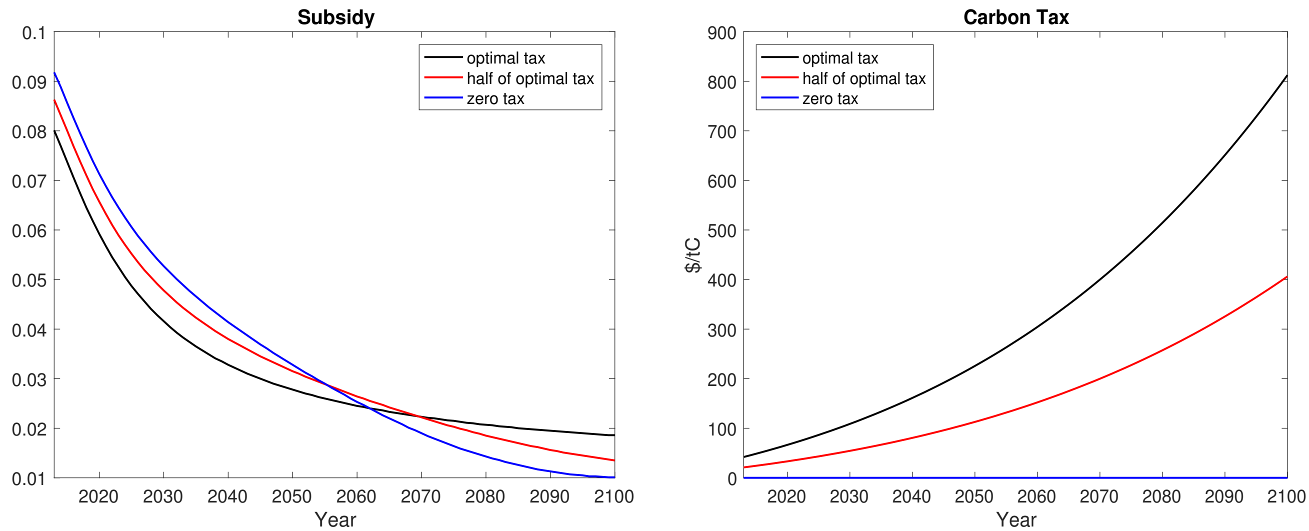


Figure 8: Temperature, Emission, and Tax level under the mild or stringent climate policy targets

scenario because carbon pricing internalizes the global warming externality, and thus is better suited to target climate policy objectives.

5.4.2 Optimal Subsidy with a Pre-specified Tax

Section 5.4.1 analyzed two extreme cases. In practice, an intermediate situation may hold. It may be possible to have both a subsidy, and a carbon tax – but it may be politically impossible to set the tax as high as its optimal level. Policy-makers must then adjust the subsidy to meet the policy target. For example, the carbon tax could be pre-specified as half of the optimal carbon tax, where the optimal carbon tax is the solution of the first-best policy with both the carbon tax and subsidy are available. We use the mild target and reversible investment scenario to study this intermediate situation. Figure 9b displays the pre-specified carbon tax, and Figure 9a provides the corresponding optimal subsidy. We see that when tax is set to be zero, the subsidy is the largest; when tax is set at its optimal level under the first-best policy, the subsidy is the smallest; and when the tax is set to be half of the first-best optimal level, the subsidy is between the previous two extreme cases. However, this pattern is only valid before 2055: after 2070, a smaller tax will be accompanied by a smaller subsidy, as larger subsidies in the earlier periods lead to a higher renewable energy stock, lessening the need for subsidies in the later periods.



a. Optimal subsidy given a pre-specified carbon tax

b. The pre-specified carbon tax

Figure 9: Optimal subsidy when the level of tax is constrained.

6 Discussion

In this paper we have studied implications of two capital stock effects – path dependence in infrastructure and learning-by-doing in the renewable sector – for the design of optimal climate policies, using both simple analytical models and simulations of the full dynamic general equilibrium climate-economy model. We define path dependence as irreversibility of investment in both the clean and dirty energy sectors (as opposed to allowing aggregate divestment). We compare the simulation results from our model that incorporates irreversibility, with those coming from a model without inertia in the energy sectors.

The simulation results contribute to the debate on the characteristics of optimal policy to combat climate change, which involve issues about the timing as well as choice of instruments to address the problem. On the timing of climate policy, the debate has centered on whether we should adopt a "gradual slope" approach to the policy, according to which we should delay investment in low-carbon emitting technologies and instead focus on a carbon price that rises gradually. An alternative approach recommends accelerating learning-by-doing and reducing abatement costs of mitigation policies.

We demonstrate that it is optimal to stop investment in the dirty sector earlier – due to the irreversibility effect. Previous literature has justified the early investment in the renewable sector on the basis of the learning-by-doing effect (see, van der Zwaan et al. 2002). We have provided a simple analytical formula relating optimal subsidies to the growth of the renewable sector and the learning rate. Thus the optimal subsidy is higher for more stringent climate policy targets and for technologies with faster learning rates. Investment displays a delicate pattern when we account for irreversibility, as the renewable energy capital stock must grow faster in the very short term but is then held back because the dirty sector is still in use.

Regarding the debate on the instrument choice for effective climate policy, our results on the relative performance of carbon pricing versus subsidies in a second-best setting reflect the broad trends in the global climate political landscape. Nowadays we observe a rapid expansion in the use of renewable energy technologies.[31] Renewable energy technologies are viewed today as tools to mitigate climate change, improve local air quality, advance economic development and create jobs. Declining costs have played a pivotal role in the expansion of renewable energy technologies in recent years. The stage for such an expansion was set more than a decade ago when a handful of countries, including Germany, Denmark, Spain, and the United States, created a critical market for renewables, which drove early economies of scale and led to the changes we witness today (REN21, 2014). During that period and effectively until 2016, when the Paris Agreement came into force, progress in the area of international climate policy had been modest at best. Although the European Union had started campaigning for the 2°C target in the mid-1990s, this target was not formally adopted until 2010 at the UN Climate Change Conference in Cancun (Geden, 2013). As such, we could characterize the international climate policy up to 2015 as having unambitious climate policy objectives. The Paris Agreement, however, renewed the climate political landscape, at least in theory, with a larger recognition of the urgency of more ambitious emissions reductions. The agreement has also revived discussion about the importance of adopting carbon pricing to implement the emissions mitigation pledges submitted by 186 countries for the December 2015 Paris Agreement,[32] which is in line with the message from simulations of our model under the second-best setting that more ambitious climate policy should adopt carbon pricing.

Our quantitative results under the second-best setting with a pre-specified tax illustrate how a subsidy can help to overcome political economy constraints. Specifically, in the short-run, the optimal subsidy needs to be higher to compensate for a lower level of taxes but subsequently it is lower as renewable technologies reach maturity.

Finally, our model has important implications for understanding the problem of stranded assets in the climate policy literature. The narrative on stranded assets that emerges from our model has one principal component: to make low-carbon alternatives widely available, investment in those technologies should start early enough to take advantage of scale effects, as well as the acceleration effect.

7 Conclusion

This paper has shown that capital stock effects of infrastructure such as coal-based power plants are important for the design of optimal climate policies. Specifically, we characterize and then quantify the optimal time to end investments in fossil fuel power plants in a dynamic general equilibrium climate-economy model with irreversible "dirty" and "clean" capital investments. We find that for temperature changes to not exceed 2°C, investments in dirty infrastructure should end in 2020.

We show that the "Green Paradox" – that future stringent climate policy raises short-term emissions – has a converse if we focus on demand side capital stock effects. If the dirty capital

[31]Renewables accounted for nearly half of all new power generation capacity in 2014, led by growth in China, the United States, Japan and Germany, with costs continuing to fall (EIA, 2015).

[32]Baranzini et al. (2017) provide a summary of the main arguments in favor of carbon pricing in a post-Paris world. See also Farid et al. (2016) who urge for carbon taxes (or equivalently carbon trading systems) for implementation of the Paris pledges.

stock cannot be converted to other forms of capital, then it is optimal to stop investing in the dirty capital stock earlier than the case where capital investments are reversible.

Learning-by-doing significantly advances the timing of investment in renewables, not only to prevent later stranding of fossil-fuel-based assets but also to accelerate the decline in the costs of clean energy.

The timing of these effects depends on the stringency of climate policy targets. Climate policy targets induce an earlier shift (within the next few decades) to clean energy and away from dirty energy only if they are stringent. Otherwise, path dependence in energy systems and low substitutability between the dirty and clean energy sources imply a prolonged period of using the dirty capital stock.

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Online Appendix for "To Build or Not to Build? Capital Stocks and Climate Policy" (For Online-Only Publication)

A Proofs of Theoretical Results: Simplified Model

To start with, we define:

$$P_t := \sum_{s=1}^{\infty} (1-\delta)^{s-1} \Delta_{t,s} \left(r_{t+s} - \delta - e_{t+s} \right). \tag{A.1}$$

This is the net present value of investment in the irreversible asset, infrastructure, relative to the opportunity cost. The following technical lemma is very illuminating:

Lemma A.1. *Given the framework above,*

1. $P_t \leq 0$ *for all* t.
2. $i_t > 0$ *only if* $P_t = 0$.
3. $i_t > 0$ *only if both* $r_t - \delta \leq e_t$ *and* $r_{t+1} - \delta \geq e_{t+1}$.
4. $i_t > 0$ *with* $r_{t+1} - \delta > e_{t+1}$ *only if* $i_{t+1} = 0$.

Proof of Lemma A.1. Write o_t for all other sources of income, net of any other investments (which may also be irreversible). We maximize

$$\sum_{t=1}^{\infty} \beta^t u(c_t) \tag{A.2}$$

subject to constraints

$$\mu_t^{bc} \qquad i_t + c_t = r_t k_t + o_t \tag{A.3}$$

$$\mu_t^{i} \qquad i_t \geq 0 \tag{A.4}$$

$$\mu_t^{k} \qquad i_t \geq k_{t+1} - (1-\delta) k_t \tag{A.5}$$

The Lagrangian is:

$$\mathcal{L}_t = \sum_{t=0}^{\infty} \beta^t \bigg(u(c_t) - \mu_t^{bc}(i_t + c_t) + \mu_t^{bc}(r_t k_t + o_t) + \mu_t^i i_t \tag{A.6}$$

$$+ \mu_t^k (i_t - (k_{t+1} - (1-\delta) k_t)) \bigg) \tag{A.7}$$

Leading to FOCs and complementary slack conditions

$$
\begin{aligned}
&c_t & u'(c_t) &= \mu_t^{bc} && \text{(A.8)}\\
&i_t & \mu_t^{bc} &= \mu_t^i + \mu_t^k && \text{(A.9)}\\
&k_{t+1} & \mu_t^k &= \beta(\mu_{t+1}^{bc} r_{t+1} + \mu_{t+1}^k(1-\delta)) && \text{(A.10)}\\
& & \mu_t^i &\geq 0 && \text{(A.11)}\\
& & \mu_t^i i_t &= 0 && \text{(A.12)}\\
& & \mu_t^k &\geq 0 && \text{(A.13)}\\
& & \mu_t^k(i_t - (k_{t+1} - (1-\delta)k_t)) &= 0 && \text{(A.14)}
\end{aligned}
$$

Substitute (A.8) and (A.9) into (A.10) and divide by $\beta u'(c_{t+1})$:

$$\frac{u'(c_t)}{\beta u'(c_{t+1})}\left(1 - \frac{\mu_t^i}{\mu_t^{bc}}\right) = r_{t+1} + \left(1 - \frac{\mu_{t+1}^i}{\mu_{t+1}^{bc}}\right)(1-\delta) \tag{A.15}$$

Write $e_{t+1} := \frac{u'(c_t)}{\beta u'(c_{t+1})} - 1$ and $\Delta_{t,s} = \prod_{s'=1}^{s} \frac{1}{1+e_{t+s'}}$. Re-arrange so that this will provide a forward-looking formula for $\frac{\mu_t^i}{\mu_t^{bc}}$:

$$
\begin{aligned}
\frac{\mu_t^i}{\mu_t^{bc}} &= \frac{e_{t+1} - (r_{t+1} - \delta)}{e_{t+1} + 1} + \frac{(1-\delta)}{(e_{t+1}+1)}\frac{\mu_{t+1}^i}{\mu_{t+1}^{bc}}\\
&= \frac{e_{t+1} - (r_{t+1} - \delta)}{e_{t+1} + 1} + \frac{1-\delta}{e_{t+1}+1}\left(\frac{e_{t+2} - (r_{t+2} - \delta)}{e_{t+2} + 1} + \frac{(1-\delta)}{(e_{t+2}+1)}\frac{\mu_{t+2}^i}{\mu_{t+2}^{bc}}\right)\\
&= \sum_{s=1}^{T}(1-\delta)^{s-1}\Delta_{t,s}\left(e_{t+s} - r_{t+s} + \delta\right) + (1-\delta)^T \Delta_{t,T}\frac{\mu_{t+T}^i}{\mu_{t+T}^{bc}}.
\end{aligned} \tag{A.16}
$$

