XMAS: An extended model for analysis and simulation*

Benjamin García[†] Sebastián Guarda[‡] Markus Kirchner[§] Rodrigo Tranamil[¶]

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Research Division
Central Bank of Chile

Abstract

The Extended Model for Analysis and Simulations (XMAS) is the Central Bank of Chile's newest dynamic stochastic general equilibrium (DSGE) model. Designed for forecasting and policy analysis, the model, that builds on the work of Medina and Soto(2007) , includes several new features, in line with recent developments in the modeling of small open economies, particularly commodity-exporting emerging economies like Chile. The improvements over the base model include the modeling of non-core inflation dynamics, an endogenous commodity sector, an augmented fiscal sector and additional real and nominal frictions like variable capital utilization and a labor market with search and matching frictions that allows for labor variation in both the intensive and extensive margins. Even with a significant increase in complexity, that allows for a more granular analysis of the Chilean economy, we show that XMAS feature comparable forecast accuracy than the simpler specification from Medina and Soto(2007).

1 Introduction

This paper presents the Central Bank of Chile's Extended Model for Analysis and Simulations (XMAS), a dynamic stochastic general equilibrium (DSGE) model of the Chilean economy for monetary policy analysis and macroeconomic forecasting purposes. The structure of this model incorporates a range of new features, most of which are directly motivated by the experience of commodity-exporting emerging economies in general, and Chile in particular, over the past several years.

The Central Bank of Chile has been using dynamic stochastic general equilibrium (DSGE) models for regular policy analysis and medium-term projections in its Monetary Policy Report since the late 2000s. The first DSGE model used at the Central Bank was the Model for Analysis and Simulations (MAS), developed by Medina and Soto (2007), "base model" hereafter. This is a quantitative New Keynesian small open economy model with several features to describe the Chilean economy, including a commodity sector and a structural balance rule for fiscal policy. The model also includes a number of other non-standard elements, such as non-Ricardian households as well as oil imports as an intermediate input for consumption and production. Otherwise, the base model incorporates all the typical elements (sticky prices and wages, habit formation in consumption, physical capital with adjustments costs

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[†]Central Bank of Chile, e-mail address: bgarcia@bcentral.cl
[‡]Central Bank of Chile, e-mail address: sguarda@bcentral.cl
[§]Central Bank of Chile, e-mail address: mkirchner@bcentral.cl
[¶]Universitat Pompeu Fabra, e-mail address: rodrigo.tranamil@upf.edu

in investment, etc.) of second-generation New Keynesian DSGE models à la Christiano et al. (2005) and Smets and Wouters (2007) and small open economy features (including a simple financial friction, a debt-elastic country premium) similar to Adolfson et al. (2007).

Since the late 2000s, the macroeconomic literature has advanced in a number of directions that could be considered for the next generation of DSGE models for policy analysis and forecasting at central banks in emerging market economies such as Chile. For instance, after the global financial crisis, models with extended financial-real connections or financial frictions have been developed (for a review, see Taylor and Uhlig, 2016). However, most of these models, such as Christiano et al. (2015), have been developed for economically and financially advanced economies. It is therefore not obvious that the financial frictions embedded in those models are also useful in the context of emerging economies with significant amounts of funding from abroad, for instance, to better understand the transmission of foreign shocks (see García-Cicco et al., 2015). Instead (and considering that the base model already includes a simple financial friction), other advances of the literature may be more of a priority in the context of emerging economies, especially those related to developments that have directly affected commodity-exporting economies.

Hence, Xmas considers several extensions that are closely linked to the experience of commodity-exporting emerging economies in general, and Chile in particular, over the past decade. Through these extensions, Xmas allows a more detailed analysis and more comprehensive forecasts of the Chilean economy. In this paper, we describe each extension in detail and discuss its relevance for monetary policy analysis and forecasting purposes.

A first major extension of Xmas is a commodity sector with endogenous production. In the base model, commodity exports are modelled as a stochastic endowment. In Xmas, production in the commodity sector is conducted through sector-specific capital, subject to adjustment costs and time-to-build frictions in investment. This extension allows to capture the important role of commodity price changes for investment fluctuations in commodity-exporting economies (see Fornero et al., 2014). Adjustment costs and time to build reflect the difficulty of adjusting commodity-specific capital in the short run.

A second main extension is an augmented fiscal block including additional structure on both the spending side and the income side of the government budget. The base model includes government consumption as the only item on the spending side, and non-distortionary lump-sum taxes (in addition to one-period debt) on the income side. In Xmas, the government investment and transfers are added on the spending side, while on the income side a set of distortionary taxes on consumption as well as labor and capital income is included. This extension allows to capture the importance of all of these fiscal instruments for government income and spending in Chile, where spending items are determined jointly under the structural balance rule.[1] A related extension is government smoothing of oil prices, which is introduced to capture the administrated character of oil prices in Chile.

A third main extension is the incorporation of search and matching frictions and involuntary unemployment in the labor market. This extension is thought to resolve a major potential weakness in the base model, which is that all fluctuations in labor supply originate from the intensive margin (hours per worker) but there is no variation in the extensive margin (number of employed workers). This stands in stark contrast to actual data for Chile which shows that a significant fraction of the variation in total hours is explained by the extensive margin.

Finally, Xmas incorporates some other features from the literature, including variable capacity utilization and delayed pass-through of global prices and productivity.

As we show in detail in the following, despite the significantly added complexity of Xmas, its empirical fit and out-of-sample forecast accuracy is comparable or better compared to the base model. The main dimensions where Xmas outperforms the base model are consumption, nominal interest rates, and labor intensity.

Overall, our results are particularly relevant for economic modellers at central banks, especially from commodity-exporting emerging economies.

[1] For related studies for Chile, see Kumhof and Laxton (2010) and Medina and Soto (2016). Other related studies include Coenen et al. (2012) and Leeper et al. (2010).

The rest of the paper is structured as follows. Section 2 presents a comprehensive description of the extended model. Section 3 describes the estimation strategy. Section 4 describes the main additional channels with respect to Medina and Soto (2007) and show some forecasting exercises. Section 5 concludes, highlighting a number of directions for future work.

2 The Model

Following Medina and Soto (2007), García-Cicco et al. (2015), and Guerra-Salas et al. (2018), we present a small open economy model with nominal and real rigidities, Ricardian and non-Ricardian households, search and matching frictions in the labor market and involuntary unemployment. Domestic goods are produced with capital and labor, there is habit formation in consumption, there are adjustment costs in investment, firms face a Calvo-pricing problem with partial indexation, and there is imperfect exchange rate pass-through into import prices in the short run due to local currency price stickiness. The economy also exports a commodity good. The economy is subject to shocks to preferences, labor market, technology (for the home, commodity and investment production sectors), government expenditures on consumption, investment and transfers, monetary policy, foreign GDP, foreign inflation, foreign interest rates and the international price of the import basket, oil and the commodity good.

2.1 Households

There is a continuum of infinitely lived households of two types: non-Ricardian and Ricardian, with mass ω and $1-\omega$ respectively. Each type of household has identical asset endowments and identical preferences. Instantaneous utility in period $t = \{0, 1, 2, \ldots\}$ depends on consumption $(\widehat{C}_t)$ and the number of hours worked (h_t) by the household's employed members (n_t). As in Merz (1995), there is full risk-sharing within each household, so that consumption is equal among its members, independent of employment status.

Similar to Coenen et al. (2013), but with habits in private consumption instead of the aggregate consumption bundle, the consumption for households of type $j = \{R, NR\}$ is specified as a constant elasticity of substitution (CES) aggregate of the households' purchases for consumption purposes (C_t^j) and government consumption (C_t^G):

$$\widehat{C}_t^j \equiv \widehat{C}\left(C_t^j - \varsigma \check{C}_{t-1}^j, C_t^G\right) = \left[(1-o_{\widehat{C}})^{\frac{1}{\eta_{\widehat{C}}}}\left(C_t^j - \varsigma \check{C}_{t-1}^j\right)^{\frac{\eta_{\widehat{C}}-1}{\eta_{\widehat{C}}}} + o_{\widehat{C}}^{\frac{1}{\eta_{\widehat{C}}}}\left(C_t^G\right)^{\frac{\eta_{\widehat{C}}-1}{\eta_{\widehat{C}}}}\right]^{\frac{\eta_{\widehat{C}}}{\eta_{\widehat{C}}-1}}$$

Where and $\check{C}_t^j$ denotes average consumption across households of type j (with $C_t^j = \check{C}_t^j$ in equilibrium), and $1 \geq \varsigma \geq 0$.[2]

Expected discounted utility of a representative household[3] of type $j \in \{R, NR\}$ is given by[4]

$$E_t \sum_{s=0}^{\infty} \beta^s \varrho_{t+s}\left[\frac{1}{1-\sigma}\left(\widehat{C}_{t+s}^j\right)^{1-\sigma} - n_{t+s} G_{t+s}\right], \qquad j \in \{R, NR\} \tag{1}$$

Where $G_t = \Theta_t^j \kappa_t \left(A_{t-1}^H\right)^{1-\sigma} \frac{h_t^{1+\phi}}{1+\phi}$ is the disutility of work of an employed household member[5], ϱ_t is an exogenous preference shock, κ_t is an exogenous disutility shock (common to all households), $\beta \in (0,1)$, $\sigma > 0$, $\kappa > 0$, and $\phi \geq 0$.

[2]Throughout, uppercase letters denote nominal variables containing a unit root in equilibrium (either due to technology or to long-run inflation) while lowercase letters indicate variables with no unit root. Real variables are constructed using the domestic consumption good as the numeraire. In the appendix we describe how each variable is transformed to achieve stationarity in equilibrium. Variables without time subscript denote non-stochastic steady state values in the stationary model.

[3]The household instantaneous utility is defined as the sum of every member i's utility:$U_t^j = \int U_t^j(i)\, di$.

[4]The variable A_t^H (with $a_t^H \equiv A_t^H / A_{t-1}^H$) is a non-stationary technology disturbance in home goods, and A_t (with $a_t \equiv A_t / A_{t-1}$) is its global counterpart, see below.

[5]By assumption, unemployed members do not derive any labor related disutility.

As in Galí et al. (2012) we introduce the variable Θ_t^j as an endogenous preference shifter that satisfies[6]

$$\Theta_t^j = \widetilde{\chi}_t^j \left(A_{t-1}^H\right)^{\sigma} \left(\widehat{C}\left(\check{C}_t^j - \varsigma\check{C}_{t-1}^j, \overline{C_t^G}\right)\right)^{-\sigma}, \quad \widetilde{\chi}_{t+s}^j = \left(\widetilde{\chi}_{t-1}^j\right)^{1-\nu} \left(A_{t-1}^H\right)^{-\sigma\nu} \left(\widehat{C}\left(\check{C}_t^j - \varsigma\check{C}_{t-1}^j, \overline{C_t^G}\right)\right)^{\sigma\nu}, \quad \nu \in [0,1]$$

The employed members of the representative household earn a total real wage of $W_t h_t n_t$, where W_t denote the hourly wage of employed members. Each one of the unemployed members earn $UB_t = A_{t-1}ub$ of unemployment benefits which are paid out by an unemployment funds administrator (UFA). Households also derive income from lump sum transfers from the government amounting TR_t^j.

Households pay lump sum taxes in amount of T_t^j, a tax rate of τ_t^C over the purchase of consumption goods, and a tax rate of τ_t^L on their labor income. The labor tax is composed by a general wage tax and a forced contribution to an unemployment insurance fund (i.e $\tau_t^L \equiv \tau_t^W + \tau_t^{UFA}$). The first component is collected by the government while the second goes to the UFA.

2.1.1 Ricardian Households

Only Ricardian households can save and borrow by purchasing domestic currency denominated government bonds (B_t^R) and by trading foreign currency bonds (B_t^{R*}) with foreign agents, both being non-state contingent assets. They also purchase an investment good, (I_t^R) which determines their physical capital stock for next period (K_t^R), and receive dividends$\left(D_t^R\right)$ from the ownership of domestic firms as well as rents$\left(REN_t^{R*}\right)$ due to ownership of firms abroad (the latter are assumed to evolve stochastically according to $ren_t^{R*} = \overline{ren}^{R*}\xi_t^{ren}$, where $\overline{ren}^{R*} \geq 0$ and ξ_t^{ren} is an exogenous process). They pay a tax rate of τ_t^D on dividends and τ_t^K on capital income.

Let r_t, r_t^* and r_t^K denote the gross real returns on B_{t-1}^R, B_{t-1}^{R*} and $K_t^{S,R}$ respectively, and let rer_t be the real exchange rate (i.e. the price of foreign consumption goods in terms of domestic consumption goods). We allow for the distinction between capital services (denoted as $K_t^{S,R}$) used in the production of goods, and physical units of capital (K_t^R), owned by the households, with a law of motion governed by the investment and depreciation rates. The former is defined as the productive potential of the available physical capital stock for a given utilization rate $\bar{u}_t$ chosen by the households, where

$$K_t^{S,R} = \bar{u}_t K_{t-1}^R \tag{2}$$

We follow Christiano et al. (2011) by introducing $\phi_{\bar{u}}\left(\bar{u}_t\right) K_{t-1}$, the investment goods used for private capital maintenance, as a part of the total private investment, alongside with the investment goods used for increasing the households physical capital. By assumption, these maintenance costs are deducted from capital taxation, and follow the same structure as in García-Cicco et al. (2015):

$$\phi_{\bar{u}}\left(\bar{u}_t\right) = \frac{r^k}{\Phi_{\bar{u}}}\left(e^{\Phi_{\bar{u}}(\bar{u}_t - 1)} - 1\right) \tag{3}$$

Where the parameter $\Phi_{\bar{u}} \equiv \phi_{\bar{u}}''(1)/\phi_{\bar{u}}'(1) > 0$ governs the importance of these utilization costs. The physical capital stock evolves according to the law of motion:

$$K_t^R = (1-\delta)K_{t-1}^R + \left[1 - \phi_I\left(\frac{I_t^R}{I_{t-1}^R}\right)\right]\varpi_t I_t^R, \tag{4}$$

With depreciation rate $\delta \in (0,1]$, where ϖ_t is an investment shock that captures changes in the efficiency of the investment process (see Justiniano et al., 2011), I_t^R denotes capital augmenting investment expenditures, and $\phi_I\left(I_t^R/I_{t-1}^R\right) \equiv (\Phi_I/2)\left(I_t^R/I_{t-1}^R - a\right)^2$are convex investment adjustment costs with elasticity $\Phi_I = \phi_I''(a) \geq 0$.

[6]In order to avoid unintended fluctuations in the labor supply due to shifts in the government consumption path, the consumption measure that enters the preference shifter is defined as the average consumption bundle across households of type j if government consumption were at its long term level.

The period-by-period budget constraint of the representative Ricardian household is then given by

$$\begin{aligned}\left(B_t^R + rer_t B_t^{R*}\right) - \left(B_{t-1}^R + rer_t B_{t-1}^{R*}\right) \quad &= \quad rer_t REN_t^{R*} + TR_t^R + (1-\tau_t^L) W_t h_t n_t + (1-n_t)\, UB_t \\ &\quad + (r_t - 1) B_{t-1}^R + (r_t^* - 1)\, rer_t B_{t-1}^{R*} + (1-\tau_t^D) D_t^R \\ &\quad + K_{t-1}^R \left[r_t^K \bar{u}_t \left(1-\tau_t^K\right) + \tau_t^K p_t^I \left(\delta + \phi_{\bar{u}}\left(\bar{u}_t\right)\right)\right] \\ &\quad - (1+\tau_t^C) C_t^R - p_t^I \left(I_t^R + K_{t-1}^R \phi_{\bar{u}}\left(\bar{u}_t\right)\right) - T_t^R \end{aligned} \tag{5}$$

The household chooses C_t^R, I_t^R, K_t^R, B_t^R, B_t^{R*}, and $\bar{u}_t$ to maximize (1) subject to (2)-(5), taking r_t, r_t^*, r_t^K, rer_t, T_t^R, REN_t^{R*}, TR_t^R, D_t^R and $\check{C}_t^R$ as given. This intertemporal decision problem is associated with the following Lagrangian:

$$\mathcal{L}_t^R = E_t \sum_{s=0}^{\infty} \beta^s \varrho_{t+s} \left\{ \begin{array}{l} \frac{1}{1-\sigma} \left(\widehat{C}_{t+s}^R\right)^{1-\sigma} - \Theta_{t+s}^R \kappa_{t+s} \frac{1}{1+\phi} \left(A_{t+s-1}^H\right)^{1-\sigma} h_{t+s}^{1+\phi} \\ +\Lambda_{t+s}^R \left[\begin{array}{l} rer_{t+s} REN_{t+s}^{R*} + TR_{t+s}^R + (1-\tau_{t+s}^L) W_{t+s} h_{t+s} n_{t+s} + (1-n_{t+s})\, UB_{t+s} \\ +K_{t+s-1}^R \left[r_{t+s}^K \bar{u}_{t+s} \left(1-\tau_{t+s}^K\right) + \tau_{t+s}^K p_{t+s}^I \left(\delta + \phi_{\bar{u}}\left(\bar{u}_{t+s}\right)\right)\right] \\ +(1-\tau_{t+s}^D) D_{t+s}^R + rer_{t+s} r_{t+s}^* B_{t+s-1}^{R*} + r_{t+s} B_{t+s-1}^R - B_{t+s}^R - rer_{t+s} B_{t+s}^{R*} \\ -(1+\tau_{t+s}^C) C_{t+s}^R - p_{t+s}^I \left(I_{t+s}^R + K_{t+s-1}^R \phi_{\bar{u}}\left(\bar{u}_{t+s}\right)\right) - T_{t+s}^R \end{array} \right] \\ +\Lambda_{t+s}^R q_{t+s} \left[(1-\delta) K_{t+s-1}^R + \left(1 - \phi_I\left(I_{t+s}^R / I_{t+s-1}^R\right)\right) \varpi_{t+s} I_{t+s}^R - K_{t+s}^R\right] \end{array} \right\}$$

Where Λ_t^R denotes the Lagrange multiplier associated with the budget constraint and $\Lambda_t^R q_t$ denotes the multiplier associated with the law of motion for capital. The corresponding first-order optimality conditions are:

$$C_t^R \quad : \quad \Lambda_t^R = \frac{1}{(1+\tau_t^C)} \left(\widehat{C}_t^R\right)^{-\sigma} \left(\frac{\left(1-o_{\widehat{C}}\right) \widehat{C}_t^R}{C_t^R - \varsigma \check{C}_{t-1}^R}\right)^{\frac{1}{\eta_{\widehat{C}}}}, \tag{6}$$

$$B_t^R \quad : \quad \Lambda_t^R = \beta E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \Lambda_{t+1}^R r_{t+1} \right\}, \tag{7}$$

$$B_t^{R*} \quad : \quad \Lambda_t^R = \beta E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \Lambda_{t+1}^R \frac{rer_{t+1}}{rer_t} r_{t+1}^* \right\}, \tag{8}$$

$$K_t^R \quad : \quad \Lambda_t^R q_t = \beta E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \Lambda_{t+1}^R \left[\begin{array}{c} r_{t+1}^K \bar{u}_{t+1} \left(1-\tau_{t+1}^K\right) + q_{t+1}(1-\delta) \\ +p_{t+1}^I \left[\tau_{t+1}^K \delta - \phi_{\bar{u}}\left(\bar{u}_{t+1}\right)\left(1-\tau_{t+1}^K\right)\right] \end{array} \right] \right\}, \tag{9}$$

$$I_t^R \quad : \quad \frac{p_t^I}{q_t} = \left\{ 1 - \phi_I \left(\frac{I_t^R}{I_{t-1}^R}\right) - \phi_I' \left(\frac{I_t^R}{I_{t-1}^R}\right) \frac{I_t^R}{I_{t-1}^R} \right\} \varpi_t \tag{10}$$

$$+ \beta E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\Lambda_{t+1}}{\Lambda_t} \frac{q_{t+1}}{q_t} \phi_I' \left(\frac{I_{t+1}^R}{I_t^R}\right) \left(\frac{I_{t+1}^R}{I_t^R}\right)^2 \varpi_{t+1} \right\} \tag{11}$$

$$\bar{u}_t \quad : \quad r_t^K = p_t^I \phi_{\bar{u}}'\left(\bar{u}_t\right) \tag{12}$$

Notice that from (3) and (12), we can express the optimal utilization rate as a function with a standard deviation inversely proportional to $\Phi_{\bar{u}}$:

$$\bar{u}_t = 1 + \frac{\log\left(\frac{r_t^K}{r^K}\right) - \log\left(p_t^I\right)}{\Phi_{\bar{u}}} \tag{13}$$

The nominal interest rates are implicitly defined as

$$
\begin{aligned}
r_t &= R_{t-1}\left(\pi_t\right)^{-1}, \\
\pi_t &= \left(\frac{P_t}{P_{t-1}}\right)\frac{1+\tau_t^C}{1+\tau_{t-1}^C} \\
r_t^* &= R_{t-1}^*\xi_{t-1}\left(\pi_t^*\right)^{-1}, \\
\pi_t^* &= \frac{P_t^*}{P_{t-1}^*}
\end{aligned}
$$

Where π_t and π_t^* denote the gross inflation rates of the domestic and foreign consumption-based price indices, after tax in the domestic case. The variable ξ_{t-1} denotes a country premium given by (see Adolfson et al., 2008; Schmitt-Grohé and Uribe, 2003):

$$
\xi_t = \bar{\xi}\exp\left[-\psi\left(\frac{rer_t B_t^*}{p_t^Y Y_t} - \frac{rerb^*}{p^Y y}\right) + \frac{\zeta_t^O - \zeta^O}{\zeta^O} + \frac{\zeta_t^U - \zeta^U}{\zeta^U}\right], \qquad \psi > 0, \qquad \bar{\xi} \geq 1,
$$

Where ζ_t^O and ζ_t^U are respectively observed and unobserved exogenous shocks to the country premium. The foreign nominal interest rate R_t^* evolves exogenously, whereas the domestic central bank sets R_t. The country net asset position(B_t^*), is composed of private$\left(B_t^{Pr*}\right)$ and government$\left(B_t^{G*}\right)$ net foreign asset holdings:

$$
B_t^* = B_t^{Pr*} + B_t^{G*}
$$

2.1.2 Non-Ricardian Households

The subset of households that don't have access to asset markets face the following budget constraint:

$$
(1+\tau_t^C)C_t^{NR} = (1-\tau_t^L)W_t h_t n_t + (1-n_t)\,UB_t + TR_t^{NR} - T_t^{NR} \tag{14}
$$

Thus they solve a much simpler period by period problem associated with the following Lagrangian and optimality condition:

$$
\mathcal{L}^{NR} = \frac{1}{1-\sigma}\left(\widehat{C}_t^{NR}\right)^{1-\sigma} - \Theta_t^{NR}\kappa_t\frac{1}{1+\phi}\left(A_{t-1}^H\right)^{1-\sigma}h_t^{1+\phi} + \Lambda_t^{NR}\left[\begin{array}{l}(1-\tau_t^L)W_t h_t n_t + (1-n_t)\,UB_t + TR_t^{NR} \\ -T_t^{NR} - (1+\tau_t^C)C_t^{NR}\end{array}\right]
$$

$$
\Lambda_t^{NR} = \frac{1}{(1+\tau_t^C)}\left(\widehat{C}_t^{NR}\right)^{-\sigma}\left(\frac{\left(1-o_{\widehat{C}}\right)\widehat{C}_t^{NR}}{C_t^{NR} - \varsigma\check{C}_{t-1}^{NR}}\right)^{\frac{1}{\eta_{\widehat{C}}}}
$$

2.2 Labor Market

Similar to Kirchner and Tranamil (2016); Guerra-Salas et al. (2018), the labor market is modeled with search and matching frictions as in Mortensen and Pissarides (1994), allowing for both exogenous and endogenous separations, as in Cooley and Quadrini (1999) and den Haan et al. (2000).

By assumption, Ricardian and non-Ricardian workers have the same productivity. Additionally, as in Boscá et al. (2011), a labor union negotiates a unique labor contract for both types of households. This implies that firms are indifferent between different kind of workers, and thus all workers have the same wages, work the same number of hours and have the same probability of being employed. The matching function, $\mathcal{M}_\mathrm{t} = m_t v_t^{1-\mu}u_t^{\mu}$, gives the number of new employment relationships which are productive in period $t+1$. The variable u_t is the number of unemployed workers searching for a job, v_t is the number of vacancies posted by the firms, and m_t is the match efficiency at time t. μ is the match elasticity parameter. At the beginning of each period, a fraction ρ_t^x of employment relationships is assumed to terminate exogenously. After this and before production starts, the surviving workers may separate endogenously at rate ρ_t^n. This occurs if the worker´s operating cost $\widetilde{c}_t$ is greater than an endogenously determined threshold $\overline{c}_t$. The operating cost is assumed to be a random variable which is i.i.d across workers and time with c.d.f. F, which implies that $\rho_t^n = P(\widetilde{c}_t > \overline{c}_t) = 1 - F(\overline{c}_t)$. The evolution of employment is given by $n_t = (1-\rho_t)\left[n_{t-1} + \mathcal{M}_{\text{t-1}}\right]$ where ρ_t is the total separation rate which is given by $\rho_t^x + (1-\rho_t^x)\rho_t^n$. Normalizing the total population of workers to 1, we have that $n_t = 1 - u_t$. The probability that a searching worker is matched to a new job at the end of period t is $s_t = \mathcal{M}_{\text{t}}/u_t$, and the probability that a firm fills a vacancy is $e_t = \mathcal{M}_{\text{t}}/v_t$. The number of vacancies posted, as well as the job termination threshold $\overline{c}_t$ is optimally determined by profit maximizing firms. The wage earned by employed members, as well as their labor effort (hours worked), is the outcome of a bargaining process between firms and a union that represent the households interests.

2.3 Production and Pricing

The supply side of the economy is composed by 5 different types of firms: First, there is a perfectly competitive representative firm producing homogeneous home wholesale goods $(Y_t^{\widetilde{H}})$, with oil and a core wholesale good $(Y_t^{\tilde{Z}})$ (which is produced by the same firm with labor and capital). Second, two sets of monopolistically competitive firms turn home wholesale goods into differentiated varieties of the home and exportable goods $(Y_t^H(j)Y_t^{H*}(j))$ and a third set turns imported goods into the differentiated varieties of the foreign good $(Y_t^F(j))$ in the same fashion. Third, there are three perfectly competitive aggregators packing the different varieties of the home, exportable and foreign goods into corresponding composite goods (Y_t^H, Y_t^{H*}, Y_t^F). The fourth type consists of two more competitive aggregators: one that bundles the composite home and foreign goods to create different types of goods for consumption (core, agricultural and government, C_t^Z, C_t^A, C_t^G) and investment (private, government and commodity, $I_t^f, I_t^G, I_t^{Co,f}$), and another that bundles the core and agricultural goods with oil to produce a final consumption good for the households (C_t). And finally, a competitive representative firm produces commodity goods for export (Y_t^{Co}) using only sector-specific capital. The different types of final goods are purchased by the households (C_t, I_t^f), the government (C_t^G, I_t^G) and the commodity exporting firm $(I_t^{Co,f})$. All firms are owned by the Ricardian households, with the exception of the commodity-exporting firm that is owned by the government and foreign agents.

Capital Stock	I^f	$\rightarrow$	K	;	I^G	$\rightarrow$	K^G
	I^{Cof}	$\rightarrow$	K^{Co}				
Home Wholesale Good	$\{n \times h, K, K^G\}$	$\rightarrow$	$Y^{\tilde{Z}}$	;	$\left\{X^{\tilde{Z}}, X^O\right\}$	$\rightarrow$	$Y^{\widetilde{H}}$
Differentiated Varieties	$X^{\widetilde{H}}$	$\rightarrow$	$\left\{\begin{array}{c} Y^H(j) \\ Y^{H*}(j) \end{array}\right.$	; ;	M	$\rightarrow$	$Y^F(j)$
Composite Goods	$\int X^H(j)dj$	$\rightarrow$	Y^H	;	$\int X^F(j)dj$	$\rightarrow$	Y^F
	$\int X^{H^*}(j)dj$	$\rightarrow$	Y^{H*}	;			
Final Goods and Services	$\{X^H, X^F\}$	$\rightarrow$	$\left\{\begin{array}{c} C^Z \\ C^A \\ C^G \\ I^f \\ I^G \\ I^{Co,f} \end{array}\right.$	;	$\{C^Z, C^O, C^A\}$	$\rightarrow$	C
Commodity Good	K^{Co}	$\rightarrow$	Y^{Co}				

Figure 1: Input/Output characterization of the model's real economy

2.3.1 Final Goods

For the final consumption good, a representative final goods firm combines a core consumption good with an agricultural good and oil. Another representative final goods firm demands composite home and foreign goods in the amounts $X_t^{J,H}$ and $X_t^{J,F}$, in order to produce core (C_t^Z), agricultural (C_t^A) and government (C_t^G) consumption goods, and private ($I_t^f \equiv I_t + \phi_{\bar{u}}(\bar{u}_t) K_{t-1}$), commodity ($I_t^{Co,f} \equiv I_t^{Co} + \phi_{\bar{u}}^{Co}\left(\bar{u}_t^{Co}\right) K_{t-1}^{Co}$), and government ($I_t^G$) investment goods, where $J \in \left\{C^Z, C^G, I^f, I^{Co,f}, I^G\right\}$. The respective CES technologies are given by:

$$C_t = \left[(1-\kappa_O-\kappa_A)^{\frac{1}{\eta_C}} \left(C_t^Z\right)^{\frac{\eta_C-1}{\eta_C}} + \kappa_O^{\frac{1}{\eta_C}} \left(C_t^O\right)^{\frac{\eta_C-1}{\eta_C}} + \kappa_A^{\frac{1}{\eta_C}} \left(C_t^A\right)^{\frac{\eta_C-1}{\eta_C}}\right]^{\frac{\eta_C}{\eta_C-1}} \tag{15}$$

$$J_t = \left[(1-o_J)^{\frac{1}{\eta_J}} \left(X_t^{J,H}\right)^{\frac{\eta_J-1}{\eta_J}} + o_J^{\frac{1}{\eta_J}} \left(X_t^{J,F}\right)^{\frac{\eta_J-1}{\eta_J}}\right]^{\frac{\eta_J}{\eta_J-1}} \tag{16}$$

With $\kappa_O, \kappa_A, o_J \in [0,1]$, $\kappa_O + \kappa_A \leq 1$ and $\eta_C, \eta_J > 0$. Let p_t^J,p_t^O, p_t^H and p_t^F denote respectively the relative prices of the good J, oil, and the composite home and foreign goods with respect to the final consumption good (with $p_t^C = 1$). Subject to the technology constraints (15), (16), and (24), the firms maximize their profits $\Pi_t^C = C_t - p_t^Z C_t^Z - p_t^O C_t^O - p_t^A C_t^A$ and $\Pi_t^J = p_t^J J_t - p_t^H X_t^{J,H} - p_t^F X_t^{J,F}$ over the input demands taking the respective prices as given. That is, the firms solve the following optimization problems:

$$\max_{C_t^Z, C_t^O, C_t^A} \left\{ \begin{array}{l} \left[(1-\kappa_O-\kappa_A)^{\frac{1}{\eta_C}} \left(C_t^Z\right)^{\frac{\eta_C-1}{\eta_C}} + \kappa_O^{\frac{1}{\eta_C}} \left(C_t^O\right)^{\frac{\eta_C-1}{\eta_C}} + \kappa_A^{\frac{1}{\eta_C}} \left(C_t^A\right)^{\frac{\eta_C-1}{\eta_C}}\right]^{\frac{\eta_C}{\eta_C-1}} \\ -p_t^Z C_t^Z - p_t^O C_t^O - p_t^A C_t^A \end{array} \right\}$$

$$\max_{X_t^{J,H}, X_t^{J,F}} \left\{ p_t^J \left[(1-o_J)^{\frac{1}{\eta_J}} \left(X_t^{J,H}\right)^{\frac{\eta_J-1}{\eta_J}} + o_J^{\frac{1}{\eta_J}} \left(X_t^{J,F}\right)^{\frac{\eta_J-1}{\eta_J}}\right]^{\frac{\eta_J}{\eta_J-1}} - p_t^H X_t^{J,H} - p_t^F X_t^{J,F} \right\}$$

The first-order conditions determining the optimal input demands are

$$
\begin{aligned}
C_t^Z &= (1-\kappa_O-\kappa_A)\left(p_t^Z\right)^{-\eta_C} C_t &(17)\\
C_t^O &= \kappa_O\left(p_t^O\right)^{-\eta_C} C_t &(18)\\
C_t^A &= \kappa_A\left(p_t^A\right)^{-\eta_C} C_t &(19)\\
X_t^{J,H} &= (1-o_J)\left(\frac{p_t^H}{p_t^J}\right)^{-\eta_J} J_t &(20)\\
X_t^{J,F} &= o_J\left(\frac{p_t^F}{p_t^J}\right)^{-\eta_J} J_t &(21)
\end{aligned}
$$

Substituting (17)-(21) into (15)-(16) yields the following relations:

$$1=(1-\kappa_O-\kappa_A)\left(p_t^Z\right)^{1-\eta_C}+\kappa_O\left(p_t^O\right)^{1-\eta_C}+\kappa_A\left(p_t^A\right)^{1-\eta_C} \tag{22}$$

$$p_t^J=\left[(1-o_J)\left(p_t^H\right)^{1-\eta_J}+o_J\left(p_t^F\right)^{1-\eta_J}\right]^{\frac{1}{1-\eta_J}} \tag{23}$$

Showing that the firm earns zero profits in each period:

$$
\begin{aligned}
\Pi_t^C &= C_t-p_t^Z C_t^Z-p_t^O C_t^O-p_t^A C_t^A\\
&= C_t\left[1-(1-\kappa_O-\kappa_A)\left(p_t^Z\right)^{1-\eta_C}-\kappa_O\left(p_t^O\right)^{1-\eta_C}-\kappa_A\left(p_t^A\right)^{1-\eta_C}\right]\\
&= 0\\
\Pi_t^J &= p_t^J J_t-p_t^H X_t^{J,H}-p_t^F X_t^{J,F}\\
&= J_t\left(p_t^J\right)^{\eta_J}\left[\left(p_t^J\right)^{1-\eta_J}-(1-o_J)\left(p_t^H\right)^{1-\eta_J}-o_J\left(p_t^F\right)^{1-\eta_J}\right]\\
&= 0
\end{aligned}
$$

Similar to Medina and Soto (2007), the technology for the agricultural consumption good has an additional stochastic disturbance z_t^A in order to model the higher volatility of the sector that is observed in the data. Accordingly, the sector's technology, factor demand and prices are given by:

$$C_t^A=z_t^A\left[(1-o_A)^{\frac{1}{\eta_A}}\left(X_t^{A,H}\right)^{\frac{\eta_A-1}{\eta_A}}+o_A^{\frac{1}{\eta_A}}\left(X_t^{A,F}\right)^{\frac{\eta_A-1}{\eta_A}}\right]^{\frac{\eta_A}{\eta_A-1}} \tag{24}$$

$$
\begin{aligned}
X_t^{A,H} &= \left(z_t^A\right)^{\eta_A-1}(1-o_A)\left(\frac{p_t^H}{p_t^A}\right)^{-\eta_A} C_t^A &(25)\\
X_t^{A,F} &= \left(z_t^A\right)^{\eta_A-1} o_A\left(\frac{p_t^F}{p_t^A}\right)^{-\eta_A} C_t^A &(26)
\end{aligned}
$$

$$p_t^A=\frac{1}{z_t^A}\left[(1-o_A)\left(p_t^H\right)^{1-\eta_A}+o_A\left(p_t^F\right)^{1-\eta_A}\right]^{\frac{1}{1-\eta_A}} \tag{27}$$

2.3.2 Composite Goods

Three groups of competitive packing firms demand all varieties $j\in[0,1]$ of foreign, home, and exportable goods in amounts $X^F(j)$, $X^H(j)$ and $X^{H^*}(j)$, and combines them in order to produce composite foreign$\left(Y_t^F\right)$, composite home$\left(Y_t^H\right)$, and exportable$\left(Y_t^{H^*}\right)$ goods. With $J=\{F,H,H*\}$ the CES technologies that transform inputs $X_t^J(j)$

into the respective outputs Y_t^J are given by:

$$Y_t^J = \left[\int_0^1 X_t^J(j)^{\frac{\epsilon_J - 1}{\epsilon_J}} dj\right]^{\frac{\epsilon_J}{\epsilon_J - 1}}, \qquad \epsilon_J > 0. \tag{28}$$

Let $P_t^J(j)$ denote the price of the good of variety j. Subject to the technology constraint (28), the firm maximizes its profits $\Pi_t^J = P_t^J Y_t^J - \int_0^1 P_t^J(j) X_t^J(j) dj$ over the input demands taking the relative prices as given. That is, the firm solves the following optimization problem:

$$\max_{X_t^J(j)} \left\{ P_t^J \left[\int_0^1 X_t^J(j)^{\frac{\epsilon_J - 1}{\epsilon_J}} dj\right]^{\frac{\epsilon_J}{\epsilon_J - 1}} - \int_0^1 P_t^J(j) X_t^J(j) dj \right\}, \quad \text{for all } j.$$

The first-order conditions determining the optimal input demands for each variety are

$$X_t^J(j) = \left(\frac{P_t^J(j)}{P_t^J}\right)^{-\epsilon_J} Y_t^J, \quad \text{for all } j. \tag{29}$$

Substituting (29) into (28) yields

$$\left(Y_t^J\right)^{\frac{\epsilon_J - 1}{\epsilon_J}} = \int_0^1 \left(X_t^J(j)\right)^{\frac{\epsilon_J - 1}{\epsilon_J}} dj = \left(Y_t^J\right)^{\frac{\epsilon_J - 1}{\epsilon_J}} \int_0^1 \left(\frac{P_t^J(j)}{P_t^J}\right)^{1-\epsilon_J} dj,$$

Or

$$1 = \int_0^1 \left(\frac{P_t^J(j)}{P_t^J}\right)^{1-\epsilon_J} dj, \tag{30}$$

Showing that the firm earns zero profits in each period:

$$\Pi_t^J = P_t^J Y_t^J - \int_0^1 P_t^J(j) X_t^J(j) dj = P_t^J Y_t^J \left[1 - \int_0^1 \left(\frac{P_t^J(j)}{P_t^J}\right)^{1-\epsilon_J} dj\right] = 0.$$

According to (30) the price level P_t^J satisfies

$$P_t^J = \left[\int_0^1 P_t^J(j)^{1-\epsilon_J} dj\right]^{\frac{1}{1-\epsilon_J}}. \tag{31}$$

2.3.3 Differentiated Varieties

Two sets of monopolistically competitive firms demand home wholesale goods in quantities $X_t^{\tilde{H}}$ and $X_t^{\tilde{H}*}$, respectively, and differentiate them into domestically sold (home) and exportable varieties, $Y_t^H(j)$ and $Y_t^{H*}(j)$. It takes one unit of the input good to produce one unit of variety j, such that $\int_0^1 Y_t^H(j)dj = X_t^{\tilde{H}}$ and $\int_0^1 Y_t^{H*}(j)dj = X_t^{\tilde{H}*}$. Another set of monopolistically competitive importing firms demands a quantity M_t of an imported good at the price P_t^{M*} and differentiates it into varieties $Y_t^F(j)$ that are sold domestically, such that $\int_0^1 Y_t^F(j)dj = M_t$. The firm producing variety j of the respective good satisfies the demand given by (29) but it has monopoly power for its variety. Domestically sold varieties ($Y_t^F(j)$ and $Y_t^H(j)$) are invoiced in local currency, while exported varieties ($Y_t^{H*}(j)$), following Adolfson et al. (2007), Medina and Soto (2007) and others, are priced in foreign currency and indexed to foreign inflation.

