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Heterogeneous Information, Diverse Higher-Order Beliefs and Business Cycles: Propagation Mechanisms and Empirical Performance

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Preliminary. Please Do Not Quote.

Abstract

This paper augments a standard DSGE model to capture production externalities, firm-specific cost shocks and heterogeneous information across firms. It is shown that this model structure leads to firms having to forecast the forecasts of other firms, and thus the presence of diverse higher-order beliefs. The model solution differs from that of the corresponding model with no firm-specific cost shocks (and thus homogeneous information) by a propagation component reflecting the belief diversity. The paper studies how this belief diversity affects model dynamics using parameter values that are based on Euro Area data. In comparison to the underlying standard DSGE model, the model featuring diverse higher-order beliefs can predict deeper and more prolonged recessions, with the magnitude of the differences in the recession dynamics being a function most prominently of the magnitude of the firm-specific costs shocks, relative to the aggregate costs shocks. It is thus argued that ceteris paribus the Great Recession of 2008/2009 that followed the global financial crisis can be captured better in a business cycle model with diverse higher-order beliefs than in a corresponding business cycle model without belief diversity as a propagation mechanism.

JEL classification: C63, D84, E32, E37.
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1 Introduction

One of the historical milestone contributions to business cycle research has been Pigou's (1929) description of recessions and recoveries being linked to waves of pessimism and optimism that decision makers at firms operate through. In his descriptive approach, Pigou detailed how firms' decision makers in the presence of limited information about other firms, production lags as well as partial interconnectedness would propagate forecast errors within and across industries, leading to output levels mirroring decision makers' waves of optimism and pessimism.

The representative agent approach that, despite notable criticisms such as Kirman (1992) and Carroll (2000),[1] is underlying much of the modern business cycle research,[2] clearly abstracts from the diverse beliefs-induced dynamics that Pigou had in mind. In this paper, we augment a standard, representative agent-based dynamic stochastic general equilibrium (DSGE) model to capture production externalities (mirroring Pigou's partial interconnectedness of firms), firm-specific cost shocks (leading to the initial firm-specific forecast errors in Pigou's account) and heterogeneous information across firms (reflecting the limited information of firms' decision makers about other firms that Pigou noted). We show in this paper how in such an augmented DSGE model the information that firms base their decisions on is partially determined by the expectations of other firms, with there being common knowledge that every firm is in the same situation of perpetual learning about the other firms, leading to firms having to "forecast the forecasts of others", that is, an infinite regress in expectations in which higher-order beliefs matter. Our model solution, based on the approach of Binder and Pesaran (1998), relative to the solution of the standard, representative agent-based DSGE model features an additional term capturing the belief diversity of the firms. Parameterizing our model on the basis of Euro Area data, we document how this belief diversity affects business cycle dynamics. We find, in particular, that in comparison to the underlying standard DSGE model, our model featuring diverse higher-order beliefs can predict deeper and more prolonged recessions, with the magnitude of the differences in the recession dynamics being a function most prominently of the magnitude of the firm-specific costs shocks, relative to the aggregate costs shocks. Part of the business cycles generated by our model are thus due to the waves of optimism and pessimism that Pigou had in mind.

Our work is related to important strands of research on business cycle dynamics that

[1] This criticism notes that while bottom-up modelling regularly predicts aggregate-level impact of heterogeneity, the representative agent approach has caused many to believe that heterogeneity is unimportant for modelling macroeconomic outcomes.

[2] For important exceptions, that typically have focused on the heterogeneity of households see, for example, Rios-Rull (1995) and Heathcote, Storesletten and Violante (2009).

has emphasized the (potential) importance of information frictions for macroeconomic outcomes: Keynes (1936), arguing that diverse-belief transmission channels would work through asset markets, wrote about investors' concern with short-term movements in market prices, inducing even relatively informed investors to be concerned with average opinion (the "animal spirits" of average opinion itself being the subject of speculation). Lucas (1975) constructed a model with heterogeneous information of firms, in which a geographical segmentation of firms (on "islands") prevents market data from fully revealing the private information of other firms, and thus economy-wide average beliefs become a state variable in production decision rules. Lucas' (1975) model did not feature waves of optimism and pessimism, though, as for tractability reasons he assumed that firms at the end of every period would pool their individual forecasts, so that belief diversity would never last beyond the end of each period. Models maintaining Lucas' (1975) geographical segmentation of firms but in which diverse beliefs can propagate over time, leading to the presence of infinite-dimensional state vectors of diverse beliefs in firms' production decision rules, were constructed and solved inter alia by Townsend (1983), Taub (1989), Sargent (1991) and Kasa (2000). The most recent work involving a Lucas (1975)-inspired geographical segmentation of firms, but modelling firms as strategically interacting in the presence of heterogeneous information about aggregate fundamentals is Angeletos and La'O (2009, 2013). Also, modelling approaches quite different from those in Lucas (1975), Townsend (1983) and Angeletos and La'O (2009) have been advanced. Among these are business cycle models with news shocks, including Beaudry and Portier (2004) as well as Christiano, Ilut, Motto and Rostagno (2008). While these models feature "noisy" cyclical fluctuations, they do not go beyond the representative agent paradigm and do not involve diverse beliefs. Another class of models are those involving the notion of Rational Beliefs Equilibria advanced by Kurz (1994). In these models, one business-cycle example of which is Kurz, Motolese, Piccillo and Wu (2015), individuals form beliefs that are different from the ones based on rational expectations, but rather (due to on-going regime changes that cannot be learned about due to the short duration of each regime) form subjective expectations that are compatible with past data. Finally, business cycle models under rational inattention (see, in particular, Sims, 2004; Maćkowiak, and Wiederholt, 2015) generate "noisy" cyclical fluctuations in response to individuals choosing to limit themselves in their processing and absorption of all information available to them.[3]

Our model set-up, as in Angeletos and La'O (2009, 2013), features multiple intermediate good producing firms as one source of propagation. Inter alia based on the work of

[3]This listing is not all-inclusive. Other models include "sticky information" models following the set-up of Mankiw and Reis (2002). See also the monograph by Veldkamp (2011) for a useful discussion of models in macroeconomics and finance involving informational frictions.

Hall (1986, 1988),[4] we view it as a modelling strength that the intermediate good producing firms are depicted as operating in an imperfectly competitive industry. In line with Vogel (2008), the intermediate good producing firms in our model have only a limited amount of information about their competitors' production cost structure, but continuously try to gain further information about the latter. The key information heterogeneity in our model does not refer to aggregate fundamentals (as in Angeletos and La'O, 2009, 2013), but rather refers to private information about firm-specific cost shocks. Following Binder and Pesaran (1998), we do invoke a behavioral restriction, invoking the separation of information into public and private information.[5] In contrast to the Rational Beliefs Equilibria-based literature, firms in our model set-up form their expectations as mathematical conditional expectations, given the model structure and the information sets specified. Overall, we view our model structure as a novel, important alternative to those advanced in the previous literature, without setting out in this paper to empirically discriminate between the assumptions underlying the various non-nested model structures.

The reminder of this paper is organized as follows: Section 2 describes the baseline DSGE model that serves as the starting point (and measure of comparison) for our model set-up. Our augmentation of the baseline DSGE model by production externalities, firm-specific cost shocks and heterogeneous information across firms is described in Section 3, that also discusses model solution. Section 4 provides various essential details on our model simulation and presents our empirical findings. Section 5 concludes.

2 General setup

The baseline DSGE model on which our model set-up is based is Fernández-Villaverde and Rubio-Ramirez (2006), as this model, beyond featuring multiple intermediate good producing firms as one source of propagation, also captures nominal frictions (rendering it meaningful to assess its empirical fit for Euro Area data both for real and nominal variables) and allows for some analysis of policy decisions by including a monetary policy decision rule.

A representative household consumes, saves, holds money, supplies labor, and sets its

[4]Hall found that a substantial number of (two-digit-SIC level) U.S. industries exhibit important non-competitive characteristics in that marginal cost in these industries is well below output price, and conjectured that aggregate U.S. output fluctuations are closely linked to industries' deviations from perfectly competitive market structures.