Next we will show that the final term in (A.16) tends to zero as $T \to \infty$. Since we assumed that there exist $\epsilon > 0$ and $R \gg 0$ with $-\delta + \epsilon < e_t < R$ for all t, it follows that $\frac{1-\delta}{1+e_t} < 1 - \frac{\epsilon}{1+e_t} < 1 - \frac{\epsilon}{R+1}$ for all t and so that $(1-\delta)^T \Delta_{t,T} \to 0$ as $T \to \infty$. Finally, $0 \leq \mu_T^i \leq \mu_T^{bc}$ for all T, by consideration of (A.9) and (A.13). It follows that $0 \leq \frac{\mu_T^i}{\mu_T^{bc}} \leq 1$, and hence the final term in (A.16) tends to 0 as $T \to \infty$, and we conclude:

$$\frac{\mu_t^i}{\mu_t^{bc}} = \sum_{s=1}^{\infty}(1-\delta)^{s-1}\Delta_{t,s}\left(e_{t+s} - r_{t+s} + \delta\right) =: -P_t \tag{A.17}$$

with per-period equation:

$$\Delta_{t,1}^{-1}\frac{\mu_t^i}{\mu_t^{bc}} = (e_{t+1} - r_{t+1} + \delta) + (1-\delta)\frac{\mu_{t+1}^i}{\mu_{t+1}^{bc}} \tag{A.18}$$

Part 1 of Lemma A.1 follows from (A.17). Next, if $i_t > 0$, complementary slackness (A.12) tells us $\mu_t^i = 0$ and so Part 2 follows from (A.17).

If $i_t > 0$ then by (A.12) $\mu_t^i = 0$, and since $\mu_{t+1}^i \geq 0$ and $\mu_{t-1}^i \geq 0$, (A.18) implies $r_{t+1} - \delta \geq e_{t+1}$ and $r_t - \delta \leq e_t$. In addition, Part 4 follows, in the same way as the previous result: if $i_t > 0$ with $r_{t+1} - \delta > e_{t+1}$, then (A.18) implies $\mu_{t+1}^i > 0$ and then $i_{t+1} = 0$ from (A.12). □

Proof of Proposition 2.1. Immediate from Lemma A.1 Part 3. □

Proof of Proposition 2.2. If $r_{s_1} - \delta < e_{s_1}$ then $i_{s_1 - 1} = 0$ (by Lemma A.1 Part 3). However, by assumption, $i_0 > 0$. Let t_0 be maximal such that $t_0 < s_1$ and $i_{t_0} > 0$. Now, by Lemma A.1 Part 2, $P_{t_0} = 0$. So:

$$\begin{aligned} 0 = P_{t_0} &= \sum_{s=1}^{s_1 - t_0} (1-\delta)^{s-1} \Delta_{t_0,s}(r_{t_0+s} - \delta - e_{t_0+s}) + \sum_{s=s_1-t_0+1}^{\infty} (1-\delta)^{s-1} \Delta_{t_0,s}(r_{t_0+s} - \delta - e_{t_0+s}) \\ &= \sum_{s=t_0+1}^{s_1} (1-\delta)^{s-t_0-1} \Delta_{t_0,s-t_0}(r_s - \delta - e_s) + \sum_{s=1}^{\infty} (1-\delta)^{s_1-t_0+s-1} \Delta_{t_0,s_1-t_0+s}(r_{s_1+s} - \delta - e_{s_1+s}) \end{aligned} \tag{A.19}$$

It is easy to show that, for any t_1, t_2, we have $\Delta_{0,t_1}\Delta_{t_1,t_2} = \Delta_{0,t_1+t_2}$. Thus $\Delta_{0,t_0}\Delta_{t_0,s_1-t_0+s} = \Delta_{0,s_1+s}$. It also follows that $\Delta_{0,s_1}\Delta_{s_1,s} = \Delta_{0,s_1+s}$, and that $\Delta_{0,t_0}\Delta_{t_0,s_1-t_0} = \Delta_{0,s_1}$. Putting these facts together we see that $\Delta_{t_0,s_1-t_0+s} = \Delta_{t_0,s_1-t_0}\Delta_{s_1,s}$ So, continuing from (A.19), we see

$$\begin{aligned} P_{t_0} &= \sum_{s=t_0+1}^{s_1} (1-\delta)^{s-t_0-1} \Delta_{t_0,s-t_0}(r_s - \delta - e_s) \\ &\quad + (1-\delta)^{s_1-t_0} \Delta_{t_0,s_1-t_0} \sum_{s=1}^{\infty} (1-\delta)^{s-1} \Delta_{s_1,s}(r_{s_1+s} - \delta - e_{s_1+s}) \\ &= \sum_{s=t_0+1}^{s_1} (1-\delta)^{s-t_0-1} \Delta_{t_0,s-t_0}(r_s - \delta - e_s) + (1-\delta)^{s_1-t_0} \Delta_{t_0,s_1-t_0} P_{s_1}. \end{aligned} \tag{A.20}$$

But $P_{s_1} \leq 0$ by Lemma A.1 Part 1. And $i_{t_0} > 0$ so $r_{t_0} - \delta \leq e_{t_0}$ by Lemma A.1 Part 3. Thus:

$$\sum_{s=t_0+1}^{s_1} (1-\delta)^{s-t_0-1} \Delta_{t_0,s-t_0}(r_s - \delta - e_s) \geq 0.$$

Since $r_{s_1} - \delta - e_{s_1} < 0$ it follows that there exists $s \in \{t_0 + 1, \dots, s_1 - 1\}$ such that $r_s - \delta > e_s$. Letting s_0 be the minimal such s, it is clear that this meets our requirements.

Next, by exactly the same arguments as those used to prove (A.20), and by $P_{s_2} \leq 0$, it follows that

$$\begin{aligned} 0 = P_0 &= \sum_{s=1}^{s_2} (1-\delta)^{s-1} \Delta_{0,s}(r_s - \delta - e_s) + (1-\delta)^{s_2} \Delta_{0,s_2} P_{s_2} \\ &\leq \sum_{s=1}^{s_2} (1-\delta)^{s-1} \Delta_{0,s}(r_s - \delta - e_s) \end{aligned}$$

By splitting the sum into terms with $s \in \{1, \dots, s_1 - 1\}$ and $s \in \{s_1, \dots, s_2\}$, and rearranging, we obtain the expression given. □

Proof of Corollary 2.3. First, see that without the constraint $I_t \geq 0$ we have $\tilde{r}_t - \delta = e_t$ for all t.

Next, since $I_0 > 0$ we know $P_0 = 0$ by Lemma A.1 Part 2. If $r_t - \delta = e_t = \tilde{r}_t - \delta$ for all t then $K_t = \tilde{K}_t$ for all t, but this is not possible since $\tilde{I}_{t_1} < 0$ and $I_{t_1} \geq 0$. If we assume $r_t - \delta \geq e_t$ for all t we must conclude also $r_t - \delta > e_t$ for some t, whence $P_0 > 0$, which is a contradiction. So there exist some minimal s_1 such that $r_{s_1} - \delta < e_{s_1}$ and some maximal $s_2 \in \mathbb{R} \cup \{\infty\}$ such that $s_2 \geq s_1$ and $r_t - \delta < e_t$ for $t \in \{s_1, \dots, s_2\}$. Applying Proposition 2.2 we conclude that there exists

$s_0 \leq s_1 - 1$ such that $r_{s_0} - \delta > e_{s_0}$ and such that $I_t = 0$ for $t \in \{s_0, \ldots, s_2 - 1\}$. Pick s_0 minimal with these properties.

We show that s_0 is minimal such that $r_t - \delta \neq e_t$. First, by definition of s_1, there is no $t < s_0$ with $r_t - \delta < e_t$. Next, if $r_t - \delta > e_t$ for $t < s_0$ then there exists $t' \in \{t, \ldots, s_0 - 1\}$ such that $I_{t'} > 0$ (for otherwise s_0 is not minimal as defined). But $P_0 = 0$ and $P_{t'} = 0$ imply that there must also exist $t'' \in \{1, \ldots, t'\}$ such that $r_{t''} - \delta < e_{t''}$, and we already know this is not so.

Since $r_t - \delta = e_t = \tilde{r}_t - \delta$ for $t \in \{0, \ldots, s_0 - 1\}$, it follows that $K_t = \tilde{K}_t$ for $t \in \{0, \ldots, s_0 - 1\}$ and so that $I_{t-1} = \tilde{I}_{t-1} \geq 0$ for $t \in \{0, \ldots, s_0 - 1\}$. So we know $t_1 \geq s_0$.

Next, $r_{s_0} - \delta > e_{s_0} = \tilde{r}_{s_0} - \delta$ so $K_{s_0} < \tilde{K}_{s_0}$; but $K_{s_0 - 1} = \tilde{K}_{s_0 - 1}$, so $I_{s_0 - 1} < \tilde{I}_{s_0 - 1}$. So set $t_0 := s_0 - 1$.

Finally, by definition $r_{s_1} - \delta < e_{s_1} = \tilde{r}_{s_1} - \delta$, which implies $K_{s_1} > \tilde{K}_{s_1}$. But $K_{t_0+1} < \tilde{K}_{t_0+1}$ and so, since $I_t = 0$ for $t \in \{t_0 + 1, \ldots, s_2 - 1\}$ we conclude that $K_t < \tilde{K}_t$ for $t \leq \{t_0 + 1, \ldots, \min(s_2 - 1, t_1)\}$. Since $s_1 \leq s_2 - 1$ and since $K_{s_1} > \tilde{K}_{s_1}$ we conclude that $\min(s_2 - 1, t_1) = t_1$, i.e. that $K_t < \tilde{K}_t$ for $t \in \{t_0 + 1, \ldots, t_1\}$ as required. □

The Social Planner's problem for Section 3.1 The planner optimizes

$$\sum_{t=0}^{\infty} \beta^t L_t u \left(\frac{C_t}{L_t} \right) \tag{A.21}$$

subject to the constraints:

$$\Lambda_t^s \qquad I_t + C_t = f_t(H_t, O_t) \tag{A.22}$$

$$\mu_t^I \qquad I_t \geq 0 \tag{A.23}$$

$$\mu_t^H \qquad I_t = p_t^H (H_{t+1} - (1 - \delta) H_t) \tag{A.24}$$

$$\mu_t^p \qquad p_t^H = G(H_t) \tag{A.25}$$

where $O_t = L_t o_t$ represents all other factors of production in the economy. In our model the planner treats this as exogenous.

At time t, the Lagrangian is

$$\mathcal{L}_t = \sum_{t=0}^{\infty} \beta^t \Bigg(L_t u \left(\frac{C_t}{L_t} \right) - \Lambda_t^s \Big(I_t + C_t - f_t(H_t, O_t) \Big) + \mu_t^I I_t + \mu_t^H (I_t - p_t^H (H_{t+1} - (1 - \delta) H_t)) + \mu_t^p (p_t^H - G(H_t)) \Bigg)$$

the first order conditions are:

$$\partial C_t : \qquad \Lambda_t^s = u' \left(\frac{C_t}{L_t} \right) \tag{A.26}$$

$$\partial H_{t+1} : \qquad p_t^H \mu_t^H = \beta \left(\Lambda_{t+1}^s \frac{\partial f_{t+1}}{\partial H_{t+1}} + \mu_{t+1}^H p_{t+1}^H (1 - \delta) \right) - \beta \mu_{t+1}^p G'(H_{t+1}) \tag{A.27}$$

$$\partial I_t : \qquad \Lambda_t^s = \mu_t^H + \mu_t^I \tag{A.28}$$

$$\partial p_t^H : \qquad \mu_t^p = \mu_t^H (H_{t+1} - (1 - \delta) H_t) \tag{A.29}$$

together with the constraints above and the inequality $\mu_t^I \geq 0$, which is complementary slack with (A.23).