Domestically sold varieties For varieties priced in local currency, with $J^X = \{M, \tilde{H}\}$ and $J^Y = \{F, H\}$, the nominal marginal cost in terms of the associated composite good price is $P_t^{J^Y} mc_t^{J^Y}(j)$. As every firm buys their inputs from the same market of homogeneous goods, they all face the same marginal cost: $P_t^{J^Y} mc_t^{J^Y}(j) = P_t^{J^Y} mc_t^{J^Y} = P_t^{J^X}$

for all j.[7] In the case of imported goods, $P_t^M \equiv S_t P_t^{M*}$ where S_t is the nominal exchange rate, implicitly defined by $rer_t = S_t P_t^*/P_t$. Given marginal costs, the firm producing variety j chooses its price $P_t^{J^Y}(j)$ to maximize profits. In setting prices, the firm faces a Calvo-type problem, whereby each period it can change its price optimally with probability $1-\theta_{J^Y}$, and if it cannot optimally change its price, it indexes its previous price according to a weighted product of past and steady state inflation with weights $\vartheta_{J^Y} \in [0,1]$ and $1-\vartheta_{J^Y}$. A firm reoptimizing in period t will choose the price $\tilde{P}_t^{J^Y}(j)$ that maximizes the current market value of the profits generated until it can reoptimize. As the firms are owned by the Ricardian households, they discount profits by their stochastic discount factor for nominal payoffs: $\chi_{t,t+s} \equiv \beta^s(\varrho_{t+s}/\varrho_t)(\Lambda_{t+s}^R/\Lambda_t^R)\left(P_t(1+\tau_t^C)/P_{t+s}(1+\tau_{t+s}^C)\right)$, for $s \geq 0$. A reoptimizing firm producing J^Y with inputs J^X therefore solves

$$
\begin{aligned}
\max_{\tilde{P}_t^{J^Y}(j)} \quad & E_t \sum_{s=0}^{\infty} \theta_{J^Y}^s \chi_{t,t+s} \left(\tilde{P}_t^{J^Y}(j)\Gamma_{t,s}^{J^Y} - P_{t+s}^{J^X} \right) Y_{t+s}^{J^Y}(j), \\
\text{s.t.} \quad & Y_{t+s}^{J^Y}(j) = \left(\frac{\tilde{P}_t^{J^Y}(j)\Gamma_{t,s}^{J^Y}}{P_{t+s}^{J^Y}} \right)^{-\epsilon_{J^Y}} Y_{t+s}^{J^Y}, \\
& P_t^{J^X} = P_t^{J^Y} mc_t^{J^Y},
\end{aligned}
$$

where $\Gamma_{t,s}^{J^Y} = \left(\frac{P_{t+s-1}(1+\tau_{t+s-1}^C)}{P_{t-1}(1+\tau_{t-1}^C)}\right)^{\vartheta_{J^Y}} \pi^{(1-\vartheta_{J^Y})s}$ is an indexation variable defined as a weighted average between past and steady state inflation that satisfies $\Gamma_{t,0}^{J^Y} = 1$, $\Gamma_{t,s}^{J^Y} = \Gamma_{t,s-1}^{J^Y} g_{t+s-1}^{\Gamma^{J^Y}}$ and $\Gamma_{t,s+1}^{J^Y} = g_t^{\Gamma^{J^Y}} \Gamma_{t+1,s}^{J^Y}$with $g_{t+s}^{\Gamma^{J^Y}} \equiv \pi_{t+s}^{\vartheta_{J^Y}} \pi^{1-\vartheta_{J^Y}}$ for $s \geq 1$ is the gross growth rate of the indexation variable. Substituting out the demand constraint in the objective function yields

$$
\max_{\tilde{P}_t^{J^Y}(j)} E_t \sum_{s=0}^{\infty} \theta_{J^Y}^s \chi_{t,t+s} \left[\tilde{P}_t^{J^Y}(j)^{1-\epsilon_{J^Y}} \left(\Gamma_{t,s}^{J^Y}\right)^{1-\epsilon_{J^Y}} - \tilde{P}_t^{J^Y}(j)^{-\epsilon_{J^Y}} \left(\Gamma_{t,s}^{J^Y}\right)^{-\epsilon_{J^Y}} P_{t+s}^{J^Y} mc_{t+s}^{J^Y} \right] \left(P_{t+s}^{J^Y}\right)^{\epsilon_{J^Y}} Y_{t+s}^{J^Y}.
$$

The first-order condition determining the optimal price is given by[8]

$$
\begin{aligned}
& E_t \sum_{s=0}^{\infty} \theta_{J^Y}^s \chi_{t,t+s} (-\epsilon_{J^Y}) \left(\tilde{P}_t^{J^Y}\right)^{-\epsilon_{J^Y}-1} \left(\Gamma_{t,s}^{J^Y}\right)^{-\epsilon_{J^Y}} P_{t+s}^{J^Y} mc_{t+s}^{J^Y} \left(P_{t+s}^{J^Y}\right)^{\epsilon_{J^Y}} Y_{t+s}^{J^Y} \\
= \quad & E_t \sum_{s=0}^{\infty} \theta_{J^Y}^s \chi_{t,t+s} (1-\epsilon_{J^Y}) \left(\tilde{P}_t^{J^Y}\right)^{-\epsilon_{J^Y}} \left(\Gamma_{t,s}^{J^Y}\right)^{1-\epsilon_{J^Y}} \left(P_{t+s}^{J^Y}\right)^{\epsilon_{J^Y}} Y_{t+s}^{J^Y},
\end{aligned}
$$

or, multiplying by $\tilde{P}_t^{J^Y}/P_t^{J^Y}$ and dividing by $-\epsilon_{J^Y}$ on both sides:

$$
\begin{aligned}
& E_t \sum_{s=0}^{\infty} \theta_{J^Y}^s \chi_{t,t+s} \left(\tilde{P}_t^{J^Y}\right)^{-\epsilon_{J^Y}} \left(\Gamma_{t,s}^{J^Y}\right)^{-\epsilon_{J^Y}} \left(P_t^{J^Y}\right)^{-1} P_{t+s}^{J^Y} mc_{t+s}^{J^Y} \left(P_{t+s}^{J^Y}\right)^{\epsilon_{J^Y}} Y_{t+s}^{J^Y} \\
= \quad & E_t \sum_{s=0}^{\infty} \theta_{J^Y}^s \chi_{t,t+s} \left(\tilde{P}_t^{J^Y}\right)^{1-\epsilon_{J^Y}} \left(\Gamma_{t,s}^{J^Y}\right)^{1-\epsilon_{J^Y}} \left(P_t^{J^Y}\right)^{-1} \left(P_{t+s}^{J^Y}\right)^{\epsilon_{J^Y}} Y_{t+s}^{J^Y} \left(\frac{\epsilon_{J^Y}-1}{\epsilon_{J^Y}}\right).
\end{aligned}
$$

Letting $\tilde{p}_t^{J^Y} \equiv \tilde{P}_t^{J^Y}/P_t^{J^Y}$ denote the optimal price in terms of the corresponding composite good price and defining also $p_t^{J^Y} \equiv P_t^{J^Y}/P_t$, the first-order condition can be re-written in recursive form as follows:

$$
F_t^{J^Y 1} = F_t^{J^Y 2} = F_t^{J^Y},
$$

[7] Therefore $mc_t^{J^Y} = P_t^{J^X}/P_t^{J^Y}$ is the real marginal cost of these firms, in terms of the associated composite goods.

[8] By symmetry, the optimal price is identical across firms, i.e., $\tilde{P}_t^{J^Y}(j) = \tilde{P}_t^{J^Y}$ for all j.

where

$$
\begin{aligned}
F_t^{J^Y 1} &= E_t \sum_{s=0}^{\infty} (\beta\theta_{J^Y})^s \frac{\varrho_{t+s}}{\varrho_t} \frac{\Lambda_{t+s}^R}{\Lambda_t^R} \frac{P_t(1+\tau_t^C)}{P_{t+s}(1+\tau_{t+s}^C)} \tilde{P}_t^{J^Y}(j)^{-\epsilon_{J^Y}} \left(\Gamma_{t,s}^{J^Y}\right)^{-\epsilon_{J^Y}} \left(P_t^{J^Y}\right)^{-1} P_{t+s}^{J^Y} mc_{t+s}^{J^Y} \left(P_{t+s}^{J^Y}\right)^{\epsilon_{J^Y}} Y_{t+s}^{J^Y}, \\
&= \left(\frac{\tilde{P}_t^{J^Y}}{P_t^{J^Y}}\right)^{-\epsilon_{J^Y}} mc_t^{J^Y} Y_t^{J^Y} + \beta\theta_{J^Y} E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\Lambda_{t+1}^R}{\Lambda_t^R} \frac{(1+\tau_t^C)}{(1+\tau_{t+1}^C)} \frac{P_t}{P_{t+1}} \left(g_t^{\Gamma^{J^Y}} \frac{\tilde{P}_t^{J^Y}}{\tilde{P}_{t+1}^{J^Y}} \right)^{-\epsilon_{J^Y}} \left(\frac{P_t^{J^Y}}{P_{t+1}^{J^Y}} \right)^{-1} F_{t+1}^{J^Y 1} \right\}, \\
&= \left(\tilde{p}_t^{J^Y}\right)^{-\epsilon_{J^Y}} mc_t^{J^Y} Y_t^{J^Y} + \beta\theta_{J^Y} E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\Lambda_{t+1}^R}{\Lambda_t^R} \left(\frac{g_t^{\Gamma^{J^Y}}}{\pi_{t+1}} \frac{\tilde{p}_t^{J^Y}}{\tilde{p}_{t+1}^{J^Y}} \right)^{-\epsilon_{J^Y}} \left(\frac{p_t^{J^Y}}{p_{t+1}^{J^Y}} \frac{(1+\tau_{t+1}^C)}{(1+\tau_t^C)} \right)^{-1-\epsilon_{J^Y}} F_{t+1}^{J^Y 1} \right\},
\end{aligned}
$$

and

$$
\begin{aligned}
F_t^{J^Y 2} &= E_t \sum_{s=0}^{\infty} (\beta\theta_{J^Y})^s \frac{\varrho_{t+s}}{\varrho_t} \frac{\Lambda_{t+s}^R}{\Lambda_t^R} \frac{P_t(1+\tau_t^C)}{P_{t+s}(1+\tau_{t+s}^C)} \left(\tilde{P}_t^{J^Y}\right)^{1-\epsilon_{J^Y}} \left(\Gamma_{t,s}^{J^Y}\right)^{1-\epsilon_{J^Y}} \left(P_t^{J^Y}\right)^{-1} \left(P_{t+s}^{J^Y}\right)^{\epsilon_{J^Y}} Y_{t+s}^{J^Y} \left(\frac{\epsilon_{J^Y}-1}{\epsilon_{J^Y}}\right), \\
&= \left(\frac{\tilde{P}_t^{J^Y}}{P_t^{J^Y}}\right)^{1-\epsilon_{J^Y}} Y_t^{J^Y} \left(\frac{\epsilon_{J^Y}-1}{\epsilon_{J^Y}}\right) + \beta\theta_{J^Y} E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\Lambda_{t+1}^R}{\Lambda_t^R} \frac{(1+\tau_t^C)}{(1+\tau_{t+1}^C)} \frac{P_t}{P_{t+1}} \left(g_t^{\Gamma^{J^Y}} \frac{\tilde{P}_t^{J^Y}}{\tilde{P}_{t+1}^{J^Y}} \right)^{1-\epsilon_{J^Y}} \left(\frac{P_t^{J^Y}}{P_{t+1}^{J^Y}} \right)^{-1} F_{t+1}^{J^Y 2} \right\}, \\
&= \left(\tilde{p}_t^{J^Y}\right)^{1-\epsilon_{J^Y}} Y_t^{J^Y} \left(\frac{\epsilon_{J^Y}-1}{\epsilon_{J^Y}}\right) + \beta\theta_{J^Y} E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\Lambda_{t+1}^R}{\Lambda_t^R} \left(\frac{g_t^{\Gamma^{J^Y}}}{\pi_{t+1}} \frac{\tilde{p}_t^{J^Y}}{\tilde{p}_{t+1}^{J^Y}} \right)^{1-\epsilon_{J^Y}} \left(\frac{p_t^{J^Y}}{p_{t+1}^{J^Y}} \frac{(1+\tau_{t+1}^C)}{(1+\tau_t^C)} \right)^{-\epsilon_{J^Y}} F_{t+1}^{J^Y 2} \right\}.
\end{aligned}
$$

Further, let $\Psi^{J^Y}(t)$ denote the set of firms that cannot optimally choose their price in period t. By (31), the price level $P_t^{J^Y}$ evolves as follows:

$$
\begin{aligned}
\left(P_t^{J^Y}\right)^{1-\epsilon_{J^Y}} = \int_0^1 P_t^{J^Y}(j)^{1-\epsilon_{J^Y}} dj &= (1-\theta_{J^Y}) \left(\tilde{P}_t^{J^Y}\right)^{1-\epsilon_{J^Y}} + \int_{\Psi^{J^Y}(t)} \left(g_{t-1}^{\Gamma^{J^Y}} P_{t-1}^{J^Y}(j) \right)^{1-\epsilon_{J^Y}} dj, \\
&= (1-\theta_{J^Y}) \left(\tilde{P}_t^{J^Y}\right)^{1-\epsilon_{J^Y}} + \theta_{J^Y} \left(g_{t-1}^{\Gamma^{J^Y}} P_{t-1}^{J^Y} \right)^{1-\epsilon_{J^Y}}.
\end{aligned}
$$

Dividing both sides by $\left(P_t^{J^Y}\right)^{1-\epsilon_{J^Y}}$ yields

$$
1 = (1-\theta_{J^Y}) \left(\tilde{p}_t^{J^Y}\right)^{1-\epsilon_{J^Y}} + \theta_{J^Y} \left(\frac{p_{t-1}^{J^Y}}{p_t^{J^Y}} \frac{g_{t-1}^{\Gamma^{J^Y}}}{\pi_t} \frac{(1+\tau_t^C)}{(1+\tau_{t-1}^C)} \right)^{1-\epsilon_{J^Y}}.
$$

The second equality above follows from the fact that the distribution of prices among firms not reoptimizing in period t corresponds to the distribution of effective prices in period $t-1$, though with total mass reduced to θ_{J^Y}.

Exported varieties For exported varieties, priced in foreign currency, the setup is equivalent. The only differences are that both the nominal rigidities and the relevant stochastic discount factor are defined in terms of the foreign currency. The nominal marginal cost of exporter j in terms of the exportable composite good price is $S_t P_t^{H*} mc_t^{H*}(j)$. As every firm buys their inputs from the same market of homogeneous goods, they all face the same marginal costs: $S_t P_t^{H*} mc_t^{H*}(j) = S_t P_t^{H*} mc_t^{H*} = P_t^{\tilde{H}}$ for all j. Given marginal costs, the firm producing variety j chooses its price $P_t^{H*}(j)$ to maximize profits. In setting prices, the firm faces a Calvo-type problem, whereby each period it can change its price optimally with probability $1-\theta_{H*}$, and if it cannot optimally change its price, it indexes its previous price according to a weighted product of past and steady state inflation with weights $\vartheta_{H*} \in [0,1]$ and $1-\vartheta_{H*}$. A firm reoptimizing in period t will choose the price $\tilde{P}_t^{H*}(j)$ that maximizes the current market value of the profits generated until it can reoptimize. As the firms are owned by the Ricardian households, they discount profits by their stochastic discount factor for nominal payoffs in foreign currency: $\chi^*_{t,t+s} \equiv \beta^s(\varrho_{t+s}/\varrho_t)(\Lambda^R_{t+s}/\Lambda^R_t)\left(P_t(1+\tau^C_t)/P_{t+s}(1+\tau^C_{t+s})\right)(S_{t+s}/S_t)$, for $s \geq 0$. A reoptimizing firm therefore solves

$$\begin{aligned} \max_{\tilde{P}^{H*}_t(j)} \quad & E_t \sum_{s=0}^{\infty} \theta^s_{H*}\chi^*_{t,t+s}\left(\tilde{P}^{H*}_t(j)\Gamma^{H*}_{t,s} - \frac{P^{\tilde{H}}_{t+s}}{S_{t+s}}\right)Y^{H*}_{t+s}(j), \\ \text{s.t.} \quad & Y^{H*}_{t+s}(j) = \left(\frac{\tilde{P}^{H*}_t(j)\Gamma^{H*}_{t,s}}{P^{H*}_{t+s}}\right)^{-\epsilon_{H*}} Y^{H*}_{t+s}, \\ & P^{\tilde{H}}_t = S_t P^{H*}_t mc^{H*}_t, \end{aligned}$$

where $\Gamma^{H*}_{t,s} = \left(\frac{P^*_{t+s-1}}{P^*_{t-1}}\right)^{\vartheta_{H*}}(\pi^*)^{(1-\vartheta_{H*})s}$ is an indexation variable that satisfies $\Gamma^{H*}_{t,0} = 1$ and $\Gamma^{H*}_{t,s} = \Gamma^{H*}_{t,s-1}g^{\Gamma^{H*}}_{t+s-1}$ with $g^{\Gamma^{H*}}_{t+s} \equiv \left(\pi^*_{t+s}\right)^{\vartheta_{H*}}(\pi^*)^{1-\vartheta_{H*}}$ for $s \geq 1$. Substituting out the demand constraint in the objective function yields

$$\max_{\tilde{P}^{H*}_t(j)} E_t \sum_{s=0}^{\infty} \theta^s_{H*}\chi^*_{t,t+s}\left[\tilde{P}^{H*}_t(j)^{1-\epsilon_{H*}}\left(\Gamma^{H*}_{t,s}\right)^{1-\epsilon_{H*}} - \tilde{P}^{H*}_t(j)^{-\epsilon_{H*}}\left(\Gamma^{H*}_{t,s}\right)^{-\epsilon_{H*}} P^{H*}_{t+s}mc^{H*}_{t+s}\right]\left(P^{H*}_{t+s}\right)^{\epsilon_{H*}} Y^{H*}_{t+s}.$$

The first-order condition determining the optimal price is given by[9]

$$\begin{aligned} & E_t \sum_{s=0}^{\infty} \theta^s_{H*}\chi^*_{t,t+s}(-\epsilon_{H*})\left(\tilde{P}^{H*}_t\right)^{-\epsilon_{H*}-1}\left(\Gamma^{H*}_{t,s}\right)^{-\epsilon_{H*}} P^{H*}_{t+s}mc^{H*}_{t+s}\left(P^{H*}_{t+s}\right)^{\epsilon_{H*}} Y^{H*}_{t+s} \\ = \; & E_t \sum_{s=0}^{\infty} \theta^s_{H*}\chi^*_{t,t+s}(1-\epsilon_{H*})\left(\tilde{P}^{H*}_t\right)^{-\epsilon_{H*}}\left(\Gamma^{H*}_{t,s}\right)^{1-\epsilon_{H*}}\left(P^{H*}_{t+s}\right)^{\epsilon_{H*}} Y^{H*}_{t+s}, \end{aligned}$$

or, multiplying by $\tilde{P}^{H*}_t/P^{H*}_t$ and dividing by $-\epsilon_{H*}$ on both sides:

$$\begin{aligned} & E_t \sum_{s=0}^{\infty} \theta^s_{H*}\chi^*_{t,t+s}\left(\tilde{P}^{H*}_t\right)^{-\epsilon_{H*}}\left(\Gamma^{H*}_{t,s}\right)^{-\epsilon_{H*}}\left(P^{H*}_t\right)^{-1} P^{H*}_{t+s}mc^{H*}_{t+s}\left(P^{H*}_{t+s}\right)^{\epsilon_{H*}} Y^{H*}_{t+s} \\ = \; & E_t \sum_{s=0}^{\infty} \theta^s_{H*}\chi^*_{t,t+s}\left(\tilde{P}^{H*}_t\right)^{1-\epsilon_{H*}}\left(\Gamma^{H*}_{t,s}\right)^{1-\epsilon_{H*}}\left(P^{H*}_t\right)^{-1}\left(P^{H*}_{t+s}\right)^{\epsilon_{H*}} Y^{H*}_{t+s}\left(\frac{\epsilon_{H*}-1}{\epsilon_{H*}}\right). \end{aligned}$$

Letting $\tilde{p}^{H*}_t \equiv \tilde{P}^{H*}_t/P^{H*}_t$ denote the optimal price in terms of the corresponding composite good price and defining also $p^{H*}_t \equiv P^{H*}_t/P^*_t$, the first-order condition can be re-written in recursive form as follows:

$$F^{H*1}_{t+1} = F^{H*2}_{t+1} = F^{H*}_{t+1},$$

where

$$\begin{aligned} F^{H*1}_t &= E_t \sum_{s=0}^{\infty} (\theta_{H*}\beta)^s \frac{\varrho_{t+s}}{\varrho_t}\frac{\Lambda^R_{t+s}}{\Lambda^R_t}\frac{P_t(1+\tau^C_t)}{P_{t+s}(1+\tau^C_{t+s})}\frac{S_{t+s}}{S_t}\left(\tilde{P}^{H*}_t\right)^{-\epsilon_{H*}}\left(\Gamma^{H*}_{t,s}\right)^{-\epsilon_{H*}}\left(P^{H*}_t\right)^{-1} P^{H*}_{t+s}mc^{H*}_{t+s}\left(P^{H*}_{t+s}\right)^{\epsilon_{H*}} Y^{H*}_{t+s}, \\ &= \left(\frac{\tilde{P}^{H*}_t}{P^{H*}_t}\right)^{-\epsilon_{H*}} mc^{H*}_t Y^{H*}_t + \beta\theta_{H*}E_t\left\{\frac{\varrho_{t+1}}{\varrho_t}\frac{\Lambda^R_{t+1}}{\Lambda^R_t}\frac{P_t(1+\tau^C_t)}{P_{t+1}(1+\tau^C_{t+1})}\frac{S_{t+1}}{S_t}\left(g^{\Gamma^{H*}}_t\frac{\tilde{P}^{H*}_t}{\tilde{P}^{H*}_{t+1}}\right)^{-\epsilon_{H*}}\left(\frac{P^{H*}_t}{P^{H*}_{t+1}}\right)^{-1} F^{H*1}_{t+1}\right\}, \\ &= \left(\tilde{p}^{H*}_t\right)^{-\epsilon_{H*}} mc^{H*}_t Y^{H*}_t + \beta\theta_{H*}E_t\left\{\frac{\varrho_{t+1}}{\varrho_t}\frac{\Lambda^R_{t+1}}{\Lambda^R_t}\frac{rer_{t+1}}{rer_t}\left(\frac{g^{\Gamma^{H*}}_t}{\pi^*_{t+1}}\frac{\tilde{p}^{H*}_t}{\tilde{p}^{H*}_{t+1}}\right)^{-\epsilon_{H*}}\left(\frac{p^{H*}_t}{p^{H*}_{t+1}}\right)^{-1-\epsilon_{H*}} F^{H*1}_{t+1}\right\}, \end{aligned}$$

[9] By symmetry, the optimal price is identical across firms, i.e., $\tilde{P}^{H*}_t(j) = \tilde{P}^{H*}_t$ for all j.

and

$$
\begin{aligned}
F_t^{H*2} &= E_t \sum_{s=0}^{\infty} (\theta_{H*}\beta)^s \frac{\varrho_{t+s}}{\varrho_t} \frac{\Lambda_{t+s}^R}{\Lambda_t^R} \frac{P_t(1+\tau_t^C)}{P_{t+s}(1+\tau_{t+s}^C)} \frac{S_{t+s}}{S_t} \left(\tilde{P}_t^{H*}\right)^{1-\epsilon_{H*}} \left(\Gamma_{t,s}^{H*}\right)^{1-\epsilon_{H*}} \left(P_t^{H*}\right)^{-1} \left(P_{t+s}^{H*}\right)^{\epsilon_{H*}} Y_{t+s}^{H*} \left(\frac{\epsilon_{H*}-1}{\epsilon_{H*}}\right), \\
&= \left(\frac{\tilde{P}_t^{H*}}{P_t^{H*}}\right)^{1-\epsilon_{H*}} Y_{t+s}^{H*} \left(\frac{\epsilon_{H*}-1}{\epsilon_{H*}}\right) + \theta_{H*}\beta E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\Lambda_{t+1}^R}{\Lambda_t^R} \frac{P_t(1+\tau_t^C)}{P_{t+1}(1+\tau_{t+1}^C)} \frac{S_{t+1}}{S_t} \left(g_t^{\Gamma^{H*}} \frac{\tilde{P}_t^{H*}}{\tilde{P}_{t+1}^{H*}}\right)^{1-\epsilon_{H*}} \left(\frac{P_t^{H*}}{P_{t+1}^{H*}}\right)^{-1} F_{t+1}^{H*2} \right\}, \\
&= \left(\tilde{p}_t^{H*}\right)^{1-\epsilon_{H*}} Y_t^{H*} \left(\frac{\epsilon_{H*}-1}{\epsilon_{H*}}\right) + \theta_{H*}\beta E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\Lambda_{t+1}^R}{\Lambda_t^R} \frac{rer_{t+1}}{rer_t} \left(\frac{g_t^{\Gamma^{H*}}}{\pi_{t+1}^*} \frac{\tilde{p}_t^{H*}}{\tilde{p}_{t+1}^{H*}}\right)^{1-\epsilon_{H*}} \left(\frac{p_t^{H*}}{p_{t+1}^{H*}}\right)^{-\epsilon_{H*}} F_{t+1}^{H*2} \right\}.
\end{aligned}
$$

Further, let $\Psi^{H*}(t)$ denote the set of firms that cannot optimally choose their price in period t. By (31), the price level P_t^{H*} evolves as follows:

$$
\begin{aligned}
\left(P_t^{H*}\right)^{1-\epsilon_{H*}} = \int_0^1 P_t^{H*}(j)^{1-\epsilon_{H*}} dj &= (1-\theta_{H*}) \left(\tilde{P}_t^{H*}\right)^{1-\epsilon_{H*}} + \int_{\Psi^{H*}(t)} \left(g_{t-1}^{\Gamma^{H*}} P_{t-1}^{H*}(j)\right)^{1-\epsilon_{H*}} dj, \\
&= (1-\theta_{H*}) \left(\tilde{P}_t^{H*}\right)^{1-\epsilon_{H*}} + \theta_{H*} \left(g_{t-1}^{\Gamma^{H*}} P_{t-1}^{H*}\right)^{1-\epsilon_{H*}}.
\end{aligned}
$$

Dividing both sides by $\left(P_t^{H*}\right)^{1-\epsilon_{H*}}$ yields

$$
1 = (1-\theta_{H*}) \left(\tilde{p}_t^{H*}\right)^{1-\epsilon_{H*}} + \theta_{H*} \left(\frac{p_{t-1}^{H*}}{p_t^{H*}} \frac{g_{t-1}^{\Gamma^{H*}}}{\pi_t^*}\right)^{1-\epsilon_{H*}}.
$$

The second equality above follows from the fact that the distribution of prices among firms not reoptimizing in period t corresponds to the distribution of effective prices in period $t-1$, though with total mass reduced to θ_{H*}.

2.3.4 Wholesale Domestic Goods

The technology of the representative firm producing the homogeneous home good requires the utilization of imported oil as a complementary input, together with capital and labor.

$$
Y_t^{\widetilde{H}} = z_t \left[(1-o_O)^{\frac{1}{\eta_O}} \left(X_t^{\tilde{Z}}\right)^{\frac{\eta_O-1}{\eta_O}} + o_O^{\frac{1}{\eta_O}} \left(X_t^O\right)^{\frac{\eta_O-1}{\eta_O}} \right]^{\frac{\eta_O}{\eta_O-1}}, \; o_O \in (0,1), \; \eta_O > 0 \tag{32}
$$

Where z_t is an exogenous stationary shock, X_t^O is the amount of oil used as an intermediate input, and $X_t^{\tilde{Z}}$ is the demand for the core (non-oil) productive input $Y_t^{\tilde{Z}}$, a composite of labor and capital produced using a Cobb-Douglas technology:

$$
Y_t^{\tilde{Z}} = \left(\widetilde{K}_t\right)^{\alpha} \left(A_t^H n_t h_t\right)^{1-\alpha}, \qquad \alpha \in (0,1), \tag{33}
$$

Where A_t^H (with $a_t^H \equiv {A_t^H}/{A_{t-1}^H}$) is a non-stationary labor-augmenting technology disturbance. Similar to Coenen et al. (2012, 2013), $\widetilde{K}_t$, the capital good used in the production of the homogeneous good, is a CES composite between private and public capital:

$$
\widetilde{K}_t = \left[(1-o_{KG})^{\frac{1}{\eta_{KG}}} \left(K_t^S\right)^{\frac{\eta_{KG}-1}{\eta_{KG}}} + o_{KG}^{\frac{1}{\eta_{KG}}} \left(K_{t-1}^G\right)^{\frac{\eta_{KG}-1}{\eta_{KG}}} \right]^{\frac{\eta_{KG}}{\eta_{KG}-1}}, \qquad o_{KG} \in (0,1), \qquad \eta_{KG} > 0 \tag{34}
$$

Denote $\widetilde{c}_t^i$ as worker i's operating cost for the firm. Recall that when this cost is too high ($\widetilde{c}_t \geq \bar{c}_t$), production does not take place. Then, the average operating cost per worker is given by $H_t^C = A_{t-1}^H \int_0^{\bar{c}_t} \widetilde{c}_t \frac{dF(\widetilde{c}_t)}{F(\bar{c}_t)}$. The threshold value $\bar{c}_t$ is optimally decided by the firm in each period. There is a posting cost per vacancy identical for all firms with the form $\Omega_t = A^H_{t-1}\Omega_v$, where Ω_v is a constant. We allow H^C_t and Ω_t to grow proportionately with the productivity trend A^H_{t-1} in order to maintain a balanced steady-state growth path. As in Christiano et al. (2011), these costs are assumed to be paid in terms of composite home goods. The firm's workforce evolves over time as

$$n_t = (1-\rho_t)(n_{t-1} + e_{t-1}v_{t-1}) \tag{35}$$

Since today's choice of v_t affects tomorrow's workforce, the firm faces an inter-temporal decision problem to maximize expected discounted profits. As firms are owned by Ricardian households, the firm's stochastic discount factor for real payoffs satisfies $\Xi^R_{t,t+s} \equiv \beta^s(\varrho_{t+s}/\varrho_t)(\Lambda^R_{t+s}/\Lambda^R_t)$, for $s \geq 0$. The wholesale firm chooses how much capital services and oil to use $(K^S_t,\, X^O_t)$, and how much labor to hire and fire $(v_t, n_t, \bar{c}_t)$ subject to (32), (33), (34) and (35). Hence, the firm's problem is

$$\begin{aligned}
\max_{K^S_t, X^O_t, v_t, n_t, \bar{c}_t} \quad & E_t \sum_{s=0}^{\infty} \Xi^R_{t,t+s} \begin{bmatrix} p^{\widetilde{H}}_{t+s} Y^{\widetilde{H}}_{t+s} - W_{t+s} n_{t+s} h_{t+s} - p^H_{t+s} H^C_{t+s} n_{t+s} \\ -p^H_{t+s}\Omega_{t+s} v_{t+s} - r^K_{t+s} K^S_{t+s} - p^O_{t+s} X^O_{t+s} \end{bmatrix} \\
s.t. \quad & Y^{\widetilde{H}}_t = z_t \left[(1-o_O)^{\frac{1}{\eta_O}} \left(X^{\tilde{Z}}_t \right)^{\frac{\eta_O - 1}{\eta_O}} + o_O{}^{\frac{1}{\eta_O}} \left(X^O_t \right)^{\frac{\eta_O-1}{\eta_O}} \right]^{\frac{\eta_O}{\eta_O - 1}} \\
& X^{\tilde{Z}}_t = Y^{\tilde{Z}}_t = \left(\widetilde{K}_t \right)^{\alpha} \left(A^H_t n_t h_t \right)^{1-\alpha} \\
& \widetilde{K}_t = \left[(1-o_{KG})^{\frac{1}{\eta_{KG}}} \left(K^S_t \right)^{\frac{\eta_{KG}-1}{\eta_{KG}}} + o_{KG}{}^{\frac{1}{\eta_{KG}}} \left(K^G_{t-1} \right)^{\frac{\eta_{KG}-1}{\eta_{KG}}} \right]^{\frac{\eta_{KG}}{\eta_{KG}-1}} \\
& n_t = (1-\rho_t)(n_{t-1} + e_{t-1} v_{t-1})
\end{aligned}$$

Where $p^{\widetilde{H}}_t$ and p^O_t are the prices of $Y^{\widetilde{H}}_t$ and X^O_t in terms of the final consumption good, respectively. The second constraint follows from the fact that as the only use of $Y^{\tilde{Z}}_t$ is to satisfy the demand from (32), the market clearing condition requires $X^{\tilde{Z}}_t = Y^{\tilde{Z}}_t$.

Given the constraints outlined above, the problem for the firm can be separated in two parts: the cost minimizing production of $Y^{\tilde{Z}}_t$(choosing $n_t, v_t, \bar{c}_t$ and K^S_t) and the profit maximizing production of $Y^{\widetilde{H}}_t$(choosing $Y^{\tilde{Z}}_t$ and X^O_t).

In the first stage, the firm wants to minimize the expected discounted cost of production of $Y^{\tilde{Z}}_t$ by solving

$$\begin{aligned}
\min_{K^S_t, v_t, n_t, \bar{c}_t} \quad & E_t \sum_{s=0}^{\infty} \Xi^R_{t,t+s} \left[W_{t+s} n_{t+s} h_{t+s} + p^H_{t+s} H^C_{t+s} n_{t+s} + p^H_{t+s} \Omega_{t+s} v_{t+s} + r^K_{t+s} K^S_{t+s} \right] \\
s.t. \quad & Y^{\tilde{Z}}_t = \left(\widetilde{K}_t \right)^{\alpha} \left(A^H_t n_t h_t \right)^{1-\alpha} \\
& n_t = (1-\rho_t)(n_{t-1} + e_{t-1} v_{t-1})
\end{aligned}$$

The Lagrangian for this minimization problem is

$$L^{Y^{\tilde{Z}}}_t = E_t \sum_{s=0}^{\infty} \Xi^R_{t,t+s} \begin{bmatrix} W_{t+s} n_{t+s} h_{t+s} + p^H_{t+s} H^C_{t+s} n_{t+s} + p^H_{t+s} \Omega_{t+s} v_{t+s} + r^K_{t+s} K^S_{t+s} \\ + mc^{\tilde{Z}}_{t+s} \left(Y^{\tilde{Z}}_{t+s} - \left(\widetilde{K}_{t+s} \right)^{\alpha} \left(A^H_{t+s} n_{t+s} h_{t+s} \right)^{1-\alpha} \right) \\ + \Upsilon_{t+s} \left(n_{t+s} - (1-\rho_{t+s})(n_{t+s-1} + e_{t+s-1} v_{t+s-1}) \right) \end{bmatrix}$$

Where $mc^{\tilde{Z}}_t$ denotes the multiplier on the technology constraint (i.e. real marginal cost of $Y^{\tilde{Z}}_t$ in terms of the final good), and Υ_t denotes the multiplier on firm's workforce. The first-order conditions are

$$K_t^S \ : \widetilde{K}_t = \alpha \left(\frac{r_t^{\widetilde{K}}}{mc_t^{\tilde{Z}}} \right)^{-1} Y_t^{\tilde{Z}} \tag{36}$$

$$v_t \ : \frac{p_t^H \Omega_t}{e_t} = E_t \Xi_{t,t+1}^R (1-\rho_{t+1}) \ \Upsilon_{t+1} \tag{37}$$

$$n_t \ : \Upsilon_t = mc_t^{\tilde{Z}} (1-\alpha) \frac{Y_t^{\tilde{Z}}}{n_t} - W_t h_t - p_t^H H_t^C + E_t \Xi_{t,t+1}^R (1-\rho_{t+1}) \ \Upsilon_{t+1} \tag{38}$$

$$\overline{c}_t \ : \frac{\Upsilon_t}{1-\rho_t} \frac{\partial \rho_t}{\partial \overline{c}_t} = -h_t \frac{\partial W_t}{\partial \overline{c}_t} - \frac{\partial H_t^C}{\partial \overline{c}_t} p_t^H \tag{39}$$

Where $r_t^{\widetilde{K}}$, the cost of acquiring an additional unit of $\widetilde{K}_t$, is defined as

$$r_t^{\widetilde{K}} = r_t^K \left(\frac{\partial \widetilde{K}_t}{\partial K_t^S} \right)^{-1} = r_t^K \left(\frac{K_t^S}{(1-o_{KG}) \widetilde{K}_t} \right)^{\frac{1}{\eta_{KG}}}$$

Combining (37) and (38) yields the job creation condition:

$$\frac{p_t^H \Omega_t}{e_t} = E_t \Xi_{t,t+1}^R \ (1-\rho_{t+1}) \left(mc_{t+1}^{\tilde{Z}} (1-\alpha) \frac{Y_{t+1}^{\tilde{Z}}}{n_{t+1}} - W_{t+1} h_{t+1} - p_{t+1}^H H_{t+1}^C + \frac{p_{t+1}^H \Omega_{t+1}}{e_{t+1}} \right)$$

Firms post vacancies to expand employment until the effective cost of posting an additional vacancy $(p_t^H \Omega_t / e_t)$ equals the expected marginal product of an additional worker $(mc_{t+1}^{\tilde{Z}} (1-\alpha) Y_{t+1}^{\tilde{Z}} / n_{t+1})$ minus the wage payment to that worker $(W_{t+1} h_{t+1})$ minus the average operating cost of the firm $(p_{t+1}^H H_{t+1}^C)$ plus its expected return of next periods reduction of vacancy posting costs $(p_{t+1}^H \Omega_{t+1} / e_{t+1})$, conditional on the worker surviving job destruction in period $t+1$ with probability $1-\rho_{t+1}$.