[5]The rational inattention literature derives any behavioral restriction endogenously from an underlying optimization rationale, but does not per se model a "forecasting the forecasts of others" problem. While Angeletos and La'O (2009, 2013) do not involve any form of limited information processing, their set-up requires geographical segmentation ("islands"). Thus, none of these structures are nested within each other.

own wages subject to a demand curve and Calvo-style pricing. Final output is manufactured by a final good producer, who uses as inputs the output of multiple intermediate good producing firms that are monopolistic competitors. The intermediate good producing firms rent capital and labor to manufacture their good, facing Calvo-style pricing. The monetary authority fixes the one-period nominal interest rate through open market operations with public debt. In modelling the intermediate good producing firms, we go beyond the set-up of Fernández-Villaverde and Rubio-Ramirez (2006), and capture production externalities (mirroring Pigou's partial interconnectedness of these firms),[6] firm-specific cost shocks (leading to the initial firm-specific forecast errors in Pigou's account) and heterogeneous information across these firms (reflecting the limited information of these firms' decision makers about other firms that Pigou noted). The individual information set of intermediate goods producer i is denoted as Ω_{it}. The individiual information set of intermediate goods producer i is composed of the public information known in period t, Ψ_t, and the period t private information of this intermediate goods producer, Φ_{it} :

$$\Omega_{it} = \Psi_t \cup \Phi_{it}. \tag{1}$$

The period t private information of intermediate goods producer$i$ contains one component only, namely (the realization of) his/her firm-specific cost shock, ν_{it}. The current realization of the aggregate state variables that are affected by the firm-specific shocks are not contained in Ω_{it}. Each intermediate goods producer assigns a conditional probability to the current (and future) decisions of all households, firms and the monetary authority in the economy. Only at the beginning of period $t+1$ will the period t aggregate state variables that are affected by the firm-specific shocks be fully revealed. In line with Binder and Pesaran (1998), we make the following assumption:

Beliefs-Formation Assumption: Intermediate good producing firm i's belief about intermediate good producing firm j's current and future decisions ($i \neq j$) is given by

$$E\left[y_{j,t+s}|\Omega_{it}\right] = E\left[y_{j,t+s}|\Psi_t\right], \quad \text{for } i,j = 1,2,....N \quad \text{,and s = 0,1,....} \tag{2}$$

This beliefs-formation assumption allows partialling-out the mean beliefs of intermediate good producing firm i:

$$\mathbb{E}\left(y_{t+s}|\Omega_{it}\right) = \mathbb{E}\left(y_{t+s}|\Psi_t\right) + \omega_i \qquad \underbrace{\cdot\left[\mathbb{E}\left(y_{i,t+s}|\Omega_{it}\right) - \mathbb{E}\left(y_{i,t+s}|\Psi_t\right)\right]}_{\text{expectation wedge for intermediate good producing firm } i} \tag{3}$$

[6]When optimizing, the intermediate goods producers believe that their decisions are not affecting the aggregate capital stock.

for $i = 1, 2, ...N$ and $s = 0, 1, ...$, where ω_i is the relative weight of intermediate good producing firm i.

In what follows, optimal decision rules based on the public information set and in the absence of firm-specific cost shocks will be called the homogeneous information model,[7] while an optimal decision rule based on the individual information sets will be called firms' model, and finally the aggregated model inter alia involving aggregation across firms' models will be called the heterogeneous information model.

2.1 Households

As in Fernández-Villaverde and Rubio-Ramírez (2007) there exists a continuum of households, each household maximizing a lifetime utility separable in consumption,c_{jt}, real money balances, $\frac{m_{jt}}{p_t}$, and hours worked, l_{jt}:

$$E_0 \sum_{t=0}^{\infty} \beta^t \cdot d_t \cdot \left\{ \log \left(c_{jt} - h \cdot c_{jt-1} \right) + \vartheta \cdot \log \left(\frac{m_{jt}}{p_t} \right) - \varphi_t \cdot \psi \cdot \frac{l_{jt}^{1+\gamma}}{1+\gamma} \right\}, \tag{4}$$

where β denotes the discount factor, d_t an inter-temporal preference shock, h the habit persistence parameter, and γ the inverse Frisch labor elasticity. Furthermore, the model features the following laws of motion for the inter-temporal preference shock d_t and the labor supply shock φ_t:

$$\log d_t = \rho_d \cdot \log d_{t-1} + \varepsilon_{d,t} \text{ where } \varepsilon_{d,t} \sim \mathcal{N}(0, 1); \tag{5}$$

$$\log \varphi_t = \rho_\varphi \cdot \log \varphi_{t-1} + \varepsilon_{\varphi,t} \text{ where } \varepsilon_{\varphi,t} \sim \mathcal{N}(0, 1). \tag{6}$$

Every period, each household consumes,c_{jt}, invests,x_{jt}, saves by holding real balances, $\frac{m_{jt}}{p_t}$, government bonds,$\frac{b_{j,t+1}}{p_t}$, and purchases Arrow-Debreu securities, $a_{j,t+1}$. Households have access to a complete market of Arrow-Debreu securities. Similarly to Fernández-Villaverde and Rubio-Ramírez (2007), we denote the securities that pay one unit of consumption good in the state of the economy when event $\omega_{j,t+1,t}$ realized with $a_{j,t+1}$. Household j purchases this security at (real) price $q_{j,t+1,t}$ in period t.

Household j receives real income from labor,$w_{jt} \cdot l_{jt}$, earns income on capital net of costs depending on the utilization rate, $\left(r_t \cdot u_{jt} - \mu_t^{-1} \cdot a(u_{jt}) \right) \cdot k_{j,t}$,[8] where $\mu_t^{-1} \cdot \Phi(u_{jt})$

[7]Note that the homogeneous information model results in a solution that is that for the representative-agent-paradigm model.

[8]Note that in comparison to Fernández-Villaverde and Rubio-Ramírez (2007), we use the end-

is a capital utilization cost and μ_t is an investment-specific technology shock. As usual, we assume that $\Phi[1] = 0, \Phi'$ and $\Phi'' > 0$. Furthermore the household receives lump-sum transfers and profits from the labor-packing firms of the economy.

Thus the j-th household's per-period budget constraint is given by:

$$c_{jt} + x_{jt} + \frac{m_{jt}}{p_t} + \frac{b_{j,t+1}}{p_t} + \int q_{j,t+1} \cdot a_{j,t+1} \cdot d\omega_{j,t+1,t} =$$

$$= w_{jt} \cdot l_{jt} + \left(r_t \cdot u_{jt} - \mu_t^{-1} \cdot \Phi(u_{jt})\right) \cdot k_{j,t} + \frac{m_{j,t-1}}{p_t} + R_{t-1} \cdot \frac{b_{jt}}{p_t} + a_{jt} + T_t + F_t. \tag{7}$$

The law of motion for the capital stock involves quadratic investment adjustment costs:

$$k_{jt+1} = (1 - \delta) \cdot k_{jt} + \mu_t \cdot \left(1 - S\left[\frac{x_{jt}}{x_{j,t-1}}\right]\right) \cdot x_{jt}, \tag{8}$$

where δ denotes the rate of depreciation and $S[\cdot]$ the quadratic adjustment cost function such that $S[\Lambda_x] = 0$, where Λ_x is the growth rate of investment along the balanced growth path. The investment-specific technology shock follows an autoregressive process:

$$\mu_t = \mu_{t-1} \cdot \exp\left(\Lambda_\mu + z_{\mu,t}\right) \text{ where } z_{\mu t} = \sigma_\mu \cdot \varepsilon_{\mu t} \text{ and } \varepsilon_{\mu t} \sim \mathcal{N}(0, 1). \tag{9}$$

This implies the first-order conditions with respect to $c_{jt}, b_{jt}, u_{jt}, k_{jt+1}$ and x_{jt} :

$$d_t \cdot (c_{jt} - h \cdot c_{jt-1})^{-1} - h \cdot \beta \cdot E_t\left[d_{t+1} \cdot (c_{jt+1} - h \cdot c_{jt})^{-1}\right] = \lambda_{jt}, \tag{10}$$

$$\lambda_{jt} = \beta \cdot E_t\left[\lambda_{jt+1} \cdot \frac{R_t}{\Pi_{t+1}}\right], \tag{11}$$

$$r_t = \mu_t^{-1} \cdot \Phi'[u_{jt}], \tag{12}$$

of-period-notation for capital, as it makes the relation of information sets and timing easier to follow.

$$q_{jt} = \beta \cdot E_t \left[\frac{\lambda_{j,t+1}}{\lambda_{jt}} \cdot \left((1-\delta) \cdot q_{j,t+1} + r_{t+1} \cdot u_{j,t+1} - \mu_{t+1}^{-1} \cdot \Phi \left[u_{j,t+1} \right] \right) \right], \tag{13}$$

$$1 = q_{jt} \cdot \mu_t \left(1 - S \left[\frac{x_{jt}}{x_{j,t-1}} \right] - S' \left[\frac{x_{jt}}{x_{j,t-1}} \right] \cdot \frac{x_{jt}}{x_{j,t-1}} \right) +$$

$$+\beta \cdot E_t \left[q_{j,t+1} \cdot \frac{\lambda_{j,t+1}}{\lambda_{jt}} \cdot \mu_{t+1} \cdot S' \left[\frac{x_{jt}}{x_{j,t-1}} \right] \cdot \left(\frac{x_{jt}}{x_{j,t-1}} \right)^2 \right], \tag{14}$$

where λ_{jt} denotes the Lagrange multiplier of the budget constraint, and Q_{jt} the marginal Tobin's q, $q_{j,t} = \frac{Q_{jt}}{\lambda_{jt}}$, the Lagrange multiplier associated with the investment adjustment constraint normalized by λ_{jt}.