Proof of Proposition 3.1. Divide (A.27) through by $p_t^H \beta \Lambda_{t+1}^s$, substitute in (A.29) and re-arrange to obtain:

$$R_{t+1} = \frac{\mu_t^H - \beta(1-\delta)\mu_{t+1}^H}{\beta \Lambda_{t+1}^s} \tag{A.30}$$
$$= \frac{1}{p_t^H}\frac{\partial f_{t+1}}{\partial H_{t+1}} + \left(1 - \frac{\mu_{t+1}^I}{\Lambda_{t+1}^s}\right)\frac{p_{t+1}^H - p_t^H}{p_t^H}(1-\delta) - \left(1 - \frac{\mu_{t+1}^I}{\Lambda_{t+1}^s}\right)\frac{H_{t+2} - (1-\delta)H_{t+1}}{p_t^H}G'(H_{t+1})$$

if $I_{t+1} > 0$ then, by complementary slackness, $\mu_{t+1}^I = 0$. Thus, multiplying both sides by $\frac{p_t^H}{p_{t+1}^H}$, and substituting in the definition for direct returns we obtain the expression given.

Finally, in the case $I_{t+1} > 0$, the defining formula for R_{t+1} becomes just:

$$R_{t+1} = \frac{\Lambda_t^s - \beta(1-\delta)\Lambda_{t+1}^s}{\beta \Lambda_{t+1}^s} = \frac{u'(C_t/L_t)}{\beta u'(C_t/L_t)} - 1 + \delta = e_{t+1} + \delta.$$

(where we substitute also from (A.26)), as required. □

Corollary A.2. *Assume that $G'(H) < 0$ and $H_{t+1} > (1-\delta)H_t$. If $\delta = 1$ then $\frac{p_t^H}{p_{t+1}^H}R_{t+1} > r_{t+1}^s$. If $\delta = 0$, if G is convex, and if $H_{t+2} - H_{t+1} = H_{t+1} - H_t$ then $\frac{p_t^H}{p_{t+1}^H}R_{t+1} < r_{t+1}^s$.*

Regarding the case in which the price effect dominates the learning effect: the assumption of convexity for the function G giving the decline in prices, is very natural. The assumption that capital is increasing by a constant *amount*, rather than a constant *factor*, is less so; and increases in $H_{t+2} - H_{t+1}$ relative to $H_{t+1} - H_t$ will increase $\frac{p_t^H}{p_{t+1}^H}R_{t+1}$ relative to r_{t+1}^s. In general we expect the learning effect to dominate, but it is worth noting that when capital is very persistent, and when there is a considerably delay in realizing the benefits of learning, then the extent to which the learning effect pushes the shadow return above the direct return, is mitigated by the price effect.

Proof of Proposition 3.2. Considering first the firm, there is no inter-temporal element to their objective function or constraints and so we can consider their optimization period-by-period; obviously the relevant first-order condition is that

$$\frac{\partial f_t}{\partial H_t} = r_t p_t^H. \tag{A.31}$$

Meanwhile, the household maximizes:

$$\sum_{t=0}^{\infty} \beta^t \frac{L_t}{L_0} u\left(\frac{L_0}{L_t} c_t\right) \tag{A.32}$$

subject to the constraints:

$$\Lambda_t \qquad i_t + c_t = (r_t + \tau_t) p_t^H h_t + o_t \tag{A.33}$$

$$\mu_t^i \qquad i_t \geq 0 \tag{A.34}$$

$$\mu_t^h \qquad i_t = p_t^H (h_{t+1} - (1-\delta) h_t) \tag{A.35}$$

Additionally, the price is constrained by $p_t^H = G(H_t)$, but the household does not take this into account. At time t, the Lagrangian is

$$\mathcal{L}_t = \sum_{t=0}^{\infty} \beta^t \Bigg(\frac{L_t}{L_0} u\left(\frac{L_0}{L_t} c_t\right) - \Lambda_t \Big(i_t + c_t - (r_t + \tau_t) p_t^H h_t - o_t \Big) + \mu_t^i i_t \\ + \mu_t^h (i_t - p_t^H (h_{t+1} - (1-\delta) h_t)) \Bigg)$$

the first order conditions are:

$$\partial c_t : \qquad \Lambda_t = u'\left(\frac{L_0}{L_t} c_t\right) = u'\left(\frac{C_t}{L_t}\right) \tag{A.36}$$

$$\partial h_{t+1} : \qquad p_t^H \mu_t^h = \beta \Lambda_{t+1} (r_{t+1} + \tau_{t+1}) p_{t+1}^H + \beta \mu_{t+1}^h p_{t+1}^H (1-\delta) \tag{A.37}$$

$$\partial i_t : \qquad \Lambda_t = \mu_t^h + \mu_t^i \tag{A.38}$$

together with the constraints above and the inequality $\mu_t^i \geq 0$, which is complementary slack with (A.34).

Substitute (A.31) into (A.37) and rearrange: now this first order condition reads:

$$p_t^H \mu_t^h = \beta \left(\Lambda_{t+1} \frac{\partial f_{t+1}}{\partial H_{t+1}} + \mu_{t+1}^h p_{t+1}^H (1-\delta) \right) + \beta \Lambda_{t+1} \tau_{t+1} p_{t+1}^H \tag{A.39}$$

We seek the equation for τ_{t+1} that will lead to the same solution as in the social planner's problem; as derived above, this is defined by constraints (A.22)–(A.25), first order conditions (A.26)–(A.29) and the inequality $\mu_t^I \geq 0$, which is complementary slack with (A.23). Those equations are all counterparts to the equations of this model, with the exception of A.39: we wish this to imply (A.27). But this will be the case if we set (substituting in also (A.29))

$$\Lambda_{t+1} \tau_{t+1} p_{t+1}^H = -\mu_{t+1}^h (H_{t+2} - (1-\delta) H_{t+1}) G'(H_{t+1})$$

$$\Leftrightarrow \tau_{t+1} = -\frac{\mu_{t+1}^h}{\Lambda_{t+1}} \frac{H_{t+2} - (1-\delta) H_{t+1}}{p_{t+1}^H} G'(H_{t+1}) \tag{A.40}$$

So if $i_t > 0$, which implies $\mu_{t+1}^h = \Lambda_{t+1}$, then the two models are defined by the same first-order conditions in variables C_t, H_t and I_t. In each case p_t^H is defined by H_t, so if $O_t = L_0 o_t$ for all t then the solutions are equal – that is, this level of subsidy achieves the social optimum (subject to O_t).

We have treated o_t and O_t as exogenous for both the household and the social planner. More generally, a model will allow optimization in all factors of production and sources of income. However, if all externalities except for the learning-by-doing in p_t^H have been internalized, then by the Coase Theorem and the First Welfare Theorem, it follows that the optimal O_t^* for the planner satisfies $O_t^* = L_0 o_t^*$, where o_t^* is optimal for the household, so the solutions to the models coincide. □

Proof of Corollary 3.3. If $p_t^H = G(H_t) = p_0^H \left(\frac{H_t}{H_0}\right)^{-\lambda}$, then

$$G'(H_t) = -\lambda \frac{p_0^H}{H_0} \left(\frac{H_t}{H_0}\right)^{-\lambda-1} = -\lambda \frac{p_0^H}{H_t} \left(\frac{H_t}{H_0}\right)^{-\lambda} = -\frac{\lambda p_t^H}{H_t}$$

Hence, in this case,

$$\tau_t = \lambda \left(\frac{H_{t+1}}{H_t} - (1-\delta)\right)$$

B Calibration

This section describes calibration of the model. We build on the seminal "DICE 2013" climate-economy model of Nordhaus (2014a), which serves as benchmark in the literature and policy applications. Some of the parameter values are drawn from the existing studies, in particular, from Hassler et al. (2012), Papageorgiou et al. (2017) and Rezai and van der Ploeg (2017). All the parameter values are summarized in Table A.1. Details of the calibration are as follows:

B.1 Production

Labor L_0 is given for 2012 using United Nations data. We assume it continues to evolve as in DICE 2013. We set the value of elasticity of substitution between general output, Y^g, and electricity, E, in the final-goods production function, $\kappa = 0.46$, following Rezai and van der Ploeg (2017), as a compromise between short-term insubstitutability (Hassler et al. (2012)) and longer-term substitutability. We take the value of θ from Papageorgiou et al. (2017) to be 0.003. The technology weightings A_0^g and A_t^E will be set to match other data. Subsequently, A_t^g evolves as in DICE, and A_t^E evolves in step with it. We set $\alpha = 0.4$ as an approximation of the values Papageorgiou et al. (2017) get in their various specifications, but this is also commonly-used value in the literature. We set the deprecation rate of the general capital stock at $\delta^g = 0.05$ following Rezai and van der Ploeg (2017).

In modeling the electricity sector we follow Papageorgiou et al. (2017): we set the value of w at 0.32 (across various specifications, they find $w = 0.19$ to 0.70, with a mean of 0.32). We set the value of the substitution parameter $\xi = 0.46$, in line with their estimates. We find the initial generating capital stocks for the dirty and renewable generating capacity from EIA data.[33] We set A_0^E so that electricity output in the first period matches the EIA data on electricity output in 2012.

In calibrating the prices of fossil and renewable energy capital p_t^D, p_t^H, we set p_t^D to be constant and to match the current price of new coal-fired power stations in China, as these may be the marginal new plants in consideration.[34] For p_t^H, see the section below. Exponential depreciation for fossil fuel and renewable energy capital is calculated so that the net lifetime availability of capital is equal to the general expected lifetime of plants in this sector: 40 and 25 years respectively.

We know the initial value of K_t^D from EIA data for 2012, and D_t from European Union data. We assume that initially $\zeta_t = 1$.

[33] All fossil generating capacity has been included on the "dirty" side. For renewables, we exclude hydropower, because it is a relatively mature source of electricity (costs are not falling very fast) and its use is constrained by physical geography, with a large fraction of suitable sites already in use (so its use cannot expand fast), so this technology does not represent the features of interest in the model. Since extensive hydropower capacity already exists, the inclusion of existing capacity would severely bias the trajectory of the equation relating renewable capital to cost of renewable capital.

[34] Numbers taken from Energy and Environmental Economics, Inc. (2012).

The function form of fossil fuel extraction cost is taken from Rezai and van der Ploeg (2017), but we calibrate it differently because we are more concerned with the price of coal than oil. So we set γ_1 to represent the cost of coal in 2012 (IEA2014 data), which we have converted to give this cost as a price per GtC of CO_2 pollution (so that fuel and pollution will be in a straightforward 1:1 ratio), to give a cost of 0.09 trillion 2010$ / GtC. We take $S_0 = 2000$.[35] Using the IEA estimate of the cost of coal in 2040 along a given trajectory, and the additional fractional fossil stock use that this would represent, the second parameter of the resource cost equation is calculated to be $\gamma_2 = 1.64$.

We set the value of ϕ_2 in the mitigation expenditure function Ψ_t from DICE2013.

[35]The proven resources of all fossil fuels may be estimated as 1003 GtC using EIA data. However continued exploration will enlarge these stocks. We use the stock figure of 2000 GtC.

Parameter	Value	Units	Definition
L_0	7.10	billion people	Population
A_0^g	2.53		Productivity
K_0^g	150.00	trillion 2010$	Initial 'general' capital stock
θ	0.003		Energy share parameter, global output
α	0.4		Share of capital, global output
κ	0.46		Elas of substitution btw energy and capital/labor
ξ	0.46		Elas of subs between clean and dirty electricity capital
ω	0.32		Weight on renewable capital in electricity output
D_0	9.4	GtC	CO_2 Emissions in year 2012
D_0^{land}	0.90	GtC	Land-use CO_2 emissions in year 2012
D_0^E	3.30	GtC	Electricity CO_2 emissions in year 2012
D_0^g	5.22	GtC	General economy CO_2 emissions in 2012
ν	0.91	GtC/(tW capacity)	Fuel use & emissions from dirty electricity production
S_0	2000	GtC	Existing stock of fossil fuel (as of 2012)
Y_0	60.11	trillion 2010$	Initial gross world output
K_0^D	3.61	tW	Initial capital stock of fossil technology
H_0	0.46	tW	Initial renewable-energy-knowledge capital stock
p^D	0.57	trillion 2010$/tW	Price of dirty electricity capital
p_0^H	2.11	trillion 2010$/tW	Initial price of clean electricity capital
δ^g	0.05	year^{-1}	Capital stock depreciation rate
δ^D	0.025	year^{-1}	Fossil energy capital depreciation
δ^H	0.04	year^{-1}	Renewable energy capital depreciation
γ_1	0.09	trillion 2010$/GtC	Parameter of fuel extraction costs
γ_2	1.64		Parameter of fuel extraction costs
A_0^E	6.93		Productivity of energy production
λ	0.295		Rate of learning.
ς_1	0.00267		Damage function parameter.
ς_2	2		Damage function parameter.
ς_3	0.001		Damage function parameter.
ς_4	50		Damage function parameter.
ϕ_2	2.8		Mitigation expenditure parameter.
ϕ_3	0.01		Mitigation expenditure parameter.
σ_0	0.0904	GtC/trillion 2010$	the carbon-equivalent emissions to output ratio.
$\phi_{1,0}$	0.041		Backstop costs.