Combining (37), (38), (39), and using the fact that $(1-\rho_t^x) / (1-\rho_t) = 1/F(\overline{c}_t)$, we can get the job destruction condition:[10]

$$p_t^H A_{t-1}^H \overline{c}_t = mc_t^{\tilde{Z}} (1-\alpha) \frac{Y_t^{\tilde{Z}}}{n_t} - W_t h_t + \frac{p_t^H \Omega_t}{e_t}$$

This expression defines the critical threshold $\overline{c}_t$ above which jobs are separated.

In the second stage, firms maximize the profits from producing $Y_t^{\widetilde{H}}$ as if $Y_t^{\tilde{Z}}$ was produced by a vertically integrated subsidiary selling it at marginal cost. Note that there are no intertemporal decisions in this stage. The firm's problem is:

$$\max_{X_t^{\tilde{Z}}, X_t^O} p_t^{\widetilde{H}} z_t \left[(1-o_O)^{\frac{1}{\eta_O}} \left(X_t^{\tilde{Z}} \right)^{\frac{\eta_O - 1}{\eta_O}} + o_O{}^{\frac{1}{\eta_O}} \left(X_t^O \right)^{\frac{\eta_O - 1}{\eta_O}} \right]^{\frac{\eta_O}{\eta_O - 1}} - p_t^O X_t^O - mc_t^{\tilde{Z}} X_t^{\tilde{Z}}$$

[10] Derivatives $\frac{\partial W_t}{\partial \overline{c}_t} = 0$, $\frac{\partial \rho_t}{\partial \overline{c}_t} = -(1-\rho_t^x) f(\overline{c}_t)$ and $\frac{\partial H_t^C}{\partial \overline{c}_t} = A_{t-1}^H \left(\overline{c}_t - \frac{H_t^C}{A_{t-1}^H} \right) \frac{f(\overline{c}_t)}{F(\overline{c}_t)}$ were calculated using the Leibniz's rule for differentiation under the integral sign.

The optimal demands for $X_t^{\tilde{Z}}$ and X_t^{O} are then given by

$$X_t^{\tilde{Z}} = (z_t)^{\eta_O - 1} (1 - o_O) \left(\frac{mc_t^{\tilde{Z}}}{p_t^{\widetilde{H}}} \right)^{-\eta_O} Y_t^{\widetilde{H}} \tag{40}$$

$$X_t^{O} = (z_t)^{\eta_O - 1} o_O \left(\frac{p_t^{O}}{p_t^{\widetilde{H}}} \right)^{-\eta_O} Y_t^{\widetilde{H}} \tag{41}$$

Substituting (40) and (41) into (32) yields the following expression for the price of $Y_t^{\widetilde{H}}$ in terms of the final consumption good:

$$p_t^{\widetilde{H}} = \frac{1}{z_t} \left[(1 - \alpha_O) \left(mc_t^{\tilde{Z}} \right)^{1-\eta_O} + \alpha_O \left(p_t^{O} \right)^{1-\eta_O} \right]^{\frac{1}{1-\eta_O}} \tag{42}$$

2.3.5 Wages and Hours

The wages and hours bargaining process between the union and the representative firm begin after the endogenous separation process has finished. At the time the bargaining takes place, both the number of workers (n_t) and the operating cost associated with each one of those workers $\left(\widetilde{c}_t^{i}\right)$ is known. With a unique contract to be bargained for all workers, both the firm and the union consider an aggregate weighted surplus across all households.

As in Gertler et al. (2008) we assume the union and the firms agree to an efficient allocation of hours where the marginal value product of a worker-hour equals the marginal cost of work (in terms of consumption goods and adjusted by taxes) for an employed household member.

$$\left(1 - \omega^U\right) \frac{\partial G_t^R / \partial h_t^R}{\Lambda_t^R \left(1 - \tau_t^W\right)} + \omega^U \frac{\partial G_t^{NR} / \partial h_t^{NR}}{\Lambda_t^{NR} \left(1 - \tau_t^W\right)} = mc_t^{\tilde{Z}} \frac{\partial^2 Y_t^{\tilde{Z}}}{\partial h_t \partial n} \Rightarrow h_t = \left[\frac{mc_t^{\tilde{Z}} (1 - \alpha)^2 \frac{Y_t^{\tilde{Z}}}{n_t}}{\frac{\Psi_t^U \kappa_t}{(1-\tau_t^L)} \left(A_{t-1}^H\right)^{1-\sigma}} \right]^{\frac{1}{1+\phi}}$$

Where $\Psi_t^U = \left(1 - \omega^U\right) \frac{\Theta_t^R}{\Lambda_t^R} + \omega^U \frac{\Theta_t^{NR}}{\Lambda_t^{NR}}$ and ω^U is the weight given to the non Ricardian members by the union. To get to the expression to the right we use the fact that Ricardian and not Ricardian workers are indistinguishible to the firm and therefore demand the same labor intensity from Ricardian and non Ricardian workers.

Given the amount of hours per worker defined above, a notional contract specifying hourly wages (W_t^n) is negotiated through Nash bargaining. The outcome of the bargaining process is a wage that maximizes a weighted average between the firm's and union's surpluses:

$$\max_{W_t^n} \left[\varphi^U \log \mathcal{S}_t^U + \left(1 - \varphi^U\right) \log \mathcal{S}_t^F \right]$$

where $\mathcal{S}_t^U$ is the union's surplus, $\mathcal{S}_t^F$ is the firm's surplus, and $\varphi^U \in (0, 1)$ is the union's relative bargaining power. The first-order condition are

$$W_t^n \ : \varphi^U \mathcal{S}_t^F \frac{\partial \mathcal{S}_t^U}{\partial W_t^n} = -\left(1 - \varphi^U\right) \mathcal{S}_t^U \frac{\partial \mathcal{S}_t^F}{\partial W_t^n} \tag{43}$$

On the union side, the union's surplus is a weighted average surplus of Ricardian and non-Ricardian households

$$\mathcal{S}_t^U = \left(1 - \omega^U\right) \mathcal{S}_t^{R,U} + \omega^U \mathcal{S}_t^{NR,U}$$

For a member of type-i household, with $i = \{R, NR\}$, the union's surplus $\left(\mathcal{S}_t^{i,U}\right)$ is defined as the difference between the value of being employed ($\mathcal{V}_t^{i,E}$) and the value of being unemployed ($\mathcal{V}_t^{i,U}$). The value of being employed is equal to the current-period benefit from the job (after tax wage income minus disutility of work), plus the discounted continuation value of remaining employed next period plus the discounted continuation value of being separated:

$$\mathcal{V}_t^{i,E} = (1-\tau_t^L) W_t^n h_t - \frac{\Theta_t^i \kappa_t \left(A_{t-1}^H\right)^{1-\sigma} \frac{(h_t^n)^{1+\phi}}{1+\phi}}{\Lambda_t^i} + E_t \Xi_{t,t+1}^i \left[(1-\rho_{t+1}) \mathcal{V}_{t+1}^{i,E} + \rho_{t+1} \mathcal{V}_{t+1}^{i,U}\right]$$

The value of being unemployed is equal to the current unemployed benefit which is paid out by the UFA, plus the discounted continuation value of being employed with probability $s_t (1-\rho_{t+1})$ plus the discounted continuation value of remaining unemployed:

$$\mathcal{V}_t^{i,U} = UB_t + E_t \Xi_{t,t+1}^i \left[s_t (1-\rho_{t+1}) \mathcal{V}_{t+1}^{i,E} + (1 - s_t (1-\rho_{t+1})) \mathcal{V}_{t+1}^{i,U}\right]$$

Hence, the type-i household's surplus is

$$\begin{aligned} \mathcal{S}_t^{i,U} &= \mathcal{V}_t^{i,E} - \mathcal{V}_t^{i,U} \\ &= (1-\tau_t^L) W_t^n h_t^n - \frac{\Theta_t^i \kappa_t \left(A_{t-1}^H\right)^{1-\sigma} (h_t^n)^{1+\phi}}{\Lambda_t^i (1+\phi)} - UB_t + (1-s_t) E_t \Xi_{t,t+1}^i (1-\rho_{t+1}) \left[\mathcal{V}_{t+1}^{i,E} - \mathcal{V}_{t+1}^{i,U}\right] \end{aligned}$$

Finally, the union's surplus is given by

$$\begin{aligned} \mathcal{S}_t^U &= \left(1-\omega^U\right) \mathcal{S}_t^{R,U} + \omega^U \mathcal{S}_t^{NR,U} \\ &= \begin{array}{l} \left(1-\omega^U\right) \left((1-\tau_t^L) W_t^n h_t - \frac{\Theta_t^R \kappa_t \left(A_{t-1}^H\right)^{1-\sigma} (h_t^n)^{1+\phi}}{\Lambda_t^R (1+\phi)} - UB_t + (1-s_t) E_t \Xi_{t,t+1}^R (1-\rho_{t+1}) \left[\mathcal{V}_{t+1}^{R,E} - \mathcal{V}_{t+1}^{R,U}\right]\right) \\ + \omega^U \left((1-\tau_t^L) W_t^n h_t - \frac{\Theta_t^{NR} \kappa_t \left(A_{t-1}^H\right)^{1-\sigma} (h_t^n)^{1+\phi}}{\Lambda_t^{NR} (1+\phi)} - UB_t + (1-s_t) E_t \Xi_{t,t+1}^{NR} (1-\rho_{t+1}) \left[\mathcal{V}_{t+1}^{NR,E} - \mathcal{V}_{t+1}^{NR,U}\right]\right) \end{array} \\ &= (1-\tau_t^L) W_t^n h_t - \Psi_t^U \kappa_t \left(A_{t-1}^H\right)^{1-\sigma} \frac{(h_t^n)^{1+\phi}}{1+\phi} - UB_t + (1-s_t) E_t \Xi_{t,t+1}^U (1-\rho_{t+1}) \mathcal{S}_{t+1}^U \end{aligned}$$

Where $\Xi_{t,t+s}^{NR} \equiv \beta^s (\varrho_{t+s}/\varrho_t)(\Lambda_{t+s}^{NR}/\Lambda_t^{NR})$ and the union's discount factor $\Xi_{t,t+1}^U$ is implicitly defined by

$$\begin{aligned} \Sigma_t^U = E_t \Xi_{t,t+1}^U (1-\rho_{t+1}) \mathcal{S}_{t+1}^U &= \left(1-\omega^U\right) E_t \Xi_{t,t+1}^R (1-\rho_{t+1}) \left[\mathcal{V}_{t+1}^{R,E} - \mathcal{V}_{t+1}^{R,U}\right] \\ &+ \omega^U E_t \Xi_{t,t+1}^{NR} (1-\rho_{t+1}) \left[\mathcal{V}_{t+1}^{NR,E} - \mathcal{V}_{t+1}^{NR,U}\right]. \end{aligned}$$

Where $\Sigma_t^U = A_{t-1} \sigma_t^U$ would be the union's expected future surplus discounted by its factor and the total separation rate.

In the firm side, the firm's surplus is defined as the difference between the expected average value functions of a vacancy posting ($\mathcal{V}_t^V$) and a filled job ($\mathcal{V}_t^J$). The value of a filled job by a worker with known operational costs of $\widetilde{c}_t$ is equal to the current-period firm's profit from one worker (productivity of labor minus operational cost plus wage payment), plus the discounted continuation value of the job next period and losing the existing job:

$$\mathcal{V}_t^J(\widetilde{c}_t) = mc_t^{\tilde{Z}} (1-\alpha) \frac{Y_t^{\tilde{Z}}}{n_t} - W_t^n h_t - p_t^H A_{t-1}^H \widetilde{c}_t + E_t \Xi_{t,t+1}^R \left[(1-\rho_{t+1}) \mathcal{V}_{t+1}^J + \rho_{t+1} \mathcal{V}_{t+1}^V\right]$$

Integrating over the density function of $\widetilde{c}_t$ over the relevant interval we obtain the average value of a filled job $\mathcal{V}_t^J$

$$\mathcal{V}_t^J = \int_0^{\bar{c}_t} \mathcal{V}_t^J(\widetilde{c}_t) \frac{dF(\widetilde{c}_t)}{F(\bar{c}_t)} = mc_t^{\tilde{Z}} (1-\alpha) \frac{Y_t^{\tilde{Z}}}{n_t} - W_t^n h_t - p_t^H A_{t-1}^H \int_0^{\bar{c}_t} \widetilde{c}_t \frac{dF(\widetilde{c}_t)}{F(\bar{c}_t)} + E_t \Xi_{t,t+1}^R \left[(1-\rho_{t+1}) \mathcal{V}_{t+1}^J + \rho_{t+1} \mathcal{V}_{t+1}^V\right]$$

$$\mathcal{V}_t^J = \int_0^{\bar{c}_t} \mathcal{V}_t^J(\widetilde{c}_t) \frac{dF(\widetilde{c}_t)}{F(\bar{c}_t)} = mc_t^{\tilde{Z}} (1-\alpha) \frac{Y_t^{\tilde{Z}}}{n_t} - W_t^n h_t - p_t^H H_t^C + E_t \Xi_{t,t+1}^R \left[(1-\rho_{t+1}) \mathcal{V}_{t+1}^J + \rho_{t+1} \mathcal{V}_{t+1}^V\right] = \mathcal{V}_t^J\left(H_t^C\right)$$

The value of a vacancy posting is given by the current vacancy posting cost, plus the discounted continuation value of a filled job next period with probability $e_t(1-\rho_{t+1})$ and an open vacancy next period:

$$\mathcal{V}_t^V = -p_t^H\Omega_t + E_t\Xi_{t,t+1}^R\left[e_t(1-\rho_{t+1})\mathcal{V}_{t+1}^J + (1-e_t(1-\rho_{t+1}))\mathcal{V}_{t+1}^V\right]$$

A free entry condition implies for firms that $\mathcal{V}_t^V = 0$ for all t, and thus the value of a vacancy posting produces

$$\frac{p_t^H\Omega_t}{e_t} = E_t\Xi_{t,t+1}^R(1-\rho_{t+1})\mathcal{V}_{t+1}^J$$

And the value of a filled job becomes

$$\mathcal{V}_t^J = mc_t^{\tilde{Z}}(1-\alpha)\frac{Y_t^{\tilde{Z}}}{n_t} - W_t^n h_t^n - p_t^H H_t^C + \frac{p_t^H\Omega_t}{e_t}$$

Hence, the firm's surplus is given by

$$\mathcal{S}_t^F = \mathcal{V}_t^J - \mathcal{V}_t^V = \mathcal{V}_t^J = mc_t^{\tilde{Z}}(1-\alpha)\frac{Y_t^{\tilde{Z}}}{n_t} - W_t^n h_t - p_t^H H_t^C + \frac{p_t^H\Omega_t}{e_t}$$

Thef irst orde condition (43) implies that

$$\varphi^U\mathcal{S}_t^F\frac{\partial\mathcal{S}_t^U}{\partial W_t^n} = -\left(1-\varphi^U\right)\mathcal{S}_t^U\frac{\partial\mathcal{S}_t^F}{\partial W_t^n} \Longrightarrow \frac{\mathcal{S}_t^U}{(1-\tau_t^L)} = \frac{\varphi^U}{1-\varphi^U}\mathcal{S}_t^F$$

Replacing the union's surplus and the firm's surplus in the previous relation and rearranging terms, yields the notional wage

$$\begin{aligned} W_t^n h_t^n = \varphi^U & \left(mc_t^{\tilde{Z}}(1-\alpha)\tfrac{Y_t^{\tilde{Z}}}{n_t} - p_t^H H_t^C + \tfrac{p_t^H\Omega_t}{e_t} \right) \\ & + \frac{(1-\varphi^U)}{(1-\tau_t^L)}\left(\Psi_t^U\kappa_t\left(A_{t-1}^H\right)^{1-\sigma}\tfrac{(h_t^n)^{1+\phi}}{1+\phi} + UB_t - (1-s_t)\Sigma_t^U\right) \end{aligned} \quad (44)$$

As in Hall (2005), we introduce wage stickiness by assuming the nominal wage paid to the individual worker(P_tW_t) is the weighted average of the notional nominal wage$(P_tW_t^n)$ and the indexed nominal wage norm$\left(\Gamma_{t-1}^W P_{t-1}W_{t-1}\right)$:

$$P_tW_t = \varkappa_W\Gamma_{t-1}^W P_{t-1}W_{t-1} + (1-\varkappa_W)P_tW_t^n, \qquad \varkappa_W \in (0,1)$$

where Γ_t^W is the wage indexation variable that satisfies $\Gamma_t^W = a\pi_t^{\vartheta_W}\pi^{1-\vartheta_W}$, with $\vartheta_W \in (0,1)$.

2.3.6 Commodity sector investment and output

The production of the commodity sector follows the framework described in Fornero et al. (2014). As in Medina and Soto (2007), there is a representative firm in the commodity sector that produces a homogeneous commodity good. The entire production is exported. A fraction χ^{Co} of the assets of that firm is owned by the government and the remaining fraction is owned by foreign investors. The cash flows generated in the commodity sector are shared accordingly, but the government levies taxes on the profits that accrue to foreign investors.

The commodity firm's production function uses capital specific to the commodity sector, K_t^{Co}, at intensity $\bar{u}_t^{Co}$, and a fixed production factor, $\bar{L}$. The latter, which is thought to represent the mineral content of land, is subject to a long-run technology trend denoted by A_t^{Co}. That trend is assumed to be cointegrated with the economy's overall technology trend A_t so as to be consistent with balanced growth in the long run, but we allow for short- to medium-term deviations deviations from that common growth path due to sectoral technology shocks. Specifically, commodity production satisfies

$$Y_t^{Co} = z_t^{Co}\left(\bar{u}_t^{Co}K_{t-1}^{Co}\right)^{\alpha_{Co}}\left(A_t^{Co}\bar{L}\right)^{1-\alpha_{Co}} \tag{45}$$

where z_t^{Co} is a stationary technology shock specific to commodity production.[11] Capital utilization is subject to a maintenance cost of $\phi_{\bar{u}}^{Co}\left(\bar{u}_t^{Co}\right)K_{t-1}^{Co}$ investment goods, with utilization cost function

$$\phi_{\bar{u}}^{Co}\left(\bar{u}_t^{Co}\right) \equiv \frac{r^{k,Co}}{\Phi_{\bar{u}}^{Co}}\left(e^{\Phi_{\bar{u}}^{Co}\left(\bar{u}_t^{Co}-1\right)}-1\right)$$

with $\Phi_{\bar{u}}^{Co} = \phi_{\bar{u}}^{Co\prime\prime}(1)/\phi_{\bar{u}}^{Co\prime}(1) > 0$ and where $r^{k,Co}$ denotes the steady state value of the return on utilized capital to be defined below.

The commodity sector's capital stock accumulates according to the following law of motion:

$$K_t^{Co} = (1-\delta_{Co})K_{t-1}^{Co} + \left(1-\phi_I^{Co}\left(\frac{I_{t-N_{Co}+1}^{ACo}}{I_{t-N_{Co}}^{ACo}}\right)\right)I_{t-N_{Co}+1}^{ACo}\varpi_{t-N_{Co}+1}^{Co}$$

Where $I_{t-N_{Co}+1}^{ACo}$ are sector's investment projects authorized in period $t-N_{Co}+1$, N_{Co} denotes the number of periods it takes until these projects become productive and augment the capital stock, $\phi_I^{Co}\left(I_t^{ACo}/I_{t-1}^{ACo}\right) \equiv \left(\Phi_I^{Co}/2\right)\left(I_t^{ACo}/I_{t-1}^{ACo}-a\right)^2$ are convex investment adjustment costs with elasticity $\Phi_I^{Co} = \phi_I^{Co\prime\prime}(a) \geq 0$, and ϖ_t^{Co} captures changes in the efficiency of the investment projects. The sector's total effective investment (not including maintenance costs) is distributed between new and old authorized projects according to

$$I_t^{Co} = \sum_{j=0}^{N_{Co}-1}\varphi_j^{Co}I_{t-j}^{ACo}, \tag{46}$$

Where φ_j^{Co} – the fraction of projects authorized in period $t-j$ that is outlaid in period t– satisfy $\sum_{j=0}^{N_{Co}-1}\varphi_j^{Co} = 1$ and $\varphi_j^{Co} = \rho^{\varphi Co}\varphi_{j-1}^{Co}$. Also, total investment goods purchased include maintenance costs, $I_t^{Co,f} \equiv I_t^{Co} + \phi_{\bar{u}}^{Co}\left(\bar{u}_t^{Co}\right)K_{t-1}^{Co}$.

Gross profits and before-tax cash flows of the firm, in units of final consumption goods, are respectively given by:

$$\Pi_t^{Co} = rer_t p_t^{Co*}Y_t^{Co}$$

$$CF_t^{Co} = \Pi_t^{Co} - p_t^{ICo}I_t^{Co,f} \tag{47}$$

Where p_t^{Co*} is the international price of the commodity in terms of the foreign consumption good. The firm maximizes its cash flow subject to the tax τ_t^{Co} on a fraction $1-\chi^{Co}$ of its gross profits, and by assumption discounts real cash flows at the same rate as the households $\Xi_{t,t+s}^R$.[12] The commodity producer's problem is then given by

$$\mathcal{L}_t^{Co} = E_t\sum_{s=0}^{\infty}\Xi_{t,t+s}^R\left\{\begin{array}{c}\left[1-\tau_t^{Co}\left(1-\chi^{Co}\right)\right]rer_{t+s}p_{t+s}^{Co*}Y_{t+s}^{Co} - p_{t+s}^{ICo}\left(I_{t+s}^{Co}+\phi_{\bar{u}}^{Co}\left(\bar{u}_{t+s}^{Co}\right)K_{t+s-1}^{Co}\right)\\ +\lambda_{t+s}^{Co}\left[z_{t+s}^{Co}\left(\bar{u}_{t+s}^{Co}K_{t+s-1}^{Co}\right)^{\alpha_{Co}}\left(A_{t+s}^{Co}\bar{L}\right)^{1-\alpha_{Co}} - Y_{t+s}^{Co}\right]\\ +q_{t+s}^{Co}\left[(1-\delta_{Co})K_{t+s-1}^{Co} + \left(1-\phi_I^{Co}\left(I_{t+s-N_{Co}+1}^{ACo}/I_{t+s-N_{Co}}^{ACo}\right)\right)I_{t+s-N_{Co}+1}^{ACo}\varpi_{t+s-N_{Co}+1}^{Co} - K_{t+s}^{Co}\right]\end{array}\right\}$$

Where λ_t^{Co} and q_t^{Co} denote the Lagrange multipliers associated with the technology constraint and the capital's law of motion.

The corresponding first order optimality conditions are

[11] Our specification focuses on capital-intensive commodity production and neglects labor inputs, since the labor share of commodity production is low.

[12] The stochastic discount factor for domestic currency payoffs of foreign investors is identical to the one of the Ricardian households if foreign investors have unrestricted access to domestic currency bonds.

$$
\begin{aligned}
Y_t^{Co}: \quad & \lambda_t^{Co} = \left[1-\tau_t^{Co}\left(1-\chi^{Co}\right)\right] rer_t p_t^{Co*} \\
K_t^{Co}: \quad & q_t^{Co} = E_t\left\{\Xi_{t,t+1}^{R}\left[\lambda_{t+1}^{Co}\alpha^{Co}\frac{Y_{t+1}^{Co}}{K_t^{Co}} + q_{t+1}^{Co}(1-\delta_{Co}) - p_{t+1}^{ICo}\phi_{\bar{u}}^{Co}\left(\bar{u}_{t+1}^{Co}\right)\right]\right\} \\
I_t^{ACo}: \quad & 0 = E_t\left\{\frac{\sum_{j=0}^{N_{Co}-1}\Xi_{t,t+j}^{R}\varphi_j^{Co}p_{t+j}^{ICo}}{\Xi_{t,t+N_{Co}-1}^{R}q_{t+N_{Co}-1}^{Co}}\right\} \\
& -\left[\left(1-\phi_I^{Co}\left(\frac{I_t^{ACo}}{I_{t-1}^{ACo}}\right)\right) - \Phi_I^{Co}\left(\frac{I_t^{ACo}}{I_{t-1}^{ACo}}-a\right)\frac{I_t^{ACo}}{I_{t-1}^{ACo}}\right]\varpi_t^{Co} \\
& - E_t\left\{\frac{\Xi_{t,t+N_{Co}}^{R}}{\Xi_{t,t+N_{Co}-1}^{R}}\frac{q_{t+N_{Co}}^{Co}}{q_{t+N_{Co}-1}^{Co}}\Phi_I^{Co}\left(\frac{I_{t+1}^{ACo}}{I_t^{ACo}}-a\right)\left(\frac{I_{t+1}^{ACo}}{I_t^{ACo}}\right)^2\varpi_{t+1}^{Co}\right\} \\
\bar{u}_t^{Co}: \quad & r_t^{k,Co} = p_t^{ICo}\phi_{\bar{u}}^{Co\prime}\left(\bar{u}_t^{Co}\right)
\end{aligned}
$$

Where $r_t^{k,Co}$ denotes the marginal return on utilized capital:

$$
r_t^{k,Co} \equiv \left[1-\tau^{Co}\left(1-\chi^{Co}\right)\right] rer_t p_t^{Co*}\alpha_{Co}\frac{Y_t^{Co}}{\bar{u}_t^{Co}K_{t-1}^{Co}}.
$$

The optimal utilization rate is therefore given by

$$
\bar{u}_t^{Co} = 1 + \frac{\log\left(\frac{r_t^{k,Co}}{r^{k,Co}}\right) - \log\left(p_t^{ICo}\right)}{\Phi_{\bar{u}}^{Co}}.
$$

2.4 Fiscal Policy

The government spends a stream of resources G_t, levies lump-sum and distortionary taxes, issues one-period bonds and receives a share of the income generated in the commodity sector. Aggregate spending is allocated to consumption of final goods $\left(C_t^G\right)$, investment in public goods $\left(I^G\right)$, transfers to households $\left(TR_t^G\right)$, and to finance a policy of domestic oil price stabilization by buying O_t units of oil (see below):

$$
G_t = p_t^{CG}C_t^G + p_t^{IG}I_t^G + TR_t^G + \left(rer_t p_t^{O^*} - p_t^O\right)O_t
$$

2.4.1 Taxes

Part of the government spending is financed with time varying distortionary taxes, where τ_t^C, τ_t^W, τ_t^K, τ_t^D and τ_t^{Co} denote respectively the tax rates on consumption, labor income, capital income, dividend income and private commodity profits. Additionally, we assume lump-sum taxes to be a constant share of nominal GDP:[13] $T_t = \alpha^T p_t^Y Y_t$. Total tax revenue is then $\Pi_t^\tau \equiv \tau_t A_{t-1} = \tau_t^C C_t + \tau_t^W W_t n_t h_t + \tau_t^K\left[r_t^K\bar{u}_t - p_t^I\left(\delta+\phi_{\bar{u}}\left(\bar{u}_t\right)\right)\right]K_{t-1} + \tau_t^D D_t + \tau_t^{Co}(1-\chi^{Co})\Pi_t^{Co} + T_t$. Lump-sum taxes are levied from non-Ricardian and Ricardian households with shares given respectively by ω^G and $1-\omega^G$. The taxes that each type of household has to pay to government therefore satisfy $\omega T_t^{NR} = \omega^G T_t$ and $(1-\omega)\,T_t^R = \left(1-\omega^G\right)T_t$.

[13]When government spending doesn't follow a structural balance rule (i.e $I_{rule}=0$, see below), lump sum taxes are defined as $\frac{T_t}{p_t^Y Y_t} = \alpha^T + \epsilon^T\left(\frac{A_{t-1}\left(rer\, b^{G^*}+b^G\right) - rer_t B_t^{G^*} - B_t^G}{p_t^Y Y_t}\right)$, where ϵ^T is a small constant that prevents indeterminacy of the steady state government debt level.

2.4.2 Government debt

In addition to issuing domestic currency denominated bonds, the government finances a share α^D of its debt in foreign currency. The government asset position is determined by the budget constraint and the rule for the government asset's currency diversification:

$$B_t^G + rer_t B_t^{G^*} = r_t^* rer_t B_{t-1}^{G^*} + r_t B_{t-1}^G + \Pi_t^\tau + \chi^{Co} CF_t^{Co} - G_t \tag{48}$$

$$rer_t B_t^{G^*} = \alpha^D \left(rer_t B_t^{G^*} + B_t^G \right) \tag{49}$$

Where B_t^G and B_t^{G*} are the government's net asset position in domestic and foreign currency.

2.4.3 Spending Rule

The desired amount of government spending$\left(\widetilde{G}_t\right)$ follows either a structural balance rule$\left(\widetilde{G}_t^{rule}\right)$ or an exogenous random path$\left(\widetilde{G}_t^{exo}\right)$:

$$\widetilde{G}_t = \left(\widetilde{G}_t^{rule}\right)^{I_{rule}} \left(\widetilde{G}_t^{exo}\right)^{1-I_{rule}}$$

Where I_{rule} is and indicator function with value 1 if the government follows a rule and 0 otherwise.

The exogenous process for the desired level of government spending $\widetilde{G}_t^{exo}$ evolves with a law of motion given by

$$\widetilde{G}_t^{exo} = \overline{G}_t \xi_t^G \tag{50}$$

Where ξ_t^G is an exogenous process with unconditional mean equal to one that captures deviations of desired government expenditure from the long term balanced growth path spending $\overline{G}_t \equiv A_{t-1} g$.

On the other hand, $\widetilde{G}_t^{rule}$ is determined through a structural balance fiscal rule, which is a simplified version of the rule described in Marcel et al. (2001) and Marcel et al. (2003), and similar to Medina and Soto (2007) and Kumhof and Laxton (2010) . Let $\tilde{B}_t^G + rer_t \tilde{B}_t^{G*}$ be the government asset position associated to the desired spending:

$$\tilde{B}_t^G + rer_t \tilde{B}_t^{G*} = r_t^* rer_t B_{t-1}^{G^*} + r_t B_{t-1}^G + \Pi_t^\tau + \chi^{Co} CF_t^{Co} - \widetilde{G}_t^{rule} \tag{51}$$

According to the rule, the deviation of the desired government surplus as a ratio of GDP from a structural balance target($\bar{s}_B$), depends on the deviation of tax revenue from potential and the deviation of government income from the commodity sector from a long-run reference value:

$$\frac{\tilde{B}_t^G + rer_t \tilde{B}_t^{G*} - B_{t-1}^G \pi_t^{-1} - rer_t B_{t-1}^{G*} \left(\pi_t^*\right)^{-1}}{p_t^Y Y_t} - \bar{s}_B = \frac{\gamma^D \left[\left(\Pi_t^\tau - \widetilde{\Pi}_t^\tau \right) + \chi^{Co} (CF_t^{Co} - \widetilde{CF}_t^{Co}) \right]}{p_t^Y Y_t} \tag{52}$$

Where $\widetilde{\Pi}_t^\tau$ denotes tax revenue at potential, i.e. current tax rates multiplied by the tax base in steady state: $\widetilde{\Pi}_t^\tau = \widetilde{\tau}_t A_{t-1} = \left[\tau_t^C c + \tau_t^W wnh + \tau_t^K \left(r^K \overline{u} - p^I \left(\delta + \phi_{\bar{u}} \left(\bar{u} \right) \right) \right) k/a + \tau_t^D d + (1 - \chi^{Co}) \tau_t^{Co} (cf^{Co} + p^{ICo} i^{Co,f}) + t \right] A_{t-1}$. Further, $\widetilde{CF}_t^{Co} = rer_t \tilde{p}_t^{Co*} Y_t^{Co} - p_t^{ICo} I_t^{Co,f}$ denotes the commodity sector cash flows that would prevail with the long-run reference price $\tilde{p}_t^{Co*} = E\left[\prod_{j=1}^{40} p_{t+j}^{Co*} \right]^{\frac{1}{40}}$, which is calculated as the forecast of the effective commodity price averaged over a 10 years horizon. The parameter γ^D governs the elasticity of the desired surplus to the cyclical part of government income. If $\gamma^D < 1$, the government spending (inclusive of interest payments) follows a procyclical pattern of higher spending in economic booms. A parameter equal to one is representative of a spending path that remains unaffected by cyclical fluctuations, while $\gamma^D > 1$ corresponds to a government that actively tries to offset the economic fluctuations.[14]

[14] The exogenous stochastic process from (50) is equivalent to an acyclical path for the government spending *excluding interest payments*

Substituting out $\tilde{B}_t^G + rer_t\tilde{B}_t^{G*}$ in (52) using (51) yields the rule for the desired government spending that follows a structural balance rule.

$$\frac{\widetilde{G}_t^{rule}}{p_t^Y Y_t} = \frac{(R_{t-1}-1)}{\pi_t}\frac{B_{t-1}^G}{p_t^Y Y_t} + \frac{\left(R_{t-1}^*\xi_{t-1}-1\right)}{\pi_t^*}\frac{rer_t B_{t-1}^{G*}}{p_t^Y Y_t} + \frac{\Pi_t^\tau - \gamma^D\breve{\Pi}_t^\tau}{p_t^Y Y_t} + \chi^{Co}\frac{CF_t^{Co} - \gamma^D \breve{CF}_t^{Co}}{p_t^Y Y_t} - \bar{s}_B \tag{53}$$

Where $\breve{\Pi}_t^\tau = \Pi_t^\tau - \widetilde{\Pi}_t^\tau$ and $\breve{CF}_t^{Co} = CF_t^{Co} - \widetilde{CF}_t^{Co}$ are respectively the cyclical fluctuation of the tax base and the commodity sector's cash flows.

2.4.4 Spending components

The individual expenditure components are assumed to be time-varying fractions of total desired expenditure, where α_{CG}, α_{IG}, and $1-\alpha_{CG}-\alpha_{IG}$ denote respectively the long term shares for consumption, investment and transfers[15]. The terms ξ_t^{CG}, ξ_t^{IG} and ξ_t^{TR} are shocks meant to capture deviations from such shares. Consumption and transfers are given by

$$p_t^{CG}C_t^G = \alpha_{CG}\widetilde{G}_t\xi_t^{CG}$$
$$TR_t^G = (1-\alpha_{CG}-\alpha_{IG}-\alpha_{UFA})\widetilde{G}_t\xi_t^{TR} + TR_t^{UFA}$$

TR_t^{UFA} are transfers from the government to the unemployement funds administrator (see below). In the long run, these transfers amount to a share α_{UFA} of the total spending. The remainder transfers are received by non-Ricardian and Ricardian households with shares given by ω^G and $1-\omega^G$. The transfers that each type of household receive from the government therefore satisfy $\omega\, TR_t^{NR} = \omega^G\left(TR_t^G - TR_t^{UFA}\right)$ and $(1-\omega)\,TR_t^R = \left(1-\omega^G\right)\left(TR_t^G - TR_t^{UFA}\right)$.

Regarding the government investment, we follow Kydland and Prescott (1982) and Leeper et al. (2010), where the stock of public capital accumulates according to the following law of motion:

$$K_t^G = (1-\delta_G)K_{t-1}^G + I_{t-N^G+1}^{AG}, \qquad \delta_G \in (0,1], \tag{54}$$

Where $I_{t-N^G+1}^{AG}$ are government investment projects authorized in period $t-N^G+1$, and N^G denotes the number of periods it takes until these projects become productive and augment the public capital stock. The production of government investment goods is done by a representative firm as described in section (2.3.1), and bought by the government at price p_t^{IG}. The investment goods acquired by the government are then distributed between new and old authorized projects according to

$$I_t^G = \sum_{j=0}^{N^G-1}\varphi_j I_{t-j}^{AG}, \tag{55}$$

Where φ_j – the fraction of projects authorized in period $t-j$ that is outlaid in period t– satisfy $\sum_{j=0}^{N^G-1}\varphi_j = 1$ and $\varphi_j = \rho^\varphi\varphi_{j-1}$.

The budget for government spending in public goods is set in terms of authorizations of new projects, taking the expenditure on previously authorized projects as given:

$$p_t^{IG}I_t^{AG} = \alpha_{IG}E_t\left[\sum_{j=0}^{N^G-1}\varphi_j\widetilde{G}_{t+j}\right]\xi_t^{IG} \tag{56}$$

By assumption, firms and consumers buy oil through the government, that sets the domestic price according to

[15] As the oil price stabilization policy is neutral to the the long term budget , its share its 0

the following rule

$$P_t^O = \left(\left(P_t p^O\right)^{1-\alpha_O}\left(P_t p_{t-1}^O\right)^{\alpha_O}\right)^{\rho_O}\left(P_t^* p_t^{O^*} S_t\right)^{1-\rho_O}\xi_t^O \quad \Leftrightarrow \quad p_t^O = \left(\left(p^O\right)^{1-\alpha_O}\left(p_{t-1}^O\right)^{\alpha_O}\right)^{\rho_O}\left(rer_t p_t^{O^*}\right)^{1-\rho_O}\xi_t^O$$

Where α_O andρ_O are smoothing parameter, ξ_t^O is an exogenous process with unconditional mean equal to one that captures deviations of the domestic oil price from the rule, and $p_t^O = P_t^O/P_t$ and $p_t^{O^*} = P_t^{O^*}/P_t^*$ are the domestic and foreign real prices of oil. The engagement in price stabilization imposes a quarterly net government spending of $O_t\left(rer_t p_t^{O*} - p_t^O\right)$.