The labor employed by the intermediate good producers is supplied by the "labor packer". He/she aggregates the labor of households with a Dixit-Stiglitz production function, where the aggregate labor demand can be expressed as :

$$l_t^d = \left(\int_0^1 l_{jt}^{\frac{\eta-1}{\eta}} \cdot dj \right)^{\frac{\eta}{\eta-1}}. \tag{15}$$

Taking all wages as given, the optimal labor demand is given by:

$$l_{jt} = \left(\frac{w_{jt}}{w_t} \right)^{-\eta} \cdot l_t^d, \quad \forall j. \tag{16}$$

The optimal aggregated wage is therefore:

$$w_t = \left(\int_0^1 w_{jt}^{1-\eta} \cdot dj \right)^{\frac{1}{1-\eta}}. \tag{17}$$

Households exhibit a Calvo-style wage setting. A fraction θ_w of households cannot re-optimize their wages, but partially index their wages to past inflation dynamics. This indexation is captured by the parameter $\chi_w \in [0, 1]$. So if the household cannot change its wage for τ periods, its real wage will be given by $\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \cdot w_{jt}$.

We assume a symmetric equilibrium on the household level, and thus in equilibrium the wage index evolves as:

$$w_t^{1-\eta} = \theta_w \cdot \left(\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t}\right)^{1-\eta} \cdot w_{t-1}^{1-\eta} + (1-\theta_w) \cdot (w_t^*)^{1-\eta} \tag{18}$$

where w_t^* denotes the optimal wage set by all households following the recursive optimal wage setting conditions:

$$f_t = \frac{\eta-1}{\eta} \cdot (w_t^*)^{1-\eta} \cdot \lambda_t \cdot w_t^{\eta} \cdot l_t^d + \beta \cdot \theta_w \cdot E_t \left[\left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}}\right)^{1-\eta} \cdot \left(\frac{w_{t+1}^*}{w_t^*}\right)^{\eta-1} \cdot f_{t+1} \right], \tag{19}$$

$$f_t = \psi \cdot d_t \cdot \varphi_t \left(\frac{w_t}{w_t^*}\right)^{\eta\cdot(1+\vartheta)} \cdot \left(l_t^d\right)^{1+\vartheta} + \beta \cdot \theta_w \cdot E_t \left[\left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}}\right)^{-\eta\cdot(1+\vartheta)} \cdot \left(\frac{w_{t+1}^*}{w_t^*}\right)^{\eta\cdot(1+\vartheta)} \cdot f_{t+1} \right]. \tag{20}$$

2.2 The Final and Intermediate Good Producers

The final good producers operate under perfect competition and aggregate the intermediate good output using the following production function:

$$y_t^d = \left(\int_0^1 y_{it}^{\frac{\epsilon-1}{\epsilon}} \cdot di\right)^{\frac{\epsilon}{\epsilon-1}},$$

where y_t^d denotes the aggregate demand and ϵ controls the elasticity of substitution. The final good producers maximize their per-period profits:

$$\max_{y_{it}} p_t \cdot y_t^d - \sum_{i=1}^{N} p_{it} \cdot y_{it}. \tag{21}$$

where we are assuming that there are$N$ intermediate good producers. The demand function for each intermediate good produce is given by:

$$y_{it} = \left(\frac{p_{it}}{p_t}\right)^{-\epsilon} \cdot y_t^d \quad \forall i. \tag{22}$$

The production technology of the intermediate good producing firms reflect that both individual and aggregate stocks of capital matter:

$$f(K,L) = A_{it} \cdot L_{it}^{1-\alpha} \cdot K_{it}^{\alpha} \cdot K_t^{\alpha_k} - \Phi \cdot Z_{it}, \tag{23}$$

subject to

$$A_{it} \cdot L_{it}^{1-\alpha} \cdot K_{it}^{\alpha} \cdot K_t^{\alpha_k} \geq \Phi \cdot Z_{it}, \tag{24}$$

where K_t denotes the aggregate level of capital: $K_t = \sum_i^N K_{it}$.

The heterogeneity in intermediate good producing firms' cost (technology) shocks is captured as follows:

$$log(A_{it}) = \Lambda_A + z_{At} + \nu_{it}, \tag{25}$$

where ν_{it} is the firm-specific technology shock, that features an autoregressive structur:e $\nu_{it} = \rho \cdot \nu_{i,t-1} + \epsilon_{\nu it}$, with $\epsilon_{\nu it} \sim \mathcal{N}\left(-\frac{\sigma_\nu^2}{2\cdot(1+\rho)}, \sigma_\nu^2\right)$, and $v_{i0} = 0$, $\forall i$.[9]

Given the zero-profit-condition on the final good producers, the aggregate price level is given by:

$$p_t = \left(\sum_{i=1}^{N} p_{it}^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}. \tag{26}$$

2.3 Monetary and Fiscal Authority

The monetary authority sets the nominal interest on the basis of the following Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \cdot \quad \left(\left(\frac{\Pi_t}{\Pi^*}\right)^{\gamma_\Pi} \cdot \left(\frac{\frac{y_t^d}{y_{t-1}^d}}{\Lambda_{y^d}}\right)^{\gamma_y} \right)^{1-\rho_R} \quad \cdot \exp\left(m_t\right), \tag{27}$$

where $\Pi*$ denotes the target level of inflation, R the nominal steady-state gross return, and Λ_{y^d} the steady-state level of output growth. The monetary policy shock follows $m_t = \sigma_m \cdot \varepsilon_{mt}$, with $\varepsilon_{mt} \sim \mathcal{N}(0,1)$. Interest rate smoothing is governed by the parameter ρ_R.

The per-period budget constraint of the fiscal authority reads:

$$T_t = \frac{\int_0^1 m_{jt} \cdot dj}{p_t} - \frac{\int_0^1 m_{j,t-1} \cdot dj}{p_t} + \frac{\int_0^1 b_{jt+1} \cdot dj}{p_t} - R_{t-1} \cdot \frac{\int_0^1 b_{jt} \cdot dj}{p_t}.$$

[9]We choose the expected value and the variance of these innovations such the expected value of $e^\nu = e^{\frac{E[\epsilon_{\nu,i,t}]}{1-\rho_i} - \frac{Var(\epsilon_{\nu,i,t})}{2(1-\rho_i^2)}} = e^0$ that is 1, that is, firm specific-shocks, on average, are neither explosive nor implosive in nature.

3 Model Solution

Let us first derive the equations characterizing the firm-specific optimality conditions, and then discuss two cases, the homogeneous information model and the heterogenenous information model.

Intermediate good producing firms maximize profits, that is, minimize costs subject to their production technology:

$$\min_{L_{it},K_{it}} w_t \cdot L_{it} + r_t \cdot K_{it}, \tag{28}$$

$$Y_{it} = \begin{cases} A \cdot L_{it}^{1-\alpha} \cdot K_{it}^{\alpha} \cdot K_t^{\alpha_k} \text{ if } f(K,L) \geq 0, \\ 0, \text{ otherwise.} \end{cases} \tag{29}$$

Given an interior solution, the first-order conditions are given by:

$$w_t = \mu_t \cdot (1-\alpha) \cdot A_{it} \cdot E\left[K_t^{\alpha_k} | \, \Omega_{it}\right] \cdot K_{it}^{\alpha} \cdot L_{it}^{-\alpha}, \tag{30}$$

$$r_t = \mu_t \cdot \alpha \cdot A_{it} \cdot E\left[K_t^{\alpha_k} | \Omega_{it}\right] \cdot K_{it}^{\alpha-1} \cdot L_{it}^{1-\alpha}, \tag{31}$$

where $\mu_t$denotes the Lagrange multiplier on the constraint. Using that the optimal ratio of capital to labor is given by:

$$\frac{w_t}{r_t} \cdot \frac{\alpha}{1-\alpha} = \frac{K_{it}}{L_{it}} \tag{32}$$

we obtain the expression for the real marginal costs:

$$mc_{it} = \left(\frac{1}{\alpha}\right)^{\alpha} \cdot \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \cdot w_t^{1-\alpha} \cdot r_t^{\alpha} \frac{1}{A_{it} \cdot E\left[K_t^{\alpha_k} | \Omega_{it}\right]}. \tag{33}$$