Table A.1: Parameter values

Variable	Definition
c_t	Per-household consumption
L_t	Population at period t
K_t^g	Aggregate capital stock in general economy
K_t^D	Aggregate dirty capital stock
H_t	Aggregate clean (renewable) capital stock
I_t^g	Aggregate investment in general economy
I_t^D	Aggregate investment in dirty capital stock
I_t^H	Aggregate investment in clean (renewable) capital stock
Ψ_t	Abatement
S_t	Fossil fuel stock at period t
$G^D(S_t)$	Fossil fuel extraction costs
r_t^D	Rate of return on fossil (dirty) capital
r_t^H	Rate of return on renewable (clean) capital
r_t^g	Rate of return on general capital
Π_t^g	Total profits from sale of the final goods
Π_t^D	Total profits from sale of the dirty fuel based electricity
Π_t^H	Total profits from sale of the clean electricity
Π_t^{DE}	Total profits from sale of the fossil fuel
Π_t^E	Total profits from sale of the aggregate electricity
Π_t	Sum of all profits
π_t	Total profits per-household
p_t^D	Cost of fossil fuel capital
p_t^H	Cost of renewable energy capital
p_t^{EH}	Price of electricity generated by clean power stations
p_t^{ED}	Price of electricity generated by fossil fuel based power plants
p_t^e	Price of aggregate electricity
p_t^{fuel}	Price of dirty fossil fuel
Γ_t^{ED}	Electricity generated by fossil-fuel based power plants
$Y_t = f(Y_t^g, E_t)$	Total output before damages
Y_t^g	Output of the general economy
$E_t = f_t^E(H_t, \Gamma_t^{ED})$	Aggregate electricity
ζ_t	Utilization rate of dirty capital stock
η_t	Emission control rate in the general sector
w_t	Wage
D_t^E	Fossil fuel (e.g., coal) used in production of electricity
D_t^g	Fossil fuel used in the general economy

Table A.2: Variables notation and definition

C The Setup of Social Planner's Problem

We will consider two alternative perspectives for returns on investment, which will be relevant in different contexts. First, as in Section 3.1, we define:

Definition C.1. The *shadow returns on investment in the general, dirty and renewable capital* *stocks* are defined to be respectively R_t^g, R_t^D and R_t^H so that:

$$R_{t+1}^g := \frac{\mu_t^{Kg} - \beta(1-\delta^g)\mu_{t+1}^{Kg}}{\beta u'\left(C_{t+1}/L_{t+1}\right)} \tag{A.41}$$

$$R_{t+1}^D := \frac{\mu_t^{KD} - \beta(1-\delta^D)\mu_{t+1}^{KD}}{\beta u'\left(C_{t+1}/L_{t+1}\right)} \tag{A.42}$$

$$R_{t+1}^H := \frac{\mu_t^{KH} - \beta(1-\delta^H)\mu_{t+1}^{KH}}{\beta u'\left(C_{t+1}/L_{t+1}\right)} \tag{A.43}$$

where μ_t^{Kg}, μ_t^{KD} and μ_t^H are the shadow prices on the capital accumulation constraints as below.

On the other hand, one might consider the more immediate definitions for direct economic returns to investment:

Definition C.2. The *direct economic returns on investment in the general, dirty and renewable capital stocks* are defined respectively to be r_t^g, r_t^D and r_t^H so that:

$$r_{t+1}^g := \frac{\partial}{\partial K_{t+1}^g}\left(Y_{t+1} - \Psi_{t+1}\right) \tag{A.44}$$

$$r_{t+1}^D := \frac{1}{p_{t+1}^D}\frac{\partial}{\partial K_{t+1}^D}\left(Y_{t+1} - \Psi_{t+1}\right) \tag{A.45}$$

$$r_{t+1}^H := \frac{1}{p_{t+1}^H}\frac{\partial}{\partial H_{t+1}}\left(Y_{t+1} - \Psi_{t+1}\right) \tag{A.46}$$

Here we measure the direct effects of investment on output net of mitigation costs, and the output is

$$Y_t = \Omega(T_t) f(Y_t^g, E_t) \tag{A.47}$$

with $Y_t^g = f_t^g(K_t^g, L_t)$.

The social planner's problem is outlined below. Specifically, the social planner maximizes the social welfare function:

$$\sum_{t=0}^{\infty} \beta^t L_t u\left(\frac{C_t}{L_t}\right) \tag{A.48}$$

subject to constraints:

$$
\begin{aligned}
Y_t &= I_t^g + I_t^D + I_t^H + C_t + G^D(S_t)(D_t^E + D_t^g) + \frac{\phi_{1,t}\eta_t^{\phi_2} Y_t^g}{(1-\eta_t)^{\phi_3}} && \mu_t^{BC} && \text{(A.49)}\\
S_{t+1} &= S_t - D_t^E - D_t^g && \mu_t^S && \text{(A.50)}\\
D_t &= D_t^E + D_t^{\text{land}} + D_t^g && \mu_t^D && \text{(A.51)}\\
T_t &= \mathcal{W}_t(D_0, \dots, D_{t-1}) && \mu_t^W && \text{(A.52)}\\
E_t &= f_t^E(H_t, \zeta_t K_t^D) = A_t^E \left(\omega (H_t)^\xi + (1-\omega)(\zeta_t K_t^D)^\xi\right)^{1/\xi} && \mu_t^E && \text{(A.53)}\\
D_t^E &= \nu \zeta_t K_t^D && \mu_t^{DE} && \text{(A.54)}\\
D_t^g &= \sigma_t (1-\eta_t) Y_t^g && \mu_t^{Dg} && \text{(A.55)}\\
\zeta_t &\le 1 && \mu_t^\zeta && \text{(A.56)}\\
p_t^H &= G(H_t) && \mu_t^{pH} && \text{(A.57)}\\
I_t^g &= K_{t+1}^g - (1-\delta^g) K_t^g && \mu_t^{Kg} && \text{(A.58)}\\
I_t^D &= p^D (K_{t+1}^D - (1-\delta^D) K_t^D) && \mu_t^{KD} && \text{(A.59)}\\
I_t^H &= p_t^H (H_{t+1} - (1-\delta^H) H_t) && \mu_t^{KH} && \text{(A.60)}\\
I_t^D &\ge 0 && \mu_t^{ID} && \text{(A.61)}\\
I_t^H &\ge 0 && \mu_t^{IH} && \text{(A.62)}
\end{aligned}
$$

(We do not need to specify $\zeta_t \ge 0$ as this will never be violated in the optimum.) So we calculate the Lagrangian $\mathcal{L}$ as

$$
\begin{aligned}
\mathcal{L} \;=\; & \sum_{t=0}^{\infty} \beta^t \left[L_t u\left(\frac{C_t}{L_t}\right) - \mu_t^S (S_{t+1} - S_t + D_t^E + D_t^g) + \mu_t^D (D_t - D_t^E - D_t^{\text{land}} - D_t^g) \right] \\
& + \sum_{t=0}^{\infty} \beta^t \mu_t^W (T_t - \mathcal{W}_t(D_0, \dots, D_{t-1})) \\
& + \sum_{t=0}^{\infty} \beta^t \mu_t^{BC} \left[\Omega(T_t) f(Y_t^g, E_t) - I_t^g - I_t^D - I_t^H - C_t - G^D(S_t)(D_t^E + D_t^g) - \frac{\phi_{1,t}\eta_t^{\phi_2} Y_t^g}{(1-\eta_t)^{\phi_3}} \right] \\
& - \sum_{t=0}^{\infty} \beta^t \left[\mu_t^E (E_t - f_t^E(H_t, \zeta_t K_t^D)) \right] \\
& + \sum_{t=0}^{\infty} \beta^t \left[\mu_t^{DE} (D_t^E - \nu \zeta_t K_t^D) + \mu_t^{Dg} (D_t^g - \sigma_t (1-\eta_t) Y_t^g) + \mu_t^{pH} (p_t^H - G(H_t)) + \mu_t^\zeta (1-\zeta_t) \right] \\
& + \sum_{t=0}^{\infty} \beta^t \left[\mu_t^{Kg} \left(I_t^g - K_{t+1}^g + (1-\delta^g) K_t^g \right) \right] \\
& + \sum_{t=0}^{\infty} \beta^t \left[\mu_t^{KD} \left(I_t^D - p^D K_{t+1}^D + p^D (1-\delta^D) K_t^D \right) + \mu_t^{ID} I_t^D \right] \\
& + \sum_{t=0}^{\infty} \beta^t \left[\mu_t^{KH} \left(I_t^H - p_t^H H_{t+1} + p_t^H (1-\delta^H) H_t \right) + \mu_t^{IH} I_t^H \right]
\end{aligned}
$$

We obtain the following first order conditions (using shorthand f_t for $f(Y_t^g, E_t)$, f_t^g for $f_t^g(K_t^g, L_t)$, etc.)

$$\partial C_t: \quad u'\left(\frac{C_t}{L_t}\right) = \mu_t^{BC} \tag{A.63}$$

$$\partial S_{t+1}: \quad \beta\mu_{t+1}^S = \mu_t^S + \beta\mu_{t+1}^{BC}\frac{dG^D}{dS}(S_{t+1})(D_{t+1}^E + D_{t+1}^g) \tag{A.64}$$

$$\partial D_t^E: \quad \mu_t^{DE} = \mu_t^S + \mu_t^D + \mu_t^{BC}G^D(S_t) \tag{A.65}$$

$$\partial D_t^g: \quad \mu_t^{Dg} = \mu_t^S + \mu_t^D + \mu_t^{BC}G^D(S_t) \tag{A.66}$$

$$\partial D_t: \quad \mu_t^D = \sum_{m=0}^{\infty}\beta^m\mu_{t+m}^W\frac{\partial \mathcal{W}_{t+m}}{\partial D_t} \tag{A.67}$$

$$\partial T_t: \quad \mu_t^W = -\mu_t^{BC}\Omega'(T_t)f_t \tag{A.68}$$

$$\partial E_t \quad \mu_t^E = \mu_t^{BC}\Omega(T_t)\frac{\partial f_t}{\partial E_t} \tag{A.69}$$

$$\partial K_{t+1}^g: \quad \mu_t^{Kg} = \beta\mu_{t+1}^{Kg}(1-\delta^g) + \beta\mu_{t+1}^{BC}\left(\Omega(T_{t+1})\frac{\partial f_{t+1}}{\partial Y_{t+1}^g} - \frac{\phi_{1,t+1}\eta_{t+1}^{\phi_2}}{(1-\eta_{t+1})^{\phi_3}}\right)\frac{\partial f_{t+1}^g}{\partial K_{t+1}^g} \tag{A.70}$$

$$\partial I_t^g \quad \mu_t^{Kg} = \mu_t^{BC} \tag{A.71}$$

$$\partial K_{t+1}^D \quad p^D\mu_t^{KD} = \beta p^D\mu_{t+1}^{KD}(1-\delta^D) + \beta\zeta_{t+1}\left(\mu_{t+1}^E\frac{\partial f_{t+1}^E}{\partial(\zeta_{t+1}K_{t+1}^D)} - \mu_{t+1}^{DE}\nu\right) \tag{A.72}$$

$$\partial I_t^D \quad \mu_t^{KD} = \mu_t^{BC} - \mu_t^{ID} \tag{A.73}$$

$$\partial H_{t+1} \quad p_t^H\mu_t^{KH} = \beta p_{t+1}^H\mu_{t+1}^{KH}(1-\delta^H) + \beta\mu_{t+1}^E\frac{\partial f_{t+1}^E}{\partial H_{t+1}} - \beta\mu_{t+1}^{pH}G'(H_{t+1}) \tag{A.74}$$

$$\partial I_t^H \quad \mu_t^{KH} = \mu_t^{BC} - \mu_t^{IH} \tag{A.75}$$

$$\partial p_t^H: \quad \mu_t^{pH} = \mu_t^{KH}(H_{t+1} - (1-\delta^H)H_t) \tag{A.76}$$

$$\partial\zeta: \quad \mu_t^{\zeta} = K_t^D\left(\mu_t^E\frac{\partial f_t^E}{\partial(\zeta_t K_t^D)} - \mu_t^{DE}\nu\right) \tag{A.77}$$

$$\partial\eta_t: \quad \sigma_t\mu_t^{Dg} = \mu_t^{BC}\frac{\phi_{1,t}\eta_t^{\phi_2-1}}{(1-\eta_t)^{1+\phi_3}}[\phi_2(1-\eta_t) + \eta_t\phi_3] \tag{A.78}$$

together with constraints (A.49)–(A.62) and inequalities $\mu_t^{\zeta} \geq 0$, $\mu_t^{ID} \geq 0$, $\mu_t^{IH} \geq 0$ which are complementary slack with corresponding equations (A.56) and (A.61)-(A.62).