2.5 Monetary Policy

Monetary policy is carried out according to a Taylor rule of the form

$$R_t = (R_{t-1})^{\rho_R}\left[\overline{R}_t\left(\frac{\widetilde{\pi}_t}{\overline{\pi}_t}\right)^{\alpha_\pi}\left(\frac{Y_t/Y_{t-1}}{a_{t-1}}\right)^{\alpha_y}\right]^{1-\rho_R}\exp(\varepsilon_t^R), \qquad \rho_R \in (0,1), \qquad \alpha_\pi > 1, \qquad \alpha_y \geq 0, \qquad \bar{\pi} \geq 1,$$

Where ε_t^R is an AR(1) exogenous process that captures deviations from the rule, and $\widetilde{\pi}_t$ is the inflation rate monitored by the central bank, an average between the present and expected total and core inflation rates:

$$\widetilde{\pi}_t = \left[\left(\pi_t^Z\right)^{\alpha_{\pi Z}}\left(\pi_t\right)^{1-\alpha_{\pi Z}}\right]^{1-\alpha_{\pi E}}\left[\left(E_t\pi_{t+4}^Z\right)^{\alpha_{\pi Z}}\left(E_t\pi_{t+4}\right)^{1-\alpha_{\pi Z}}\right]^{\alpha_{\pi E}}$$

Where $\pi_t^Z = \frac{P_t^Z}{P_{t-1}^Z}\left(\frac{1+\tau_t^C}{1+\tau_{t-1}^C}\right)$ is the after tax core inflation rate.

Following Del Negro et al. (2015), the inflation targeted by the bank and the corresponding target nominal interest rate ($\overline{\pi}_t$ and $\overline{R}_t$) can potentially depart in the short run from their steady state values:

$$\log\left(\overline{\pi}_t/\overline{\pi}\right) = \rho_{\overline{\pi}}\log\left(\overline{\pi}_{t-1}/\overline{\pi}\right) + \varepsilon_t^{\overline{\pi}}$$

$$\log\left(\overline{R}_t/\overline{R}\right) = \rho_{\overline{\pi}}\log\left(\overline{R}_{t-1}/\overline{R}\right) + \varepsilon_t^{\overline{\pi}}$$

Where $\varepsilon_t^{\overline{\pi}}$ is a normally distributed i.i.d shock with zero mean and standard deviation equal to $\sigma_{\overline{\pi}}$ that affect the central bank's targets for the inflation level and the nominal interest rate.

2.6 Unemployment Funds Administrator

The unemployment funds administrator manages a domestic currency denominated fund. The UFA collects resources from employed households and pays to households currently unemployed. Besides, a lump sum transfer of TR_t^{UFA} is received each period from the government[16]. The UFA budget constraint is then given by

$$B_t^{UFA} - B_{t-1}^{UFA} = \tau_t^{UFA} W_t h_t n_t - (1-n_t)\,UB_t + (r_t - 1)\,B_{t-1}^{UFA} + TR_t^{UFA}$$

2.7 The Rest of the World

Foreign agents demand home composite goods and buy the domestic commodity production. There are no transaction costs or other barriers to trade. The structure of the foreign economy is identical to the domestic economy, but the

[16]Lump sum transfers are given by $TR_t^{UFA} = A_{t-1}\overline{tr}^{UFA} + \epsilon^{UFA}\left(A_{t-1}b^{UFA} - B_t^{UFA}\right)$, where ϵ^{UFA} is a small constant that prevents indeterminacy of the steady state UFA net asset position, and $\overline{tr}^{UFA}$ is a constant transfer from the government, calibrated to attain a long run balanced operational budget: $\overline{tr}^{UFA} = (1-n)\,ub - \tau^{UFA} whn$.

domestic economy is assumed to be small relative to the foreign economy. We also have the relation

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^S \pi_t^*}{\pi_t},$$

Where $\pi_t^S = S_t/S_{t-1}$. Further, foreign demand for the exportable composite good X_t^{H*} is given by the reduced form schedule

$$X_t^{H*} = \left[a_{t-1}X_{t-1}^{H*}\right]^{\rho_{XH*}} \left[o^* \left(\frac{P_t^{H*}}{P_t^*}\right)^{-\eta^*} Y_t^*\right]^{1-\rho_{XH*}} \xi_t^{XH*}, \qquad o^* \in (0,1), \qquad \eta^* > 0,$$

Where ρ_{XH*} reflects some habit persistence in the foreign demand for domestic goods, ξ_t^{XH*} is an i.i.d. shock with mean one, and Y_t^* denotes foreign aggregate demand or GDP and is given by:

$$Y_t^* = A_t z_t^* \tag{57}$$

Where z_t^* follows an AR(1) process. Similar to Schmitt-Grohé and Uribe (2012), A_t^J for $J \in \{H, Co\}$ (with $a_t^J \equiv A_t^J/A_{t-1}^J$) cointegrate with the global productivity A_t according to:

$$A_t^J = \left(aA_{t-1}^J\right)^{1-\Gamma^J} (A_t)^{\Gamma^J},$$

Where the parameters $\Gamma^J \in [0,1]$ govern the speed of convergence towards the global productivity level. Defining $\nabla_t^J \equiv A_t^J/A_t$ as the wedges with respect to the global productivity levels, we obtain the following relation:

$$\nabla_t^J = \left(\frac{a}{a_t}\nabla_{t-1}^J\right)^{1-\Gamma^J}.$$

$$\frac{A_t^J}{A_{t-1}^J} \equiv a_t^J = \frac{\nabla_t^J}{\nabla_{t-1}^J}a_t$$

This setup nests the common specification with $A_t^H = A_t^{Co} = A_t$ when $\Gamma^H = \Gamma^{Co} = 1$.

Furthermore, as in García-Schmidt and García-Cicco (2017), the international price levels for exported commodities, oil, the home import basket, and the foreign CPI are allowed to co-integrate with a common trend F_t^* according to

$$P_t^J = \left(\pi^* P_{t-1}^J\right)^{1-\Gamma^J} (F_t^*)^{\Gamma^J} \xi_t^J,$$

for $J \in \{Co*, M*, O*, *\}$, where P_t^J (with $\pi_t^J \equiv P_t^J/P_{t-1}^J$) are the respective non-stationary price levels, the parameters $\Gamma^J \in (0,1]$ govern the speed of convergence towards the common trend, and the ξ_t^J are exogenous processes. The growth rate of the common trend $\pi_t^{F*} \equiv F_t^*/F_{t-1}^*$ also evolves exogenously. Defining $f_t^* \equiv F_t^*/P_t^*$ and the relative prices with respect to the foreign CPI $p_t^J \equiv P_t^J/P_t^*$ (such that $p_t^* = 1$), we can re-write the above equations in terms of stationary variables:

$$p_t^J = \left(\frac{\pi^*}{\pi_t^*}p_{t-1}^J\right)^{1-\Gamma^J} (f_t^*)^{\Gamma^J} \xi_t^J,$$

with $\pi_t^{F*} = \left(f_t^*/f_{t-1}^*\right)\pi_t^*$. This setup nests the common specification where $F_t^* = P_t^*$ and where p_t^{Co*}, p_t^{M*}, p_t^{O*} and π_t^* evolve as independent exogenous processes when $\Gamma^{Co*} = \Gamma^{M*} = \Gamma^{O*} = \Gamma^* = \xi^* = 1$ and $var(\xi_t^*) = 0$.

2.8 Aggregation and Market Clearing

2.8.1 Aggregation across households

Aggregate variables add up the per-capita amounts from non-Ricardian and Ricardian households considering their respective demographic mass ω and $1-\omega$:

$$C_t = \omega C_t^{NR} + (1-\omega)\, C_t^R \tag{58}$$

$$TR_t^G - TR_t^{UFA} = \omega\, TR_t^{NR} + (1-\omega)\, TR_t^R \tag{59}$$

$$T_t = \omega T_t^{NR} + (1-\omega)\, T_t^R \tag{60}$$

$$K_t = (1-\omega)\, K_t^R \tag{61}$$

$$K_t^S = (1-\omega)\, K_t^{S,R} \tag{62}$$

$$I_t = (1-\omega)\, I_t^R \tag{63}$$

$$B_t^{Pr} = (1-\omega)\, B_t^R \tag{64}$$

$$B_t^{Pr*} = (1-\omega)\, B_t^{R*} \tag{65}$$

$$D_t = (1-\omega)\, D_t^R \tag{66}$$

2.8.2 Goods market clearing

Defining labor costs in terms of home goods $X_t^L \equiv H_t^C n_t + \Omega_t v_t$, the market clearing conditions for the different composite goods varieties are

$$Y_t^H = X_t^H = X_t^{Z,H} + X_t^{A,H} + X_t^{CG,H} + X_t^{I,H} + X_t^{ICo,H} + X_t^{IG,H} + X_t^L \tag{67}$$

$$Y_t^F = X_t^F = X_t^{Z,F} + X_t^{A,F} + X_t^{CG,F} + X_t^{I,F} + X_t^{ICo,F} + X_t^{IG,F} \tag{68}$$

$$Y_t^{H*} = X_t^{H*} \tag{69}$$

And for the corresponding varieties:

$$Y_t^H(j) = X_t^H(j), \qquad Y_t^F(j) = X_t^F(j), \qquad Y_t^{H*}(j) = X_t^{H*}(j).$$

The market clearing condition for oil is

$$O_t = C_t^O + X_t^O. \tag{70}$$

Aggregate output of home wholesale goods is then given by

$$\begin{aligned} Y_t^{\tilde{H}} &= X_t^{\tilde{H}} + X_t^{\tilde{H}*} = \int_0^1 Y_t^H(j)dj + \int_0^1 Y_t^{H*}(j)dj = \int_0^1 X_t^H(j)dj + \int_0^1 X_t^{H*}(j)dj \\ &= Y_t^H \int_0^1 \left(\frac{P_t^H(j)}{P_t^H}\right)^{-\epsilon_H} dj + Y_t^{H*} \int_0^1 \left(\frac{P_t^{H*}(j)}{P_t^{H*}}\right)^{-\epsilon_{H*}} dj = Y_t^H \Delta_t^H + Y_t^{H*} \Delta_t^{H*}, \end{aligned}$$

while for imported goods we have

$$M_t = \int_0^1 Y_t^F(j)dj = \int_0^1 X_t^F(j)dj = Y_t^F \int_0^1 \left(\frac{P_t^F(j)}{P_t^F}\right)^{-\epsilon_F} dj = Y_t^F \Delta_t^F,$$

where Δ_t^H, Δ_t^{H*} and Δ_t^F are price dispersion terms satisfying

$$\begin{aligned}
\Delta_t^H &= \int_0^1 \left(\frac{P_t^H(j)}{P_t^H}\right)^{-\epsilon_H} dj = (1-\theta_H)\left(\tilde{p}_t^H\right)^{-\epsilon_H} + \theta_H \left(\frac{p_{t-1}^H}{p_t^H}\frac{g_{t-1}^{\Gamma^H}}{\pi_t}\frac{\left(1+\tau_t^C\right)}{\left(1+\tau_{t-1}^C\right)}\right)^{-\epsilon_H} \Delta_{t-1}^H, \\
\Delta_t^{H*} &= \int_0^1 \left(\frac{P_t^{H*}(j)}{P_t^{H*}}\right)^{-\epsilon_{H*}} dj = (1-\theta_{H*})\left(\tilde{p}_t^{H*}\right)^{-\epsilon_{H*}} + \theta_{H*} \left(\frac{p_{t-1}^{H*}}{p_t^{H*}}\frac{g_{t-1}^{\Gamma^{H*}}}{\pi_t^*}\right)^{-\epsilon_{H*}} \Delta_{t-1}^{H*}, \\
\Delta_t^F &= \int_0^1 \left(\frac{P_t^F(j)}{P_t^F}\right)^{-\epsilon_F} dj = (1-\theta_F)\left(\tilde{p}_t^F\right)^{-\epsilon_F} + \theta_F \left(\frac{p_{t-1}^F}{p_t^F}\frac{g_{t-1}^{\Gamma^F}}{\pi_t}\frac{\left(1+\tau_t^C\right)}{\left(1+\tau_{t-1}^C\right)}\right)^{-\epsilon_F} \Delta_{t-1}^F.
\end{aligned}$$

2.8.3 Aggregate demand

Aggregate demand or GDP is defined as the sum of domestic absorption$\left(Y_t^C\right)$ and the trade balance(TB_t) . In units of the final consumption good, those components are given by

$$Y_t^C = C_t + p_t^{CG}C_t^G + p_t^I I_t^f + p_t^{ICo} I_t^{Co,f} + p_t^{IG} I_t^G + p_t^H X_t^L, \tag{71}$$

$$TB_t = rer_t \left(p_t^{H*}Y_t^{H*} + p_t^{Co*}Y_t^{Co} - p_t^{M*}M_t - p^{O*}O_t\right). \tag{72}$$

We define real GDP as follows:

$$Y_t \equiv C_t + C_t^G + I_t^f + I_t^{Co,f} + I_t^G + X_t^L + Y_t^{H*} + Y_t^{Co} - M_t - O_t.$$

Then, the GDP deflator (p_t^Y, expressed as a relative price in terms of the final consumption good) is implicitly defined as

$$p_t^Y Y_t = Y_t^C + TB_t. \tag{73}$$

2.8.4 Aggregate profits

Aggregate profits of the household owned firms(Π_t^{HHOF}), in units of final goods, are given by

$$\begin{aligned}
\Pi_t^{HHOF} &= \underbrace{Y_t^C - p_t^H X_t^H - p_t^F X_t^F - p_t^O C_t^O}_{\Pi_t^C+\Pi_t^Z+\Pi_t^A+\Pi_t^{CG}+\Pi_t^I+\Pi_t^{IG}+\Pi_t^{ICo}} \\
&\quad \underbrace{+p_t^H Y_t^H - \int_0^1 X_t^H(j)\left[p_t^H p_t^H(j)\right] dj}_{\Pi_t^H} + \underbrace{\int_0^1 Y_t^H(j)\left[p_t^H p_t^H(j) - p_t^{\widetilde{H}}\right] dj}_{\int_0^1 \Pi_t^H(j)dj} \\
&\quad \underbrace{+p_t^F Y_t^F - \int_0^1 X_t^F(j)\left[p_t^F p_t^F(j)\right] dj}_{\Pi_t^F} + \underbrace{\int_0^1 Y_t^F(j)\left[p_t^F p_t^F(j) - rer_t p_t^{M*}\right] dj}_{\int_0^1 \Pi_t^F(j)dj} \\
&\quad \underbrace{+rer_t p_t^{H*}Y_t^{H*} - \int_0^1 rer_t X_t^{H*}(j)\left[p_t^{H*}p_t^{H*}(j)\right] dj}_{\Pi_t^{H*}} + \underbrace{\int_0^1 Y_t^{H*}(j)\left[rer_t p_t^{H*}p_t^{H*}(j) - p_t^{\widetilde{H}}\right] dj}_{\int_0^1 \Pi_t^{H*}(j)dj} \\
&\quad \underbrace{+p_t^{\widetilde{H}}Y_t^{\widetilde{H}} - r_t^K K_t^S - W_t h_t n_t - p_t^H X_t^L - p_t^O X_t^O}_{\Pi_t^{\widetilde{H}}} \\
&= Y_t^C + rer_t p_t^{H*}Y_t^{H*} - rer_t p_t^{M*}M_t - p_t^O O_t - r_t^K K_t^S - W_t h_t n_t - p_t^H X_t^L \\
&= p_t^Y Y_t - rer_t p_t^{Co*}Y_t^{Co} + \left(rer_t p_t^{O*} - p_t^O\right) O_t - r_t^K K_t^S - W_t h_t n_t - p_t^H X_t^L
\end{aligned} \tag{74}$$

At the end of each period, all profits are returned to the Ricardian households in the form of dividends: $D_t = \Pi_t^{HHOF}$.

2.8.5 Domestic bonds

Participating agents in the domestic bond market are the Ricardian households, the government, and the UFA. Their aggregate net holdings are in zero net supply:

$$B_t^{Pr} + B_t^G + B_t^{UFA} = 0$$

2.8.6 Foreign asset position

Summing over the Ricardian and non-Ricardian households budgets from (5) and (14) by using the aggregation conditions (58)-(66) yields the aggregate household budget:

$$\begin{aligned}(1+\tau_t^C)C_t + p_t^I I_t^f + B_t^{Pr} + rer_t B_t^{Pr*} + T_t = {} & rer_t (1-\omega) REN_t^{R*} + (1-\tau_t^W)W_t h_t n_t + (1-n_t)UB_t \\ & + K_{t-1}\left[r_t^K \bar{u}_t \left(1-\tau_t^K\right) + \tau_t^K p_t^I \left(\delta + \phi_{\bar{u}}\left(\bar{u}_t\right)\right)\right] \\ & + (1-\tau_t^D)D_t + \left(TR_t^G - TR_t^{UFA}\right) + r_t B_{t-1}^{Pr} + r_t^* rer_t B_{t-1}^{Pr*}\end{aligned} \tag{75}$$

Combined with the commodity profits, the government asset position, the UFA budget, and the private firms profits given by (47), (48) and (74), the current account, or the change in the net foreign asset position, can be expressed as the sum of the net asset transfers from the rest of the world due to interest on debt, trade of goods, and rents:

$$rer_t \left(B_t^* - \frac{B_{t-1}^*}{\pi_t^*}\right) = rer_t \frac{B_{t-1}^*}{\pi_t^*}\left(R_{t-1}^*\xi_{t-1} - 1\right) + TB_t + rer_t REN_t^*$$

Where $REN_t^* = (1-\omega) REN_t^{R*} - \left(1-\chi^{Co}\right)\frac{CF_t^{Co} - \tau_t^{Co}\left(\Pi_t^{Co}\right)}{rer_t}$ are the aggregate net rents denominated in foreign currency.

3 Parameterization strategy and estimation results

Most of the model parameters are calibrated and estimated, while other parameters are endogenously determined in steady state. The calibrated parameters include those characterizing exogenous processes for which we have a data counterpart, those that are drawn from related studies for Chile or other countries, and those that are chosen to match sample averages or long-run ratios for the Chilean economy. The estimated parameters are obtained by means of Bayesian techniques as discussed below. We now describe the details of data sources and values of the calibrated and estimated parameters.

3.1 Calibrated parameters

Table 1 presents the values of those parameters that are either chosen to match observed statistics and available evidence for Chile, or following related studies for other countries. The parameters σ, ω, δ, α, o_{CG}, o_O, ϵ_H, ϵ_F and ϵ_{H*} are set as in Medina and Soto (2007). We assume capital depreciates at the same rate in all three sectors ($\delta_G = \delta_{Co} = \delta$). Mining sector parameters α^{Co}, N^{Co}, φ_j^{Co}, and o_{Co} are set as in Fornero et al. (2014). Fiscal parameters $o_{\widehat{C}}$ and o_{KG} are set as in Coenen et al. (2013) to equalize the marginal utility between public and households investment and consumption levels. We assume the government spending does not react to cyclical fluctuations, then $\gamma^D = 1$. As in Medina and Soto (2007), the government debt is financed totally in foreign currency ($\alpha^D = 1$). The shares of domestic goods in core consumption, o_Z, and investment in the domestic goods sector, o_I, are set to 21% and 33%, respectively, based on input-output tables from national accounts between 2008 and 2016. We assume complete home bias in government consumption and investment($o_{CG} = o_I = 0$). The share of agricultural goods and energy consumption in the household's consumption basket, κ_A and κ_O, are chosen to match their weight in the Chilean CPI. For simplicity we assume that the shares ω^U, ω^G are equal to the relative size of Non-Ricardian households, calibrated at 50% ($\omega = 0.5$). Finally, we assume one period time-to-build for public capital, then N^G is set to one.

Table 1: Calibrated deep parameters.

Parameter	Description	Value	Source
σ	Inverse elasticity of intertemporal substitution	1	Medina and Soto (2007)
ω	Share of non-Ricardian households	0.5	Medina and Soto (2007)
ω^U,ω^G	Weights of non-Ricardians in union and government's decisions	0.5	Equal to ω
δ	Quarterly depreciation rate	0.01	Medina and Soto (2007)
δ_G,δ_{Co}	Quarterly depreciation rate of public and comm. capital	0.01	Equal to δ
α	Share of labor in wholesale domestic goods	0.33	Medina and Soto (2007)
γ^D	Gov. spending reaction to cyclical fluctuations	1	Normalization
α^D	Share of gov. debt in foreign currency	1	Normalization
N^G	Time to build (quarters), public sector	1	Normalization
N^{Co}	Time to build (quarters), commodity sector	6	Fornero et al. (2014)
φ_j^{Co}	Financing profile of comm. invest. projects	1/6	Fornero et al. (2014)
o_Z	Share of foreign goods in core consumption	0.21	Average (2008-2016)
o_I	Share of foreign goods in investment	0.33	Average (2008-2016)
o_{CG}	Share of foreign goods in gov. consumption	0	Medina and Soto (2007)
o_{IG}	Share of foreign goods in gov. investment	0	Coenen et al. (2013)
o_{Co}	Share of foreign goods in comm. investment	0.43	Fornero et al. (2014)
o_A	Share of foreign goods in agricultural goods	0.21	Equal to o_{XC}
o_O	Share of oil in home good production	0.01	Medina and Soto (2007)
κ_A	Share of agricultural goods in consumption	0.19	CPI: 2013 weighting factor
κ_O	Share of energy goods in consumption	0.19	CPI: 2013 weighting factor
$o_{\widehat{C}}$	Share of gov. consumption in $\tilde{c}_t$	0.33	Coenen et al. (2013)
o_{KG}	Share of public capital in $\tilde{k}_t$	0.16	Coenen et al. (2013)
ϵ_H	Elast. of substitution among domestic varieties	11	Medina and Soto (2007)
ϵ_F	Elast. of substitution among imported varieties	11	Medina and Soto (2007)
ϵ_{H*}	Elast. of substitution among exported varieties	11	Medina and Soto (2007)

Table 2 shows the parameters that are chosen to match some long-run trend data in the Chilean economy. We assume a steady state labor productivity growth rate, a, of 1.5% on an annual basis.[17] The steady state inflation target, π, is set to the central bank's CPI inflation target of 3% since 2001. The household's subjective discount factor, β, is equal to 0.99997 to match a steady state real interest rate of 2%, in line with existing estimates of the neutral real interest rate for Chile. The steady state nominal exchange rate is assumed constant, in line with CLP/USD dynamics during the sample period. The average country premium (2001-2016), ξ^*, is set to 150 basis points. The steady state values of p^H, p^O and p^{Co} are normalized to one, while h is set to 0.3 as its common in the literature.The share of physical capital to quarterly output in the commodity sector, $s^{k^{Co}}$, is set to 8 as in Fornero et al. (2014). The government share in the commodity sector, χ, is set to 0.33, consistent with the average share of production of the state-owned copper mining company (Codelco) relative to total copper production since 2001 to 2016. The fiscal-deficit-to-GDP ratio, s^{def} is set to 0, consistent with a structural balance of fiscal accounts.The unemployement rate(u), country premium(ξ^*) and steady state shares with respect to nominal GDP for trade balance(s^{tb}), government spending in invenstment(s^{iG}), consumption(s^{cG}) and transfers(s^{trG}), and commodity

[17]This is consistent with an observed long run GDP growth of approximately 3.3% and labor force growth growth of around 1.8% for Chile.

investment(s^{iCo}) and output(s^{Co}) are set to match the corresponding data averages between 2001 and 2016. The current account-to-GDP ratio, s^{CA}, is set to -0.5% in order to match a steady state net foreign asset position of 14% of annual GDP, the average between 2003-2016. The tax rate on consumption, τ^{C}, is set to 19%.[18] The tax rates on capital and dividends, τ^{K} and τ^{D}, are set to 20%.[19] The tax rate on wages, τ^{W}, is set to 7%.[20] Finally, the tax rate on the foreign mining profits, τ^{Co}, is set to 35% as in Fornero et al. (2014).

Table 2: Targeted steady state values.

Parameter	Description	Value
a	Annual balanced growth path	1.015
π	Annual inflation target	1.03
u	Unemployment rate	0.08
e	Quarterly vacancy filling probability	0.8
ρ	Quarterly job destruction probability	0.04
β	Quarterly subjective discount factor	0.999
π^{S}	Nominal exchange rate depreciation	0
ξ^{*}	Country premium, annual base	1.015
p^{H}	Home good price	1
p^{O}	Domestic oil price	1
p^{Co}	Domestic commodity price	1
h	Hours per worker	0.3
χ	Government share in mining sector	0.33
s^{def}	Fiscal deficit to GDP	0
s^{tb}	Trade balance to GDP	0.03
s^{CA}	Current account to GDP	-0.013
s^{iG}	Government investment to GDP	0.022
s^{tr}	Government transfers to GDP	0.06
s^{cG}	Government consumption to GDP	0.11
s^{Co}	Commodity GDP to GDP	0.14
s^{iCo}	Commodity investment to GDP	0.04
s^{Hc}	Workers administrative costs to GDP	0.001
s^{ρ_x}	Exogenous separations to total	0.66
s^{OC}	Oil to total consumption	0.04
$s^{k^{Co},y^{Co}}$	Mining capital to Mining GDP	12
τ^{C}	Tax rate on consumption	0.19
τ^{K}, τ^{D}	Tax rate on capital and dividends	0.20
τ^{W}	Tax rate on wages	0.07
τ^{Co}	Tax rate on foreign foreign profits	0.35

3.2 Estimation, prior distributions and posterior estimates

The model is solved by a linear approximation around the non-stochastic steady state. The model is then estimated by Bayesian methods, as described in An and Schorfheide (2007). We now briefly describe the estimation strategy before discussing the details of the data, prior distributions, and posterior results.

The whole set of the linearized equations of the model forms a linear rational expectation system whose solution can be expressed as

$$x_t = A(\theta)x_{t-1} + B(\theta)\varepsilon_t, \qquad \varepsilon_t \sim N(0, \Sigma_\varepsilon), \tag{76}$$

[18] See http://www.oecd.org/tax/tax-policy/Table-4.1-VAT-GST-Rates-June-2014.xlsx.
[19] See http://www.oecd.org/tax/tax-policy/tax-database.htm#C_CorporateCaptial.
[20] See http://www.oecd.org/ctp/tax-policy/taxing-wages-tax-burden-trends-latest-year.htm.

where x_t contains the model's variables, θ collects the structural parameters of the model to be estimated, and ε_t contains white noise innovations to the exogenous shocks of the model. The state matrix A and the input matrix B are non-linear functions of θ. Equation (76) is called the transition equation. Let x_t^{obs} be a vector of several time series to estimate the model, which are referred to as observable variables[21]. This vector is related to the model's variables through as

$$x_t^{obs} = Hx_t + u_t, \qquad u_t \sim N(0, \Sigma_u), \tag{77}$$

where H is called the output matrix that selects elements from x_t, and u_t are measurement errors which are included to avoid stochastic singularity. Equation (77) is called the measurement equation. Note that the state equation (76) and observation equation (77) constitute a linear state-space representation for the dynamic of x_t^{obs}. Let $P(\theta)$ a prior density on the structural parameters and $L(Y^T|\theta)$ the conditional likelihood function for the observed data $Y^T = [x_1^{obs}, \ldots, x_T^{obs}]'$. Using Bayes theorem, the joint posterior distribution, $P(\theta|Y^T)$, is defined as

$$P(\theta|Y^T) = \frac{L(Y^T|\theta) \times P(\theta)}{\int L(Y^T|\theta) \times P(\theta)d\theta}. \tag{78}$$

We use central tendency measures (in particular the mean) of the distribution function given by (78) as estimates of θ. We now describe the details of Y^T and $P(\theta)$.

Our data Y^T consists of 23 macroeconomic variables covering the period between 2001Q3 and 2016Q2.[22] We choose the following Chilean and foreign quarterly data: mining and non mining GDP, an indicator of export-weighted real GDP of Chile's main trading partners as a proxy for foreign aggregate demand , private and government consumption, aggregate, government and commodity related investment , government transfers, total employment as a fraction of the labor force, total hours per employee, nominal wages, core, food , and energy components of CPI, an external price index, price of West Texas Intermediate crude oil in dollars per barrel, London Metal Exchange price of refined copper in dollars per metric pound,the imports deflator, short-term central bank target rate, London Interbank Offered Rate as a proxy for the foreign interest rate , real exchange rate (rer_t), J.P. Morgan Emerging Market Bond Index Global (EMBIG) spread for Chile as a proxy for the country premium, and the trade balance to total GDP (tb_t/y_t). With the exception of the interest rates the risk spread, the real exchange rate and trade balance, all variables are log-differentiated with respect to the previous quarter. All variables are demeaned. Our estimation strategy also includes i.i.d. measurement errors for all local observables with the exception of the interest rate. Additionally we incorporate observed news shocks on the tax reform of 2014[23]. The variance of the measurement errors is calibrated to be 10% of the variance of the corresponding observables.

The posterior estimates are obtained from a random walk Metropolis–Hasting chain with 125,000 draws after a burn-in of 125,000 draws. As in Christiano et al. (2011), we scale the parameters, in particular the shocks standard deviations, to have similar order of magnitude to facilitate optimization.

For the prior selection, we follow the endogenous prior strategy used in Christiano et al. (2011) and Coenen et al. (2013), where the joint prior distribution of the estimated parameters $P(\theta)$ is computed as the product of the initial prior distribution and the likelihood that the model generated standard deviations match the volatility of the observed variables. For the choice of the initial priors $P(\theta^{ini})$, we specify independent univariate prior distributions for $\theta^{ini} = \theta^S \cup \theta^{exo}$ with θ^S containing the structural parameters of the model and θ^{exo} the parameter governing the law of motion of the exogenous processes. The prior distributions are documented in column 3 of Tables 3 and 4. The types of the priors are chosen according to the support on which the individual parameters are defined, while

[21]Adequately transformed to map them to the model.

[22]The starting point of our sample is set at the time the Chilean Central Bank started conducting monetary policy by using a nominal reference interest rate.

[23]The reform was approved on September, 2014, and specified a time-table for a staggered rise of the corporate tax to be completed over 4 years.

the means and standard deviations of the priors are selected according to our beliefs on plausible regions for the parameters or were set to match the priors of related papers for the Chilean economy and international literature. These values are presented in columns 4 and 5 of Tables 3 and 4.

Table 3: Prior and posterior distributions. Structural parameters, θ^S.

Parameter	Description	Initial Prior			Posterior		
		distr.	mean	s.d.	mean	pct. 5	pct. 95
ς	Habit formation	B	0.75	0.1	0.63	0.53	0.73
ν	Preferences endogenous shifter	B	0.5	0.2	0.08	0.02	0.23
ϕ	Inverse Frisch elasticity	G	5.0	1.0	1.47	0.83	2.46
μ	Match elasticity parameter	B	0.85	0.1	0.91	0.86	0.96
$\sigma_{\widetilde{C}}$	Worker's adm. costs dispersion	IG	0.25	Inf	6.02	4.88	7.79
100ψ	Country premium debt elast.	IG	1.0	Inf	0.24	0.18	0.31
ρ_R	Taylor rule smoothing parameter	B	0.85	0.025	0.74	0.71	0.77
α_π	Taylor rule response to total inflation	N	1.7	0.1	1.95	1.81	2.09
α_Y	Taylor rule response to GDP growth	N	0.125	0.05	0.13	0.05	0.20
α_{π^z}	Taylor rule response to core inflation	B	0.75	0.2	0.74	0.57	0.88
η_Z	Elast. of subst. H/F in core cons.	G	1.5	0.25	2.25	1.74	2.79
η^*	Elasticity of foreign demand	G	0.25	0.05	0.32	0.23	0.41
η_A	Elast. of subst. H/F in agriculture	G	1.0	0.25	1.18	0.74	1.68
η_O	Elast. of subst. Z,O in H prod.	G	0.5	0.25	0.56	0.18	1.09
η_C	Elast. of subst. CZ,O,A goods	G	1.0	0.25	0.81	0.51	1.15
η_I	Elast. of subst. H/F in investment	G	1.0	0.25	1.50	0.94	2.13
η_{Co}	Elast. of subst. H/F in comm. inv.	G	1.0	0.25	1.10	0.68	1.59
η_{KG}	Elast. of subst. priv. and pub. capital	G	1.0	0.25	0.85	0.32	1.66
$\eta_{\hat{C}}$	Elast. of subst. priv. and pub. cons.	G	1.0	0.25	2.30	1.35	3.54
ρ_O	Oil price smoothing param. 1	B	0.5	0.2	0.74	0.71	0.78
α_O	Oil price smoothing param. 2	B	0.5	0.2	0.41	0.26	0.57
ϑ_W	Indexation wages	B	0.25	0.1	0.19	0.08	0.33
$\varkappa_W$	Wage Smoothing	B	0.75	0.025	0.84	0.79	0.90
θ_H	Calvo probability domestic prices	B	0.75	0.025	0.81	0.79	0.84
ϑ_H	Indexation domestic prices	B	0.25	0.1	0.36	0.23	0.49
θ_F	Calvo probability import prices	B	0.75	0.075	0.77	0.73	0.80
ϑ_F	Indexation import prices	B	0.25	0.1	0.18	0.07	0.32
θ_{H*}	Calvo probability export prices	B	0.75	0.075	0.72	0.64	0.81
ϑ_{H*}	Indexation export prices	B	0.25	0.1	0.25	0.10	0.41
$\Phi_{\bar{u}}$	Capital utilization cost, non-mining	G	1.5	0.25	1.08	0.74	1.45
$\Phi_{\bar{u}}^{Co}$	Capital utilization cost, mining	G	1.5	0.25	2.77	2.37	3.19
Φ_I	Inv. adjustment cost elast.	G	5.0	1 .0	3.05	1.86	4.36
Φ_{ICo}	Inv. adjustment cost elast., mining	G	0.5	0.25	0.47	0.25	0.75
Γ^{Co}	Global pass through, mining prod.	B	0.5	0.2	0.49	0.15	0.84
Γ^H	Global pass through, home prod.	B	0.5	0.2	0.56	0.25	0.84
Γ^{Co*}	Global pass through, copper price.	B	0.5	0.2	0.59	0.34	0.79
Γ^{O*}	Global pass through, oil price.	B	0.5	0.2	0.77	0.53	0.93
Γ^*	Global pass through, foreign prices.	B	0.5	0.2	0.75	0.64	0.87
Γ^{M*}	Global pass through, imports prices.	B	0.5	0.2	0.37	0.28	0.46

Note: The prior distributions are: beta distribution (B) on the open interval $(0, 1)$, inverse gamma distribution (IG) on $\mathbb{R}^+$, gamma distribution (G) on $\mathbb{R}_0^+$, normal distribution (N) on $\mathbb{R}$.

Table 4: Prior and posterior distributions. Exogenous processes parameters, θ^{exo}.

Parameter	Description	Initial Prior			Posterior		
		distribution	mean	s.d.	mean	pct. 5	pct. 95
AR(1) coefficient							
ρ_{ϱ}	Preference shock	B	0.5	0.2	0.83	0.73	0.91
ρ_{ϖ}	Inv. tech. shock, non-mining	B	0.75	0.075	0.44	0.34	0.53
$\rho_{\varpi^{Co}}$	Inv. tech. shock, mining	B	0.5	0.2	0.10	0.02	0.22
ρ_{z}	Transitory tech. shock, non-mining	B	0.85	0.075	0.77	0.62	0.90
ρ_{zCo}	Transitory tech. shock, mining	B	0.85	0.075	0.78	0.64	0.89
ρ_{z*}	Transitory tech. shock, foreign	B	0.85	0.075	0.89	0.82	0.96
ρ_{z^A}	Transitory tech. shock, agriculture	B	0.75	0.075	0.90	0.86	0.95
ρ_{a}	Global unit root tech. shock	B	0.3	0.075	0.29	0.16	0.43
ρ_{ζ^o}	Obs. country premium shock	B	0.75	0.075	0.74	0.66	0.80
ρ_{ζ^u}	Unobs. country premium shock	B	0.75	0.075	0.67	0.58	0.75
$\rho_{\xi^{CG}}$	Public consumption shock	B	0.75	0.075	0.79	0.68	0.88
$\rho_{\xi^{TR}}$	Public transfer shock	B	0.5	0.2	0.80	0.71	0.89
$\rho_{\xi^{IG}}$	Public investment shock	B	0.5	0.2	0.19	0.05	0.37
ρ_{κ}	Labor supply shock	B	0.5	0.2	0.59	0.27	0.91
ρ_{ρ_x}	Job separation shock	B	0.5	0.2	0.71	0.61	0.82
$\rho_{\xi^{Po}}$	Domestic oil price shock	B	0.75	0.075	0.69	0.55	0.82
ρ_{e^R}	Monetary policy shock	B	0.5	0.2	0.27	0.15	0.39
ρ_{π^f}	Price global factor shock	B	0.5	0.2	0.17	0.07	0.28
$\rho_{p^{Co}}$	Copper price shock	B	0.5	0.2	0.70	0.51	0.80
ρ_{p^O}	Oil price shock	B	0.5	0.2	0.77	0.56	0.90
$\rho_{p^{M*}}$	Imports price shock	B	0.5	0.2	0.83	0.75	0.89
ρ_{p^*}	Foreign economy price shock	B	0.5	0.2	0.51	0.18	0.83
ρ_{R^*}	Foreign interest rate shock	B	0.5	0.2	0.89	0.86	0.91
Innovation s.d.							
$100\sigma_{\varrho}$	Preference shock	IG	0.5	Inf	2.39	1.92	2.94
$100\sigma_{\varpi}$	Inv. tech. shock, non-mining	IG	0.5	Inf	8.60	4.95	12.92
$100\sigma_{\varpi^{Co}}$	Inv. tech. shock, mining	IG	0.5	Inf	10.59	5.59	17.07
$100\sigma_{z}$	Transitory tech. shock, non-mining	IG	0.5	Inf	0.61	0.52	0.71
$100\sigma_{zCo}$	Transitory tech. shock, mining	IG	0.5	Inf	2.32	2.07	2.57
$100\sigma_{z*}$	Transitory tech. shock, foreign	IG	0.5	Inf	0.52	0.45	0.58
$100\sigma_{z^A}$	Transitory tech. shock, agriculture	IG	0.5	Inf	1.20	1.02	1.39
$100\sigma_{a}$	Global unit root tech. shock	IG	0.5	Inf	0.25	0.16	0.34
$100\sigma_{\zeta^o}$	Obs. country premium shock	IG	0.5	Inf	0.08	0.07	0.09
$100\sigma_{\zeta^u}$	Unobs. country premium shock	IG	0.5	Inf	0.77	0.53	1.03
$100\sigma_{\xi^{CG}}$	Public consumption shock	IG	0.5	Inf	1.18	1.07	1.30
$100\sigma_{\xi^{TR}}$	Public transfer shock	IG	0.5	Inf	3.15	2.71	3.58
$100\sigma_{\xi^{IG}}$	Public investment shock	IG	0.5	Inf	10.18	9.07	11.29
$100\sigma_{\kappa}$	Labor supply shock	IG	0.5	Inf	2.74	1.55	4.88
$100\sigma_{\rho_x}$	Job separation shock	IG	0.5	Inf	0.27	0.24	0.30
$100\sigma_{\xi^{Po}}$	Domestic oil price shock	IG	0.5	Inf	2.59	2.11	3.10
$100\sigma_{e^R}$	Monetary policy shock	IG	0.5	Inf	0.14	0.12	0.16
$100\sigma_{\pi^f}$	Price global factor shock	IG	0.5	Inf	3.50	2.98	4.08
$100\sigma_{p^{Co}}$	Copper price shock	IG	0.5	Inf	11.79	10.49	13.21
$100\sigma_{p^O}$	Oil price shock	IG	0.5	Inf	14.95	13.55	16.40
$100\sigma_{p^{M*}}$	Imports price shock	IG	0.5	Inf	1.49	1.14	1.88
$100\sigma_{p^*}$	Foreign economy price shock	IG	0.5	Inf	0.30	0.08	0.70
$100\sigma_{R^*}$	Foreign interest rate shock	IG	0.5	Inf	0.12	0.10	0.13

Note: The prior distributions are: beta distribution (B) on the open interval $(0, 1)$, inverse gamma distribution (IG) on $\mathbb{R}^+$, gamma distribution (G) on $\mathbb{R}_0^+$, normal distribution (N) on $\mathbb{R}$.