Using that $\frac{1}{A_{it} \cdot E\left[K_t^{\alpha_k} | \Omega_{it}\right]} = \frac{1}{A_t} \cdot \frac{1}{E\left[K_t^{\alpha_k} | \Psi_t\right]} \cdot \frac{1}{e^{\nu_{it}}} \cdot \frac{E\left[K_t^{\alpha_k} | \Psi_t\right]}{E\left[K_t^{\alpha_k} | \Omega_{it}\right]}$, we can express the real marginal costs of intermediate good producing firm i in terms of the corresponding real marginal costs under homogeneous information:

$$mc_{it} = \underbrace{\left(\frac{1}{\alpha}\right)^{\alpha} \cdot \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \cdot w_t^{1-\alpha} \cdot r_t^{\alpha} \frac{1}{A_t} \cdot \frac{1}{E\left[K_t^{\alpha_k} | \Psi_t\right]}}_{mc_t} \cdot \frac{1}{e^{\nu_{it}}} \cdot \frac{E\left[K_t^{\alpha_k} | \Psi_t\right]}{E\left[K_t^{\alpha_k} | \Omega_{it}\right]}, \tag{34}$$

$$mc_{it} = mc_t \cdot \frac{1}{e^{\nu_{it}}} \cdot \frac{E\left[K_t^{\alpha_k} | \Psi_t\right]}{E\left[K_t^{\alpha_k} | \Omega_{it}\right]}. \tag{35}$$

In the second stage of the optimization problem of the intermediate good producing firms, the latter choose the price they set to maximize their discounted real profits. The prices are set with partial indexation in Calvo-style form. Each period, a fraction of firms, θ , cannot change their prices. All other firms get to index their prices by past inflation. The second stage problem of the intermediate good producing firms involves choosing prices for period t that maximize expected profits for the complete horizon for which prices cannot be updated and are only indexed to aggregate inflation:

$$\max_{p_{it}} E_t \left[\sum_{\tau=0}^{\infty} (\beta \cdot \theta)^{\tau} \cdot \frac{\lambda_{t+\tau}}{\lambda_t} \cdot \left[\left(\prod_{s=1}^{\tau} \Pi_{t+s-1}^{\chi} \cdot \frac{p_{it}}{p^{t+\tau}} - mc_{t+\tau} \right) \cdot y_{i,t+\tau} \right] |\Omega_{it} \right] \tag{36}$$

$$\text{subject to: } y_{i,t+\tau} = \left(\prod_{s=1}^{\tau} \Pi_{t+s-1}^{\chi} \cdot \frac{p_{it}}{p^{t+\tau}} \right)^{-\epsilon} \cdot y_{t+\tau}^{d}. \tag{37}$$

Simplifying the above expression by denoting the indexation parameters as: $\Xi = \prod_{s=1}^{\tau} \Pi_{t+s-1}^{\chi}$, using that the marginal value of a unit of currency is set by the household's optimal decision rule and thus is equal across all intermediate good producing firms, and dropping constants we obtain the following first-order condition:

$$E\left[\left[\sum_{\tau=0}^{\infty} (\beta \cdot \theta)^{\tau} \cdot \lambda_{t+\tau} \left((1-\epsilon) \cdot \Xi^{1-\epsilon} \cdot \frac{p_{it}^{*}}{p_t} + \epsilon \cdot \Xi^{-\epsilon} \cdot mc_{i,t+\tau} \right) \cdot y_{t+\tau}^{d} \right] |\Omega_{it} \right] \quad = \quad 0.(38)$$

We can rewrite the optimal pricing condition as:

$$\begin{aligned} 0 &= (1-\epsilon) \cdot \underbrace{E\left[\left[\sum_{\tau=0}^{\infty} \beta^{\tau} \cdot \lambda_{t+\tau} \cdot \Xi^{1-\epsilon} \cdot \frac{p_{it}^{*}}{p_t} \cdot y_{t+\tau}^{d} \right] |\Omega_{it} \right]}_{:=g_{1it}} + \\ &+ \epsilon \cdot \underbrace{E\left[\left[\sum_{\tau=0}^{\infty} \beta^{\tau} \cdot \lambda_{t+\tau} \cdot \Xi^{-\epsilon} \cdot mc_{i,t+\tau} \cdot y_{t+\tau}^{d} \right] |\Omega_{it} \right]}_{:=g_{2it}}, \end{aligned} \tag{39}$$

$$0 = (1-\epsilon) \cdot g_{2it} + \epsilon \cdot g_{1it}. \tag{40}$$

We impose that g_{2it} is the same across all intermediate good producing firms. Although these firms face individual-specific marginal costs, they cannot charge different prices, as that would be creating arbitrage opportunities. The output of the intermediate good producing firms is qualitatively equivalent, and thus charging different prices would not be feasible. Note as the intermediate good producing firms set their optimal prices, they cannot observed all intermediate good producing firms' specific cost shocks, yet. One requirement for the equilibrium to exist is that firms cannot running losses indefinitely. It cannot be ruled out that this could occur as the market completeness of securities insures that the intermediate good producing firms can offset any losses incured due to deviations of their marginal cost from the long-run average. Therefore the intermediate good producing firms will set the same prices as they would in the homogeneous information model. This implies in turn that the evolution of prices follows:

$$1 = \theta_p \cdot \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t} \right)^{1-\epsilon} + (1 - \theta_p) \cdot \Pi_t^{*1-\epsilon}. \tag{41}$$

From this point onwards it is thus sufficient if we focus on the component of the optimal pricing equation that is related to the marginal costs. Re-writing g_{1it} recursively, we can study its components in detail:

$$g_{1it} = \lambda_t \cdot mc_{it} \cdot y_t^d + \beta \cdot \theta \cdot E \left[\Xi^{-\epsilon} \cdot g_{1i,t+1} | \Omega_{it} \right], \tag{42}$$

$$g_{1it} = \lambda_t \cdot y_t^d \cdot mc_t \cdot \frac{1}{e^{\nu_{it}}} \cdot \frac{E \left[K_t^{\alpha_k} | \Psi_t \right]}{E \left[K_t^{\alpha_k} | \Omega_{it} \right]} + \beta \cdot \theta \cdot E \left[\Xi^{-\epsilon} \cdot g_{1i,t+1} | \Omega_{it} \right]. \tag{43}$$

Using that optimal pricing Ξ is independent of i, we can write:

$$g_{1it} = \lambda_t \cdot y_t^d \cdot mc_t \cdot \frac{1}{e^{\nu_{it}}} \cdot \frac{E \left[K_t^{\alpha_k} | \Psi_t \right]}{E \left[K_t^{\alpha_k} | \Omega_{it} \right]} + \beta \cdot \theta \cdot \Xi^{-\epsilon} \cdot E \left[g_{1i,t+1} | \Omega_{it} \right]. \tag{44}$$

3.1 Homogeneous information

Under homogeneous information, that is, in the absence of intermediate good producing firm-specific costs shocks, we have $\Omega_{it} = \Psi_t, \forall i$. Except for $\alpha_k \neq 0$, we then revert back to the outcome of the baseline DSGE model: In particular, as $E \left[K_t^{\alpha_k} | \Psi_t \right] = E \left[K_t^{\alpha_k} | \Omega_{it} \right]$ and $e^0 = 1$, we have

$$mc_{it} = mc_t. \tag{45}$$

3.2 Heterogeneous information

Under heterogeneous information, the marginal costs across intermediate good producing firms will differ. Binder and Pesaran (1998) showed that for linear rational expectations models under social interactions and heterogeneous information, the model solution can in general be represented to inlcude, in addition to a "standard" component, a second component that is driven by expectation wegdes as appeared in Equation (3) above. It remains to obtain the expectation wedge component for the DSGE model here. Recall to this purpose the marginal cost-related component of optimal pricing:

$$g_{1it} = \lambda_t \cdot y_t^d \cdot mc_t \cdot \frac{1}{e^{\nu_{it}}} \cdot \frac{E\left[K_t^{\alpha_k}|\Psi_t\right]}{E\left[K_t^{\alpha_k}|\Omega_{it}\right]} + \beta \cdot \theta \cdot \Xi^{-\epsilon} \cdot E\left[g_{1i,t+1}|\Omega_{it}\right]. \tag{46}$$

Aggregating across N intermediate good producing firms, adding and subtracting conditional expectations about future marginal price dynamics, that is, $\beta \cdot \theta \cdot \Xi^{-\epsilon} \cdot E\left[g_{1i,t+1}|\Psi_t\right]$ we obtain:

$$\frac{1}{N} \cdot \sum_i^N g_{1it} = \lambda_t \cdot y_t^d \cdot mc_t \cdot \frac{1}{N} \cdot \sum_i^N \left(\frac{1}{e^{\nu_{it}}} \cdot \frac{E\left[K_t^{\alpha_k}|\Psi_t\right]}{E\left[K_t^{\alpha_k}|\Omega_{it}\right]} \right) + \tag{47}$$

$$+ \frac{1}{N} \cdot \sum_i^N \beta \cdot \theta \cdot \Xi^{-\epsilon} \cdot \left(E\left[g_{1i,t+1}|\Omega_t\right] - E\left[g_{1i,t+1}|\Psi_t\right] + E\left[g_{1i,t+1}|\Psi_t\right] \right). \tag{48}$$