It is useful to prove the following proposition, which gives an expression for the rates of return plotted in Figure 2.

Proposition C.3. *In the optimal social planner's solution,*

$$R_{t+1}^g = r_{t+1}^g \tag{A.79}$$

$$R_{t+1}^D = \frac{\zeta_t}{p^D}\left(\frac{\partial Y_{t+1}}{\partial(\zeta_t K_{t+1}^D)} - \nu\frac{\mu_{t+1}^{DE}}{u'(C_{t+1}/L_{t+1})}\right) = r_{t+1}^D - \frac{\nu\zeta_t}{p^D}\frac{\mu_{t+1}^{DE}}{u'(C_{t+1}/L_{t+1})} \tag{A.80}$$

where μ_{t+1}^{DE} is the shadow price on constraint (A.54), which determines emissions D_{t+1}^E from the use of dirty energy capital.

Returns on general capital take into account the mitigation expense involved in the use of general capital. Returns on dirty energy capital are given by the marginal productivity of fossil fuel capital infrastructure in output, net of the shadow price of fuel used alongside it, and scaled by the price of this capital stock. Following the results (A.79) and Definition (A.44), throughout the rest of the paper, we refer to r_t^g as the return on investment in the general capital stock.

Proof of Proposition C.3. Substitute (A.63) and (A.71) into (A.70) and use Definition C.1 to prove that $R_{t+1}^g = r_{t+1}^g$. It has been presented in a more compact form from the observations that $Y_{t+1} = Z(T_{t+1})f_{t+1}$ and $\Psi_{t+1} = \frac{\phi_{1,t+1}\eta_{t+1}^{\phi_2}}{(1-\eta_{t+1})^{\phi_3}} Y_{t+1}^g$. The form that is most useful for further derivations is (from (A.71)):

$$R_{t+1}^g - \delta^g = \frac{\mu_t^{BC}}{\beta\mu_{t+1}^{BC}} - 1. \tag{A.81}$$

For R_{t+1}^D, divide (A.72) by $\beta p^D \mu_{t+1}^{BC}$ and substitute (A.69), and then (A.73) and (A.63) to obtain

$$\frac{\mu_t^{KD}}{\beta\mu_{t+1}^{BC}} = \frac{\mu_{t+1}^{KD}}{\mu_{t+1}^{BC}}(1-\delta^D) + \frac{\zeta_{t+1}}{p^D}\left(Z(T_{t+1})\frac{\partial f_{t+1}}{\partial E_{t+1}}\frac{\partial f_{t+1}^E}{\partial(\zeta_{t+1}K_{t+1}^D)} - \frac{\mu_{t+1}^{DE}}{\mu_{t+1}^{BC}}\nu\right)$$

$$\Rightarrow \frac{\zeta_{t+1}}{p^D}\left(\frac{\partial Y_{t+1}}{\partial(\zeta_{t+1}K_{t+1}^D)} - \frac{\mu_{t+1}^{DE}}{\mu_{t+1}^{BC}}\nu\right) = \frac{\mu_t^{KD}}{\beta\mu_{t+1}^{BC}} - \frac{\mu_{t+1}^{KD}}{\mu_{t+1}^{BC}}(1-\delta^D) = R_{t+1}^D \tag{A.82}$$

as required. □

To prove Proposition 4.2 of the main text, we will use the following results.

Proposition C.4. ***[The social cost of carbon]*** *In an optimal solution:*

$$\chi_t = -u'\left(\frac{C_t}{L_t}\right)^{-1}\sum_{m=1}^{\infty}\beta^m u'\left(\frac{C_{t+m}}{L_{t+m}}\right)\frac{\partial Y_{t+m}}{\partial D_t}. \tag{A.83}$$

Proof of Proposition C.4 (Social Cost of Carbon). Substitute (A.68) into (A.67), and divide through by μ_t^{BC}, to obtain:

$$\frac{\mu_t^D}{\mu_t^{BC}} = -\sum_{m=0}^{\infty}\beta^m\left(\frac{\mu_{t+m}^{BC}}{\mu_t^{BC}}\Omega'(T_{t+m})f_{t+m}\right)\frac{\partial \mathcal{W}_{t+m}}{\partial D_t} \tag{A.84}$$

$$= -\sum_{m=1}^{\infty}\beta^m\left(\frac{\mu_{t+m}^{BC}}{\mu_t^{BC}}\Omega'(T_{t+m})f_{t+m}\right)\frac{\partial \mathcal{W}_{t+m}}{\partial D_t} \tag{A.85}$$

where the sum is from $m = 1$ because $\frac{\partial \mathcal{W}_t}{\partial D_t} = 0$. Next, note that

$$\frac{\partial Y_{t+m}}{\partial D_t} = \Omega'(T_{t+m})\frac{\partial \mathcal{W}_{t+m}}{\partial D_t}f_{t+m} \tag{A.86}$$

Substituting (A.86), as well as (A.63), into (A.85), we obtain and write this as

$$\chi_t := \frac{\mu_t^D}{u'(C_t/L_t)} = \frac{\mu_t^D}{\mu_t^{BC}} = -u'\left(\frac{C_t}{L_t}\right)^{-1}\sum_{m=1}^{\infty}\beta^m u'\left(\frac{C_{t+m}}{L_{t+m}}\right)\frac{\partial Y_{t+m}}{\partial D_t}. \tag{A.87}$$

Since $\Omega'(T_{t+m}) < 0$, we have $\partial Y_{t+m}/\partial D_t < 0$, then $\chi_t > 0$. We call this term the social cost of carbon. It represents the marginal future welfare effect of emissions in terms of current welfare. □

Proposition C.5. ***[Hotelling with fossil stocks]*** *Write μ_t^S for the shadow price on Equation (A.50) constraining the stock of fossil fuel. Then:*

$$\frac{\mu_{t+1}^S}{u'\left(C_{t+1}/L_{t+1}\right)} = \frac{\mu_t^S}{u'\left(C_t/L_t\right)}(1-\delta^g+r_{t+1}^g) + \frac{dG^D}{dS}(S_{t+1})(D_{t+1}^E + D_{t+1}^g) \tag{A.88}$$

and so

$$\frac{\mu_t^S}{u'\left(C_t/L_t\right)} = -\sum_{s=1}^{\infty} \Delta_{t,s}(G^D)'(S_{t+s})(D_{t+s}^E + D_{t+s}^g) \tag{A.89}$$

where $\Delta_{t,s} = \prod_{s'=1}^{s} \frac{1}{1-\delta^g+r_{t+s'}^g}$ is the compound discount factor.

That is, the return on extracting a unit of fossil fuels tomorrow should be equal to the return on extracting an extra unit today, selling it and getting a return on it at the rate of interest, less the increase in future extraction cost.

Proof of Proposition C.5 (Hotelling with fossil stocks). Divide (A.64) through by μ_{t+1}^{BC}:

$$\beta\frac{\mu_{t+1}^S}{\mu_{t+1}^{BC}} = \frac{\mu_t^S}{\mu_t^{BC}}\frac{\mu_t^{BC}}{\mu_{t+1}^{BC}} + \beta\frac{dG^D}{dS}(S_{t+1})\left(D_{t+1}^E + D_{t+1}^g\right)$$

Substitute in (A.81) and divide by β, to obtain the Hotelling rule:

$$\frac{\mu_{t+1}^S}{\mu_{t+1}^{BC}} = \frac{\mu_t^S}{\mu_t^{BC}}(1-\delta^g+r_{t+1}^g) + \frac{dG^D}{dS}(S_{t+1})\left(D_{t+1}^E + D_{t+1}^g\right) \tag{A.90}$$

That is, we proved Equation (A.88) as $\mu_t^{BC} = u'(C_t/L_t)$ from (A.63). To get the infinite sum, repeatedly substitute:

$$\frac{\mu_t^S}{\mu_t^{BC}} = \frac{1}{1-\delta^g+r_{t+1}^g}\left(\frac{\mu_{t+1}^S}{\mu_{t+1}^{BC}} - (G^D)'(S_{t+1})(D_{t+1}^E + D_{t+1}^g)\right) \tag{A.91}$$

$$= \frac{1}{1-\delta^g+r_{t+1}^g}\left(\frac{1}{1-\delta^g+r_{t+2}^g}\left(\frac{\mu_{t+2}^S}{\mu_{t+2}^{BC}} - (G^D)'(S_{t+2})(D_{t+2}^E + D_{t+2}^g)\right)\right. \tag{A.92}$$

$$\left. - (G^D)'(S_{t+1})(D_{t+1}^E + D_{t+1}^g)\right) \tag{A.93}$$

$$= -\sum_{s=1}^{\infty} \Delta_{t,s}(G^D)'(S_{t+s})\left(D_{t+s}^E + D_{t+s}^g\right) \tag{A.94}$$

where

$$\Delta_{t,s} = \prod_{s'=1}^{s} \frac{1}{1-\delta^g+r_{t+s'}^g} \tag{A.95}$$

That is, we proved Equation (A.89). □

Proposition C.6. ***[Returns on dirty fuel]***

$$\frac{\partial Y_t}{\partial D_t^E} = \frac{\mu_t^S}{u'\left(C_t/L_t\right)} + \chi_t + G^D(S_t) + \frac{p^D R_t^D}{\zeta_t \nu}. \tag{A.96}$$

That is, in an optimal solution, the marginal productivity of fossil fuels in the final output is equal to the shadow value of fossil fuel stocks plus the social cost of carbon, the extraction cost, and the fraction of the rate of return on investment in K^D (gross of depreciation) which represents fuel use.