To obtain a view of the estimated model's ability to account for the data, Table (5) reports the standard deviations, the correlation with non-commodity GDP and the first-order autocorrelation coefficients of selected domestic variables implied by the posterior mean of the parameters, and compares these statistics with the corresponding empirical moments and the ones obtained from a specification analogous to Medina and Soto (2007) to which we will refer as MAS . The third to fourth columns of the table show that the model matches most variables well, in terms of variable volatility, with exceptions including real non-commodity GDP and private consumption. The best matches are achieved with the trade-balance-to-GDP ratio and the monetary policy rule. In terms of the business cycle correlations, the fifth and sixth columns of the table show that the model captures most of the direction's relationships, but weakly in the case of real wages growth. The seventh to eighth columns of the table show that the model matches most of the persistence of the variables, with the exception of investment growth. Overall, this goodness-of-fit analysis yields as a main conclusion that the model performs significantly well in terms of fitting second moments of the data.

Table 5: Second Moments.

		s.d. (%)			Corr. with $\Delta \log Y^{NCo}$			AC order 1		
	Description	data	XMAS	MAS	data	XMAS	MAS	data	XMAS	MAS
$\Delta \log Y$	GDP growth[c]	0.98	1.08	0.90	0.92	0.92	0.92	0.31	0.05	0.37
$\Delta \log Y^{NCo}$	Non-Mining GDP growth	1.07	1.11	0.92	1.00	1.00	1.00	0.29	0.09	0.46
$\Delta \log Y^{Co}$	Mining GDP growth	3.04	3.36	2.95	-0.05	0.05	0.03	-0.14	-0.02	-0.06
$\Delta \log C$	Private consumption growth	1.11	1.10	1.12	0.74	0.66	0.71	0.41	0.26	0.23
$\Delta \log C^G$	Gov. consumption growth	1.35	1.60	1.23	-0.12	0.17	0.27	-0.19	0.00	-0.10
$\Delta \log TR^G$	Gov. real transfers growth[a]	3.17	3.07	-	-0.08	0.13	-	-0.43	-0.07	-
$\Delta \log I$	Total investment growth	3.75	3.88	3.22	0.55	0.83	0.73	0.36	0.37	0.64
$\Delta \log I^G$	Gov. investment growth[a]	13.6	12.2	-	-0.18	0.59	-	-0.46	-0.40	-
$\Delta \log I^{Co}$	Mining investment growth[a]	8.80	8.34	-	0.22	0.12	-	0.42	0.78	-
TB/Y	Nom. trade balance/GDP	5.17	3.31	2.95	0.37	-0.04	-0.03	0.78	0.88	0.88
$\Delta \log N$	Employment growth[a]	0.42	0.37	-	0.46	0.42	-	0.25	0.37	-
$\Delta \log H$	Hours per employee growth[a]	1.35	1.82	-	0.34	0.84	-	-0.63	-0.06	-
$\Delta \log HN$	Total Hours growth[b]	1.37	1.95	1.62	0.47	0.86	0.79	-0.56	0.00	0.19
$\Delta \log WN$	Nominal wage growth	0.38	0.55	0.54	-0.11	0.12	0.15	0.52	0.66	0.73
$\Delta \log W$	Real wage growth[c]	0.63	0.45	0.81	-0.08	0.11	0.02	0.43	0.35	0.21
π	Headline inflation[c]	0.67	0.63	0.78	-0.19	0.03	0.08	0.59	0.66	0.31
π^Z	Core inflation	0.49	0.52	0.46	-0.19	0.03	0.16	0.59	0.80	0.70
π^A	Food inflation[a]	1.39	1.34	-	-0.24	-0.04	-	0.53	0.13	-
π^O	Energy inflation[a]	5.37	5.27	15.6	0.32	0.04	-0.05	0.08	0.26	-0.07
R	Nominal interest rate	0.40	0.41	0.44	-0.26	-0.13	-0.19	0.88	0.91	0.89
rer	Real exchange rate	5.01	5.68	5.37	-0.22	0.05	0.02	0.75	0.82	0.84
π^S	Nominal depreciation[c]	5.12	5.12	4.70	-0.27	0.15	0.12	0.22	-0.03	-0.03

Note: The model moments are the theoretical moments at the posterior mean. MAS corresponds to a specification analogous to Medina and Soto (2007), estimated with the same sample and priors as the baseline XMAS model.

[a]: Not observed in MAS specification ;[b]: Not observed in XMAS specification ; [c]: Not observed in either specification

4 XMAS as an extended version of Medina and Soto (2007)

In this section we present a description of some transmission channels that are present in the baseline XMAS model but not in Medina and Soto (2007) that allow for more flexibility in the modeling of several economic relationships: the interaction between the external sector, the commodity sector and the rest of the economy; the interaction between fiscal and private spending in consumption and investment goods; the response of capital use intensity to economic shocks; the relationship between the extensive and intensive margins of the labor sector and the rest of the economy; the relationship between domestic and foreign oil prices; the relationship between domestic, commodity and foreign technology disturbances; and the relationship between the foreign, imports, oil and commodity prices.

In addition, in subsection 4.6 we present a forecasting comparison between the baseline specification and one analogous to Medina and Soto (2007).

4.1 Endogenous commodity sector

Endogeneizing the commodity sector (as in Fornero et al. (2014)) is especially relevant for a country where the mining sector accounts for roughly 10% of GDP and copper for 40% of exports, because shocks affecting particularly this sector will have relevant effects on the aggregate results of the economy, and thus correctly quantifying the response is relevant. First of all, a positive shock to the international price of the commodity should generate higher investment and output in that sector, and in the rest of the economy though the increase in input utilization. Also, the use of time to build in commodity investment, which nests the idea that most mining projects are rather long term, raises the importance of forecasted prices and more persistent shocks over temporary fluctuations. And finally, taxing foreign profits properly is important due to the high presence of international investors in this sector, which extract rents and influence key open economy variables. These ideas are tested in Figure 2, which shows the effects of a 50% shock to commodity prices, where the baseline scenario is compared with 3 counterfactuals: one with a simpler commodity sector, akin to MAS ($\alpha_{Co} = s_{ICo} = 0$ and $\Phi_{I_{Co}} = 10^5$), a second one with no time to build in the commodity sector ($N_{CO} = 1$ instead of 6), and a third with no taxes on the commodity sector profits ($\tau_{Co} = 0$).

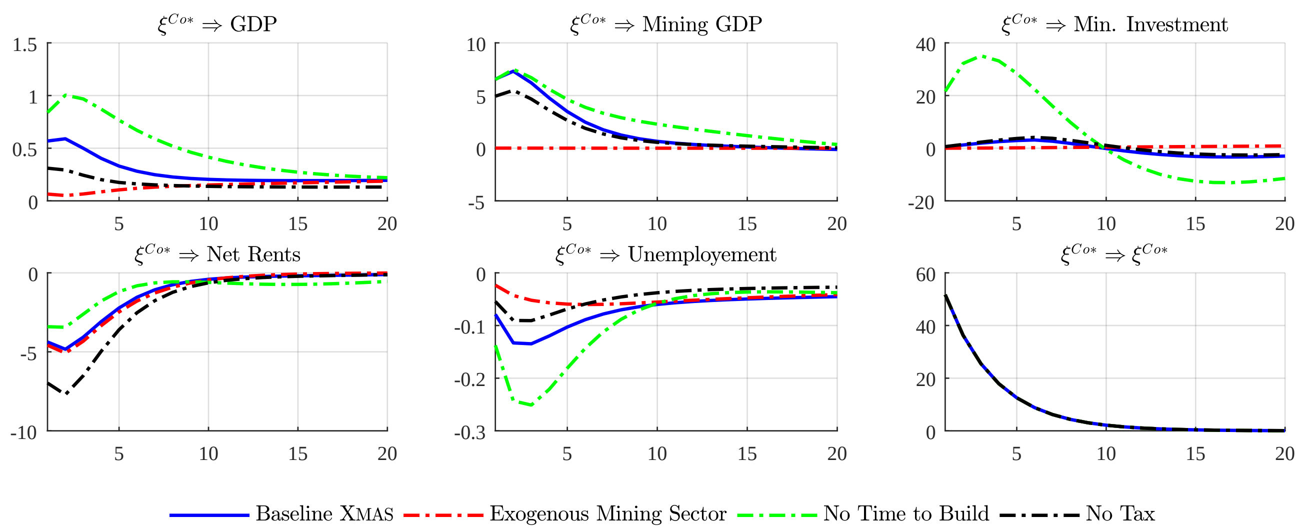


Figure 2: Commodity price shock.

Figure 2 supports the ideas previously discussed. Especially noticeable is the overreaction of commodity investment over a temporary price shock if there is no time to build, which transmits to the rest of the economy through the greater demand of home and foreign inputs for the creation of the commodity investment good. One may also notice that the tax on foreign profits reduces the rents of international investors, which means a greater benefit for the national economy in the particular case of this shock.

The endogenous commodity production also has the advantage of providing greater insight over aggregate investment fluctuations. In particular, greater commodity investment should not significantly raise demand for labor as this input has a very small participation in this sector (in our model, zero). This means that it is relevant to distinguish the source of an increase in aggregate investment. In the following figure we compare analogous investment shocks for the private and commodity sectors:

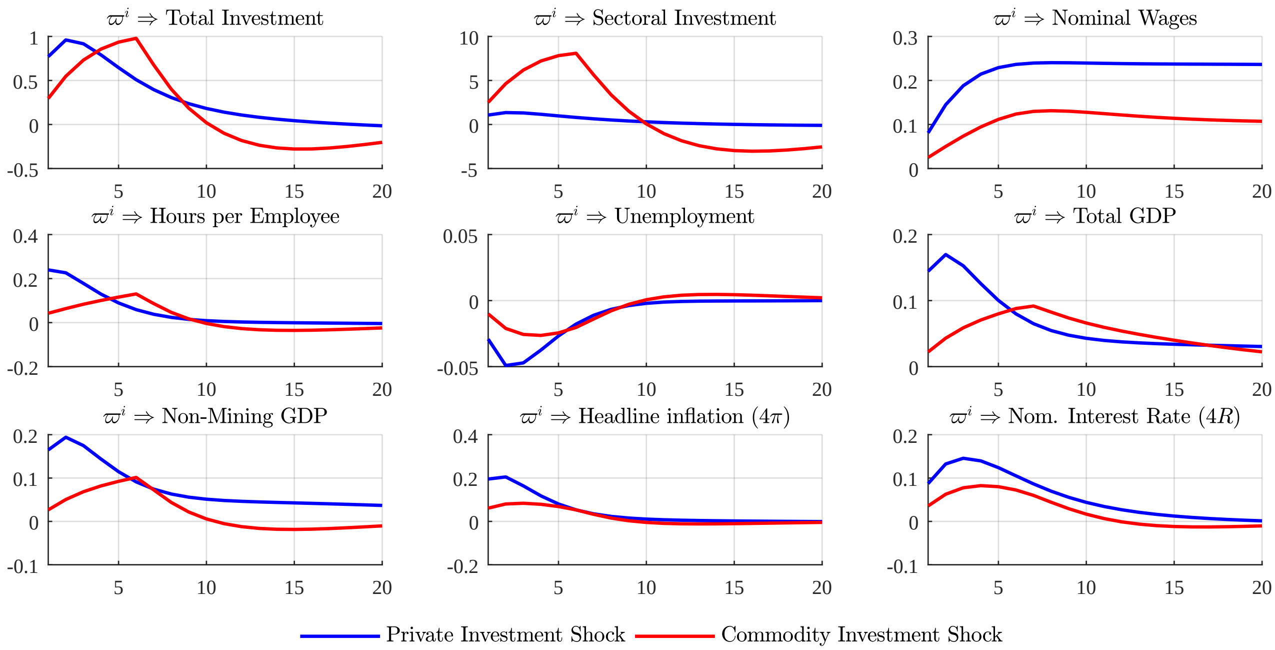


Figure 3: Effects of a non-commodity private investment shock, and a commodity investment shock

Having similar effects over aggregate investment (the commodity shock being much larger due to the smaller size of the sector), a shock in non-commodity investment has a stronger effect over hours worked, unemployment and wages through the rise of labor demand. Also, the commodity shock does not transmit as strongly to the rest of the economy due to the absence of the use of labor, implying softer effects over inflation, interest rates and also wages.

4.2 Augmented Fiscal Policy

Government expenditure is decomposed into consumption, investment and transfers in order to better understand the impact of different kinds of fiscal policy. Two novel mechanisms are incorporated following Coenen et al. (2012, 2013): the value households give to government consumption and the interaction between public and private capital in the production of goods.

The valuable government consumption channel allows for more flexibility when modeling the comovement between government and private consumption by managing the level of crowding out(in) following a change in the level of fiscal spending in consumption goods. The key parameter is $\eta_{\widehat{C}}$, the elasticity of substitution between private and government consumption in the final consumption good bundle. When private and government are complements, an increase in government consumption would increase the household's desire to spend in private consumption goods. On the other hand, if they are substitutes, an increase in government consumption decreases the desire to consume. It is important to keep in mind that only Ricardian households will be able to adjust their consumption accordingly, non-Ricardian consmption will only exhibit general equilibrium effects of expansionary fiscal policy which will ease their budget constraint.

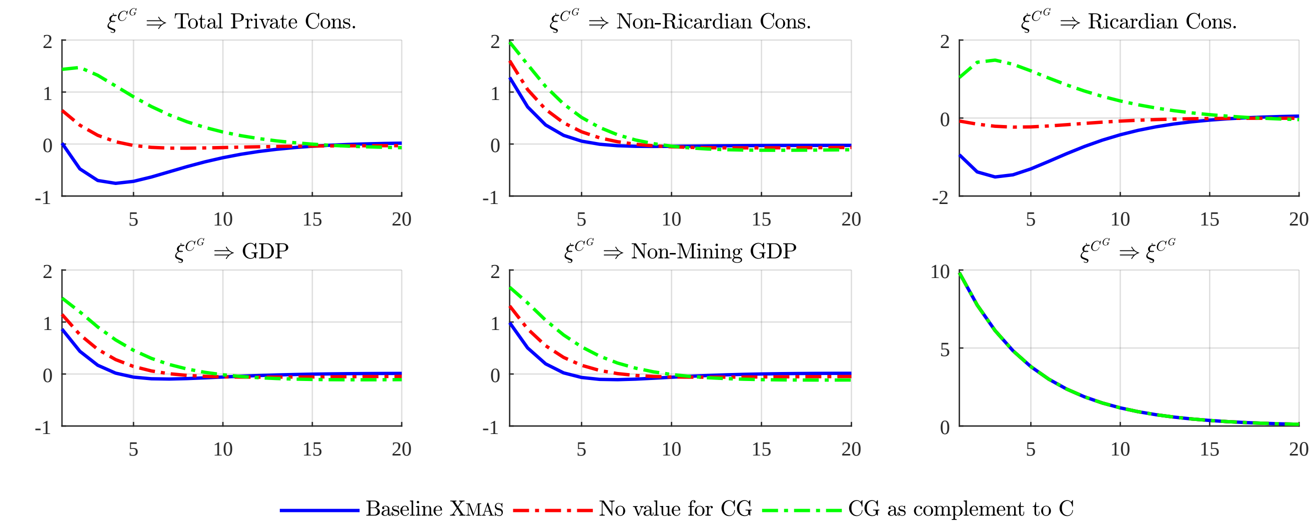


Figure 4: Government consumption shock.

Figure 4 shows the effects of a transitory 10% increase in government consumption spending. The model estimated elasticity of 2.30 suggest that the average fiscal spending in consumption goods tend to act as a substitute to private consumption, decreasing the overall fiscal multiplier that were to be present if both goods acted as complements (the green line, where the parameter $\eta_{\widehat{C}}$ is set as $\eta^{'}_{\widehat{C}} = \frac{1}{\eta_{\widehat{C}}}$). This can be appreciated in the difference that a fiscal consumption stimulus makes over GDP, the first graph. Non-Ricardians always increase their consumption with this shock as their budget constraint eases, but the response of Ricardians depends heavily on the elasticity of substitution. If government consumption is not valued, then ricardians will not respond and the overall consumption response will only reflect non-Ricardian budget constraints, which is why incorporating this channel and then estimating $\eta_{\widehat{C}}$ is relevant.

In the case of government investment, the very low persistence of these shocks gives us a different insight. We compute the impulse responses for private and government investment shocks, both of which increase total investment by 1% on impact:

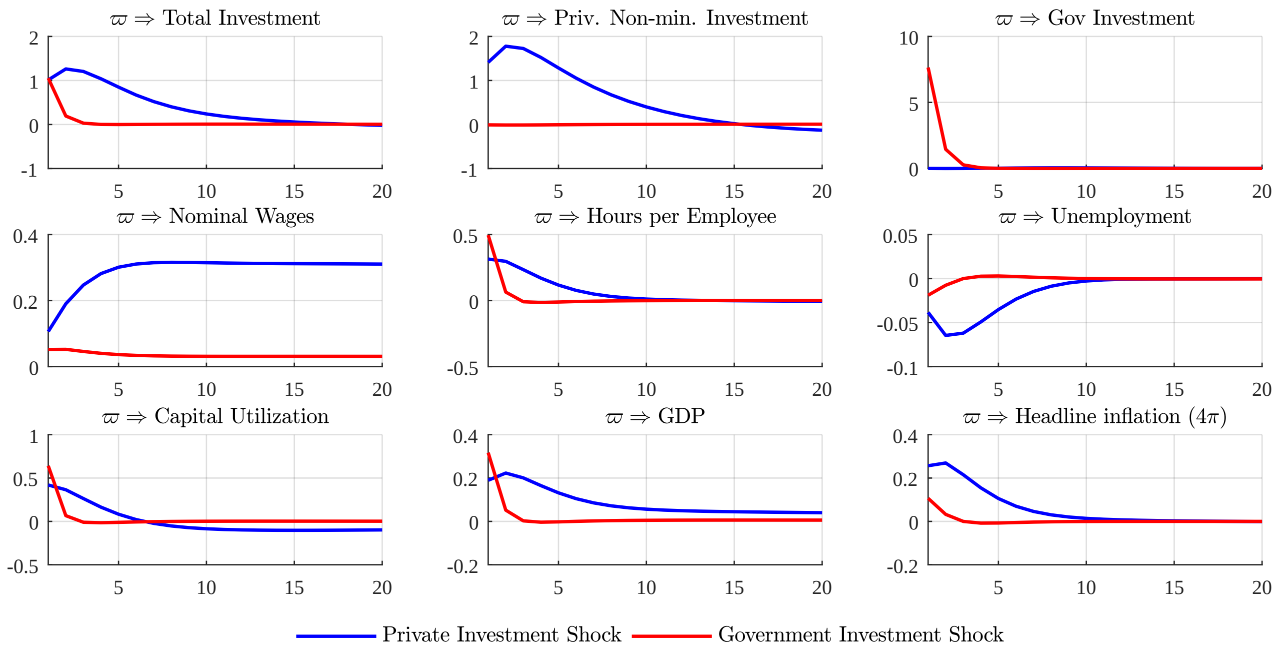


Figure 5: Private vs government investment shock.

Even though private and public capital are estimated to be complements as $\eta_{KG} < 1$, an exogenous, surprise increase in government investment will not trigger a significant increase in private investment. The increase in private capital services will be taken care mainly through a higher utilization rate, as is shown in the above graphs by the red line. The low persistence of the shock also implies that the labor market will adjust through more hours of work and little to no change in unemployment, and forward looking variables like wages and inflation will barely react. The blue line tells a very different story, as private investment shocks are highly persistent and will generate large responses from almost all variables in the expected directions. It is important to note that these two shocks, even though they might look the same on impact over aggregate investment, have very different consequences for GDP and inflation, making a very relevant piece of information whether sudden changes in investment come from the private or government (or commodity) sectors.

The importance of public capital for private investment can be appreciated better if we assume government investment shocks are more persistent. In particular, we calculate impulse responses for these shocks in three additional cases: first assuming assuming these shocks are just as persistent as government consumption shocks, second by assuming $\eta_{KG} > 1$[24], and third by assuming both previous claims. In the following figure we display all cases. A small but positive response of private investment to the government investment shock can be appreciated in the baseline scenario, which disappears in the case of both kinds of capital being substitutes. Introducing higher persistence into the shock brings into action another mechanism: an increase in the interest rate which unambigously lowers private investment in the short run. In this case however, η_{KG} plays a large role in the size of the response, and can even mean an increase in private investment in the medium run if both capitals are complements.

[24] In particular, the new elasticity is set as $\eta'_{KG} = \frac{1}{\eta_{KG}}$

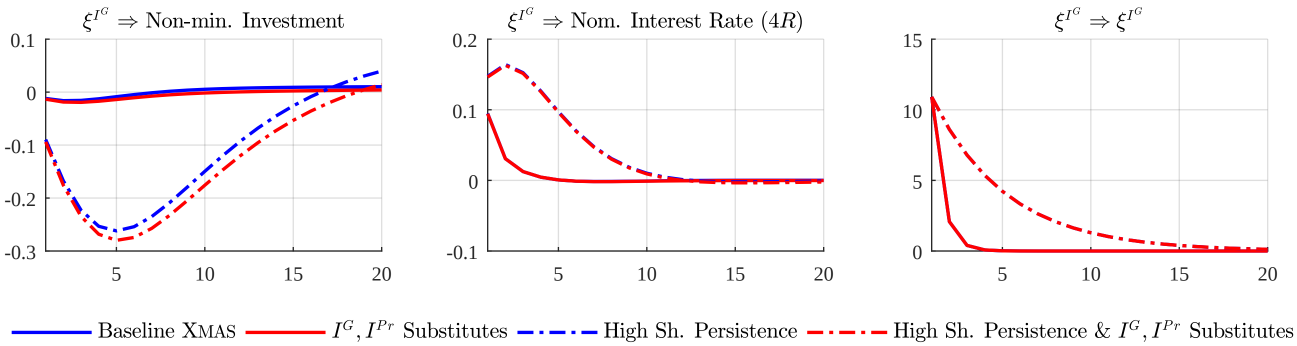


Figure 6: Government investment shock.

Finally, the decomposition of government expenditure into consumption, investment and transfers is useful in itself to better understand the impact of expenditure changes. Figure 7 shows the effects of similar increases in expenditure, arising from each of the three sources:

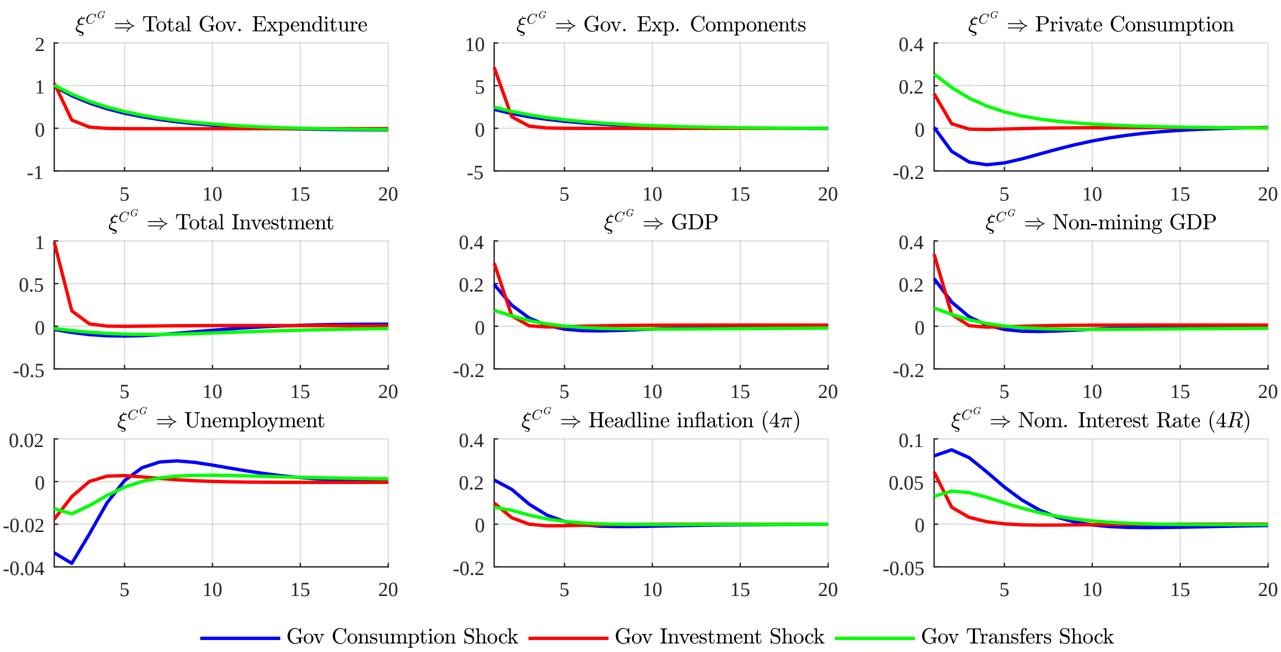


Figure 7: Government expenditure shocks.

All shocks are sized to produce the same effect on government expenditure on impact, where the investment shocks exhibits a much lower persistence than its counterparts, arising from the estimation procedure. Greater effects on GDP arise from government consumption or investment, while transfers have a smaller effect due to the presence of ricardian households (for whom the ricardian equivalence applies) and the absence of the use of inputs in production which is present in both other types of expenditure. The private consumption response is in line with the discussion above, where $\eta_{\widehat{C}} > 1$ is the cause of a very different response of private consumption to either transfers or government consumption, transfers acting through a greater consumption of non-ricardian households. Unemployment and inflation differences between government consumption and investment can be explained in the following way: both increase aggregate demand, but government investment raises private sector productivity through capital complementarity, and therefore also acts partly as a positive supply shock. That is why inflationary pressures are lower for the investment shock as costs are partly lowered by the greater capital productivity. And in the case of unemployment, the increase in aggregate demand is partly offset by this greater productivity in the case of the investment shock, while the consumption shock requires greater labor input to achieve equilibrium, therefore lowering unemployment temporarily.

4.3 Real and Nominal rigidities

4.3.1 Variable capital utilization

Allowing for variable capital utilization gives more flexibility on the modeling of the investment and capital rental rate dynamics. As noted by Christiano et al. (2005), by allowing the services of capital to respond contemporaneously to a shock, variable capital utilization helps dampen the larger change in the rental rate of capital that would otherwise occur. The smaller response of the rental rates induce smaller responses of marginal costs and therefore less volatile inflation dynamics. The degree in which capital utilization moves around the steady state level of utilization depends entirely on the utilization cost parameter Φ_u, when the parameter is *large*, the model converges to the case with no variable capital utilization.

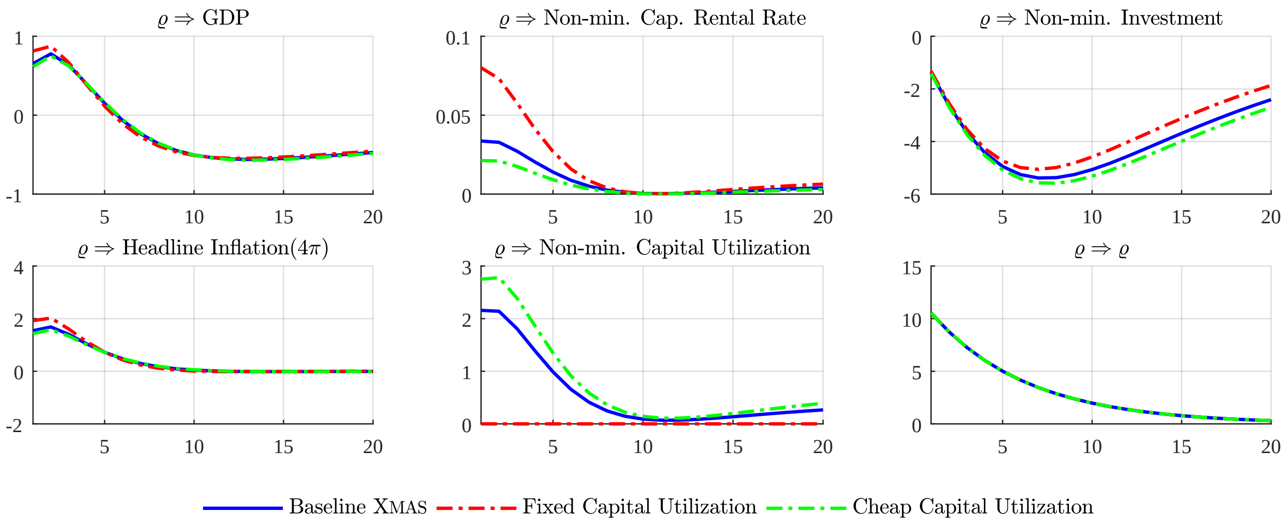


Figure 8: Demand shock with variable capital utilization.

Figure 8 shows the effects of a demand shock that causes a 1% increase in GDP. Two counterfactuals are implemented, one with large utilization costs ($\Phi_u = 10^5$), approaching the case with no variation in capital utilization and another with half the posterior estimate for Φ_u, which implies quite cheap costs. With lower utilization costs, the economy becomes more flexible, and is able to respond to the shock with more movement in quantities and less variation in prices, as is evidenced by responses of the rental rate of capital and inflation. A practical example of this sort would be the greater use of electricity and other non-fixed capital inputs in production, while companies face a surprise increase in their sales, and have to intensify their use of machinery in order to produce enough to satisfy a growing demand. In fact, a historical comparison of the smoothed value of capital utilization in the model and an index for electricity consumption for the same period in Chile will yield a positive correlation between both series.

4.3.2 Non core inflation

The model introduces both agricultural and oil goods separated from the rest of the consumption bundle because these goods exhibit significantly different price behavior from the rest of the consumption goods, which are labeled as core consumption goods. In particular, agricultural and oil prices are highly dependent on the specific supply and demand conditions in their spot markets, and therefore exhibit significant volatility and are quite insensitive to the macroeconomic conditions of a small open economy. That is why core inflation is of higher interest to the model's central bank, as expressed in the Taylor rule, since it is the inflation rate over which monetary policy has an impact, and also the inflation rate that will likely prevail as non-core shocks will dissipate quicker. In order to test this second hypothesis we run a variance decomposition exercise to understand the importance of agricultural and oil shocks over price volatility at different horizons:

Table 6: Annual inflation unconditional variance decomposition

	Agricultural Shock (z^A)	Oil Shocks (ξ^O,ξ^{O*})	Other Shocks
P_t/P_{t-4}	8.30%	12.81%	78.89%
$E\left[P_{t+1}/P_{t-3}\right]$	7.28%	12.28%	80.44%
$E\left[P_{t+2}/P_{t-2}\right]$	5.87%	10.77%	83.36%
$E\left[P_{t+4}/P_{t}\right]$	1.44%	5.56%	93.00%

Table 6 shows that the importance of non-core shocks in inflation is generally decreasing in time, accounting for almost 25% of variation in present annual inflation but less than 10% in expected inflation one year ahead, where other shocks, more persistent and with greater propagation channels, will be of higher relevance in determining inflation volatility.

Regarding oil in particular, the smoothing of the domestic price is introduced in order to match the volatility differential between the domestic price of oil based products and the international oil price in domestic currency (the latter being much more volatile than the former). Its implementation mimics the MEPCO program, a Chilean fiscal fund that acts as a buffer against the short term volatility of the foreign oil price. The introduction of this channel reduces the unconditional standard deviation of the core inflation rate from 0.65% to 0.58% , and for the total inflation rate from 0.99% to 0.68%, much closer to the empirical moments of 0.51% and 0.69% respectively. External oil price shocks have significantly different effects as well:

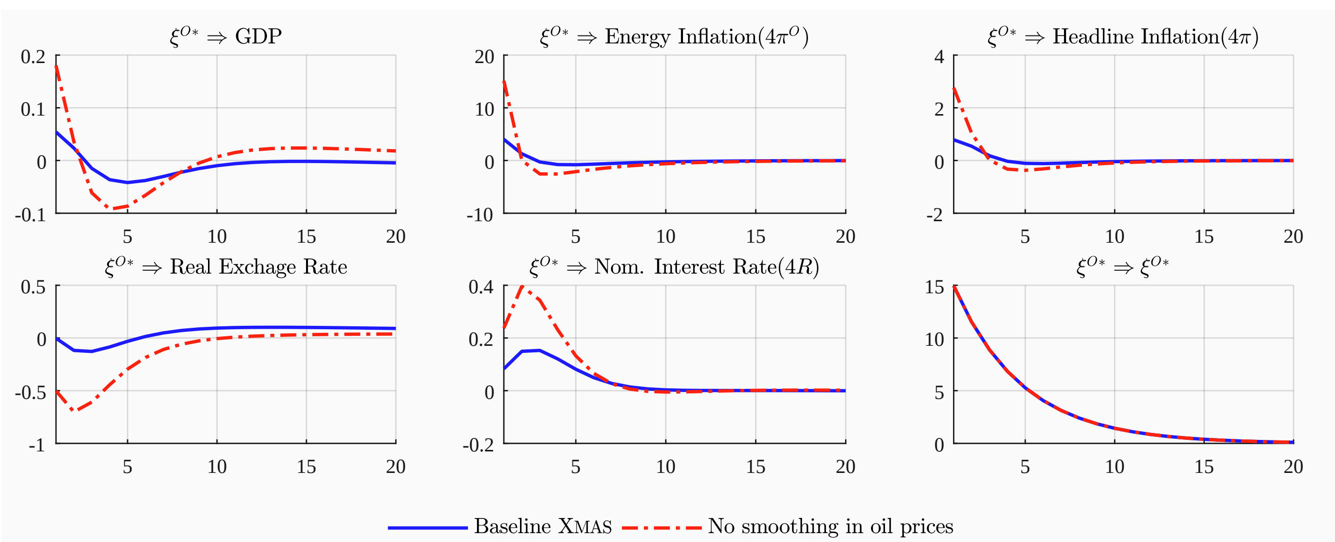


Figure 9: International oil price shock.

Figure 9 shows the effects of a large shock to the foreign oil price. The alternative counterfactual of no smoothing of the domestic oil price corresponds to the case of $\rho_O = \alpha_O = 0$. In the baseline case, the smoothing of the local oil price has significant consequences in most variables of interest: GDP, inflation and interest rate all exhibit a more moderate response. Additionally, by avoiding an overreaction of the monetary policy, the real exchange rate depreciates most of the relevant horizon instead of appreciating, in line with the empirical correlations.

4.4 Global prices and productivity delayed pass through

We introduce a potentially delayed pass through from a global price factor to oil and commodity prices and from global technology towards local and commodity sector productivity. While maintaining a long term cointegration process, this process allows for a reduction of the excessive propagation of short term foreign variables fluctuations into the economy. The speed of propagation for each process, dictated by the Γ parameters listed below, is estimated to provide the model the proper flexibility.

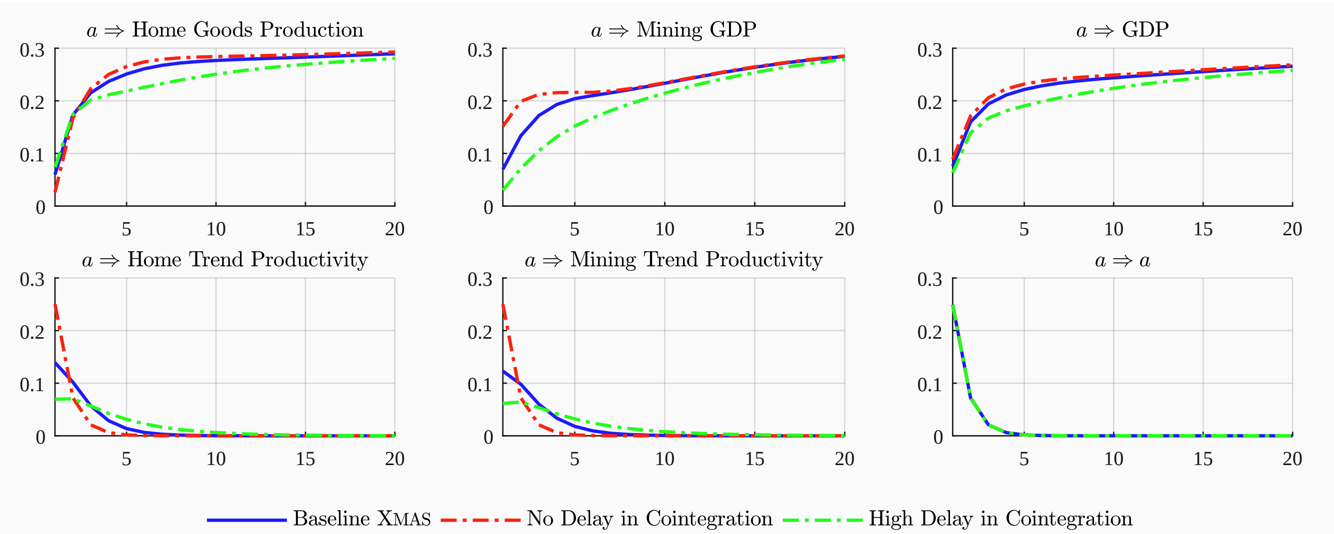


Figure 10: Global productivity shock.