We can decompose the conditional expectations about the aggregate capital stock as follows:

$$\frac{E\left[K_t^{\alpha_k}|\Psi_t\right]}{E\left[K_t^{\alpha_k}|\Omega_{it}\right]} = \frac{E\left[K_t^{\alpha_k}|\Psi_t\right]}{E\left[K_t^{\alpha_k}|\Psi_t\right] + \omega_i \underbrace{\cdot \left(E\left[K_t^{\alpha_k}|\Omega_{it}\right] - E\left[K_t^{\alpha_k}|\Psi_t\right]\right)}_{\text{expectation wedge of intermediate good producing firm } i}} = \tag{49}$$

$$= \frac{1}{1 + \omega_i \cdot \frac{E\left[K_t^{\alpha_k}|\Omega_{it}\right] - E\left[K_t^{\alpha_k}|\Psi_t\right]}{E\left[K_t^{\alpha_k}|\Psi_t\right]}} \tag{50}$$

We can argue that $\omega_i \cdot \frac{E\left[K_t^{\alpha_k}|\Omega_{it}\right] - E\left[K_t^{\alpha_k}|\Psi_t\right]}{E\left[K_t^{\alpha_k}|\Psi_t\right]}$ is small, as both the relative expectation wedge as well as the individual weight of any individual intermediate good producing firm is small, and thus it is a good approximation to use:

$$\frac{1}{1+x} \approxeq 1 - x, \tag{51}$$

impying for Equations (49) and (50) that:

$$\frac{E\left[K_t^{\alpha_k}|\Psi_t\right]}{E\left[K_t^{\alpha_k}|\Omega_{it}\right]} \approxeq 1 - \omega_i \cdot \frac{E\left[K_t^{\alpha_k}|\Omega_{it}\right] - E\left[K_t^{\alpha_k}|\Psi_t\right]}{E\left[K_t^{\alpha_k}|\Psi_t\right]}. \tag{52}$$

Therefore, the aggregated optimal marginal cost related optimal pricing condition becomes:

$$\frac{1}{N}\cdot\sum_{i}^{N} g_{1it} \approxeq \lambda_t \cdot y_t^d \cdot mc_t \cdot \frac{1}{N}\cdot\sum_{i}^{N}\left(\frac{1}{e^{\nu_{it}}}\cdot\left(1-\omega_i\cdot\frac{E\left[K_t^{\alpha_k}|\Omega_{it}\right]-E\left[K_t^{\alpha_k}|\Psi_t\right]}{E\left[K_t^{\alpha_k}|\Psi_t\right]}\right)\right)+ \tag{53}$$

$$+\frac{1}{N}\cdot\sum_{i}^{N}\beta\cdot\theta\cdot\Xi^{-\epsilon}\cdot\left(E\left[g_{1i,t+1}|\Omega_{it}\right]-E\left[g_{1i,t+1}|\Psi_t\right]+E\left[g_{1i,t+1}|\Psi_t\right]\right). \tag{54}$$

Note that the aggregated expectations regarding future marginal cost dynamics based on public information in period t are not involving information about the intermediate good producing firm-specific shock in period t. Therefore, conditional on public information we obtain the homogeneous information dynamics for future marginal cost, and can drop the i subscript:

$$E\left[g_{1i,t+1}|\Psi_t\right] = E\left[g_{1i,t+1}|\Psi_t\right]. \tag{55}$$

Collecting terms we thus arrive at:

$$\frac{1}{N}\cdot\sum_{i}^{N} g_{1it} \approxeq \lambda_t \cdot y_t^d \cdot mc_t \cdot \frac{1}{N}\cdot\sum_{i}^{N}\frac{1}{e^{\nu_{it}}}+\frac{1}{N}\cdot\sum_{i}^{N}\beta\cdot\theta\cdot\Xi^{-\epsilon}\cdot E_t\left[g_{1i,t+1}|\Psi_t\right]- \tag{56}$$

$$-\frac{1}{N}\cdot\sum_{i}^{N}\frac{1}{e^{\nu_{it}}}\cdot\lambda_t\cdot y_t^d\cdot mc_t\cdot\omega_i\cdot\frac{E\left[K_t^{\alpha_k}|\Omega_{it}\right]-E\left[K_t^{\alpha_k}|\Psi_t\right]}{E\left[K_t^{\alpha_k}|\Psi_t\right]}+ \tag{57}$$

$$+\frac{1}{N}\cdot\sum_{i}^{N}\beta\cdot\theta\cdot\Xi^{-\epsilon}\cdot\left(E\left[g_{1i,t+1}|\Omega_{it}\right]-E\left[g_{1i,t+1}|\Psi_t\right]\right) \tag{58}$$

Using that $\lim\limits_{N\mapsto\infty}\dfrac{1}{N}\cdot\sum_{i}^{N}\frac{1}{e^{\nu_{it}}}\xrightarrow{p}1$, due to the law of large numbers, we can recover the homogeneous information equilibrium value of g_{1t}.[10]

$$\frac{1}{N}\cdot\sum_{i}^{N} g_{1it} \approxeq \underbrace{\lambda_t \cdot y_t^d \cdot mc_t + \beta\cdot\theta\cdot\Xi^{-\epsilon}\cdot E\left[g_{1,t+1}|\Psi_t\right]}_{\text{optimal solution for } g_{1t} \text{ in the homogenous information model}} - \tag{59}$$

$$-\frac{1}{N}\cdot\sum_{i}^{N}\frac{1}{e^{\nu_{it}}}\cdot\lambda_t\cdot y_t^d\cdot mc_t\cdot\omega_i\cdot\frac{E\left[K_t^{\alpha_k}|\Omega_{it}\right]-E\left[K_t^{\alpha_k}|\Psi_t\right]}{E\left[K_t^{\alpha_k}|\Psi_t\right]}+ \tag{60}$$

$$+\frac{1}{N}\cdot\sum_{i}^{N}\beta\cdot\theta\cdot\Xi^{-\epsilon}\cdot\left(E\left[g_{1i,t+1}|\Omega_{it}\right]-E\left[g_{1i,t+1}|\Psi_t\right]\right). \tag{61}$$

The result in Equation (61) establishes that neglecting information heterogeneity and disparate beliefs/expectations in the presence of production externalities would ignore part of the model dyanmics.[11] Under production externalities, there is however, an expectation wedge arising from the contemporaneous effect of heterogeneous information on the optimal policy function for the capital stock:

$$E[K_t]_{wedge} = -e^{-\nu_{it}}\cdot\frac{1}{N}\cdot\sum_{i}^{N}\lambda_t\cdot y_t^d\cdot mc_t\cdot\omega_i\cdot\frac{E\left[K_t^{\alpha_k}|\Omega_{it}\right]-E\left[K_t^{\alpha_k}|\Psi_t\right]}{E\left[K_t^{\alpha_k}|\Psi_t\right]}, \tag{62}$$

and an expectation wedge related to the future path of the real marginal cost dynamics conditioned on the private vs. public information sets, captured by:

$$E[g_{1,t+1}]_{bias} = \frac{1}{N}\cdot\sum_{i}^{N}\beta\cdot\theta\cdot\Xi^{-\epsilon}\cdot\left(E_t\left[g_{1i,t+1}|\Omega_{it}\right]-E_t\left[g_{1,t+1}\Psi_t\right]\right). \tag{63}$$

3.3 Overall Model Equilibrium

The overall equilibrium path of the model depends on the specification of the information sets. Under homogeneous information, the equilibrium path is described by the following equations: the first-order conditions with respect to consumption, Equation (10), the Euler equation, Equation (11), the first-order condition with respect to capital utilization, Equation (12), Tobin's q equation, Equation (13), the first-order condition setting the optimal investment level, and Equation (14) from the household side. These are then complemented by the two equations describing the optimal wage setting, Equations (19) and (20), as well as the law of motion for wages, Equation (18). The problem of the

[10]Note that the sum only converges to its expected value, 1, under homogeneous information with $o_p(1)$.

[11]Note that setting $\alpha_k = 0$ would eliminate both expectation wedges and render the heterogeneous information model equivalent to the homogeneous information one. Such an equivalence would disappear in case higher-order methods were used to approximate the model's first-order conditions; exploring this is beyond the scope of this paper, though.

intermediate output producing firm is characterized by the optimal inputs, Equation (32), the optimal law of motion for prices, Equation (41), and the equation defining the marginal costs, Equation (34). Finally, the market clearing conditions affect the equilibrium path. Under homogeneous information, we also need to add in the expectation wedges.

4 Model Parameterization, Simulation and Empirical Findings

In what follows we describe the procedure to simulate the heterogeneous model with social interactions and a finite number of agents. First the distributional properties of the expectation biases are established, then the simulation steps are discussed, finally an approximate simulation technique is introduced for practical applicability purposes, providing a straightforward and easy way to account for heterogeneity and disparate expectations with social interactions in a rational expectations framework.