Proof of Proposition C.6 (Returns on dirty fuel). Now take (A.65), divide by μ_t^{BC} and substitute in (A.83):

$$\frac{\mu_t^{DE}}{\mu_t^{BC}} = \frac{\mu_t^S}{\mu_t^{BC}} + \chi_t + G^D(S_t)$$

For R_{t+1}^D, divide (A.72) by $\beta p^D \mu_{t+1}^{BC}$ and substitute (A.69), and then (A.73) and (A.63) to obtain

$$\frac{\mu_t^{KD}}{\beta\mu_{t+1}^{BC}} = \frac{\mu_{t+1}^{KD}}{\mu_{t+1}^{BC}}(1-\delta^D) + \frac{\zeta_{t+1}}{p^D}\left(\Omega(T_{t+1})\frac{\partial f_{t+1}}{\partial E_{t+1}}\frac{\partial f_{t+1}^E}{\partial(\zeta_{t+1}K_{t+1}^D)} - \frac{\mu_{t+1}^{DE}}{\mu_{t+1}^{BC}}\nu\right)$$

$$\Rightarrow \frac{\zeta_{t+1}}{p^D}\left(\frac{\partial Y_{t+1}}{\partial(\zeta_{t+1}K_{t+1}^D)} - \frac{\mu_{t+1}^{DE}}{\mu_{t+1}^{BC}}\nu\right) = \frac{\mu_t^{KD}}{\beta\mu_{t+1}^{BC}} - \frac{\mu_{t+1}^{KD}}{\mu_{t+1}^{BC}}(1-\delta^D) = R_{t+1}^D \tag{A.97}$$

which could be written as:

$$R_{t+1}^D = \frac{\zeta_{t+1}}{p^D}\left(\Omega(T_{t+1})\frac{\partial f_{t+1}}{\partial E_{t+1}}\frac{\partial f_{t+1}^E}{\partial(\zeta_{t+1}K_{t+1}^D)} - \nu\left(\frac{\mu_{t+1}^S}{\mu_{t+1}^{BC}} + \chi_{t+1} + G^D(S_{t+1})\right)\right). \tag{A.98}$$

Now, differentiating (A.47) by D_t^E and multiplying by ν:

$$\nu\frac{\partial Y_{t+1}}{\partial D_{t+1}^E} = \Omega(T_{t+1})\frac{\partial f_{t+1}}{\partial E_{t+1}}\frac{\partial f_{t+1}^E}{\partial(\zeta_{t+1}K_{t+1}^D)} \tag{A.99}$$

So:

$$R_{t+1}^D = \frac{\nu\zeta_{t+1}}{p^D}\left(\frac{\partial Y_{t+1}}{\partial D_{t+1}^E} - \frac{\mu_{t+1}^S}{\mu_{t+1}^{BC}} - \chi_{t+1} - G^D(S_{t+1})\right) \tag{A.100}$$

$$\Rightarrow \frac{\partial Y_t}{\partial D_t^E} = \frac{\mu_t^S}{\mu_t^{BC}} + \chi_t + G^D(S_t) + \frac{p^D R_t^D}{\zeta_t \nu} \tag{A.101}$$

□

Lemma C.7. *In the optimal social planner's solution, If $I_t^H > 0$ and $I_{t+1}^H > 0$ then:*

$$\frac{p_{t+1}^H}{p_t^H} r_{t+1}^H = 1 + r_{t+1}^g - \delta^g - \frac{p_{t+1}^H}{p_t^H}(1-\delta^H) + \frac{(H_{t+2} - (1-\delta^H)H_{t+1})}{p_t^H} G'(H_{t+1}) \tag{A.102}$$

Proof of Lemma C.7. Consider the equation for renewable capital (A.74). Dividing by $\beta p_t^H \mu_{t+1}^{BC}$, and substituting in equations (A.69) and (A.76) as well as (A.75), we see

$$\begin{aligned}
\frac{\mu_t^{KH}}{\beta\mu_{t+1}^{BC}} &= \frac{p_{t+1}^H}{p_t^H}\frac{(\mu_{t+1}^{BC}-\mu_{t+1}^{IH})}{\mu_{t+1}^{BC}}(1-\delta^H) + \frac{\Omega(T_{t+1})}{p_t^H}\frac{\partial f_{t+1}}{\partial E_{t+1}}\frac{\partial f_{t+1}^E}{\partial H_{t+1}} \\
&\quad - \frac{(\mu_{t+1}^{BC}-\mu_{t+1}^{IH})}{\mu_{t+1}^{BC}}\frac{(H_{t+2}-(1-\delta^H)H_{t+1})}{p_t^H}G'(H_{t+1}). \\
&= \left(1+\frac{p_{t+1}^H - p_t^H}{p_t^H}\right)\left(1-\frac{\mu_{t+1}^{IH}}{\mu_{t+1}^{BC}}\right)(1-\delta^H) + \frac{1}{p_t^H}\frac{\partial Y_{t+1}}{\partial H_{t+1}} \\
&\quad - \left(1-\frac{\mu_{t+1}^{IH}}{\mu_{t+1}^{BC}}\right)\frac{(H_{t+2}-(1-\delta^H)H_{t+1})}{p_t^H}G'(H_{t+1}).
\end{aligned}$$

From (A.75) and (A.81), we have

$$\frac{\mu_t^{KH}}{\beta\mu_{t+1}^{BC}} = \frac{\mu_t^{BC}-\mu_t^{IH}}{\beta\mu_{t+1}^{BC}} = (1+r_{t+1}^g-\delta^g)\left(1-\frac{\mu_t^{IH}}{\mu_t^{BC}}\right) \tag{A.103}$$

Combining the above two equations and Definition C.2, we have

$$\begin{aligned}
(1+r_{t+1}^g-\delta^g)\left(1-\frac{\mu_t^{IH}}{\mu_t^{BC}}\right) &= \left(1+\frac{p_{t+1}^H-p_t^H}{p_t^H}\right)\left(1-\frac{\mu_{t+1}^{IH}}{\mu_{t+1}^{BC}}\right)(1-\delta^H) + r_{t+1}^H\frac{p_{t+1}^H}{p_t^H} \\
&\quad - \left(1-\frac{\mu_{t+1}^{IH}}{\mu_{t+1}^{BC}}\right)\frac{(H_{t+2}-(1-\delta^H)H_{t+1})}{p_t^H}G'(H_{t+1}).
\end{aligned}$$

This gives the more general form; when $I_t^H > 0$ and $I_{t+1}^H > 0$, implying $\mu_t^{IH} = \mu_{t+1}^{IH} = 0$, then the version given in the lemma follows. □

D Decentralized Equilibrium

A representative household maximizes:

$$\sum_{t=0}^{\infty}\beta^t\frac{L_t}{L_0}u\left(\frac{L_0}{L_t}c_t\right) \tag{A.104}$$

subject to the constraints:

$$\Lambda_t \qquad i_t^g + i_t^D + i_t^H + c_t = \frac{L_t}{L_0} w_t + \pi_t + r_t^g k_t^g + r_t^D p_t^D k_t^D + r_t^H p_t^H h_t + \frac{1}{L_0}\left(\tau_t^D (D_t^E + D_t^g) - \tau_t^H p_t^H H_t\right) \tag{A.105}$$

$$\mu_t^{iD} \qquad i_t^D \geq 0 \tag{A.106}$$

$$\mu_t^{iH} \qquad i_t^H \geq 0 \tag{A.107}$$

$$\mu_t^{kg} \qquad i_t^g = k_{t+1}^g - (1-\delta^g) k_t^g \tag{A.108}$$

$$\mu_t^{kD} \qquad i_t^D = p_t^D (k_{t+1}^D - (1-\delta^D) k_t^D) \tag{A.109}$$

$$\mu_t^{kH} \qquad i_t^H = p_t^H (k_{t+1}^H - (1-\delta^H) k_t^H) \tag{A.110}$$

At time t, the Lagrangian is

$$\mathcal{L}_t = \sum_{t=0}^{\infty} \beta^t \Bigg(\frac{L_t}{L_0} u\left(\frac{L_0}{L_t} c_t\right) - \Lambda_t \left(i_t^g + i_t^D + i_t^H + c_t\right) + \Lambda_t \left(\frac{L_t}{L_0} w_t + \pi_t + r_t^g k_t^g + r_t^D p_t^D k_t^D + r_t^H p_t^H h_t\right) + \frac{\Lambda_t}{L_0}\left(\tau_t^D (D_t^E + D_t^g) - \tau_t^H p_t^H H_t\right) + \mu_t^{iD} i_t^D + \mu_t^{iH} i_t^H + \mu_t^{kg}\left(i_t^g - (k_{t+1}^g - (1-\delta^g) k_t^g)\right) + \mu_t^{kD}(i_t^D - p_t^D (k_{t+1}^D - (1-\delta^D) k_t^D)) + \mu_t^{kH}(i_t^H - p_t^H (h_{t+1} - (1-\delta^H) h_t)) \Bigg) \tag{A.111}$$

the first order conditions are:

$$\partial c_t: \qquad \Lambda_t = u'\left(\frac{L_0}{L_t} c_t\right) = u'\left(\frac{C_t}{L_t}\right) \tag{A.112}$$

$$\partial k_{t+1}^g: \qquad \mu_t^{kg} = \beta(\Lambda_{t+1} r_{t+1}^g + \mu_{t+1}^{kg}(1-\delta^g)) \tag{A.113}$$

$$\partial k_{t+1}^D: \qquad p_t^D \mu_t^{kD} = \beta\left(\Lambda_{t+1} p_{t+1}^D r_{t+1}^D + \mu_{t+1}^{kD} p_{t+1}^D (1-\delta^D)\right) \tag{A.114}$$

$$\partial h_{t+1}: \qquad p_t^H \mu_t^{kH} = \beta\left(\Lambda_{t+1} p_{t+1}^H r_{t+1}^H + \mu_{t+1}^{kH} p_{t+1}^H (1-\delta^H)\right) \tag{A.115}$$

$$\partial i_t^g: \qquad \Lambda_t = \mu_t^{kg} \tag{A.116}$$

$$\partial i_t^D: \qquad \Lambda_t = \mu_t^{kD} + \mu_t^{iD} \tag{A.117}$$

$$\partial i_t^H: \qquad \Lambda_t = \mu_t^{kH} + \mu_t^{iH} \tag{A.118}$$

together with the constraints above and the inequalities $\mu_t^{iD} \geq 0$, $\mu_t^{iH} \geq 0$, which are complementary slack with (A.106) and (A.107).

As usual we combine (A.113) with (A.116) to write:

$$\frac{\Lambda_t}{\beta \Lambda_{t+1}} = 1 - \delta^g + r_{t+1}^g \tag{A.119}$$

Substitute (A.118) into (A.115), divide by Λ_{t+1}, and then substitute in (A.119) and divide by

β:

$$p_t^H \left(\frac{\Lambda_t}{\Lambda_{t+1}} - \frac{\mu_t^{iH}}{\Lambda_{t+1}} \right) = \beta \left(p_{t+1}^H r_{t+1}^H + \left(1 - \frac{\mu_{t+1}^{iH}}{\Lambda_{t+1}} \right) p_{t+1}^H (1 - \delta^H) \right) \tag{A.120}$$

$$\Leftrightarrow p_t^H (1 - \delta^g + r_{t+1}^g) \left(1 - \frac{\mu_t^{iH}}{\Lambda_t} \right) = p_{t+1}^H r_{t+1}^H + \left(1 - \frac{\mu_{t+1}^{IH}}{\Lambda_{t+1}} \right) p_{t+1}^H (1 - \delta^H) \tag{A.121}$$

$$\Leftrightarrow p_{t+1}^H r_{t+1}^H = p_t^H (1 - \delta^g + r_{t+1}^g) \left(1 - \frac{\mu_t^{iH}}{\Lambda_t} \right) - p_{t+1}^H (1 - \delta^H) \left(1 - \frac{\mu_{t+1}^{iH}}{\Lambda_{t+1}} \right) \tag{A.122}$$

Recall that also $\mu_t^{iH} i_t^H = 0$. So we will be able to combine this result with others below to obtain equations determining i_t^H, and thus we will be able to scale up the household's problem.

Similarly, considering dirty capital, we can substitute (A.117) into (A.114), then substitute in (A.119) to obtain:

$$p_{t+1}^D r_{t+1}^D = p_t^D (1 - \delta^g + r_{t+1}^g) \left(1 - \frac{\mu_t^{iD}}{\Lambda_t} \right) - p_{t+1}^D (1 - \delta^D) \left(1 - \frac{\mu_{t+1}^{iD}}{\Lambda_{t+1}} \right) \tag{A.123}$$

And, again, $\mu_t^{iD} i_t^D = 0$.

Of course, if investment is ongoing ($\mu_t^{iH} = \mu_{t+1}^{iH} = \mu_t^{iD} = \mu_{t+1}^{iD} = 0$) then these two equations are identities between variables we are claiming are "exogenous". In that case, these provide necessary conditions on investment being non-zero (and non-infinite).