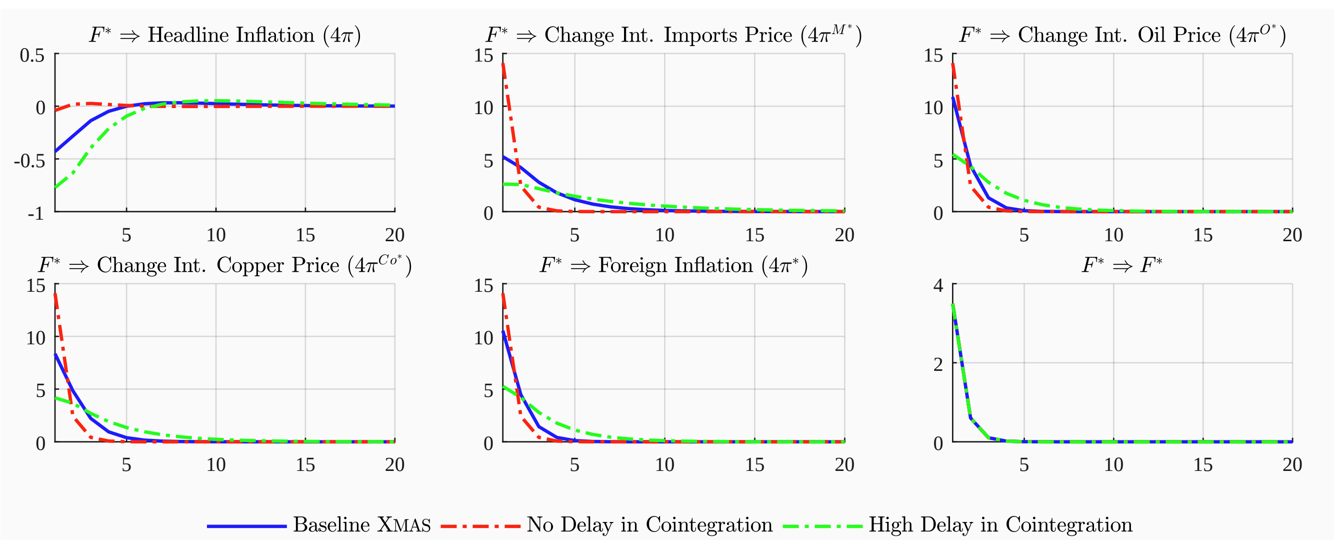


Figure 11: Global price shock.

Figures 10 and 11 show the effect of shocks to global productivity and foreign prices, comparing the baseline parameterization with a counterfactual with no delayed pass through ($\Gamma^H = \Gamma^{Co} = \Gamma^{pCo} = \Gamma^{pM} = \Gamma^{pO} = \Gamma^* = 1$)

and with an increased delay (with $\Gamma^H, \Gamma^{Co}, \Gamma^{pCo}, \Gamma^{pM}, \Gamma^{pO}$ and Γ^* set to half their posterior estimates). In the case of the productivity shock, delayed pass-through in productivity can be appreciated in the bottom graphs, while their expected effects (delayed increases in production) are apparent in the top graphs. In figure 11, we present a shock to the common factor F^*influencing all external prices. This one shock affects different prices in different magnitudes, as the pass-through delay is estimated for each price, presenting different posterior estimates. In all cases, no delay in the cointegration implies a larger response of the particular price on impact, but dying out more quickly that in the base model. Of course, one might rightly think that international prices do not always move altogether, for example because oil and copper prices are determined by independent supply (and demand) factors. For each of the external prices (including the general foreign inflation) there are individual shocks ensuring enough flexibility for the model to interpret short term divergence in prices.

4.5 Labor market with search and matching

The introduction of search and matching in the labor market is a significant departure from Medina and Soto (2007). Our specification enables the analysis of the extensive margin of work (unemployment vs employment) as well as the intensive margin (hours worked). This is shown to be important as the extensive margin accounts for a significant part of the aggregate labor supply volatility. Furthermore, as shown in figure 12 and table 7, they don't seem to follow the same trajectories across the business cycle: the extensive margin follows output and inflation more closely than the intensive one. For this reason, observing the evolution of employment can give a less noisy signal about the state of the economy than total worked hours.

Note: GDP, hours and employment in levels are transformed to log difference from a log-linear trend, and inflation is obtained from q/q log variation of prices. All variables are then standardized as standard deviation differences from means, and a moving average of the present and past 3 quarters is taken.

Figure 12: Labor Variables and Business Cycle

Table 7: Correlation between labor variables and business cycle variables

	Real GDP	Inflation
Hours worked	-0.170	-0.081
Employment	0.822	0.365

Note: All variables are transformed as in the previous graph, except for the moving average transformation.

We compare the baseline specification with one with Calvo-type frictions in the labor market. In this specification, the extensive margin is assumed to be absent, with every household actively participating in the production of goods ($n_t = n = 1$). Therefore, the aggregate labor supply is entirely driven by fluctuation of the intensive margin.[25] This can be appreciated in our first exercise, where we compare the empirical standard deviation of our hours worked and employment data (in logs) with the model's theoretical moments:

Table 8: Volatility of intensive and extensive margins

	Data	Baseline Model	Calvo Wages
$\sigma_{\log(h)}$	1.006	2.373	3.950
$\sigma_{\log(n)}$	0.848	0.793	0
$\sigma_{\log(h\times n)}$	1.322	2.745	3.950

Table 8 shows how the introduction of search and matching significantly closes the gap between the model and the data's moments, first by reducing volatility in hours worked, and second by the introduction of the extensive margin of labor in an excellent match with data. The latter is particularly important, as we have seen that employment has a higher correlation with the business cycle than hours worked. It should also be noted that the improvement in fit is not just through a reallocation of volatility across margins of labor, because the model's aggregate labor volatility (the sum of both margins) is also decreased thanks to the search & matching framework, bringing it closer to data.

Search and matching also gives us additional insight into the mechanisms at play in the labor market which affect the rest of the economy. We compute impulse response functions for three shocks: lower investment prices, a looser monetary policy and an expansive government consumption. In all cases, we expect them to cause an increase in labor demand leading to higher real wages. While this happens with all three shocks using a search and matching framework, the impulse-response functions for the Calvo wage model show only very small increases in the case of the investment and government consumption shocks, and even a fall in the case of expansive monetary policy. This can be due to the employment frictions present while using search and matching, which prevent the clearing of the labor market and hamper the adjustment in quantities of labor desired to respond to the shock.[26] This in turn leads to an equilibrium where prices make a greater adjustment.

[25] A complete description of the model with Calvo type wages is given in the appendix A. For the following exercises, we calibrate the parameter θ_W to the posterior estimate of $\varkappa_W = 0.80$ in order to produce the comparable wage inertia in both specifications. Following Medina and Soto (2007) the elasticity of substitution among labor varieties is set as $\epsilon_W = \epsilon_H = \epsilon_F = \epsilon_{H*} = 11$. Finally, we set $\omega^U = \omega = 0.5$ assuming unbiased unions.

[26] Here $H \times N$ is taken as the total quantity of labor, as only the employed households provide hours of work.

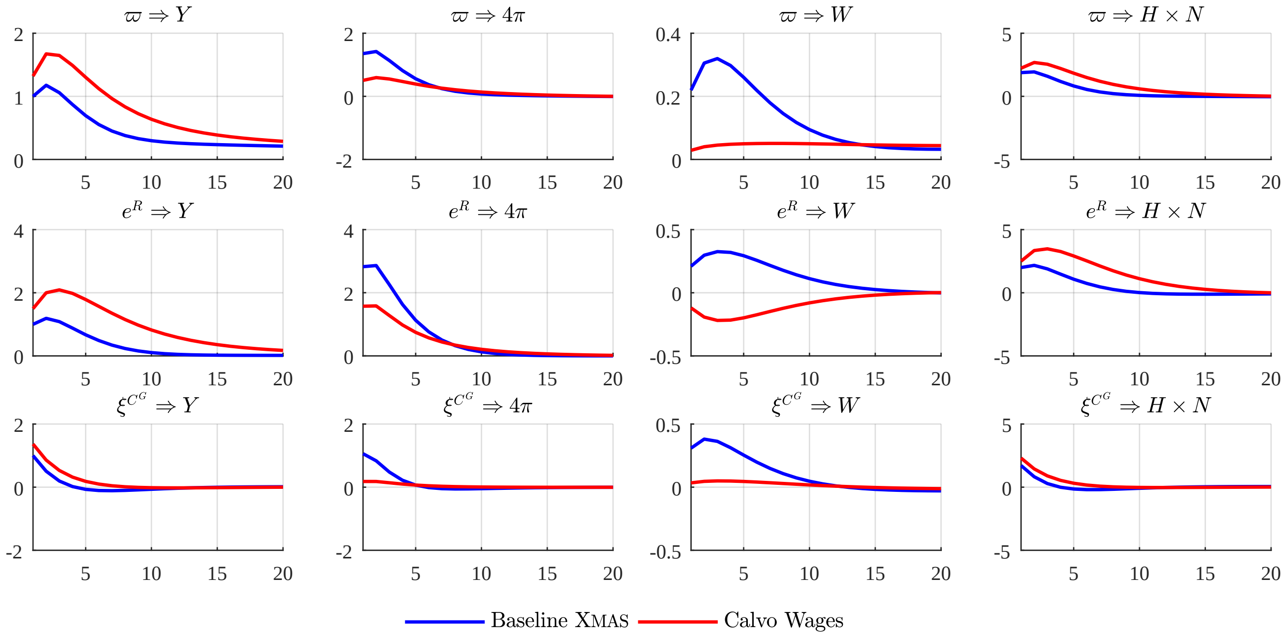
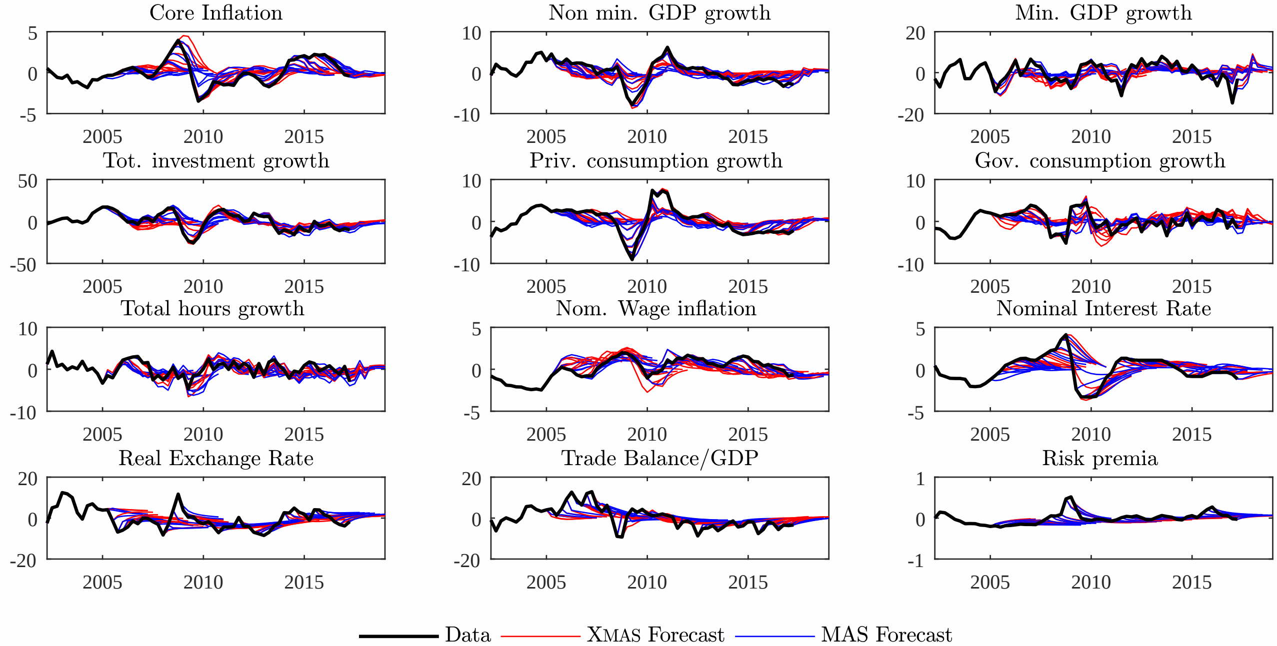


Figure 13: Demand shocks and the labor market: effects of a shock to investment efficiency(ϖ), monetary policy(e^R), and government consumption(η^{CG})

4.6 Model Forecast

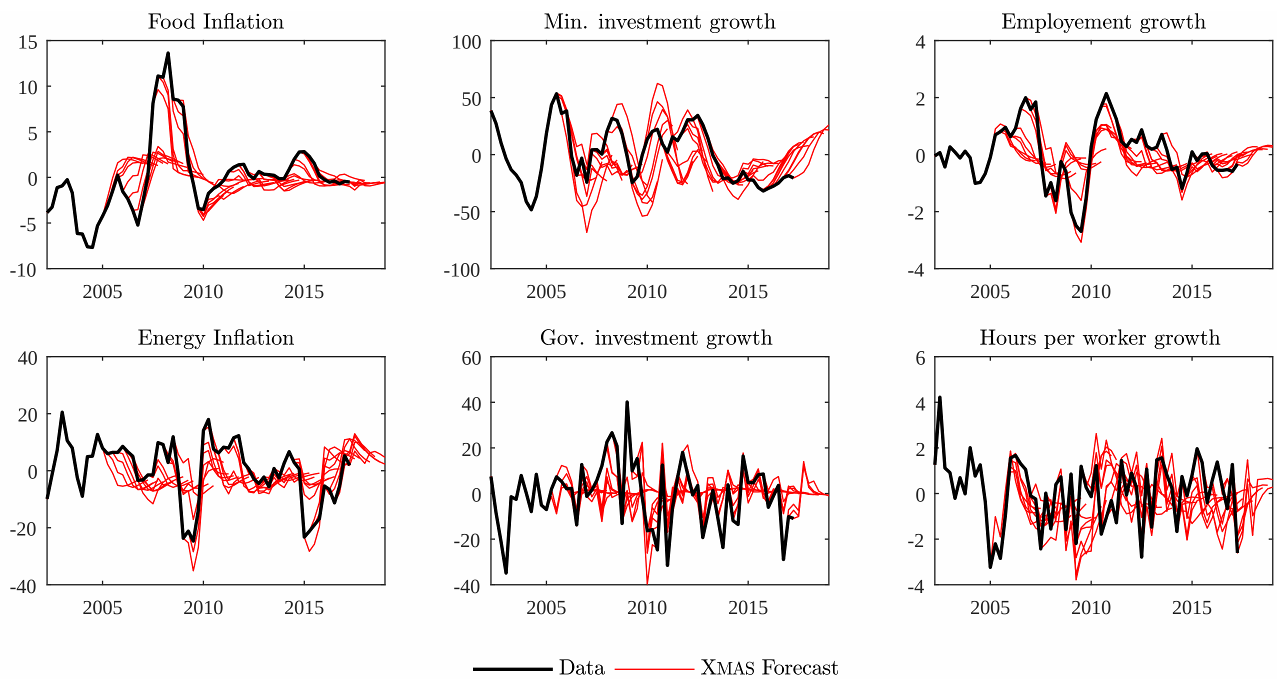
We now compare the baseline model with the MAS specification in terms of forecast accuracy. We do pseudo-out of sample forecasting, where all the variables are dynamically forecasted at different horizons, but always utilizing the full sample posterior parameters[27]. Figures 14 and 16 show respectively the models forecasts and forecast errors for the endogenous variables that both specifications observe. Despite the significantly added complexity of the XMAS specification, its out of sample forecast accuracy is fairly comparable to the simpler specification, even outperforming the MAS specification in some dimensions. In addition, as shown in figures 15 and 17 , the structure of XMAS allows for an assessment on individual components of investment, inflation, and labor market variables. It is reassuring that the increased granularity of the model does not come at the expense of a decrease in the precision of its forecasts.

[27]A full out of sample forecast comparison with meaningful number of forecast evaluations would require the use of very short samples for the initial estimations. Given the size of the model and the number of parameters to be estimated, the use of shorter samples would require increasingly tight priors in order to guarantee sensible results. This effect is magnified with the utilization of the endogenous priors approach, due to the mid-sample fluctuations in overall volatility generated in the aftermath of the 2008 financial crisis. For those reasons, we abstain to attempt a recursive model estimation.



Note: growth and inflation variables transformed to y/y log-variation. Nominal interest rate transformed to annual base. All variables in deviation from sample mean.

Figure 14: Model forecasts of variables present on both XMAS and MAS specifications.



Note: growth and inflation variables transformed to y/y log-variation. All variables in deviation from sample mean.

Figure 15: Model forecasts of variables only present on XMAS specification..

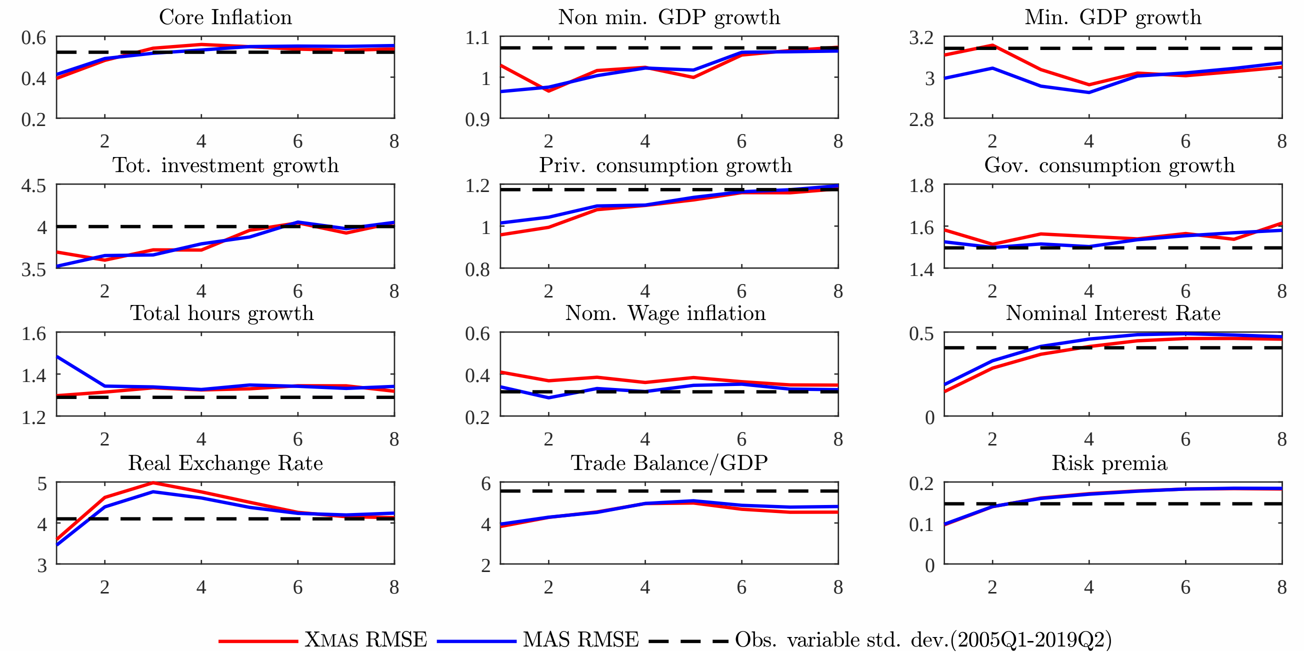


Figure 16: Root mean square error 1 to 8 periods ahead for variables present on both XMAS and MAS specifications.

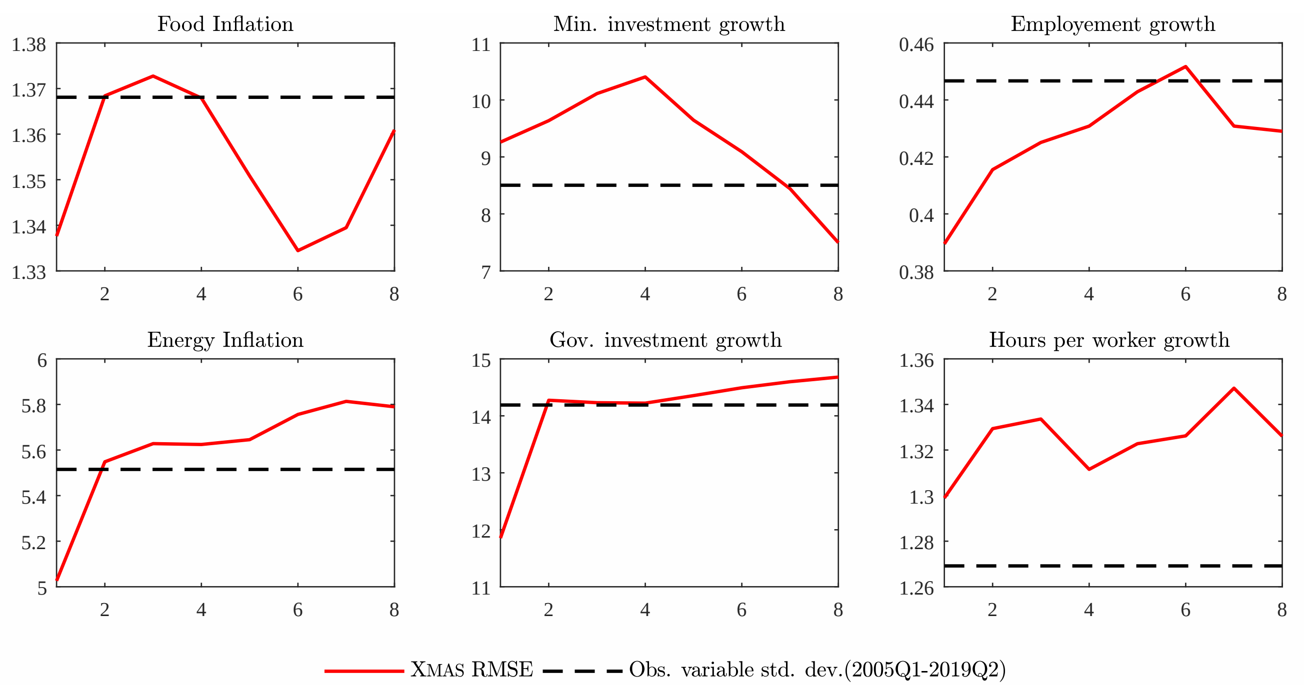


Figure 17: Root mean square error 1 to 8 periods ahead for variables only present on XMAS specification.

5 Concluding Remarks

We present XMAS, a DSGE model with a focus on monetary policy analysis and macroeconomic forecasting that, building on Medina and Soto (2007), implements a range of new features, motivated by the experience of commodity-exporting emerging economies in general, and Chile in particular. The improvements over the base model include the modeling of non-core inflation dynamics, an endogenous commodity sector, an augmented fiscal sector and additional real and nominal frictions like variable capital utilization and a labor market with search and matching frictions that allows for labor variation in both the intensive and extensive margins. We show that Xmas feature comparable forecast accuracy than the simpler specification from Medina and Soto (2007). This despite the significantly increased complexity of the model, that allows for economic analysis at a more granular level. Future work includes the introduction of an endogenous productivity fluctuation channel through inter-firm labor transitions, the inclusion of an optimal monetary policy framework instead of the Taylor rule approach currently in place, the addition of financial frictions and a banking sector, and the exploration of different expectational frameworks that may permit the modeling of anticipated monetary shocks with realistic outcomes, minimizing the forward guidance puzzle effect as described by Del Negro et al. (2012).

Appendix

A Calvo wages in the XMAS

In a standard New Keynesian model with Calvo wages, since the extensive margin is non-existent, $n_t = 1$ and $u_t = 0$ at all times. Since there is no unemployment, there is no role for the UFA, therefore we set $B_t^{UFA} = TR_t^{UFA} = \tau_t^{UFA} = 0$. Finally, there is no matching function, no exogenous or endogenous separations, and no vacancy postings, allowing us to dispense of vacancy and operating costs, setting $X_t^L = 0$.

Wages and hours are not determined by Nash bargaining. Instead both Ricardian and non-Ricardian households supply differentiated labor services to a continuum of unions which act as wage setters on behalf of the households in monopolistically competitive markets. The unions pool the wage income of all households and then distribute the aggregate wage income in equal proportions among the latter.[28] Once wages are set, the unions satisfy all labor demand at that wage.

Labor demand is given by the wholesale goods firm's cost minimization problem, which becomes much simpler:

$$\min_{\widetilde{h}_t^d, K_t^S} L_t^{Y^{\tilde{Z}}} = W_t \widetilde{h}_t^d + r_t^K K_t^S + mc_t^{\widetilde{Z}} \left[Y_t^{\widetilde{Z}} - \left(\widetilde{K}_t\right)^{\alpha} \left(A_t^H \widetilde{h}_t^d\right)^{1-\alpha} \right] \tag{79}$$

Where $\widetilde{h}_t^d$ denotes the demand for units of composite labor. The optimal labor demand is given by

$$\widetilde{h}_t^d = (1-\alpha) \left(\frac{W_t}{mc_t^{\widetilde{Z}}} \right)^{-1} Y_t^{\widetilde{Z}} \tag{80}$$

This demand is satisfied by perfectly competitive packing firms which demand all varieties $j \in [0,1]$ of labor services in amounts $h^d(j)$ and combines them in order to produce composite labor services (h), much like the firms in 2.3.2, with the production function, variety j demand, and aggregate nominal wage respectively given by:

$$\widetilde{h}_t = \left[\int_0^1 h_t^d(j)^{\frac{\epsilon_W - 1}{\epsilon_W}} dj \right]^{\frac{\epsilon_W}{\epsilon_W - 1}}, \qquad \epsilon_W > 0. \tag{81}$$

$$h_t^d(j) = \left(\frac{W_t^n(j)}{W_t^n} \right)^{-\epsilon_W} \widetilde{h}_t \tag{82}$$

$$W_t^n = \left[\int_0^1 W_t^n(j)^{1-\epsilon_W} dj \right]^{\frac{1}{1-\epsilon_W}}. \tag{83}$$

Regarding the supply of differentiated labor, following Erceg et al. (2000) and closely following the structure of the firms in 2.3.3 there is a continuum of monopolistically competitive unions indexed by $i \in [0,1]$, which act as wage setters for the differentiated labor services supplied by households. The union supplying variety j satisfies the demand given by (82) but it has monopoly power for its variety. Wage setting is subject to a Calvo-type problem, whereby each period a union can set its nominal wage optimally with probability $1-\theta_W$, and if it cannot optimally change its wage, it indexes its past wage according to a weighted product of past and steady state inflation with weights $\vartheta_W \in [0,1]$ and $1-\vartheta_W$. A union reoptimizing in period t is assumed to choose the wage $\tilde{W}_t^n$(equal for Ricardian and non Ricardian households) that maximizes the households lifetime utility until it can reoptimize.[29]

[28] Hence, households are insured against variations in household-specific wage income.

[29] The union weights the benefits of wage income by considering the agents' marginal utility of consumption – which will usually differ between Ricardian and non-Ricardian households – and a subjective weight on their welfare. The unions take into account the fact that firms allocate their labor demand uniformly across different kind of workers, and therefore $h_t^R(i) = h_t^{NR}(i) = h_t(i)\ \forall\, i, t$.

All considered, taking the aggregate nominal wage as given, the union's i maximization problem can be expressed as

$$\max_{\tilde{W}_t^n(i)} E_t \sum_{s=0}^{\infty} (\beta\theta_W)^s \varrho_{t+s} \left[\begin{array}{l} \omega^U \left(\Lambda_{t+s}^{NR}(1-\tau_{t+s}^W) \frac{\tilde{W}_t^n \Gamma_{t,s}^W}{P_{t+s}} h_{t+s}(i) - \Theta_{t+s}^{NR} \kappa_{t+s} \left(A_{t+s-1}^H\right)^{1-\sigma} \frac{h_{t+s}^{1+\phi}(i)}{1+\phi} \right) \\ + \left(1-\omega^U\right) \left(\Lambda_{t+s}^{R}(1-\tau_{t+s}^W) \frac{\tilde{W}_t^n \Gamma_{t,s}^W}{P_{t+s}} h_{t+s}(i) - \Theta_{t+s}^{R} \kappa_{t+s} \left(A_{t+s-1}^H\right)^{1-\sigma} \frac{h_{t+s}^{1+\phi}(i)}{1+\phi} \right) \end{array} \right],$$

$$\text{s.t.} \quad h_{t+s}(i) = \left(\frac{\tilde{W}_t^n \Gamma_{t,s}^W}{W_{t+s}^n} \right)^{-\epsilon_W} \widetilde{h}_{t+s},$$

A similar derivation than the one presented in section (2.3.3) yields [30]

$$f_t^W = mc_t^W \tilde{w}_t^{-\epsilon_W(1+\phi)} \widetilde{h}_t + \beta\theta_W E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\Lambda_{t+1}^U}{\Lambda_t^U} \frac{(1-\tau_{t+1}^W)}{(1-\tau_t^W)} \frac{\pi_{t+1}^W}{\pi_{t+1}} \left(\frac{g_t^{\Gamma^W}}{\pi_{t+1}^W} \right)^{-\epsilon_W(1+\phi)} \left(\frac{\tilde{w}_t}{\tilde{w}_{t+1}} \right)^{-\epsilon_W(1+\phi)} f_{t+1}^W \right\}$$

$$f_t^W = \tilde{w}_t^{1-\epsilon_W} \widetilde{h}_t \left(\frac{\epsilon_W - 1}{\epsilon_W} \right) + \beta\theta_W E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\Lambda_{t+1}^U}{\Lambda_t^U} \frac{(1-\tau_{t+1}^W)}{(1-\tau_t^W)} \left(\frac{\pi_{t+1}^W}{\pi_{t+1}} \right)^{\epsilon_w} \left(\frac{g_t^{\Gamma^W}}{\pi_{t+1}} \right)^{1-\epsilon_W} \left(\frac{\tilde{w}_t}{\tilde{w}_{t+1}} \right)^{1-\epsilon_W} f_{t+1}^W \right\},$$

and

$$1 = (1-\theta_W)\tilde{w}_t^{1-\epsilon_W} + \theta_W \left(\frac{g_{t-1}^{\Gamma^W}}{\pi_t^W} \right)^{1-\epsilon_W}.$$

Finally, the clearing condition for the labor market is

$$h_t = \int_0^1 h_t(j)dj = \int_0^1 h_t^d(j)dj = \widetilde{h}_t \int_0^1 w_t(j)^{-\epsilon_F} dj = \widetilde{h}_t \Delta_t^W = \widetilde{h}_t^d \Delta_t^W,$$

Where Δ_t^W is a wage dispersion term that satisfies

$$\Delta_t^W = (1-\theta_W)\tilde{w}_t^{-\epsilon_W} + \theta_{_W} \left(\frac{g_{t-1}^{\Gamma^W}}{\pi_t^W} \right)^{-\epsilon_W} \Delta_{t-1}^W,$$

B Equilibrium and Steady State equations

B.1 Equilibrium Conditions

The variables in uppercase that are not prices contain a unit root in equilibrium due to the presence of the non-stationary productivity shock A_t. We need to transform these variables to have a stationary version of the model. To do this, with the exceptions we enumerate below, lowercase variables denote the uppercase variable divided by A_{t-1} (e.g. $c_t \equiv \frac{C_t}{A_{t-1}}$). There are two exceptions: first is the Lagrange multiplier Λ_t that is multiplied by A_{t-1}^{σ} (i.e. $\lambda_t \equiv \Lambda_t A_{t-1}^{\sigma}$), for it decreases along the balanced growth path. Second, we need to define the parameter $\psi_t^U = \Psi_t^U / A_{t-1}^{\sigma}$ in order to define a stationary equilibrium in the labor market.