4.1 Model Parameterization

Our parameterization of the model is summarized in Table 2. We aim to match moments of the main macroeconomic time series for the Euro Area: output growth, consumption growth, inflation (defined as the GDP deflator), investment growth and short-term interest rates. We set our data sample to commence in 1995:Q4, and, using the real time vintage data set of the European Central Bank, we set the final quarter to be 2015:Q2. We base the calibration of the model parameters concerning real and nominal rigidities on the empirical estimates by Smets and Wouters (2003), but adjust the parameterization to make sure that we can match the relevant moments in our (more recent) sample period. For instance, in order to generate relatively low volatility of consumption growth even in the presence of potentially sizable firm-specific shocks, we choose a degree of habit formation that is equal to 0.7, which is larger than the real-time estimates of Smets and Wouters (0.593), but in line with Villa (2013). Among the key aspects of the parameterization is the interplay between the firm-specific capital share in output and the aggregate capital share in output. We varied the aggregate capital share from 0.28 (coinciding with what Adolfson, Laséen, Lindé and Villani (2007) report in an open-economy setting), to 0.42 (the upper bound on the parameter in Smets and Wouters, 2003). We fixed α_k to be one seventh of α, to allow for a reasonable level of externalities, while maintaining model stability.

4.2 Distributional Properties of the Expectation Wedges

One can further simplify the expressions for the expectation wedges if one sets all firms as equal in the intermediate good producing industry:

$$\omega_i = \frac{1}{N}, \tag{64}$$

and by assuming that the model is solved using the first-order approximation around the steady state, that is, assuming that the model has a state space representation in the log-deviations format. This implies that the solution under homogeneous information is a linear combination of the exogenous shocks, rendering the endogenous variables all normally distributed.

Casting the contemporaneous policy function expectation wedges in log-deviations from steady state, we otain:

$$E[K_t]_{bias} = -\frac{\lambda_t \cdot y_t^d \cdot mc_t}{N^2} \cdot \sum_i^N e^{-\nu_{it}} \cdot \frac{E\left[e^{k_t \alpha_k}|\Omega_{it}\right] - E\left[e^{k_t \alpha_k}|\Psi_t\right]}{E\left[e^{k_t \alpha_k}|\Psi_t\right]} \approxeq \tag{65}$$

$$\approxeq -\frac{\lambda_t \cdot y_t^d \cdot mc_t}{N^2} \cdot \sum_i^N e^{-\nu_{it}} \cdot \frac{E\left[1 + k_t \cdot \alpha_k|\Omega_{it}\right] - E\left[1 + k_t \cdot \alpha_k|\Psi_t\right]}{E\left[1 + k_t \cdot \alpha_k|\Psi_t\right]} = \tag{66}$$

$$= -\frac{\lambda_t \cdot y_t^d \cdot mc_t}{N^2} \cdot \sum_i^N e^{-\nu_{it}} \cdot \frac{\alpha_k \cdot \left(E\left[k_t|\Omega_{it}\right] - E\left[k_t|\Psi_t\right]\right)}{1 + \alpha_k \cdot E\left[k_t|\Psi_t\right]}. \tag{67}$$

To understand the distributional properties of the expectation wedges, consider the following steps: We know that the first-order approximation-based solution for the policy function has the following form under homogeneous information:

$$k_t = A \cdot (k_{t-1} - k_{ss}) + k_{ss} + B \cdot \epsilon_t, \tag{68}$$

where ϵ_t contains all exogenous state variables included in the filtration created by the public information set Ψ_t. Iterating backward, assuming for simplicity that all eigenvalues of A fall inside the unit circle, we obtain:

$$k_t - k_{ss} = B \cdot \epsilon_t + A \cdot B \cdot \epsilon_{t-1} + A^2 \cdot B \cdot \epsilon_t + ... = \sum_{j=0}^{t} A^j \cdot B \cdot \epsilon_{t-j}. \tag{69}$$

Knowing that the optimal policy function is linear in the exogenous variables,[12] we also

[12] This involves choosing the initialization's distribution to be the same as the distribution of $\epsilon_t, \forall t$.

know the distribution of $E\left[k_t|\Psi_t\right]$. We know that the policy function is normally distributed. Therefore the term $E\left[k_t|\Psi_t\right]$ is normally distributed with mean k_{ss} and a known finite variance, denoted by Σ_k:[13]

$$E\left[k_t|\Psi_t\right] \sim N(k_{ss}, \Sigma_k). \tag{70}$$

Having established the distribution of the homogeneous information policy function, let us next consider the policy function of intermediate good producing firm i under heterogeneous information. Note the expectation wedge in equilibrium will be solely driven by the realization of the intermediate good producing firm-specific shock, ν_{it}:

$$E\left[k_t|\Omega_{it}\right] - E\left[k_t|\Psi_t\right] = \frac{1}{N} \cdot \sum_i^N B_i \cdot \begin{bmatrix} 0 \\ \nu_{it} \end{bmatrix} \sim N(0, b_{k\nu_t}^2 \cdot \sigma_{\nu_{it}}^2). \tag{71}$$

Including the shock ν_{it} in the law of motion for technology, note that the original coefficient on the productivity shock will be split between the shocks in accordance with the relative variance of ν_{it} compared to the variance of z_{At}, that is, ϵ_{At}. We have thus established that the expectation wedge will be a product of correlated lognormal and normal distributions, divided by another normal distribution:

$$E[K_t]_{wedge} \approxeq -\frac{\lambda_t \cdot y_t^d \cdot mc_t}{N^2} \cdot \sum_i^N \underbrace{e^{-\nu_{it}}}_{\text{Lognormal}} \cdot \frac{\overbrace{\alpha_k \cdot \left(E\left[k_t|\Omega_{it}\right] - E\left[k_t|\Psi_t\right]\right)}^{\sim N(0, \alpha_k^2 b_{k\nu_t}^2 \cdot \sigma_{\nu_{it}}^2)}}{1 + \alpha_k \cdot \underbrace{E\left[k_t|\Psi_t\right]}_{\sim N(k_{ss}, \Sigma_k)}}. \tag{72}$$

Similarly, the distribution of the wedge for the path of marginal costs will be normal, with the steady state as the first and the homogeneous information equilibrium variance of the intermediate good producing firm's policy function as the second moment:

$$E\left[g_{1i,t+1}|\Omega_{it}\right] - E\left[g_{1,t+1}|\Psi_t\right] \sim N(0, b_{g_{1,t+1},\nu_t}^2 \cdot \sigma_{\nu_{it}}^2). \tag{73}$$

Therefore $E[g_{1,t+1}]_{wedge}$ is also normally distributed:

$$E[g_{1,t+1}]_{wedge} = \frac{1}{N} \cdot \sum_i^N \beta \cdot \theta \cdot \Xi^{-\epsilon} \cdot \left(E\left[g_{1i,t+1}|\Omega_{it}\right] - E\left[g_{1i,t+1}|\Psi_t\right]\right). \tag{74}$$

[13]This variance is the linear combination of the exogenous random variables' variance, due to the first order state space representation.

4.3 Simulation Procedure

1. The intermediate good producing firm's model is solved under the assumption that the firm-specific shock is part of the information set, to find the values of $b^2_{k\nu}$ and $b^2_{g_{1,t+1},\nu_t}$. The intermediate good producing firm's model contains all information except for the aggregate K. Intermediate good producing firms assume that their decision will not influence the behavior of the aggregate, therefore intermediate good producing firms treat the aggregate K_t as given. If the aggregate K_t is a parameter, it only affects the constants in the model, and not the model dynamics. This implies that individual intermediate good producing firms wrongly believe that the number of intermediate good producing firms in the economy is very large ($N \rightarrowtail \infty$). The intermediate good producing firms' model features the technology process $log(A_{it}) = \Lambda_A + z_{At} + \nu_{it}$, and sets the capital share in output to α.

2. The model without the intermediate good producing firm-specific shocks is solved to calculate the moments of $E[k_t|\Psi_t]$ and $E[g_{1,t+1}|\Psi_t]$. The homogeneous information model is the model with full information and the technology process $log(A_{it}) = \Lambda_A + z_{At} + \nu_{it}$, and sets the capital share in output to $\alpha + \alpha_k$.

3. The heterogeneous information model is solved with a finite number, N, intermediate good producing firms, inserting the draws from the expectation wedge terms as a given sequence of shocks:

   - $E\left[k_t|\Psi_t\right]$ is drawn. The realization of this random variable will be the same for all intermediate good producing firms, as they observe and are able to evaluate the impact of the realized public shocks on the policy function.
   - Setting ρ as a parameter to 0, and choosing σ^2_ν, ν_{it} is drawn as $\nu_{it} = \epsilon_{\nu it}$ with $\epsilon_{\nu it} \sim \mathcal{N}(-\frac{\sigma^2_\nu}{2\cdot(1+\rho)}, \sigma^2_\nu)$ for each individual intermediate good producing firm. $e^{-\nu_{it}}$ is calculated for each intermediate good producing firm.
   - $E\left[k_t|\Omega_{it}\right] - E\left[k|\Psi_t\right]$ and $E_t\left[g_{1i,t+1}|\Omega_{it}\right] - E_t\left[g_{1,t+1}|\Psi\right]$ are calculated using the realized shock ν_{it}.
   - $E[g_{1,t+1}]_{bias}$ and $E[K_t]_{bias}$ are calculated.
   - Finally the heterogeneous information model with wedges in period t is solved and simulated.