Moreover, because the economy is made up of identical agents behaving in this same way, we may sum complementary slack equations over all these agents to obtain

$$\mu_t^{iH} I_t^H = 0 \tag{A.124}$$

$$\mu_t^{iD} I_t^D = 0 \tag{A.125}$$

Moreover, now we have equations for the solution to the maximization problem, we can scale up from the household level. We have determined that, given prices and rates of return (equations for which follow) aggregate consumption C_t and investments I_t^g, I_t^D, I_t^H are determined by (also using that $p_t^D = p^D$):

$$
\begin{aligned}
I_t^g + I_t^D + I_t^H + C_t &= L_t w_t + \pi_t + r_t^g K_t^g + r_t^D p_t^D K_t^D + r_t^H p_t^H H_t \\
&\quad + \left(\tau_t^D (D_t^E + D_t^g) - \tau_t^H p_t^H H_t\right) && \text{(A.126)} \\
I_t^D &\geq 0 && \text{(A.127)} \\
I_t^H &\geq 0 && \text{(A.128)} \\
I_t^g &= K_{t+1}^g - (1-\delta^g) K_t^g && \text{(A.129)} \\
I_t^D &= p^D \left(K_{t+1}^D - (1-\delta^D) K_t^D\right) && \text{(A.130)} \\
I_t^H &= p_t^H \left(K_{t+1}^H - (1-\delta^H) K_t^H\right) && \text{(A.131)} \\
\frac{u'\left(C_t/L_t\right)}{\beta u'\left(C_{t+1}/L_{t+1}\right)} &= 1 - \delta^g + r_{t+1}^g && \text{(A.132)} \\
r_{t+1}^D &= (1-\delta^g + r_{t+1}^g)\left(1 - \frac{\mu_t^{iD}}{u'\left(C_t/L_t\right)}\right) - (1-\delta^D)\left(1 - \frac{\mu_{t+1}^{iD}}{u'\left(C_{t+1}/L_{t+1}\right)}\right) && \text{(A.133)} \\
\mu_t^{iD} &\geq 0 && \text{(A.134)} \\
I_t^D \mu_t^{iD} &= 0 && \text{(A.135)} \\
r_{t+1}^H &= \frac{p_t^H}{p_{t+1}^H}(1-\delta^g + r_{t+1}^g)\left(1 - \frac{\mu_t^{iH}}{u'\left(C_t/L_t\right)}\right) - (1-\delta^H)\left(1 - \frac{\mu_{t+1}^{iH}}{u'\left(C_{t+1}/L_{t+1}\right)}\right) && \text{(A.136)} \\
\mu_t^{iH} &\geq 0 && \text{(A.137)} \\
I_t^H \mu_t^{iH} &= 0 && \text{(A.138)}
\end{aligned}
$$

D.1 Compound interest for the firms' problems

Recall our term $\Pi_t = \Pi_t^g + \Pi_t^D + \Pi_t^H + \Pi_t^{DE} + \Pi_t^E$. We treated that as a lump-sum. However, in fact the firms are owned by the households, so they choose their activity to maximize the utility pay-off to the households. Thus, for example, the final-goods firms seek to maximize

$$
\sum_{t=0}^{\infty} \beta^t \Lambda_t \Pi_t^g \qquad \text{(A.139)}
$$

subject to its production constraints, where Λ_t is exactly the shadow price on the household's budget constraint above. It is equivalent to divide by Λ_0 and so to use a compound discount factor of $q_t := \beta^t \frac{\Lambda_t}{\Lambda_0} = \beta^t \frac{u'(c_t)}{u'(c_0)}$ for the relative price of consumption in period t, expressed in period 0 units.

Moreover, recall from (A.119) that $\frac{\Lambda_t}{\Lambda_{t+1}} = \beta(1 - \delta^g + r_{t+1}^g)$. Thus

$$
\begin{aligned}
q_t = \beta^t \frac{\Lambda_t}{\Lambda_0} &= \frac{\beta \Lambda_t}{\Lambda_{t-1}} \cdot \frac{\beta \Lambda_{t-1}}{\Lambda_{t-2}} \cdots \frac{\beta \Lambda_1}{\Lambda_0} = \prod_{j=1}^{t} \frac{1}{1 - \delta^g + r_j^g} && \text{(A.140)} \\
\frac{q_{t+1}}{q_t} &= \frac{1}{1 - \delta^g + r_{t+1}^g} && \text{(A.141)}
\end{aligned}
$$

D.2 The final-goods firms' problem

The final-goods firms maximize

$$\sum_{t=0}^{\infty} q_t \Pi_t^g = \sum_{t=0}^{\infty} q_t \left(\Omega(T_t) f(Y_t^g, E_t) - r_t^g K_t^g - w_t L_t - p_t^e E_t - \frac{\phi_{1,t}\eta_t^{\phi_2}}{(1-\eta_t)^{\phi_3}} Y_t^g - p_t^{fuel} D_t^g \right) \tag{A.142}$$

(remember that $Y_t^g \equiv f_t^g(K_t^g, L_t)$) where D_t^g are fossil fuels used by these firms, p_t^e is the price of electricity and $\frac{\phi_{1,t}\eta_t^{\phi_2}}{(1-\eta_t)^{\phi_3}} Y_t^g$ is spending on abatement by these firms, so that firms face an emission constraint given in every period by:

$$D_t^g = \sigma_t(1-\eta_t)Y_t^g \tag{A.143}$$

The first order conditions are then:

$$\partial K_t^g : \qquad \Omega(T_t)\frac{\partial f}{\partial Y_t^g}\frac{\partial f_t^g}{\partial K_t^g} = r_t^g + \frac{\phi_{1,t}\eta_t^{\phi_2}}{(1-\eta_t)^{\phi_3}}\frac{\partial f_t^g}{\partial K_t^g} + p_t^{fuel}\sigma_t(1-\eta_t)\frac{\partial f_t^g}{\partial K_t^g} \tag{A.144}$$

$$\partial L_t : \qquad \Omega(T_t)\frac{\partial f}{\partial Y_t^g}\frac{\partial f_t^g}{\partial L_t} = w_t + \frac{\phi_{1,t}\eta_t^{\phi_2}}{(1-\eta_t)^{\phi_3}}\frac{\partial f_t^g}{\partial L_t} + p_t^{fuel}\sigma_t(1-\eta_t)\frac{\partial f_t^g}{\partial L_t} \tag{A.145}$$

$$\partial E_t : \qquad \Omega(T_t)\frac{\partial f_t}{\partial E_t} = p_t^e \tag{A.146}$$

$$\partial \eta_t : \qquad p_t^{fuel}\sigma_t = \frac{\phi_{1,t}\eta^{\phi_2 - 1}}{(1-\eta_t)^{1+\phi_3}}[\phi_2(1-\eta_t) + \eta_t\phi_3] \tag{A.147}$$

Equation (A.144) is an optimal condition for demand of aggregate capital and states that the return on capital is the marginal product of capital minus additional spending on abatement to clean a given fraction of extra emissions and costs of fuel. Equation (A.144) is the counterpart of equation (A.145) for labor demand. Equation (A.146) is an optimal condition for demand of electricity. Finally, equation (A.147) says that the firm reacts to the price of fuel (implicitly to carbon tax) by choosing the level of abatement (equivalently the level of emissions) such that the price of fuel would be equal to the marginal cost of emissions reduction.

D.3 Aggregate-electricity-producing firms' problem

The firms produce aggregate electricity by combining both electricity generated by fossil-fuel-based power plants and electricity generated by renewable energy based power stations. Note that we are taking the output from these two plants, in GW, as inputs priced by p_t^{EH} and p_t^{ED} respectively and so we do not need to convert by p_t^H and p^D here.

$$\sum_{t=0}^{\infty} q_t \Pi_t^E = \sum_{t=0}^{\infty} q_t \left(p_t^e f_t^E(H_t, \zeta_t K_t^D) - p_t^{EH} H_t - p_t^{ED}(\zeta_t K_t^D) \right) \tag{A.148}$$

FOCs are:

$$p_t^e \frac{\partial f_t^E}{\partial H_t} = p_t^{EH} \tag{A.149}$$

$$p_t^e \frac{\partial f_t^E}{\partial(\zeta_t K_t^D)} = p_t^{ED} \tag{A.150}$$

D.4 The dirty-electricity-producing firms' problem

The dirty electricity producing firms are fossil-fuel based power stations, which combine existing infrastructure (for example, coal-based power plants) with fossil fuel, and so maximizes:

$$\sum_{t=0}^{\infty} q_t \Pi_t^D = \sum_{t=0}^{\infty} q_t \left(p_t^{ED}(\zeta_t K_t^D) - r_t^D p^D K_t^D - p_t^{fuel} D_t^E \right) \tag{A.151}$$

where firms face the emission constraint: $D_t^E = \nu \zeta_t K_t^D$, and constraint $\zeta_t \leq 1$. So the Lagrangian is (making the obvious substitution)

$$\sum_{t=0}^{\infty} q_t \left(p_t^{ED}(\zeta_t K_t^D) - r_t^D p^D K_t^D - p_t^{fuel} \nu \zeta_t K_t^D + \mu_t^{\zeta}(1 - \zeta_t) \right) \tag{A.152}$$

And the first order conditions and constraints are

$$\partial K_t^D : \qquad r_t^D p^D = \left(p_t^{ED} - p_t^{fuel} \nu \right) \zeta_t \tag{A.153}$$

$$\partial \zeta_t : \qquad \mu_t^{\zeta} = K_t^D \left(p_t^{ED} - p_t^{fuel} \nu \right) \tag{A.154}$$

$$\mu_t^{\zeta}(1 - \zeta_t) = 0 \tag{A.155}$$

$$\mu_t^{\zeta} \geq 0 \tag{A.156}$$

where μ_t^{ζ} is Lagrangian multiplier attached to the above constraint. Thus, if $\zeta < 1$ then $p_t^{ED} = p_t^{fuel} \nu$, and $r_t^D p^D = 0$ or $r_t^D = 0$. Intuitively, when there is underutilization, the market pushes the return on dirty energy capital to zero.

D.5 The fossil-fuel-extracting firm's problem

The firm maximizes

$$\sum_{t=0}^{\infty} q_t \Pi_t^{DE} = \sum_{t=0}^{\infty} q_t [p_t^{fuel} - \tau_t^D - G^D(S_t)](D_t^E + D_t^g) \tag{A.157}$$

where τ^D is tax on production of fossil fuels. The firm faces the constraint:

$$S_{t+1} = S_t - (D_t^E + D_t^g) \tag{A.158}$$

to which we assign the shadow price $\tilde{\mu}_t$. So the Lagrangian is

$$\mathcal{L}_t = \sum_{t=0}^{\infty} q_t \Bigg([p_t^{fuel} - \tau_t^D - G^D(S_t)](D_t^E + D_t^g) \tag{A.159}$$

$$- \tilde{\mu}_t \left(S_{t+1} - S_t + (D_t^E + D_t^g) \right) \Bigg) \tag{A.160}$$

FOCs are:

$$\partial(D_t^E + D_t^g): \qquad \tilde{\mu}_t = p_t^{fuel} - \tau_t^D - G^D(S_t) \tag{A.161}$$

$$\partial S_{t+1}: \qquad q_t\tilde{\mu}_t = q_{t+1}\left(\tilde{\mu}_{t+1} - (D_{t+1}^E + D_{t+1}^g)\left(G^D\right)'(S_{t+1})\right) \tag{A.162}$$

Combining the firm's first order conditions yields the standard Hotelling condition, into which we then substitute from (A.141)

$$p_t^{fuel} - \tau_t^D - G^D(S_t) = \frac{q_{t+1}}{q_t}\left(p_{t+1}^{fuel} - \tau_{t+1}^D - G^D(S_{t+1}) - (D_{t+1}^E + D_{t+1}^g)\left(G^D\right)'(S_{t+1})\right) \tag{A.163}$$

$$= \frac{1}{1-\delta^g + r_{t+1}^g}\left(p_{t+1}^{fuel} - \tau_{t+1}^D - G^D(S_{t+1}) - (D_{t+1}^E + D_{t+1}^g)\left(G^D\right)'(S_{t+1})\right) \tag{A.164}$$

which states that the return on extracting an extra unit of fossil fuels, selling and getting a return on it must be equal to the expected capital gain from keeping an extra unit of fossil fuels in the earth, but extracting it tomorrow minus the increase in future extraction costs. As before, we may repeatedly substitute forward to obtain

$$p_t^{fuel} - \tau_t^D - G^D(S_t) = -\sum_{s=1}^{\infty}\Delta_{t,s}(D_{t+s}^E + D_{t+s}^g)\left(G^D\right)'(S_{t+s}) \tag{A.165}$$

$$\text{where } \Delta_{t,s} := \prod_{s'=1}^{s}\frac{1}{1-\delta^g + r_{t+s'}^g} \tag{A.166}$$

D.6 The renewable energy firms' problem

In contrast to other sectors, we assume that the firms in the renewable sector are small in the sense that they take the stock of accumulated knowledge about using the renewable energy H_t as given. The renewable energy firms receive subsidy of τ_t^H on their dollar-valued holdings of renewable energy capital H_t. The firms take all prices as given, so they maximize:

$$\sum_{t=0}^{\infty} q_t\Pi_t^H = \sum_{t=0}^{\infty} q_t[p_t^{EH} - p_t^H(r_t^H - \tau_t^H)]H_t. \tag{A.167}$$