The rational expectations equilibrium of the stationary version of the model is the set of sequences for the

[30] Where $\Lambda_{t+s}^U \equiv \omega^U \Lambda_{t+s}^{NR} + \left(1-\omega^U\right) \Lambda_{t+s}^R$ is the weighted marginal utility of consumption relevant for the union's decision, $\tilde{w}_t = \tilde{W}_t^n / W_t^n$ denotes the optimal nominal wage relative to the aggregate nominal wage index, $\pi_t^W = W_t^n / W_{t-1}^n$ is the wage inflation, and $mc_t^W = \frac{-U_H/U_C}{(1-\tau_t^W)W_t^n/P_t} = \frac{\Theta_t^U \kappa_t \left(A_{t-1}^H\right)^{1-\sigma} (h_t)^{\phi} / \Lambda_t^U}{(1-\tau_t^W)W_t}$ denotes the gap with the efficient allocation when wages are flexible. $\Theta_{t+s}^U \equiv \omega^U \Theta_{t+s}^{NR} + \left(1-\omega^U\right) \Theta_{t+s}^R$ is the weighted endogenous shifter.

endogenous variables such that for given initial values and exogenous variables and assuming

$$\tilde{c}_t \sim \log N\left(\mu_{\tilde{c}}, \sigma^2_{\tilde{c}}\right)$$

the following conditions are satisfied:

from Households (2.1):

$$\widehat{c}_t^R = \left[\left(1 - o_{\widehat{C}}\right)^{\frac{1}{\eta_{\widehat{C}}}} \left(c_t^R - \varsigma c_{t-1}^R / a_{t-1}\right)^{\frac{\eta_{\widehat{C}}-1}{\eta_{\widehat{C}}}} + o_{\widehat{C}}^{\frac{1}{\eta_{\widehat{C}}}} \left(c_t^G\right)^{\frac{\eta_{\widehat{C}}-1}{\eta_{\widehat{C}}}}\right]^{\frac{\eta_{\widehat{C}}}{\eta_{\widehat{C}}-1}} \tag{EE.1}$$

$$\widehat{c}_t^{NR} = \left[\left(1 - o_{\widehat{C}}\right)^{\frac{1}{\eta_{\widehat{C}}}} \left(c_t^{NR} - \varsigma c_{t-1}^{NR} / a_{t-1}\right)^{\frac{\eta_{\widehat{C}}-1}{\eta_{\widehat{C}}}} + o_{\widehat{C}}^{\frac{1}{\eta_{\widehat{C}}}} \left(c_t^G\right)^{\frac{\eta_{\widehat{C}}-1}{\eta_{\widehat{C}}}}\right]^{\frac{\eta_{\widehat{C}}}{\eta_{\widehat{C}}-1}} \tag{EE.2}$$

$$\tau_t^L = \tau_t^{UFA} + \tau_t^W \tag{EE.3}$$

$$k_t^{S,R} = \bar{u}_t \frac{k_{t-1}^R}{a_{t-1}} \tag{EE.4}$$

$$\phi_{\bar{u}}(\bar{u}_t) = \frac{r^k}{\Phi_{\bar{u}}} \left(e^{\Phi_{\bar{u}}(\bar{u}_t - 1)} - 1\right) \tag{EE.5}$$

$$k_t^R = (1-\delta)\frac{k_{t-1}^R}{a_{t-1}} + \left(1 - \frac{\Phi_I}{2}\left(\frac{i_t^R}{i_{t-1}^R} a_{t-1} - a\right)^2\right) \varpi_t i_t^R \tag{EE.6}$$

$$\left(1 + \tau_t^C\right) \lambda_t^R = \left(\widehat{c}_t^R\right)^{-\sigma} \left(\frac{\left(1 - o_{\widehat{C}}\right) \widehat{c}_t^R}{c_t^R - \varsigma c_{t-1}^R / a_{t-1}}\right)^{\frac{1}{\eta_{\widehat{C}}}} \tag{EE.7}$$

$$\lambda_t^R = \frac{\beta}{a_t^\sigma} R_t E_t \left\{\frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\pi_{t+1}}\right\} \tag{EE.8}$$

$$\lambda_t^R = \frac{\beta}{a_t^\sigma} R_t^* \xi_t E_t \left\{\frac{\varrho_{t+1}}{\varrho_t} \frac{\pi_{t+1}^S \lambda_{t+1}^R}{\pi_{t+1}}\right\} \tag{EE.9}$$

$$q_t = \frac{\beta}{a_t^\sigma} E_t \left\{\frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} \left[\begin{array}{c} r_{t+1}^K \bar{u}_{t+1}\left(1 - \tau_{t+1}^K\right) + q_{t+1}(1-\delta) \\ +p_{t+1}^I \left[\tau_{t+1}^K \delta - \phi_{\bar{u}}(\bar{u}_{t+1})\left(1 - \tau_{t+1}^K\right)\right] \end{array}\right]\right\} \tag{EE.10}$$

$$\begin{aligned} \frac{p_t^I}{q_t} \quad = \quad & \left[\left(1 - \frac{\Phi_I}{2}\left(\frac{i_t^R}{i_{t-1}^R} a_{t-1} - a\right)^2\right) - \Phi_I \left(\frac{i_t^R}{i_{t-1}^R} a_{t-1} - a\right) \frac{i_t^R}{i_{t-1}^R} a_{t-1}\right] \varpi_t \\ & + \frac{\beta}{a_t^\sigma} E_t \left\{\frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} \frac{q_{t+1}}{q_t} \Phi_I \left(\frac{i_{t+1}^R}{i_t^R} a_t - a\right) \left(\frac{i_{t+1}^R}{i_t^R} a_t\right)^2 \varpi_{t+1}\right\} \end{aligned} \tag{EE.11}$$

$$\bar{u}_t = 1 + \frac{\log\left({r_t^K}/{r^K}\right) - \log\left(p_t^I\right)}{\Phi_{\bar{u}}} \tag{EE.12}$$

$$\xi_t = \bar{\xi} \exp\left[-\psi\left(\frac{rer_t b_t^*}{p_t^Y y_t} - \frac{rer\, b^*}{p^Y y}\right) + \frac{\zeta_t^O - \zeta^O}{\zeta^O} + \frac{\zeta_t^U - \zeta^U}{\zeta^U}\right] \tag{EE.13}$$

$$b_t^* = b_t^{P^*} + b_t^{G^*} \tag{EE.14}$$

$$\left(1+\tau_t^C\right)\lambda_t^{NR} = \left(\widehat{c}_t^{NR}\right)^{-\sigma}\left(\frac{\left(1-o_{\widehat{C}}\right)\widehat{c}_t^{NR}}{c_t^{NR}-\varsigma c_{t-1}^{NR}/a_{t-1}}\right)^{\frac{1}{\eta_{\widehat{C}}}} \tag{EE.15}$$

$$\left(1+\tau_t^C\right)c_t^{NR} = \left(1-\tau_t^L\right)w_t n_t h_t + u_t ub + tr_t^{NR} - t_t^{NR} \tag{EE.16}$$

$$\theta_t^R = \widetilde{\chi}_t^R\left(\nabla_{t-1}^H\right)^{\sigma}\left(\widehat{c}\left(c_t^R-\varsigma c_{t-1}^R/a_{t-1}, c^G\right)\right)^{-\sigma} \tag{EE.17}$$

$$\theta_t^{NR} = \widetilde{\chi}_t^{NR}\left(\nabla_{t-1}^H\right)^{\sigma}\left(\widehat{c}\left(c_t^{NR}-\varsigma c_{t-1}^{NR}/a_{t-1}, c^G\right)\right)^{-\sigma} \tag{EE.18}$$

$$\widetilde{\chi}_t^R = \left(\widetilde{\chi}_{t-1}^R\right)^{1-\nu}\left(\nabla_{t-1}^H\right)^{-\sigma\nu}\left(\widehat{c}\left(c_t^R-\varsigma c_{t-1}^R/a_{t-1}, c^G\right)\right)^{\sigma\nu} \tag{EE.19}$$

$$\widetilde{\chi}_t^{NR} = \left(\widetilde{\chi}_{t-1}^{NR}\right)^{1-\nu}\left(\nabla_{t-1}^H\right)^{-\sigma\nu}\left(\widehat{c}\left(c_t^{NR}-\varsigma c_{t-1}^{NR}/a_{t-1}, c^G\right)\right)^{\sigma\nu} \tag{EE.20}$$

from Labor Market (2.2):

$$n_t = (1-\rho_t)\left(n_{t-1} + m_{t-1}v_{t-1}^{1-\mu}u_{t-1}^{\mu}\right) \tag{EE.21}$$

$$u_t = 1 - n_t \tag{EE.22}$$

$$s_t = m_t\left(\frac{v_t}{u_t}\right)^{1-\mu} \tag{EE.23}$$

$$e_t = m_t\left(\frac{v_t}{u_t}\right)^{-\mu} \tag{EE.24}$$

$$\rho_t = \rho_t^x + \left(1-\rho_t^x\right)\rho_t^n \tag{EE.25}$$

$$\rho_t^n = 1 - F(\bar{c}_t) = 1 - \Phi\left(\frac{\ln\bar{c}_t - \mu_{\tilde{c}}}{\sigma_{\tilde{c}}}\right) \tag{EE.26}$$

where Φ is the standard normal c.d.f.

from Final Goods (2.3.1):

$$c_t = \left[(1-\kappa_O-\kappa_A)^{\frac{1}{\eta_C}}\left(c_t^Z\right)^{\frac{\eta_C-1}{\eta_C}} + \kappa_O^{\frac{1}{\eta_C}}\left(c_t^O\right)^{\frac{\eta_C-1}{\eta_C}} + \kappa_A^{\frac{1}{\eta_C}}\left(c_t^A\right)^{\frac{\eta_C-1}{\eta_C}}\right]^{\frac{\eta_C}{\eta_C-1}} \tag{EE.27}$$

$$c_t^Z = (1-\kappa_O-\kappa_A)\left(p_t^Z\right)^{-\eta_C}c_t \tag{EE.28}$$

$$c_t^O = \kappa_O\left(p_t^O\right)^{-\eta_C}c_t \tag{EE.29}$$

$$c_t^A = \kappa_A\left(p_t^A\right)^{-\eta_C}c_t \tag{EE.30}$$

$$c_t^Z = \left[(1-o_Z)^{\frac{1}{\eta_Z}}\left(x_t^{Z,H}\right)^{\frac{\eta_Z-1}{\eta_Z}} + o_Z^{\frac{1}{\eta_Z}}\left(x_t^{Z,F}\right)^{\frac{\eta_Z-1}{\eta_Z}}\right]^{\frac{\eta_Z}{\eta_Z-1}} \tag{EE.31}$$

$$x_t^{Z,F} = o_Z\left(p_t^F/p_t^Z\right)^{-\eta_Z}c_t^Z \tag{EE.32}$$

$$x_t^{Z,H} = (1-o_Z)\left(p_t^H/p_t^Z\right)^{-\eta_Z}c_t^Z \tag{EE.33}$$

$$c_t^A = z_t^A\left[(1-o_A)^{\frac{1}{\eta_A}}\left(x_t^{A,H}\right)^{\frac{\eta_A-1}{\eta_A}} + o_A^{\frac{1}{\eta_A}}\left(x_t^{A,F}\right)^{\frac{\eta_A-1}{\eta_A}}\right]^{\frac{\eta_A}{\eta_A-1}} \tag{EE.34}$$

$$x_t^{A,F} = \left(z_t^A\right)^{\eta_A - 1} o_A \left(p_t^F / p_t^A\right)^{-\eta_A} c_t^A \tag{EE.35}$$

$$x_t^{A,H} = \left(z_t^A\right)^{\eta_A - 1} (1 - o_A) \left(p_t^H / p_t^A\right)^{-\eta_A} c_t^A \tag{EE.36}$$

$$c_t^G = \left[(1 - o_{CG})^{\frac{1}{\eta_{CG}}} \left(x_t^{CG,H}\right)^{\frac{\eta_{CG}-1}{\eta_{CG}}} + o_{CG}^{\frac{1}{\eta_{CG}}} \left(x_t^{CG,F}\right)^{\frac{\eta_{CG}-1}{\eta_{CG}}}\right]^{\frac{\eta_{CG}}{\eta_{CG}-1}} \tag{EE.37}$$

$$x_t^{CG,F} = o_{CG} \left(p_t^F / p_t^{CG}\right)^{-\eta_{CG}} c_t^G \tag{EE.38}$$

$$x_t^{CG,H} = (1 - o_{CG}) \left(p_t^H / p_t^{CG}\right)^{-\eta_{CG}} c_t^G \tag{EE.39}$$

$$i_t^f = \left[(1 - o_I)^{\frac{1}{\eta_I}} \left(x_t^{I,H}\right)^{\frac{\eta_I-1}{\eta_I}} + o_I^{\frac{1}{\eta_I}} \left(x_t^{I,F}\right)^{\frac{\eta_I-1}{\eta_I}}\right]^{\frac{\eta_I}{\eta_I-1}} \tag{EE.40}$$

$$x_t^{I,F} = o_I \left(p_t^F / p_t^I\right)^{-\eta_I} i_t^f \tag{EE.41}$$

$$x_t^{I,H} = (1 - o_I) \left(p_t^H / p_t^I\right)^{-\eta_I} i_t^f \tag{EE.42}$$

$$i_t^f = i_t + \phi_{\bar{u}} \left(\bar{u}_t\right) \frac{k_{t-1}}{a_{t-1}} \tag{EE.43}$$

$$i_t^{Co,f} = \left[(1 - o_{Co})^{\frac{1}{\eta_{Co}}} \left(x_t^{Co,H}\right)^{\frac{\eta_{Co}-1}{\eta_{Co}}} + o_{Co}^{\frac{1}{\eta_{Co}}} \left(x_t^{Co,F}\right)^{\frac{\eta_{Co}-1}{\eta_{Co}}}\right]^{\frac{\eta_{Co}}{\eta_{Co}-1}} \tag{EE.44}$$

$$x_t^{Co,F} = o_{Co} \left(p_t^F / p_t^{ICo}\right)^{-\eta_{Co}} i_t^{Co,f} \tag{EE.45}$$

$$x_t^{Co,H} = (1 - o_{Co}) \left(p_t^H / p_t^{ICo}\right)^{-\eta_{Co}} i_t^{Co,f} \tag{EE.46}$$

$$i_t^G = \left[(1 - o_{IG})^{\frac{1}{\eta_{IG}}} \left(x_t^{IG,H}\right)^{\frac{\eta_{IG}-1}{\eta_{IG}}} + o_{IG}^{\frac{1}{\eta_{IG}}} \left(x_t^{IG,F}\right)^{\frac{\eta_{IG}-1}{\eta_{IG}}}\right]^{\frac{\eta_{IG}}{\eta_{IG}-1}} \tag{EE.47}$$

$$x_t^{IG,F} = o_{IG} \left(p_t^F / p_t^{IG}\right)^{-\eta_{IG}} i_t^G \tag{EE.48}$$

$$x_t^{IG,H} = (1 - o_{IG}) \left(p_t^H / p_t^{IG}\right)^{-\eta_{IG}} i_t^G \tag{EE.49}$$

from Differentiated Varieties (2.3.3):

$$\begin{aligned} f_t^H \quad = \quad & \left(\tilde{p}_t^H\right)^{-\epsilon_H} y_t^H mc_t^H + \frac{\beta}{a_t^{\sigma-1}} \theta_H \\ & \times E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} \left(\frac{g_t^{\Gamma^H}}{\pi_{t+1}} \frac{\tilde{p}_t^H}{\tilde{p}_{t+1}^H} \right)^{-\epsilon_H} \left(\frac{p_t^H}{p_{t+1}^H} \frac{\left(1+\tau_{t+1}^C\right)}{\left(1+\tau_t^C\right)} \right)^{-1-\epsilon_H} f_{t+1}^H \right\} \end{aligned} \tag{EE.50}$$

$$\begin{aligned} f_t^H \quad = \quad & \left(\tilde{p}_t^H\right)^{1-\epsilon_H} y_t^H \left(\frac{\epsilon_H - 1}{\epsilon_H}\right) + \frac{\beta}{a_t^{\sigma-1}} \theta_H \\ & \times E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} \left(\frac{g_t^{\Gamma^H}}{\pi_{t+1}} \frac{\tilde{p}_t^H}{\tilde{p}_{t+1}^H} \right)^{1-\epsilon_H} \left(\frac{p_t^H}{p_{t+1}^H} \frac{\left(1+\tau_{t+1}^C\right)}{\left(1+\tau_t^C\right)} \right)^{-\epsilon_H} f_{t+1}^H \right\} \end{aligned} \tag{EE.51}$$

$$1 = (1 - \theta_H) \left(\tilde{p}_t^H\right)^{1-\epsilon_H} + \theta_H \left(\frac{p_{t-1}^H}{p_t^H} \frac{g_{t-1}^{\Gamma^H}}{\pi_t} \frac{\left(1+\tau_t^C\right)}{\left(1+\tau_{t-1}^C\right)} \right)^{1-\epsilon_H} \tag{EE.52}$$

$$mc_t^H = \frac{p_t^{\widetilde{H}}}{p_t^H} \tag{EE.53}$$

$$g_t^{\Gamma^H} = \pi_t^{\vartheta_H} \pi^{1-\vartheta_H} \tag{EE.54}$$

$$\begin{aligned} f_t^F &= \left(\tilde{p}_t^F\right)^{-\epsilon_F} y_t^F mc_t^F + \frac{\beta}{a_t^{\sigma-1}} \theta_F \\ &\times E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} \left(\frac{g_t^{\Gamma^F}}{\pi_{t+1}} \frac{\tilde{p}_t^F}{\tilde{p}_{t+1}^F} \right)^{-\epsilon_F} \left(\frac{p_t^F}{p_{t+1}^F} \frac{\left(1+\tau_{t+1}^C\right)}{\left(1+\tau_t^C\right)} \right)^{-1-\epsilon_F} f_{t+1}^F \right\} \end{aligned} \tag{EE.55}$$

$$\begin{aligned} f_t^F &= \left(\tilde{p}_t^F\right)^{1-\epsilon_F} y_t^F \left(\frac{\epsilon_F - 1}{\epsilon_F} \right) + \frac{\beta}{a_t^{\sigma-1}} \theta_F \\ &\times E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} \left(\frac{g_t^{\Gamma^F}}{\pi_{t+1}} \frac{\tilde{p}_t^F}{\tilde{p}_{t+1}^F} \right)^{1-\epsilon_F} \left(\frac{p_t^F}{p_{t+1}^F} \frac{\left(1+\tau_{t+1}^C\right)}{\left(1+\tau_t^C\right)} \right)^{-\epsilon_F} f_{t+1}^F \right\} \end{aligned} \tag{EE.56}$$

$$1 = (1-\theta_F)\left(\tilde{p}_t^F\right)^{1-\epsilon_F} + \theta_F \left(\frac{p_{t-1}^F}{p_t^F} \frac{g_{t-1}^{\Gamma^F}}{\pi_t} \frac{\left(1+\tau_t^C\right)}{\left(1+\tau_{t-1}^C\right)} \right)^{1-\epsilon_F} \tag{EE.57}$$

$$mc_t^F = \frac{p^{M*} rer_t}{p_t^F} \tag{EE.58}$$

$$g_t^{\Gamma^F} = \pi_t^{\vartheta_F} \pi^{1-\vartheta_F} \tag{EE.59}$$

$$\begin{aligned} f_t^{H*} &= \left(\tilde{p}_t^{H*}\right)^{-\epsilon_{H*}} y_t^{H*} mc_t^{H*} + \frac{\beta}{a_t^{\sigma-1}} \theta_{H*} \\ &\times E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} \frac{rer_{t+1}}{rer_t} \left(\frac{g_t^{\Gamma^{H*}}}{\pi_{t+1}^*} \frac{\tilde{p}_t^{H*}}{\tilde{p}_{t+1}^{H*}} \right)^{-\epsilon_{H*}} \left(\frac{p_t^{H*}}{p_{t+1}^{H*}} \right)^{-1-\epsilon_{H*}} f_{t+1}^{H*} \right\} \end{aligned} \tag{EE.60}$$

$$\begin{aligned} f_t^{H*} &= \left(\tilde{p}_t^{H*}\right)^{1-\epsilon_{H*}} y_t^{H*} \left(\frac{\epsilon_{H*} - 1}{\epsilon_{H*}} \right) + \frac{\beta}{a_t^{\sigma-1}} \theta_{H*} \\ &\times E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} \frac{rer_{t+1}}{rer_t} \left(\frac{g_t^{\Gamma^{H*}}}{\pi_{t+1}^*} \frac{\tilde{p}_t^{H*}}{\tilde{p}_{t+1}^{H*}} \right)^{1-\epsilon_{H*}} \left(\frac{p_t^{H*}}{p_{t+1}^{H*}} \right)^{-\epsilon_{H*}} f_{t+1}^{H*} \right\} \end{aligned} \tag{EE.61}$$

$$1 = (1-\theta_{H*})\left(\tilde{p}_t^{H*}\right)^{1-\epsilon_{H*}} + \theta_{H*} \left(\frac{p_{t-1}^{H*}}{p_t^{H*}} \frac{g_{t-1}^{\Gamma^{H*}}}{\pi_t^*} \right)^{1-\epsilon_{H*}} \tag{EE.62}$$

$$mc_t^{H*} = \frac{p_t^{\widetilde{H}}}{rer_t p_t^{H*}} \tag{EE.63}$$

$$g_t^{\Gamma^{H*}} = \left(\pi_t^*\right)^{\vartheta_{H*}} \left(\pi^*\right)^{1-\vartheta_{H*}} \tag{EE.64}$$

from Wholesale Domestic Goods (2.3.4):

$$y_t^{\widetilde{H}} = z_t \left[(1-o_O)^{\frac{1}{\eta_O}} \left(x_t^{\widetilde{Z}} \right)^{\frac{\eta_O - 1}{\eta_O}} + o_O^{\frac{1}{\eta_O}} \left(x_t^O \right)^{\frac{\eta_O - 1}{\eta_O}} \right]^{\frac{\eta_O}{\eta_O - 1}} \tag{EE.65}$$

$$x_t^{\widetilde{Z}} = (z_t)^{\eta_O - 1} (1 - o_O) \left(\frac{mc_t^{\widetilde{Z}}}{p_t^{\widetilde{H}}} \right)^{-\eta_O} y_t^{\widetilde{H}} \tag{EE.66}$$

$$x_t^O = (z_t)^{\eta_O - 1} o_O \left(\frac{p_t^O}{p_t^{\widetilde{H}}} \right)^{-\eta_O} y_t^{\widetilde{H}} \tag{EE.67}$$

$$y_t^{\widetilde{Z}} = \left(\widetilde{k}_t \right)^{\alpha} \left(a_t \nabla_t^H n_t h_t \right)^{1-\alpha} \tag{EE.68}$$

$$\widetilde{k}_t = \alpha \left(\frac{r_t^{\widetilde{K}}}{mc_t^{\widetilde{Z}}} \right)^{-1} y_t^{\widetilde{Z}} \tag{EE.69}$$

$$\widetilde{k}_t = \left[(1 - o_{KG})^{\frac{1}{\eta_{KG}}} \left(k_t^S \right)^{\frac{\eta_{KG}-1}{\eta_{KG}}} + o_{KG}{}^{\frac{1}{\eta_{KG}}} \left(\frac{k_{t-1}^G}{a_{t-1}} \right)^{\frac{\eta_{KG}-1}{\eta_{KG}}} \right]^{\frac{\eta_{KG}}{\eta_{KG}-1}} \tag{EE.70}$$

$$k_t^S = (1 - o_{KG}) \left(\frac{r_t^K}{r_t^{\widetilde{K}}} \right)^{-\eta_{KG}} \widetilde{k}_t \tag{EE.71}$$

$$\frac{\nabla_{t-1}^H p_t^H \Omega_v}{e_t} = \frac{\beta}{a_t^{\sigma-1}} E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} (1 - \rho_{t+1}) \left(mc_{t+1}^{\widetilde{Z}} (1-\alpha) \frac{y_{t+1}^{\widetilde{Z}}}{n_{t+1}} - w_{t+1} h_{t+1} - p_{t+1}^H h_{t+1}^C + \frac{\nabla_t^H p_{t+1}^H \Omega_v}{e_{t+1}} \right) \right\} \tag{EE.72}$$

$$\nabla_{t-1}^H p_t^H \bar{c}_t = mc_t^{\widetilde{Z}} (1-\alpha) \frac{y_t^{\widetilde{Z}}}{n_t} - w_t h_t + \frac{\nabla_{t-1}^H p_t^H \Omega_v}{e_t} \tag{EE.73}$$

$$h_t^C = \nabla_{t-1}^H \frac{\exp\left(\frac{\sigma_{\bar{c}}^2}{2} \right) \Phi \left(\frac{\ln \bar{c}_t - \sigma_{\bar{c}}^2}{\sigma_{\bar{c}}} \right)}{1 - \rho_t^n} \tag{EE.74}$$

from Wages and Hours (2.3.5):

$$h_t = \left[\frac{mc_t^{\widetilde{Z}} (1-\alpha)^2 \frac{y_t^{\widetilde{Z}}}{n_t}}{\frac{\psi_t^U \kappa_t}{(1-\tau_t^L)} \left(\nabla_{t-1}^H \right)^{1-\sigma}} \right]^{\frac{1}{1+\phi}} \tag{EE.75}$$

$$\begin{aligned} w_t^n h_t = {} & \varphi^U \left[mc_t^{\widetilde{Z}} (1-\alpha) \frac{y_t^{\widetilde{Z}}}{n_t} - p_t^H h_t^C + \nabla_{t-1}^H \frac{p_t^H \Omega_v}{e_t} \right] \\ & + \frac{(1-\varphi^U)}{(1-\tau_t^L)} \left[ub + \psi_t^U \kappa_t \left(\nabla_{t-1}^H \right)^{1-\sigma} \frac{h_t^{1+\phi}}{1+\phi} - (1-s_t) \sigma_t^U \right] \end{aligned} \tag{EE.76}$$

$$\sigma_t^U = \left(1 - \omega^U\right) E_t \left\{ \frac{\beta}{a_t^{\sigma}} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} (1 - \rho_{t+1}) s_{t+1}^{R,U} \right\} + \omega^U E_t \left\{ \frac{\beta}{a_t^{\sigma}} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^{NR}}{\lambda_t^{NR}} (1 - \rho_{t+1}) s_{t+1}^{NR,U} \right\} \tag{EE.77}$$

$$s_t^{R,U} = (1-\tau_t^L)w_t^n h_t - \frac{\Theta_t^R \kappa_t \left(\nabla_{t-1}^H\right)^{1-\sigma} \frac{h_t^{1+\phi}}{1+\phi}}{\lambda_t^R} - ub + (1-s_t) E_t \left\{ \frac{\beta}{a_t^\sigma} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} (1-\rho_{t+1}) s_{t+1}^{R,U} \right\} \tag{EE.78}$$

$$s_t^{NR,U} = (1-\tau_t^L)w_t^n h_t - \frac{\Theta_t^{NR} \kappa_t \left(\nabla_{t-1}^H\right)^{1-\sigma} \frac{h_t^{1+\phi}}{1+\phi}}{\lambda_t^{NR}} - ub + (1-s_t) E_t \left\{ \frac{\beta}{a_t^\sigma} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^{NR}}{\lambda_t^{NR}} (1-\rho_{t+1}) s_{t+1}^{NR,U} \right\} \tag{EE.79}$$

$$\pi_t w_t = \varkappa_W \Gamma_{t-1}^W \frac{w_{t-1}}{a_{t-1}} \frac{(1+\tau_t^C)}{(1+\tau_{t-1}^C)} + (1-\varkappa_W)\pi_t w_t^n \tag{EE.80}$$

$$\Gamma_t^W = a\pi_t^{\vartheta_W} \pi^{1-\vartheta_W} \tag{EE.81}$$

$$\psi_t^U = \left(1-\omega^U\right) \frac{\Theta_t^R}{\lambda_t^R} + \omega^U \frac{\Theta_t^{NR}}{\lambda_t^{NR}} \tag{EE.82}$$

from Commodity sector investment and output (2.3.6):

$$y_t^{Co} = z_t^{Co} \left(\bar{u}_t^{Co} \frac{k_{t-1}^{Co}}{a_{t-1}} \right)^{\alpha Co} \left(a_t \nabla_t^{Co} \overline{L} \right)^{1-\alpha Co} \tag{EE.83}$$

$$k_t^{Co} = (1-\delta_{Co}) \frac{k_{t-1}^{Co}}{a_{t-1}} + \left(1 - \frac{\Phi_I^{Co}}{2} \left(\frac{i_{t-N_{Co}+1}^{ACo}}{i_{t-N_{Co}}^{ACo}} a_{t-N_{Co}} - a \right)^2 \right) \frac{i_{t-N_{Co}+1}^{ACo} a_t}{\prod_{i=1}^{N_{Co}} a_{t+1-i}} \varpi_{t-N_{Co}+1}^{Co} \tag{EE.84}$$

$$i_t^{Co} = \varphi_0^{Co} i_t^{ACo} + \frac{\varphi_1^{Co} i_{t-1}^{ACo}}{a_{t-1}} + \frac{\varphi_2^{Co} i_{t-2}^{ACo}}{a_{t-1}a_{t-2}} + \ldots + \frac{\varphi_{N_{Co}-1}^{Co} i_{t-N_{Co}+1}^{ACo}}{a_{t-1}a_{t-2}\ldots a_{t-N_{Co}+1}} \tag{EE.85}$$

$$\phi_{\bar{u}}^{Co}\left(\bar{u}_t^{Co}\right) \equiv \frac{r^{k,Co}}{\Phi_{\bar{u}}^{Co}} \left(e^{\Phi_{\bar{u}}^{Co}\left(\bar{u}_t^{Co}-1\right)} - 1 \right) \tag{EE.86}$$

$$i_t^{Co,f} = i_t^{Co} + \phi_{\bar{u}}^{Co}\left(\bar{u}_t^{Co}\right) \frac{k_{t-1}^{Co}}{a_{t-1}} \tag{EE.87}$$

$$cf_t^{Co} = rer_t p_t^{Co*} y_t^{Co} - p_t^{ICo} i_t^{Co.f} \tag{EE.88}$$

$$\lambda_t^{Co} = \left[1 - \tau_t^{Co}\left(1-\chi^{Co}\right)\right] rer_t p_t^{Co*} \tag{EE.89}$$

$$q_t^{Co} = E_t \left\{ \frac{\pi_{t+1}}{R_t} \left[\lambda_{t+1}^{Co} \alpha^{Co} \frac{y_{t+1}^{Co}}{k_t^{Co}} a_t + q_{t+1}^{Co}(1-\delta_{Co}) - p_{t+1}^{ICo} \phi_{\bar{u}}^{Co}\left(\bar{u}_{t+1}^{Co}\right) \frac{k_t^{Co}}{a_t} \right] \right\} \tag{EE.90}$$

$$\begin{aligned} 0 = {} & E_t \left\{ \frac{\sum_{j=0}^{N^{Co}-1} \beta^j \frac{\varrho_{t+j}}{\varrho_t} \frac{\lambda_{t+j}^R}{\lambda_t^R} \left(\frac{a_{t-1}}{\prod_{i=0}^{j} a_{t+i-1}} \right)^\sigma \varphi_j^{Co} p_{t+j}^{ICo}}{\left(\beta^{N^{Co}-1}\right) \frac{\varrho_{t+N^{Co}-1}}{\varrho_t} \frac{\lambda_{t+N^{Co}-1}^R}{\lambda_t^R} \left(\frac{a_{t-1}}{\prod_{i=0}^{N^{Co}-1} a_{t+i-1}} \right)^\sigma q_{t+N^{Co}-1}^{Co}} \right\} \\ & - \left[\left(1 - \frac{\Phi_I^{Co}}{2} \left(\frac{i_t^{ACo}}{i_{t-1}^{ACo}} a_{t-1} - a \right)^2 \right) - \Phi_I^{Co} \left(\frac{i_t^{ACo}}{i_{t-1}^{ACo}} a_{t-1} - a \right) \frac{i_t^{ACo}}{i_{t-1}^{ACo}} a_{t-1} \right] \varpi_t^{Co} \\ & - E_t \left\{ \begin{array}{l} \beta \frac{\varrho_{t+N^{Co}}}{\varrho_{t+N^{Co}-1}} \frac{\lambda_{t+N^{Co}}^R}{a_{t+N^{Co}}^\sigma \lambda_{t+N^{Co}-1}^R} \frac{q_{t+N^{Co}}^{Co}}{q_{t+N^{Co}-1}^{Co}} \\ \times \Phi_I^{Co} \left(\frac{i_{t+1}^{ACo}}{i_t^{ACo}} a_t - a \right) \left(\frac{i_{t+1}^{ACo}}{i_t^{ACo}} a_t \right)^2 \varpi_{t+1}^{Co} \end{array} \right\} \end{aligned} \tag{EE.91}$$

$$r_t^{k,Co} = \lambda_t^{Co}\alpha^{Co}\frac{y_t^{Co}}{\bar{u}_t^{Co}k_{t-1}^{Co}}a_{t-1} \tag{EE.92}$$

$$\bar{u}_t^{Co} = 1 + \frac{\log\left(r_t^{k,Co}/r^{k,Co}\right) - \log\left(p_t^{ICo}\right)}{\Phi_{\bar{u}}^{Co}} \tag{EE.93}$$

from Fiscal Policy (2.4):

$$g_t = p_t^{CG}c_t^G + p_t^{IG}i_t^G + tr_t^G + \left(rer_t p_t^{O*} - p^O\right)o_t \tag{EE.94}$$

$$t_t = \alpha^T p_t^Y y + (1 - I_{rule})\,\epsilon^T\left(rer_t b^{G^*} + b^G - rer_t b_t^{G^*} - b_t^G\right) \tag{EE.95}$$

$$\omega t_t^{NR} = \omega_G t_t \tag{EE.96}$$

$$(1-\omega)\,t_t^R = (1-\omega_G)\,t_t \tag{EE.97}$$

$$\tau_t = \tau_t^C c_t + \tau_t^W w_t n_t h_t + \tau_t^K\left[r_t^K\bar{u}_t - p_t^I\left(\delta + \phi_{\bar{u}}\left(\bar{u}_t\right)\right)\right]\frac{k_{t-1}}{a_{t-1}} + \tau_t^D d_t + (1-\chi^{Co})\tau_t^{Co}(cf_t^{Co} + p_t^{ICo}i_t^{Co,f}) + t_t \tag{EE.98}$$

$$b_t^G + rer_t b_t^{G^*} = R_{t-1}\frac{b_{t-1}^G}{\pi_t a_{t-1}} + R_{t-1}^*\xi_{t-1}\frac{rer_t b_{t-1}^{G*}}{\pi_t^* a_{t-1}} + \tau_t + \chi^{Co}cf_t^{Co} - g_t \tag{EE.99}$$

$$rer_t b_t^{G^*} = \alpha^D\left(rer_t b_t^{G^*} + b_t^G\right) \tag{EE.100}$$

$$\widetilde{g}_t = \left(\widetilde{g}_t^{rule}\right)^{I_{rule}}\left(\widetilde{g}_t^{exo}\right)^{1-I_{rule}} \tag{EE.101}$$

$$\widetilde{g}_t^{exo} = \overline{g}\xi_t^G \tag{EE.102}$$

$$\widetilde{g}_t^{rule} = \frac{(R_{t-1}-1)b_{t-1}^G}{\pi_t a_{t-1}} + \frac{(R_{t-1}^*\xi_{t-1}-1)rer_t b_{t-1}^{G*}}{\pi_t^* a_{t-1}} + \tau_t - \gamma^D\breve{\tau}_t + \chi^{Co}\left(cf_t^{Co} - \gamma^D\breve{cf}_t^{Co}\right) - \bar{s}_B p_t^Y y_t \tag{EE.103}$$

$$\breve{\tau}_t = \tau_t - \widetilde{\tau}_t \tag{EE.104}$$

$$\widetilde{\tau}_t = \tau_t^C c + \tau_t^W wnh + \tau_t^K\left(r^K\overline{u} - p^I\left(\delta + \phi_{\bar{u}}\left(\bar{u}\right)\right)\right)k/a + \tau_t^D d + (1-\chi^{Co})\tau_t^{Co}(cf^{Co} + p^{ICo}i^{Co,f}) + t_t \tag{EE.105}$$

$$\breve{cf}_t^{Co} = cf_t^{Co} - \widetilde{cf}_t^{Co} \tag{EE.106}$$

$$\widetilde{cf}_t^{Co} = rer_t\widetilde{p}_t^{Co*}y_t^{Co} - p_t^{ICo}i_t^{Co,f} \tag{EE.107}$$

$$\log\left(\widetilde{p}_t^{Co*}\right) = \frac{1}{40}E_t\sum_{i=1}^{40}\log\left(p_{t+i}^{Co*}\right) \tag{EE.108}$$

$$p_t^{CG}c_t^G = \alpha_{CG}\widetilde{g}_t\xi_t^{CG} \tag{EE.109}$$

$$tr_t^G = \left(1 - \alpha_{CG} - \alpha_{IG} - \frac{tr^{UFA}}{g}\right)\widetilde{g}_t\xi_t^{TR} + tr_t^{UFA} \tag{EE.110}$$

$$\omega tr_t^{NR} = \omega_G\left(tr_t^G - tr_t^{UFA}\right) \tag{EE.111}$$

$$(1-\omega)\, tr_t^R = (1-\omega_G)\left(tr_t^G - tr_t^{UFA}\right) \tag{EE.112}$$

$$k_t^G = (1-\delta_G)\frac{k_{t-1}^G}{a_{t-1}} + \frac{i_{t-N^G+1}^{AG}a_t}{\prod_{i=1}^{N^G}a_{t+1-i}} \tag{EE.113}$$

$$i_t^G = \varphi_0 i_t^{AG} + \frac{\varphi_1 i_{t-1}^{AG}}{a_{t-1}} + \frac{\varphi_2 i_{t-2}^{AG}}{a_{t-1}a_{t-2}} + \ldots + \frac{\varphi_{N^G-1}i_{t-N^G+1}^{AG}}{a_{t-1}a_{t-2}\ldots a_{t-N^G+1}} \tag{EE.114}$$

$$p_t^{IG}i_t^{AG} = \alpha_{IG}E_t\left[\sum_{j=0}^{N^G-1}\varphi_j\frac{\widetilde{g}_{t+j}\prod_{i=0}^{j}a_{t+i-1}}{a_{t-1}}\right]\xi_t^{IG} \tag{EE.115}$$

$$p_t^O = \left(\left(p^O\right)^{1-\alpha_O}\left(p_{t-1}^O\right)^{\alpha_O}\right)^{\rho_O}\left(rer_t p_t^{O^*}\right)^{1-\rho_O}\xi_t^O \tag{EE.116}$$

from Monetary Policy (2.5):

$$R_t = (R_{t-1})^{\rho_R}\left[\overline{R}_t\left(\frac{\widetilde{\pi}_t}{\overline{\pi}_t}\right)^{\alpha_\pi}\left(\frac{y_t}{y_{t-1}}\right)^{\alpha_y}\right]^{1-\rho_R}\exp(\varepsilon_t^R) \tag{EE.117}$$

$$\widetilde{\pi}_t = \left[\left(\pi_t^Z\right)^{\alpha_{\pi Z}}(\pi_t)^{1-\alpha_{\pi Z}}\right]^{1-\alpha_{\pi E}}\left[\left(E_t\pi_{t+4}^Z\right)^{\alpha_{\pi Z}}\left(E_t\pi_{t+4}\right)^{1-\alpha_{\pi Z}}\right]^{\alpha_{\pi E}} \tag{EE.118}$$

$$\pi_t^Z = \frac{p_t^Z}{p_{t-1}^Z}\pi_t \tag{EE.119}$$

from Unemployment Funds Administrator (2.6):

$$b_t^{UFA} = \tau_t^{UFA}w_t h_t n_t - (1-n_t)\,ub + tr_t^{UFA} + R_{t-1}\frac{b_{t-1}^{UFA}}{\pi_t a_{t-1}} \tag{EE.120}$$

$$tr_t^{UFA} = \overline{tr}^{UFA} + \epsilon^{UFA}\left(b^{UFA} - b_t^{UFA}\right) \tag{EE.121}$$

from The Rest of the World (2.7):

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^S\pi_t^*}{\pi_t}, \tag{EE.122}$$

$$x_t^{H*} = \left[x_{t-1}^{H*}\right]^{\rho_{XH*}}\left[o^*\left(p_t^{H*}\right)^{-\eta^*}y_t^*\right]^{1-\rho_{XH*}}\xi_t^{XH*} \tag{EE.123}$$

$$y_t^* = a_t z_t^* \tag{EE.124}$$

$$\nabla_t^H = \left(\frac{a}{a_t}\nabla_{t-1}^H\right)^{1-\Gamma^H} \tag{EE.125}$$

$$\nabla_t^{Co} = \left(\frac{a}{a_t}\nabla_{t-1}^{Co}\right)^{1-\Gamma^{Co}} \tag{EE.126}$$

$$\pi_t^{F*} = \frac{f_t^*}{f_{t-1}^*}\pi_t^* \tag{EE.127}$$

$$1 = \left(\frac{\pi^*}{\pi_t^*}\right)^{1-\Gamma^*} (f_t^*)^{\Gamma^*} \xi_t^* \tag{EE.128}$$

$$p_t^{Co*} = \left(\frac{\pi^*}{\pi_t^*} p_{t-1}^{Co*}\right)^{1-\Gamma^{Co*}} (f_t^*)^{\Gamma^{Co*}} \xi_t^{Co*} \tag{EE.129}$$

$$p_t^{O*} = \left(\frac{\pi^*}{\pi_t^*} p_{t-1}^{O*}\right)^{1-\Gamma^{O*}} (f_t^*)^{\Gamma^{O*}} \xi_t^{O*} \tag{EE.130}$$

$$p_t^{M*} = \left(\frac{\pi^*}{\pi_t^*} p_{t-1}^{M*}\right)^{1-\Gamma^{M*}} (f_t^*)^{\Gamma^{M*}} \xi_t^{M*} \tag{EE.131}$$

$$\pi_t^{Co*} = \frac{p_t^{Co*}}{p_{t-1}^{Co*}} \pi_t^* \tag{EE.132}$$

$$\pi_t^{O*} = \frac{p_t^{O*}}{p_{t-1}^{O*}} \pi_t^* \tag{EE.133}$$

$$\pi_t^{M*} = \frac{p_t^{M*}}{p_{t-1}^{M*}} \pi_t^* \tag{EE.134}$$

from Aggregation and Market Clearing (2.8):

$$c_t = \omega c_t^{NR} + (1-\omega) c_t^R \tag{EE.135}$$

$$k_t = (1-\omega) k_t^R \tag{EE.136}$$

$$k_t^S = (1-\omega) k_t^{S,R} \tag{EE.137}$$

$$i_t = (1-\omega) i_t^R \tag{EE.138}$$

$$b_t^{Pr} = (1-\omega) b_t^R \tag{EE.139}$$

$$b_t^{Pr*} = (1-\omega) b_t^{R*} \tag{EE.140}$$

$$d_t = (1-\omega) d_t^R \tag{EE.141}$$

$$y_t^H = x_t^H \tag{EE.142}$$

$$x_t^L = h_t^C n_t + \nabla_{t-1}^H \Omega_v v_t \tag{EE.143}$$

$$x_t^H = x_t^{Z,H} + x_t^{A,H} + x_t^{CG,H} + x_t^{I,H} + x_t^{ICo,H} + x_t^{IG,H} + x_t^L, \tag{EE.144}$$

$$y_t^F = x_t^F \tag{EE.145}$$

$$x_t^F = x_t^{Z,F} + x_t^{A,F} + x_t^{CG,F} + x_t^{I,F} + x_t^{ICo,F} + x_t^{IG,F}, \tag{EE.146}$$

$$y_t^{H*} = x_t^{H*} \tag{EE.147}$$

$$y_t^{\tilde{H}} = y_t^H \Delta_t^H + y_t^{H*} \Delta_t^{H*} \tag{EE.148}$$

$$m_t = y_t^F \Delta_t^F \tag{EE.149}$$

$$\Delta_t^H = (1-\theta_H)\left(\tilde{p}_t^H\right)^{-\epsilon_H} + \theta_H \left(\frac{p_{t-1}^H}{p_t^H}\frac{g_{t-1}^{\Gamma^H}}{\pi_t}\frac{\left(1+\tau_t^C\right)}{\left(1+\tau_{t-1}^C\right)}\right)^{-\epsilon_H} \Delta_{t-1}^H \tag{EE.150}$$

$$\Delta_t^F = (1-\theta_F)\left(\tilde{p}_t^F\right)^{-\epsilon_F} + \theta_F \left(\frac{p_{t-1}^F}{p_t^F}\frac{g_{t-1}^{\Gamma^F}}{\pi_t}\frac{\left(1+\tau_t^C\right)}{\left(1+\tau_{t-1}^C\right)}\right)^{-\epsilon_F} \Delta_{t-1}^F \tag{EE.151}$$

$$\Delta_t^{H*} = (1-\theta_{H*})\left(\tilde{p}_t^{H*}\right)^{-\epsilon_{H*}} + \theta_{H*} \left(\frac{p_{t-1}^{H*}}{p_t^{H*}}\frac{g_{t-1}^{\Gamma^{H*}}}{\pi_t^*}\right)^{-\epsilon_{H*}} \Delta_{t-1}^{H*} \tag{EE.152}$$

$$o_t = c_t^O + x_t^O \tag{EE.153}$$

$$y_t^C = c_t + p_t^{CG} c_t^G + p_t^I i_t^f + p_t^{ICo} i_t^{Co,f} + p_t^{IG} i_t^G + p_t^H x_t^L \tag{EE.154}$$

$$tb_t = rer_t \left(p_t^{H*} y_t^{H*} + p_t^{Co*} y_t^{Co} - p_t^{M*} m_t - p^{O*} o_t\right) \tag{EE.155}$$

$$y_t = c_t + c_t^G + i_t^f + i_t^G + i_t^{Co,f} + x_t^L + y_t^{H*} + y_t^{Co} - m_t - o_t \tag{EE.156}$$

$$p_t^Y y_t = y_t^C + tb_t \tag{EE.157}$$

$$d_t = p_t^Y y_t - rer_t p_t^{Co*} y_t^{Co} + \left(rer_t p_t^{O*} - p_t^O\right) o_t - r_t^K k_t^S - w_t n_t h_t - p_t^H x_t^L \tag{EE.158}$$

$$rer_t \left(b_t^* - \frac{b_{t-1}^*}{\pi_t^* a_{t-1}}\right) = rer_t \frac{b_{t-1}^*}{\pi_t^* a_{t-1}} \left(R_{t-1}^* \xi_{t-1} - 1\right) + tb_t + rer_t ren_t^* \tag{EE.159}$$

$$ren_t^* = (1-\omega)\, ren_t^{R*} - \left(1-\chi^{Co}\right) \frac{cf_t^{Co} - \tau_t^{Co}(cf_t^{Co} + p_t^{ICo} i_t^{Co,f})}{rer_t} \tag{EE.160}$$

$$ren_t^{R*} = \overline{ren}^{R*} \xi_t^{ren} \tag{EE.161}$$

$$b_t^P + b_t^G + b_t^{UFA} = 0 \tag{EE.162}$$

The exogenous processes for

$$X = \left\{a, \pi^*, R^*, \varpi, \varrho, \xi_t^G, \xi_t^{CG}, \varpi^{Co}, \xi_t^{IG}, \xi_t^{pO}, \xi_t^{PCo*}, \xi_t^{PM*}, \xi_t^{PO*}, \xi_t^{TR}, \xi_t^{REN}, \xi_t^{Y^{H*}}, z, z^A, z^{Co}, \zeta^O, \zeta^U, z^*, \kappa, m, \rho^x\right\}$$

are $\log\left(X_t/\bar{X}\right) = \rho_X \log\left(X_{t-1}/\bar{X}\right) + \varepsilon_t^X$, where the ε_t^X are i.i.d. shocks, $\rho_X \in (0,1)$ and $\bar{X} > 0$.