4. Step 3. is repeated 10,000 times in a Monte Carlo experiment, to eliminate the impact of random number generator initialization.

All systems are solved using first order approximation around the steady state.

4.4 Empirical Findings

We adduce the following tables and figures to document the properties of our business cycle model under diverse higher-order beliefs:

Table 4 reports the standard deviation of output and other key model variables relative to output. When the variance of the firm-specific shocks is zero, these moments match those in baseline DSGE model analyses (as they should, given a modest-only degree of capital externalities). As the variance of the intermediate good producing firm-specific shocks rises relative to aggregate shocks in the intermediate good producing industry, the overall volatility of output rises as well. Table 2 documents, though, that the persistence of output rises even more notably as the variance of the intermediate good producing firm-specific shocks rises relative to aggregate shocks in the intermediate good producing industry. Cyclical fluctuations exhibit stronger memory under higher-order belief diversity.

Figures 1 to 7 report Burns-Mirchell diagrams (see also Bry and Boschan, 1971, who label such diagrams recession-recovery patterns) for our simulated series, to report on the typical pattern of the model series around business cycle peaks.[14] Figure 1 makes clear that as the variance of the intermediate good producing firm-specific shocks rises relative to aggregate shocks in the intermediate good producing industry, business cycles under our diverse higher-order belief structure gain in amplitude and become longer lasting than homogeneous information-based business cycles. Figure 2 traces this to (average) profits in the intermediate good producing industry, and Figure 3 shows the spectrum of profit realizations over the business cycle across all firms in the intermediate good producing industry. The heightened amplitude of profits under disparate higher-order beliefs is mirrored by the dynamics of the investment series, but much less so by the dynamics of the consumption series (Figures 4 and 5). The heightened fluctuations in output and inflation lead to stronger adjustments by the monetary authority, as Figures 6 and 7 illustrate. The fact that the intermediate good producing firms remain profitable even during waves of pessimism is further documented by Figure 8.

The remaining figures document conditional model dynamics, in response to intermediate good producing firm-specific technology/cost shocks, and in response to monetary policy shocks. While the conditional dynamics in response to intermediate good producing

[14] For the Burns-Mithcell diagrams, we identify business cycle peaks and troughs by a rule of thumb: a quarter is labeled a business cycle peak if it is immediately preceding two successive quarters with negative growth rates of output; and a quarter is labeled a business cycle trough if it is the first quarter following a business cycle peak that is immediately preceding two successive quarters with positive growth rates of output. The Burns-Mitcehll diagrams plot averages across all business cycles of simulated output and all other simulated variables of interest, in percentage deviations from overall average values, for ten quarters before and ten quarters after the cyclical peak

firm-specific technology/cost shocks reflect in more detail how higher-order diverse beliefs can generate sizable propagation, it is notable that the conditional dynamics in response to a monetary policy shock are much less affected by the presence of diverse higher-order beliefs in the intermediate good producing industry.

5 Conclusion

In this paper, we have advanced a DSGE-based businsess cycle model that closely mirrors Pigou's (1929) description of recessions and recoveries being linked to waves of pessimism and optimism that decision makers at firms operate through. To this purpose, we have augmented a standard, representative agent-based DSGE model to capture production externalities, firm-specific cost shocks and heterogeneous information across firms. We have shown in this paper how in such an augmented DSGE model the information that firms base their decisions on is partially determined by the expectations of other firms, with there being common knowledge that every firm is in the same situation of perpetual learning about the other firms, leading to firms having to "forecast the forecasts of others", that is, an infinite regress in expectations in which higher-order beliefs matter. Our model solution relative to the solution of the standard, representative agent-based DSGE model features an additional term capturing the belief diversity of the firms. Parameterizing our model on the basis of Euro Area data, we document how this belief diversity affects business cycle dynamics. We find, in particular, that in comparison to the underlying standard DSGE model, our model featuring diverse higher-order beliefs can predict deeper and more prolonged recessions, with the magnitude of the differences in the recession dynamics being a function most prominently of the magnitude of the firm-specific costs shocks, relative to the aggregate costs shocks. Based on these findings, our paper shows that ceteris paribus the Great Recession of 2008/2009 that followed the global financial crisis can be captured better in a business cycle model with diverse higher-order beliefs than in a corresponding business cycle model without belief diversity as a propagation mechanism.

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Table 1: Data series on which parameter values are based

Series	Source	Transfromation	Frequency	Sample
Real Ouput Growth	RTD.Q.S0.S.G_GDPM_TO_U.E: Nominal GDP (Seasonally adjusted, not working day adjusted, Millions of euros)	Deflated by the GDP deflator, DEFLATORVint	Quarterly	1995:Q4 to 2015:Q2
Real Consumption Growth	RTD.Q.S0.S.G_FCHI_TO_U.E: Private Consumption Nominal (PCN) (Seasonally adjusted, not working day adjusted, Final consumption of households and NPISHs, Millions of euros).	Deflated by the GDP deflator, DEFLATORVint	Quarterly	1995:Q4 to 2015:Q2
Real Investment Growth	RTD.Q.S0.S.G_GFCF_TO_U.E: Gross Nominal Fixed Capital Formation (Seasonally adjusted, not working day adjusted, Millions of euros).	Deflated by the GDP deflator, DEFLATORVint	Quarterly	1995:Q4 to 2015:Q2
Inflation	RTD.Q.S0.S.G_GDPM_TO_D.X : Price Index for GDP (Seasonally adjusted, not working day adjusted, Index level)	1/4th of annual rate	Quarterly	1995:Q4 to 2015:Q2
Policy rate	RTD.M.S0.N.C_EONIA.E: Eonia Rate	1/4th of annual rate	Quarterly	1995:Q4 to 2015:Q2
Real money balances growth rate	RTD.M.S0.Y.M_M3_V_NC.E: Monetary aggregate M3, all currencies combined - outstanding amounts at the end of the period (Millions of euros, working day and seasonally adjusted)	Deflated by the GDP deflator, DEFLATORVint	Quarterly	1995:Q4 to 2015:Q2

Table 2: Calibrated parameters

Parameter	Description	Value
α	Capital share	0.21 and 0.35
α_k	Capital externality parameter	$\frac{1}{7}$ of α
δ	Depreciation rate	0.025
ϵ	Elasticity of substitution between goods varieties	10
η	Elasticity of substitution between labor varieties	10
Φ	Firm fixed costs	0
Λ_2	Capital utilization, quadratic term	0.143
$100(\beta - 1)$	Discount factor	0.998
h	Degree of habit	0.7
γ	Inverse Frisch labor elasticity of labor supply	0.24
ψ	Labor disutility parameter	8.92
κ	Capital adjustment costs parameter	0.175
θ_p	Calvo parameter for prices	0.74
χ	Price indexation for goods	0.16
θ_w	Calvo parameter wages	0.68
χ_w	Price indexation wages	0.51
ρ_R	Interest rate smoothing coefficient	0.92
γ_y	Taylor rule coefficient on output growth rate	0.10
γ_Π	Taylor rule coefficient on output inflation growth rate	0.15
$\Pi*$	Steady state level of inflation	1.016
ρ_d	Autocorrelation of preference shock	0.12
ρ_ϕ	Autocorrelation of labor disutility shock	0.93
σ_A	Log standard deviation of neutral technology shock	-3.97
σ_d	Log standard deviation preference shock	-1.51
σ_ϕ	Log standard deviation labor disutility shock	-2.36
σ_μ	Log standard deviation investment-specific technology	-5.43
σ_m	Log standard deviation preference shock	-5.85
Λ_μ	Steady state growth rate of investment-specific technology	3.4e-3
Λ_A	Steady state neutral technology growth	2.8e-3