The first order condition is just:

$$p_t^{EH} = p_t^H(r_t^H - \tau_t^H) \tag{A.168}$$

D.7 The Principal's Problem

In this section we collect all equations we need to solve the decentralized equilibrium model and formulate it as the principal-agent problem:

$$\max_{\tau^D,\tau^H}\sum_{t=0}^{\infty}\beta^t L_t u\left(\frac{C_t}{L_t}\right) \tag{A.169}$$

subject to:

$$
\begin{aligned}
I_t^g + I_t^D + I_t^H + C_t &= L_t w_t + \Pi_t + r_t^g K_t^g + r_t^D p^D K_t^D + r_t^H p_t^H H_t \\
&\quad + \left(\tau_t^D (D_t^E + D_t^g) - \tau_t^H p_t^H H_t\right) && \text{(A.170)} \\
I_t^D &\geq 0 && \text{(A.171)} \\
I_t^H &\geq 0 && \text{(A.172)} \\
I_t^g &= K_{t+1}^g - (1-\delta^g) K_t^g && \text{(A.173)} \\
I_t^D &= p^D \left(K_{t+1}^D - (1-\delta^D) K_t^D\right) && \text{(A.174)} \\
I_t^H &= p_t^H \left(K_{t+1}^H - (1-\delta^H) K_t^H\right) && \text{(A.175)} \\
p_t^H &= G(H_t) && \text{(A.176)} \\
D_t^E &= \nu \zeta_t K_t^D && \text{(A.177)} \\
D_t^g &= \sigma_t (1-\eta_t) Y_t^g && \text{(A.178)} \\
\frac{u'(C_t/L_t)}{\beta u'(C_{t+1}/L_{t+1})} &= 1 - \delta^g + r_{t+1}^g && \text{(A.179)} \\
r_{t+1}^D &= (1-\delta^g + r_{t+1}^g)\left(1 - \frac{\mu_t^{iD}}{u'(C_t/L_t)}\right) - (1-\delta^D)\left(1 - \frac{\mu_{t+1}^{iD}}{u'(C_{t+1}/L_{t+1})}\right) \\
& && \text{(A.180)} \\
\mu_t^{iD} &\geq 0 && \text{(A.181)} \\
I_t^D \mu_t^{iD} &= 0 && \text{(A.182)} \\
r_{t+1}^H &= \frac{p_t^H}{p_{t+1}^H}(1-\delta^g + r_{t+1}^g)\left(1 - \frac{\mu_t^{iH}}{u'(C_t/L_t)}\right) - (1-\delta^H)\left(1 - \frac{\mu_{t+1}^{iH}}{u'(C_{t+1}/L_{t+1})}\right) \\
& && \text{(A.183)} \\
\mu_t^{iH} &\geq 0 && \text{(A.184)} \\
I_t^H \mu_t^{iH} &= 0 && \text{(A.185)} \\
r_t^g &= \left(\Omega(T_t)\frac{\partial f}{\partial Y_t^g} - \frac{\phi_{1,t}\eta_t^{\phi_2}}{(1-\eta_t)^{\phi_3}} - p_t^{fuel}\sigma_t(1-\eta_t)\right)\frac{\partial f_t^g}{\partial K_t^g} \\
&= \Bigg(\Omega(T_t)(1-\theta)\left[(1-\theta)(Y_t^g)^{1-1/\kappa} + \theta(E_t)^{1-1/\kappa}\right]^{\frac{1/\kappa}{1-1/\kappa}}(Y_t^g)^{-\frac{1}{\kappa}} - \frac{\phi_{1,t}\eta_t^{\phi_2}}{(1-\eta_t)^{\phi_3}} \\
&\quad - p_t^{fuel}\sigma_t(1-\eta_t)\Bigg) A_t^g \alpha (K_t^g)^{\alpha-1}(L_t)^{1-\alpha} && \text{(A.186)}
\end{aligned}
$$

$$
\begin{aligned}
w_t &= \left(\Omega(T_t)\frac{\partial f}{\partial Y_t^g} - \frac{\phi_{1,t}\eta_t^{\phi_2}}{(1-\eta_t)^{\phi_3}} - p_t^{fuel}\sigma_t(1-\eta_t)\right)\frac{\partial f_t^g}{\partial L_t} \\
&= \Bigg(\Omega(T_t)(1-\theta)\left[(1-\theta)(Y_t^g)^{1-1/\kappa} + \theta(E_t)^{1-1/\kappa}\right]^{\frac{1/\kappa}{1-1/\kappa}}(Y_t^g)^{-\frac{1}{\kappa}} - \frac{\phi_{1,t}\eta_t^{\phi_2}}{(1-\eta_t)^{\phi_3}} \\
&\qquad - p_t^{fuel}\sigma_t(1-\eta_t)\Bigg)A_t^g(1-\alpha)(K_t^g)^{\alpha}(L_t)^{-\alpha} \qquad \text{(A.187)}
\end{aligned}
$$

$$
p_t^e = \Omega(T_t)\frac{\partial f_t}{\partial E_t} = \Omega(T_t)\theta\left[(1-\theta)(Y_t^g)^{1-1/\kappa} + \theta(E_t)^{1-1/\kappa}\right]^{\frac{1/\kappa}{1-1/\kappa}} E_t^{-1/\kappa} \qquad \text{(A.188)}
$$

$$
p_t^{fuel}\sigma_t = \frac{\phi_{1,t}\eta_t^{\phi_2-1}}{(1-\eta_t)^{1+\phi_3}}[\phi_2(1-\eta_t) + \eta_t\phi_3] \qquad \text{(A.189)}
$$

$$
p_t^{EH} = p_t^e\frac{\partial f_t^E}{\partial H_t} = p_t^e A_t^E \omega H_t^{\xi-1}\left(\omega H_t^{\xi} + (1-\omega)(\Gamma_t^{ED})^{\xi}\right)^{\frac{1-\xi}{\xi}} \qquad \text{(A.190)}
$$

$$
p_t^{ED} = p_t^e\frac{\partial f_t^E}{\partial(\zeta_t K_t^D)} = p_t^e A_t^E(1-\omega)(\zeta_t K_t^D)^{\xi-1}\left(\omega H_t^{\xi} + (1-\omega)(\Gamma_t^{ED})^{\xi}\right)^{\frac{1-\xi}{\xi}} \qquad \text{(A.191)}
$$

$$
\begin{aligned}
p^D r_t^D &= \left(p_t^{ED} - p_t^{fuel}\nu\right)\zeta_t && \text{(A.192)}\\
\mu_t^{\zeta} &= K_t^D\left(p_t^{ED} - p_t^{fuel}\nu\right) && \text{(A.193)}\\
\mu_t^{\zeta}(1-\zeta_t) &= 0 && \text{(A.194)}\\
\mu_t^{\zeta} &\geq 0 && \text{(A.195)}\\
p_t^{EH} &= p_t^H(r_t^H - \tau_t^H) && \text{(A.196)}\\
p_t^{fuel} - \tau_t^D - G^D(S_t) &= -\sum_{s=1}^{\infty}\Delta_{t,s}(D_{t+s}^E + D_{t+s}^g)\left(G^D\right)'(S_{t+s}) && \text{(A.197)}\\
\Delta_{t,s} &= \prod_{s'=1}^{s}\frac{1}{1-\delta^g + r_{t+s'}^g} && \text{(A.198)}\\
D_t &= D_t^E + D_t^{\text{land}} + D_t^g && \text{(A.199)}\\
T_t &= \mathcal{W}_t(D_0,\ldots,D_{t-1}) && \text{(A.200)}\\
S_{t+1} &= S_t - (D_t^E + D_t^g) && \text{(A.201)}
\end{aligned}
$$

D.8 Social planner problem versus decentralized equilibrium

Proof of proposition 4.2 First, from (A.188) and (A.191), we note that:

$$
p_t^{ED} = \Omega(T_t)\frac{\partial f_t}{\partial E_t}\frac{\partial f_t^E}{\partial(\zeta_t K_t^D)} = \nu\frac{\partial Y_t}{\partial D_t^E} \qquad \text{(A.202)}
$$

From (A.192) it follows that:

$$
\frac{p^D r_t^D}{\zeta_t \nu} = \frac{p_t^{ED}}{\nu} - p_t^{fuel} \qquad \text{(A.203)}
$$

And substituting here from the above implies:

$$\frac{\partial Y_t}{\partial D_t^E} = \frac{p^D r_t^D}{\zeta_t \nu} + p_t^{fuel} \tag{A.204}$$

And substituting the expression for p_t^{fuel} from (A.197), we obtain:

$$\frac{\partial Y_t}{\partial D_t^E} = \frac{p^D r_t^D}{\zeta_t \nu} + \tau_t^D + G^D(S_t) - \sum_{s=1}^{\infty} \Delta_{t,s}(D_{t+s}^E + D_{t+s}^g)\left(G^D\right)'(S_{t+s}) \tag{A.205}$$

Recall that in the social planner's solution, the returns on dirty fuel are equal to (see Proposition C.6):

$$\frac{\partial Y_t}{\partial D_t^E} = \frac{\mu_t^S}{u'(C_t/L_t)} + \chi_t + G^D(S_t) + \frac{p^D R_t^D}{\zeta_t \nu} \tag{A.206}$$

where (see Proposition C.5)

$$\frac{\mu_t^S}{u'(C_t/L_t)} = -\sum_{s=1}^{\infty} \Delta_{t,s}(G^D)'(S_{t+s})(D_{t+s}^E + D_{t+s}^g) \tag{A.207}$$

Expression (A.205) is identical to (A.206) when taxes are equal to the social cost of carbon, and when $r_t^D = R_t^D$.

Next, we find the value of subsidies under which the solutions of the social planner's problem and decentralized equilibrium coincide. First, if the investment into the renewable sector continues then $\mu_t^{iH} = \mu_{t+1}^{iH} = 0$, and from (A.183) it follows that:

$$r_{t+1}^H = \frac{p_t^H}{p_{t+1}^H}(1 - \delta^g + r_{t+1}^g) - (1 - \delta^H) \tag{A.208}$$

or

$$\frac{p_{t+1}^H}{p_t^H} r_{t+1}^H = (1 - \delta^g + r_{t+1}^g) - \frac{p_{t+1}^H}{p_t^H}(1 - \delta^H) \tag{A.209}$$

Using (A.188), (A.190) and (A.196), we can also write that:

$$r_{t+1}^H = \frac{1}{p_{t+1}^H}\frac{\partial Y_{t+1}}{\partial H_{t+1}} + \tau_{t+1}^H \tag{A.210}$$

Next, we denote the return on clean investment in the social planner's case as $\tilde{r}_{t+1}^H$. Recall that in the social planner solution (Lemma C.7):

$$\frac{p_{t+1}^H}{p_t^H}\tilde{r}_{t+1}^H = (1 + r_{t+1}^g - \delta^g) - \frac{p_{t+1}^H}{p_t^H}(1 - \delta^H) + \frac{H_{t+2} - (1 - \delta^H)H_{t+1}}{p_t^H} G'(H_{t+1}) \tag{A.211}$$

and

$$\tilde{r}_{t+1}^H = \frac{1}{p_{t+1}^H}\frac{\partial Y_{t+1}}{\partial H_{t+1}} \tag{A.212}$$

Comparison of (A.210) with (A.212) yields the value of subsides:

$$\tau_{t+1}^H = r_{t+1}^H - \tilde{r}_{t+1}^H \tag{A.213}$$

But a comparison of (A.209) with (A.211), further yields that:

$$\frac{p_{t+1}^H}{p_t^H}\left(r_{t+1}^H - \tilde{r}_{t+1}^H\right) = -\frac{H_{t+2} - (1-\delta^H)H_{t+1}}{p_t^H} G'(H_{t+1}) \tag{A.214}$$

and the level of subsidies:

$$\tau_t^H = -(H_{t+1} - (1-\delta^H)H_t)\frac{G'(H_t)}{p_t^H} \tag{A.215}$$

Finally note that it is straightforward to show that the budget constraint (A.170) is identical to the economy's aggregate constraint as in the social planner's problem after substituting the expressions for profits and returns on capital and labor. □