In the specification with Calvo wages, we replace equations EE.21, EE.23, EE.24, EE.25, EE.26, EE.72, EE.73, EE.74, EE.75, EE.76, EE.77, EE.78, EE.79, EE.80, EE.81, EE.82, EE.120 and EE.121 with:

$$n_t = 1$$

$$x_t^L = 0$$

$$mc_t^W = \frac{\Theta_t^U \kappa_t \nabla_{t-1}^H \left(h_t^d\right)^\phi}{\left(1-\tau_t^W\right) w_t \lambda_t^U}$$

$$h_t^d = (1-\alpha)\left(\frac{w_t}{mc_t^{\widetilde{HZ}}}\right)^{-1} y_t^{\widetilde{HZ}}$$

$$f_t^W = mc_t^W \tilde{w}_t^{-\epsilon_W(1+\phi)} h_t^d + \frac{\beta}{a_t^{\sigma-1}} \theta_W \times E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^U}{\lambda_t^U} \left(\frac{g_t^{\Gamma^w}}{\pi_{t+1}} \frac{\tilde{w}_t}{\tilde{w}_{t+1}} \right)^{-\epsilon_W(1+\phi)} \left(\frac{w_t}{w_{t+1}} \right)^{-1-\epsilon_W(1+\phi)} f_{t+1}^W \right\}$$

$$f_t^W = \tilde{w}_t^{1-\epsilon_W} h_t^d \left(\frac{\epsilon_W - 1}{\epsilon_W} \right) + \frac{\beta}{a_t^{\sigma-1}} \theta_W \times E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^U}{\lambda_t^U} \left(\frac{g_t^{\Gamma^w}}{\pi_{t+1}} \frac{\tilde{w}_t}{\tilde{w}_{t+1}} \right)^{1-\epsilon_W} \left(\frac{w_t}{w_{t+1}} \right)^{-\epsilon_W} f_{t+1}^W \right\}$$

$$1 = (1-\theta_W)\tilde{w}_t^{1-\epsilon_W} + \theta_W \left(\frac{w_{t-1}}{w_t} \frac{g_{t-1}^{\Gamma^w}}{\pi_t} \right)^{1-\epsilon_W}$$

$$g_t^{\Gamma^w} = \pi_t^{\vartheta_W} \pi^{1-\vartheta_W}$$

$$h_t = h_t^d \Delta_t^W$$

$$\Delta_t^W = (1-\theta_W)\tilde{w}_t^{-\epsilon_W} + \theta_{{}_W} \left(\frac{g_{t-1}^{\Gamma^W}}{\pi_t^W} \right)^{-\epsilon_W} \Delta_{t-1}^W$$

$$\lambda_t^U = \omega^U \lambda_t^{NR} + \left(1-\omega^U\right) \lambda_t^R$$

$$\Theta_t^U \equiv \omega^U \Theta_t^{NR} + \left(1-\omega^U\right) \Theta_t^R$$

$$b_t^{UFA} = 0$$

$$tr_t^{UFA} = 0$$

B.2 Steady State

We show how to compute the steady state for given values of h,π^S,p^{O*},p^{Co},p^H, $s^{OC} = p^O o^C / c$,$\nabla^{k^{Co} y^{Co}} = \frac{q^{Co} k^{Co}}{rer \times p^{Co*} y^{Co}}$,$s^{iCo} = p^{ICo} i^{Co} / \left(p^Y y\right)$,$s^{Co} = rer \times p^{Co*} y^{Co} / \left(p^Y y\right)$,$s^{cg} = p^{CG} c^G / \left(p^Y y\right)$,$s^{ig} = p^{IG} i^G / \left(p^Y y\right)$,$s^{trG} = tr^G / \left(p^Y y\right)$, $s^{tb} = tb / \left(p^Y y\right)$, $def^{UFA} = \frac{(1-n)ub - \tau^{UFA} whn}{\tau^{UFA} whn}$, $s^{CA} = rer \times b^* \left(1 - \frac{1}{a\pi^*}\right) / \left(p^Y y\right)$, u, $\rho = p^{E,U} / \left(1 - p^{U,E}\right)$, $s_{\rho^x} = \rho^x / \rho$, and e and with the parameters $\overline{R}^*$,$\overline{\pi}^*$,$\overline{\chi}^{pO*}$,$\overline{\chi}^{pCo*}$,$\overline{\kappa}$,κ_O,ub,o_{KG},$\overline{z}^{Co}$,α_{Co},δ_{Co},α_{CG},α_{IG},α_T,o^*,$o_{\widehat{C}}$,$\overline{tr}^{UFA}$,ren^{R*} , ρ, ρ^x, v, $\mu_{\tilde{C}}$ and Ω_v determined endogenously, while the values of the remaining parameters are taken as given.

From the exogenous processes for

$$X^* = \left\{ a, \varpi, \varrho, \xi_t^{CG}, \xi_t^G, \varpi^{Co}, \xi_t^{IG}, \xi_t^{PM*}, \xi_t^{pO}, \xi_t^{REN}, \xi_t^{TR}, \xi_t^{Y^{H*}}, z, z^A, z^{Co}, \zeta^O, \zeta^U, z^* \right\}$$

we have that $X = \overline{X}$.

The steady state for the remaining endogenous variables is defined as the set of values for which all equations below hold. The system of equations is solved numerically. Starting from arbitrary values for $utCo$,k^G, h^C, k^S, $r^{\widetilde{K}}$, y_{va}^{h*}, o^C, and o^H, we iterate repeatedly through the set of equations until finding a fixed point.[31]

$$a^{Co} = a \tag{SS.1}$$

$$a^H = a \tag{SS.2}$$

$$\nabla^{Co} = \left(\frac{\widetilde{a}^{Co}}{a}\right)^{\frac{1-\Gamma^{Co}}{\Gamma^{Co}}} \tag{SS.3}$$

$$\nabla^H = \left(\frac{\widetilde{a}^H}{a}\right)^{\frac{1-\Gamma^H}{\Gamma^H}} \tag{SS.4}$$

$$\tau^L = \tau^W + \tau^{UFA}$$

$$\Theta_t^R = 1 \tag{SS.5}$$

$$\Theta_t^{NR} = 1 \tag{SS.6}$$

$$\tilde{p}^H = 1 \tag{SS.7}$$

$$\tilde{p}^F = 1 \tag{SS.8}$$

$$\Delta^H = (\tilde{p}^H)^{-\epsilon_H} \tag{SS.9}$$

$$\Delta^F = (\tilde{p}^H)^{-\epsilon_F} \tag{SS.10}$$

$$mc^H = \frac{\epsilon_H - 1}{\epsilon_H}\tilde{p}^H \tag{SS.11}$$

$$mc^F = \frac{\epsilon_F - 1}{\epsilon_F}\tilde{p}^F \tag{SS.12}$$

$$mc^{HZ} = \left[\frac{\left(z \times mc^H\right)^{1-\eta_{XO}} - o_{XO}\left(p^O/p^H\right)^{1-\eta_{XO}}}{1 - o_{XO}}\right]^{\frac{1}{1-\eta_{XO}}} \tag{SS.13}$$

$$mc^{\widetilde{HZ}} = p^H mc^{HZ} \tag{SS.14}$$

$$\pi = \bar{\pi}. \tag{SS.15}$$

$$g^{\Gamma^H} = \bar{\pi}. \tag{SS.16}$$

$$g^{\Gamma^F} = \bar{\pi}. \tag{SS.17}$$

$$R = a^{\sigma}\pi/\beta. \tag{SS.18}$$

$$\tilde{p}^{H*} = 1 \tag{SS.19}$$

$$\Delta^{H*} = \left(\tilde{p}^{H*}\right)^{-\epsilon_{H*}} \tag{SS.20}$$

[31] This is a rather fast process, usually converging after around 25 iterations for a tolerance of 10^{-8} on the root of the sum squared differences.

$$mc^{H*} = \frac{\epsilon_{H*} - 1}{\epsilon_{H*}} \tilde{p}^{H*} \tag{SS.21}$$

$$\kappa_O = s^{OC} \left(p^O\right)^{-\eta_C} \tag{SS.22}$$

$$p^{CA} = \frac{1}{z^A} \left[(1 - o_{XCA}) \left(p^H\right)^{1-\eta_{XCA}} + o_{XCA} \left(p^F\right)^{1-\eta_{XCA}} \right]^{\frac{1}{1-\eta_{XCA}}} \tag{SS.23}$$

$$p^{HX} = \frac{p^H}{mc^{HX}} \tag{SS.24}$$

$$p^F = \left[\frac{1}{o_{XC}} - \frac{1 - o_{XC}}{o_{XC}} \left(p^H\right)^{1-\eta_C} \right]^{\frac{1}{1-\eta_C}} \tag{SS.25}$$

$$rer = mc^F p^F \tag{SS.26}$$

$$p^{IPr} = \left[(1 - o_{XIPr}) \left(p^H\right)^{1-\eta_{XIPr}} + o_{XIPr} \left(p^F\right)^{1-\eta_{XIPr}} \right]^{\frac{1}{1-\eta_{XIPr}}} \tag{SS.27}$$

$$p^{CZ} = \left[(1 - o_{XCZ}) \left(p^H\right)^{1-\eta_{XCZ}} + o_{XCZ} \left(p^F\right)^{1-\eta_{XCZ}} \right]^{\frac{1}{1-\eta_{XCZ}}} \tag{SS.28}$$

$$p^{CG} = \left[(1 - o_{XCG}) \left(p^H\right)^{1-\eta_{XCG}} + o_{XCG} \left(p^F\right)^{1-\eta_{XCG}} \right]^{\frac{1}{1-\eta_{XCG}}} \tag{SS.29}$$

$$p^{IG} = \left[(1 - o_{XIG}) \left(p^H\right)^{1-\eta_{XIG}} + o_{XIG} \left(p^F\right)^{1-\eta_{XIG}} \right]^{\frac{1}{1-\eta_{XIG}}} \tag{SS.30}$$

$$p^{ICo} = \left[(1 - o_{XICo}) \left(p^H\right)^{1-\eta_{XICo}} + o_{XICo} \left(p^F\right)^{1-\eta_{XICo}} \right]^{\frac{1}{1-\eta_{XICo}}} \tag{SS.31}$$

$$p^{O^*} = {}^{p^O}\!/_{rer} \tag{SS.32}$$

$$p^{Co^*} = {}^{p^{Co}}\!/_{rer} \tag{SS.33}$$

$$\xi^{PCo*} = \left(p^{Co^*}\right)^{\Gamma_{PCo}} \tag{SS.34}$$

$$\xi^{PO*} = \left(p^{O^*}\right)^{\Gamma_{PO}} \tag{SS.35}$$

$$\overline{u} = 1 - \frac{\log\left(p^I\right)}{\Phi_u} \tag{SS.36}$$

$$q = p^I \varpi^{-1}. \tag{SS.37}$$

$$r^K = \frac{q\left(\frac{a^\sigma}{\beta} - 1 + \delta\right) - p^I \tau^K \delta}{(1 - \tau^K)\left(\bar{u} + \frac{1 - p^I}{\Phi_u}\right)} \tag{SS.38}$$

$$\phi^{\overline{u}} = \frac{r^K}{\Phi_u} \left[\exp\left(\Phi_u \left(\overline{u} - 1\right)\right) - 1 \right] \tag{SS.39}$$

$$R^* = a^\sigma \pi / (\beta \pi^S \xi). \tag{SS.40}$$

$$\pi^* = \pi / \pi^S \tag{SS.41}$$

$$\pi^{Co*} = \pi^* \tag{SS.42}$$

$$\pi^{O*} = \pi^* \tag{SS.43}$$

$$\pi^{M*} = \pi^* \tag{SS.44}$$

$$\tilde{\pi}^{Co*} = \pi^* \tag{SS.45}$$

$$\tilde{\pi}^{O*} = \pi^* \tag{SS.46}$$

$$\tilde{\pi}^{M*} = \pi^* \tag{SS.47}$$

$$p^{M*} = 1 \tag{SS.48}$$

$$n = 1 - u \tag{SS.49}$$

$$\rho^x = s_{\rho^x}\rho \tag{SS.50}$$

$$\rho^n = \frac{\rho - \rho^x}{1 - \rho^x} \tag{SS.51}$$

$$\overline{c} = e^{\sigma_{\bar{c}}\Phi^{-1}(1-\rho^n)} \tag{SS.52}$$

$$v = \frac{n\rho}{(1-\rho)\,e} \tag{SS.53}$$

$$m = e\left(\frac{v}{u}\right)^{\mu} \tag{SS.54}$$

$$s = m\left(\frac{v}{u}\right)^{1-\mu} \tag{SS.55}$$

$$\widetilde{k} = \left[(1-o_{KG})^{\frac{1}{\eta_{KG}}}\left(k^S\right)^{\frac{\eta_{KG}-1}{\eta_{KG}}} + o_{KG}{}^{\frac{1}{\eta_{KG}}}\left(k^G/a\right)^{\frac{\eta_{KG}-1}{\eta_{KG}}}\right]^{\frac{\eta_{KG}}{\eta_{KG}-1}} \tag{SS.56}$$

$$k = \frac{k^S a}{\overline{u}} \tag{SS.57}$$

$$k^R = \frac{k}{1-\omega} \tag{SS.58}$$

$$i = k\left(\frac{1-(1-\delta)/a}{\varpi}\right) \tag{SS.59}$$

$$i^R = \frac{i}{1-\omega} \tag{SS.60}$$

$$y^{\widetilde{HZ}} = \left(\widetilde{k}\right)^{\alpha}\left(a\nabla^H nh\right)^{1-\alpha} \tag{SS.61}$$

$$x^{\widetilde{HZ}} = y^{\widetilde{HZ}} \tag{SS.62}$$

$$y^{\tilde{H}} = z\left[(1-o_{XO})^{\frac{1}{\eta_{XO}}}\left(x^{\widetilde{HZ}}\right)^{\frac{\eta_{XO}-1}{\eta_{XO}}} + o_{XO}{}^{\frac{1}{\eta_{XO}}}\left(o^H\right)^{\frac{\eta_{XO}-1}{\eta_{XO}}}\right]^{\frac{\eta_{XO}}{\eta_{XO}-1}} \tag{SS.63}$$

$$y^H = \frac{y^{\tilde{H}}}{\Delta^H} \tag{SS.64}$$

$$x^H = y^H \tag{SS.65}$$

$$f^H = mc^H\left(\tilde{p}^H\right)^{-\epsilon_H} y^H/(1-\beta a^{1-\sigma}\theta_H) \tag{SS.66}$$

$$\nabla^F = 1 + (R-1)\, s^{WC} \tag{SS.67}$$

$$\gamma^W = a^{\alpha^W} \bar{\pi} \tag{SS.68}$$

$$h^C = \nabla^H \frac{\exp\left(\frac{\sigma_{\bar{c}}^2}{2}\right) \Phi\left(\frac{\ln \bar{c} - \sigma_{\bar{c}}^2}{\sigma_{\bar{c}}}\right)}{1-\rho^n} \tag{SS.69}$$

$$w = \frac{1}{\nabla^F h}\left[\frac{\beta}{a^{\sigma-1}}(1-\rho)\ \ p^H\left(\nabla^H \bar{c} - h^C\right) + mc^{\widetilde{HZ}}(1-\alpha)\,\frac{y^{\widetilde{HZ}}}{n}\ \ - p^H \nabla^H \bar{c}\right] \tag{SS.70}$$

$$w^n = \ \frac{1-\varkappa_W a^{\alpha^W - 1}}{1-\varkappa_W} w \tag{SS.71}$$

$$\omega_v \ \ = \frac{e}{\nabla^H}\left[\frac{\nabla^F wh}{p^H} + \nabla^H \bar{c} - \frac{mc^{\widetilde{HZ}}(1-\alpha)}{p^H}\frac{y^{\widetilde{HZ}}}{n}\right] \tag{SS.72}$$

$$\Psi^U \kappa = \frac{mc^{\widetilde{HZ}}(1-\alpha)^2\, \frac{y^{\widetilde{HZ}}}{n}}{h^{\frac{1}{1+\phi}}\left(\nabla^H\right)^{1-\sigma}} \tag{SS.73}$$

$$\bar{b} = \ \ \frac{1}{(1-\varphi^U)}\left[\nabla^F w^n h - \varphi^U\left[mc^{\widetilde{HZ}}(1-\alpha)\,\frac{y^{\widetilde{HZ}}}{n} - p^H h^C + p^H \nabla^H \frac{s}{e}\omega_v\right]\right] - \Psi^U \kappa \left(\nabla^H\right)^{1-\sigma}\ \frac{h^{1+\phi}}{1+\phi} \tag{SS.74}$$

$$o = o^C + o^H \tag{SS.75}$$

$$p^Y y = \frac{p^H y^H - p^O o^H + y_{va}^{h*} + p^F\left(1 - mc_t^F \Delta^F\right)\Theta_1}{1 - s^{Co} - p^F\left(1 - mc^F \Delta^F\right)\Theta_2}$$

where

$$\begin{aligned}
\Theta_1 = {} & o_{XIPr}\left(\frac{p^F}{p^I}\right)^{-\eta_{XIPr}} i^{Pr} \\
& - o_{XCZ}\left(\frac{p^F}{p^{CZ}}\right)^{-\eta_{XCZ}} \frac{1 - o_{CO} - o_{CA}}{\left(p^{CZ}\right)^{\eta_{COA}}} p^I i^{Pr} \\
& - o_{XCA}\left(\frac{p_t^F}{p_t^{CA}}\right)^{-\eta_{XCA}} \frac{o_{CA}}{\left(p^{CA}\right)^{\eta_{COA}}} p^I i^{Pr} \\
\Theta_2 = {} & o_{XCZ}\left(\frac{p^F}{p^{CZ}}\right)^{-\eta_{XCZ}} \frac{1 - o_{CO} - o_{CA}}{\left(p^{CZ}\right)^{\eta_{COA}}}\left(1 - s^{cg} - s^{ig} - s^{iCo} - s^{tb}\right) \\
& + o_{XCA}\left(\frac{p^F}{p^{CA}}\right)^{-\eta_{XCA}} \frac{o_{CA}}{\left(p^{CA}\right)^{\eta_{COA}}}\left(1 - s^{cg} - s^{ig} - s^{iCo} - s^{tb}\right) \\
& + o_{XICo}\left(\frac{p_t^F}{p_t^{ICo}}\right)^{-\eta_{XICo}} \frac{s_t^{iCo}}{p^{ICo}}
\end{aligned}$$

$$tb = s^{tb} p^Y y \tag{SS.76}$$

$$i^G = s^{ig}\frac{p^Y y}{p^{IG}} \tag{SS.77}$$

$$tr = s^{tr} p^y y \tag{SS.78}$$

$$c^G = s^{cg}\frac{p^Y y}{p^{CG}} \tag{SS.79}$$

$$i^{Pr} = i + \phi^u(\bar{u}) \times k/a \tag{SS.80}$$

$$i^{Co} = \frac{s^{iCo} p^Y y}{p^{ICo}} \tag{SS.81}$$

$$i^{AG} \quad = \quad \left({}^{i^G}\!/_{\varphi_0}\right) \frac{1 - \left({}^{\rho^\varphi}\!/_a\right)}{1 - \left({}^{\rho^\varphi}\!/_a\right)^{N^G}} \tag{SS.82}$$

$$i^{aCo} \quad = \quad \left({}^{i^{Co}}\!/_{\varphi_0^{Co}}\right) \frac{1 - \left({}^{\rho^{\varphi Co}}\!/_a\right)}{1 - \left({}^{\rho^{\varphi Co}}\!/_a\right)^{N^{Co}}} \tag{SS.83}$$

$$q^{Co} = p^{ICo} \varphi_0^{Co} \frac{1 - \left(\rho^{\varphi Co} \frac{\pi}{R}\right)^{N^{Co}}}{1 - \rho^{\varphi Co} \frac{\pi}{R}} \left(\frac{R}{\pi}\right)^{N^{Co}-1} \tag{SS.84}$$

$$y^{Co} = s^{Co} p^Y y / \left(rer \times p^{Co*}\right) \tag{SS.85}$$

$$k^{Co} = \nabla^{k^{Co}, y^{Co}} y^{Co} \frac{rer \times p^{Co*}}{q^{Co}} \tag{SS.86}$$

$$\delta_{Co} = \frac{i^{aCo}}{k^{Co} a^{N^{Co}-2}} - a + 1 \tag{SS.87}$$

$$\alpha_{Co} = \frac{k^{Co} q^{Co} \left(\frac{R}{\pi} - 1 + \delta_{Co}\right)}{a y^{Co} p^{Co*} rer} \tag{SS.88}$$

$$z^{Co} = y^{Co} \left({}^{k^{Co}}\!/_a\right)^{-\alpha Co} \left(a \nabla^{Co}\right)^{\alpha Co - 1} \tag{SS.89}$$

$$cf^{Co} = rer \times p^{Co*} y^{Co} - p^{ICo} i^{Co} \tag{SS.90}$$

$$ren^{R*} = \frac{p^Y y}{rer \times (1-\omega)} \left[s_{CA} \left(\frac{a\pi^* - R^* \xi}{a\pi^* - 1}\right) - s_{tb} + \frac{(1-\chi)}{p^Y y} \left(cf^{Co} - \tau^{Co} \left(cf^{Co} + p^{ICo} i^{Co}\right)\right)\right] \tag{SS.91}$$

$$ren^* = (1-\omega)\, ren^{R*} - \frac{(1-\chi)\left(cf^{Co} - \tau^{Co}\left(cf^{Co} + p^{ICo} i^{Co}\right)\right)}{rer} \tag{SS.92}$$

$$c = p^Y y - p^I i^{pr} - p^{ICo} i^{Co} - p^{CG} c^G - p^{IG} i^G - tb \tag{SS.93}$$

$$c^A = o_{CA} \left(p^{CA}\right)^{-\eta_{COA}} c \tag{SS.94}$$

$$x^{CA,H} = \left(z^A\right)^{\eta_{XCA}-1} \left(1 - o_{XCA}\right) \left(\frac{p^H}{p^{CA}}\right)^{-\eta_{XCA}} c^A \tag{SS.95}$$

$$x^{CA,F} = \left(z^A\right)^{\eta_{XCA}-1} o_{XCA} \left(\frac{p^F}{p^{CA}}\right)^{-\eta_{XCA}} c^A \tag{SS.96}$$

$$c^Z = \left(1 - o_{CO} - o_{CA}\right) \left(p^{CZ}\right)^{-\eta_{COA}} c \tag{SS.97}$$

$$y^C = c + p^I i^{Pr} + p^{ICo} i^{Co} + p^{CG} c^G + p^{IG} i^G \tag{SS.98}$$

$$x^{CZ,H} = \left(1 - o_{XC}\right) \left({}^{p^H}\!/_{p^{CZ}}\right)^{-\eta_{XCZ}} c^Z \tag{SS.99}$$

$$x^{CZ,F} = o_{XC} \left({}^{p^F}\!/_{p^{CZ}}\right)^{-\eta_{XCZ}} c^Z \tag{SS.100}$$

$$x^{IPr,F} = o_{XI} \left(\frac{p^F}{p^I}\right)^{-\eta_{XIPr}} i^{Pr} \tag{SS.101}$$

$$x^{IPr,H} = (1 - o_{XI}) \left(\frac{p^H}{p^I}\right)^{-\eta_{XIPr}} i^{Pr} \tag{SS.102}$$

$$x^{CG,H} = (1 - o_{XCG}) \left(\frac{p^H}{p^{CG}}\right)^{-\eta_{XCG}} c^G \tag{SS.103}$$

$$x^{CG,F} = o_{XCG} \left(\frac{p^F}{p^{CG}}\right)^{-\eta_{XCG}} c^G \tag{SS.104}$$

$$x^{IG,H} = (1 - o_{XIG}) \left(\frac{p^H}{p^{IG}}\right)^{-\eta_{XIG}} i^G \tag{SS.105}$$

$$x^{IG,F} = o_{XIG} \left(\frac{p^F}{p^{IG}}\right)^{-\eta_{XIG}} i^G \tag{SS.106}$$

$$x^{ICo,H} = (1 - o_{XICo}) \left(\frac{p^H}{p^{ICo}}\right)^{-\eta_{XICo}} i^{Co} \tag{SS.107}$$

$$x^{ICo,F} = o_{XICo} \left(\frac{p^F}{p^{ICo}}\right)^{-\eta_{XICo}} i^{Co} \tag{SS.108}$$

$$x^{WX} = y^H - x^{CZ,H} - x^{IPr,H} - x^{ICo,H} - x^{CG,H} - x^{IG,H} - x^{CA,H} - h^C n + \omega_v v \tag{SS.109}$$

$$y^{HX} = x^{WX} / \Delta^{HX} \tag{SS.110}$$

$$x^F = x^{CZ,F} + x^{IPr,F} + x^{ICo,F} + x^{CG,F} + x^{IG,F} + x^{CA,F} \tag{SS.111}$$

$$y^F = x^F \tag{SS.112}$$

$$f^F = mc^F \left(\tilde{p}^F\right)^{-\epsilon_F} y^F / (1 - \beta a^{1-\sigma} \theta_F) \tag{SS.113}$$

$$imp = y^F \Delta^F \tag{SS.114}$$

$$y = c + i^{Pr} + i^{Co} + c^G + i^G + y^{HX} + y^{Co} - imp - o \tag{SS.115}$$

$$p^Y = \left(y^C + tb\right) / y \tag{SS.116}$$

$$tr^{NR} = tr \frac{\omega_G}{\omega} \tag{SS.117}$$

$$tr^R = tr \frac{1 - \omega_G}{1 - \omega} \tag{SS.118}$$

$$b^* = \frac{s_{CA} \times p^Y y}{rer \left(1 - \frac{1}{a\pi^*}\right)} \tag{SS.119}$$

$$y^* = a \times z^* \tag{SS.120}$$

$$o^* = \left(y^{HX} / y^*\right) \left(p^{HX} / rer\right)^{\eta^*}. \tag{SS.121}$$

$$i_t^R = \frac{i_t}{1 - \omega} \tag{SS.122}$$

$$k_t^R = \frac{k_t}{1 - \omega} \tag{SS.123}$$

$$g = p^{CG} c^G + p^I i^G + tr \tag{SS.124}$$

$$\widetilde{g}^{rule} = g \tag{SS.125}$$

$$\widetilde{g}^{exo} = g \tag{SS.126}$$

$$\widetilde{g} = g \tag{SS.127}$$

$$\alpha^{CG} = \frac{p^{CG} c^{G}}{g \xi^{CG}} \tag{SS.128}$$

$$\alpha^{IG} \quad = \quad \frac{p^{IG} i^{G}}{g \xi^{IG}} \tag{SS.129}$$

$$\alpha^{BG} = \frac{\bar{b} u}{g} \tag{SS.130}$$

$$d = p^{Y} y - rer \times p^{Co*} y^{Co} - r^{K} k^{S} - wnh - h^{C} n - w_{v} v \tag{SS.131}$$

$$d^{R} = \frac{d}{1-\omega} \tag{SS.132}$$

$$b = \frac{s^{def}}{(1 - {}^{1}/_{a\pi}) + \frac{\alpha^{FC}}{1-\alpha^{FC}} \left(1 - {}^{1}/_{a\pi^{*}}\right)} p^{Y} y \tag{SS.133}$$

$$b^{G^{*}} = \frac{\alpha^{FC}}{1-\alpha^{FC}} \frac{b^{G}}{rer} \tag{SS.134}$$

$$b^{P^{*}} = b^{*} - b^{G^{*}} \tag{SS.135}$$

$$b_{t}^{R*} = \frac{b_{t}^{P*}}{1-\omega} \tag{SS.136}$$

$$\begin{aligned} t \quad = \quad & g - b\left(1 - \frac{R}{a\pi}\right) + rer \times b^{G*}\left(1 - \frac{\xi R^{*}}{a\pi^{*}}\right) - \tau^{C} c - \tau^{W} wnh \\ & - \tau^{K} \frac{k}{a}\left[r^{K}\left(\bar{u} + \frac{1-p^{I}}{\phi_{u}}\right) - p^{I}\delta\right] - \tau^{D} d - \chi pr^{Co} \end{aligned} \tag{SS.137}$$

$$\alpha_{T} = \frac{t}{p^{Y} y} \tag{SS.138}$$

$$\tau = \tau^{C} c + \tau^{W} wnh + \tau^{K} \frac{k}{a}\left[r^{K}\left(\bar{u} + \frac{1-p^{I}}{\phi_{\bar{u}}}\right) - p^{I}\delta\right] + \tau^{D} d + \tau^{Co}(1-\chi)\left(cf^{Co} + p^{ICo} i^{Co}\right) + t \tag{SS.139}$$

$$\widetilde{\tau} = \tau \tag{SS.140}$$

$$\widetilde{p}^{Co*} = p^{Co*} \tag{SS.141}$$

$$\bar{s}_{B} = -s^{def} \tag{SS.142}$$

$$g_{t}^{\Gamma^{HX}} = \pi^{*} \tag{SS.143}$$

$$c^{NR} = \frac{\left(1-\tau^{L}\right) whn + \bar{b} u + tr^{NR} - \frac{\omega^{G}}{\omega} t}{1+\tau^{C}} \tag{SS.144}$$

$$c_t^R = \frac{c_t - \omega c_t^{NR}}{1-\omega}$$

$$\Rightarrow c_t^R = \frac{c_t - h^C n - \nabla^H \omega_v v - \omega c_t^{NR}}{1-\omega} \tag{SS.145}$$

$$\tilde{c}^R = \left[(1-o_{CG})^{\frac{1}{\eta_{CG}}} \left(c^R\right)^{\frac{\eta_{CG}-1}{\eta_{CG}}} + o_{CG}{}^{\frac{1}{\eta_{CG}}} \left(c^G\right)^{\frac{\eta_{CG}-1}{\eta_{CG}}} \right]^{\frac{\eta_{CG}}{\eta_{CG}-1}} \tag{SS.146}$$

$$\tilde{c}^{NR} = \left[(1-o_{CG})^{\frac{1}{\eta_{CG}}} \left(c^{NR}\right)^{\frac{\eta_{CG}-1}{\eta_{CG}}} + o_{CG}{}^{\frac{1}{\eta_{CG}}} \left(c^G\right)^{\frac{\eta_{CG}-1}{\eta_{CG}}} \right]^{\frac{\eta_{CG}}{\eta_{CG}-1}} \tag{SS.147}$$

$$\lambda^R = \frac{1}{1+\tau^C} \left(\widetilde{c}^R - \varsigma \frac{\widetilde{c}^R}{a} \right)^{-\sigma} \left(\frac{(1-o_{GC})\, \widetilde{c}^R}{c^R} \right)^{\frac{1}{\eta_{CG}}} \tag{SS.148}$$

$$\lambda^{NR} = \frac{1}{1+\tau^C} \left(\widetilde{c}^{NR} - \varsigma \frac{\widetilde{c}^{NR}}{a} \right)^{-\sigma} \left(\frac{(1-o_{GC})\, \widetilde{c}^{NR}}{c^{NR}} \right)^{\frac{1}{\eta_{CG}}} \tag{SS.149}$$

$$\widetilde{\chi}^R = \left(\nabla^H\right)^{-\sigma} \left(\widetilde{c}^R - \varsigma \frac{\widetilde{c}^R}{a} \right)^{\sigma} \tag{SS.150}$$

$$\widetilde{\chi}^{NR} = \left(\nabla^H\right)^{-\sigma} \left(\widetilde{c}^{NR} - \varsigma \frac{\widetilde{c}^{NR}}{a} \right)^{\sigma} \tag{SS.151}$$

$$\Psi^U = (1-\omega) \frac{\Theta^R}{\lambda^R} + \omega \frac{\Theta^{NR}}{\lambda^{NR}} \tag{SS.152}$$

$$\kappa = \bar{\kappa} = \frac{mc^{\widetilde{HZ}} (1-\alpha)^2 \, \frac{y^{\widetilde{HZ}}}{n}}{\frac{\Psi^U h^{1+\phi}}{(1-\tau^L)} \left(\nabla^H\right)^{1-\sigma}} \tag{SS.153}$$

$$f^{HX} = \frac{\left(\tilde{p}^{HX}\right)^{-\epsilon_{HX}} mc^{HX} y^{HX}}{1 - \beta a_t^{1-\sigma} \theta_{HX}} \tag{SS.154}$$

$$utCo = \frac{\phi_{\bar{u}}^{Co} k^{Co}}{a}$$

$$k^G = \frac{i^{AG}/a^{N_G - 1}}{1 - \left(1-\delta^G\right)/a} \tag{SS.155}$$

$$k^S = \left(\frac{\left(\alpha \frac{mc^{HZ}}{r^{\widetilde{K}}} y^{\widetilde{HZ}}\right)^{\frac{\eta_{KG}-1}{\eta_{KG}}} - o_{KG}^{\frac{1}{\eta_{KG}}} \left(\frac{k^G}{a}\right)^{\frac{\eta_{KG}-1}{\eta_{KG}}}}{(1-o_{KG})^{\frac{1}{\eta_{KG}}}} \right)^{\frac{\eta_{KG}}{\eta_{KG}-1}} \tag{SS.156}$$

$$k^{S,R} = \frac{k^S}{1-\omega} \tag{SS.157}$$

$$r^{\widetilde{K}} = r^K \left(\frac{k^S/\tilde{k}}{1-o_{KG}} \right)^{\frac{1}{\eta_{KG}}} \tag{SS.158}$$

$$y_{va}^{h*} = p^{HX} y^{H*} - p^H x^{H*} \tag{SS.159}$$

$$o^C = o_{CO} \left(p^O\right)^{-\eta_{COA}} c \tag{SS.160}$$

$$o^H = o_{CO} \left(\frac{p^O}{p^H mc^H} \right)^{-\eta_{XO}} y^H \Delta^H \tag{SS.161}$$

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