Table 3: Volatilities of Varios Key Series

Relative Size of Firm Specific Shocks	**Output**	**Consumption/Output**	**Investment/Output**	**Profits/Output**	**Interest Rate/Output**	**Inflation/Output**
0,0	0,00135	0,76189	1,15494	0,26165	0,21785	0,20939
0,5	0,00135	0,76243	1,15398	0,57699	0,21742	0,20931
0,9	0,00136	0,75820	1,14641	0,66073	0,21596	0,20920
1,4	0,00140	0,73315	1,12007	0,53440	0,21319	0,20940
1,8	0,00153	0,67181	1,06592	0,28934	0,20946	0,21024
2,3	0,00178	0,57855	0,99384	0,15463	0,20601	0,21161
2,7	0,00217	0,47675	0,92630	0,18184	0,20382	0,21301
3,2	0,00270	0,38638	0,87645	0,02575	0,20286	0,21410
3,6	0,00335	0,31428	0,84417	0,17541	0,20263	0,21486
4,1	0,00412	0,25932	0,82443	0,36806	0,20274	0,21536
4,5	0,00500	0,21802	0,81254	0,39452	0,20297	0,21570
5,0	0,00600	0,18700	0,80535	0,37016	0,20323	0,21593

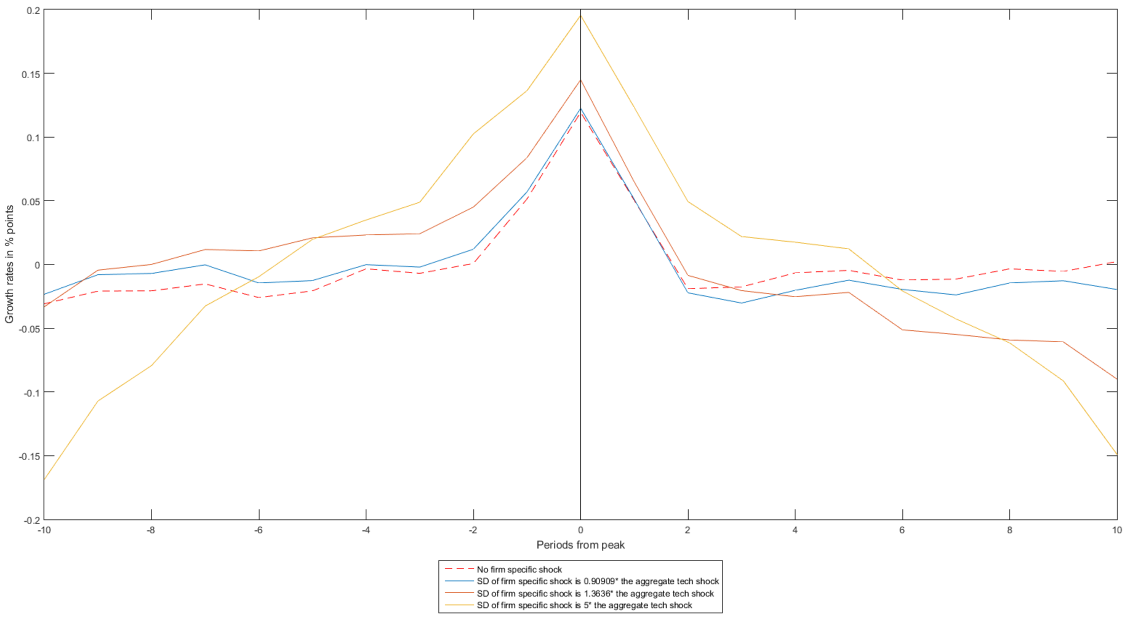
Note: The table involves $N = 100$, $\alpha = 0.21$ and $\alpha_k = 0.03$.

Table 4: Persistence of Output

Relative Size of Firm Specific Shocks	**Partial Autocorrelation Coefficients** 1st Lag	2nd Lag	3rd Lag	4th Lag	5th Lag
0,0	0,948	0,897	0,851	0,808	0,765
0,5	0,948	0,897	0,851	0,807	0,765
0,9	0,948	0,897	0,852	0,808	0,765
1,4	0,951	0,903	0,858	0,815	0,772
1,8	0,958	0,915	0,874	0,832	0,789
2,3	0,968	0,932	0,895	0,856	0,814
2,7	0,977	0,948	0,916	0,878	0,837
3,2	0,983	0,960	0,930	0,895	0,854
3,6	0,988	0,967	0,940	0,906	0,865
4,1	0,990	0,972	0,946	0,913	0,873
4,5	0,992	0,975	0,950	0,918	0,877
5,0	0,993	0,977	0,953	0,920	0,880

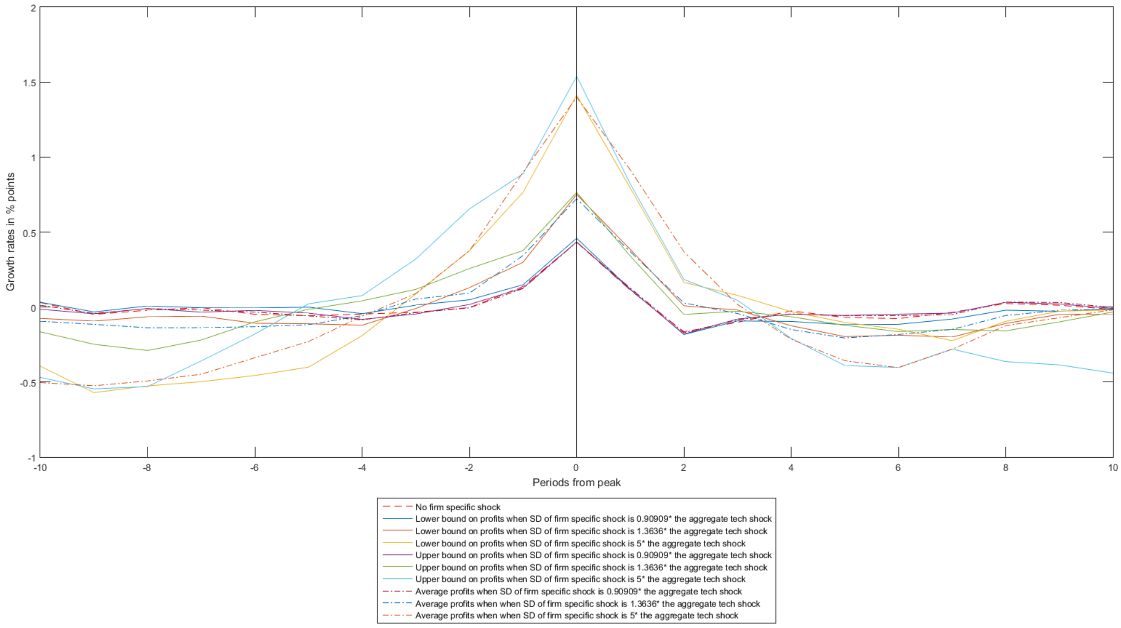
Note: The table involves $N = 100$, $\alpha = 0.21$ and $\alpha_k = 0.03$.

Figure 1: Burns-Mitchell Diagram for Output



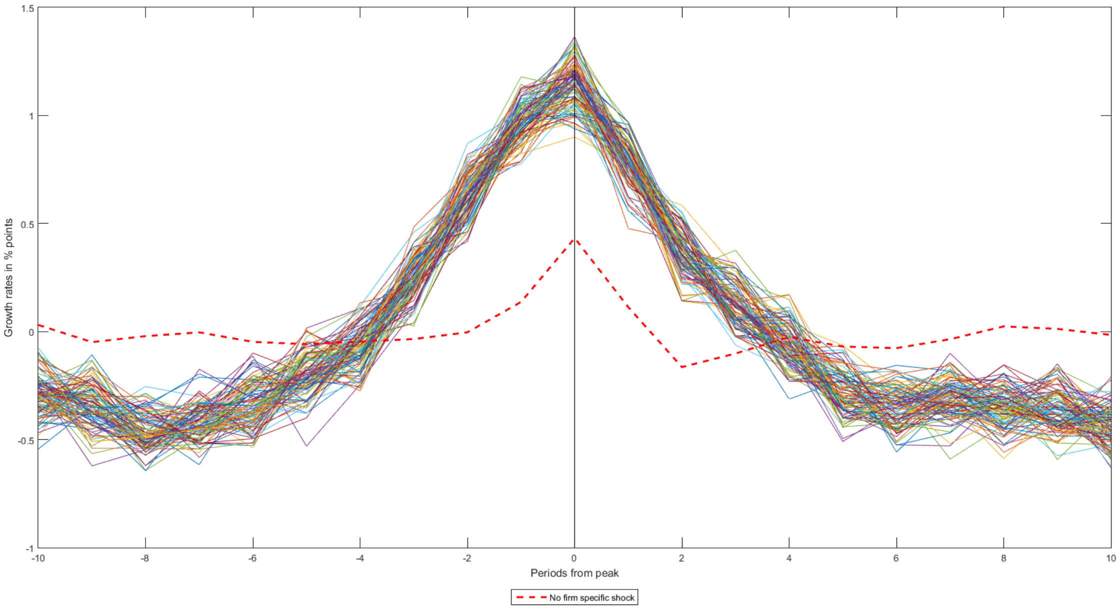
Note: This figure involves $N = 100$, $\alpha = 0,21$ and $\alpha_k = 0,03$.

Figure 2: Burns-Mitchell Diagram for Average Profits



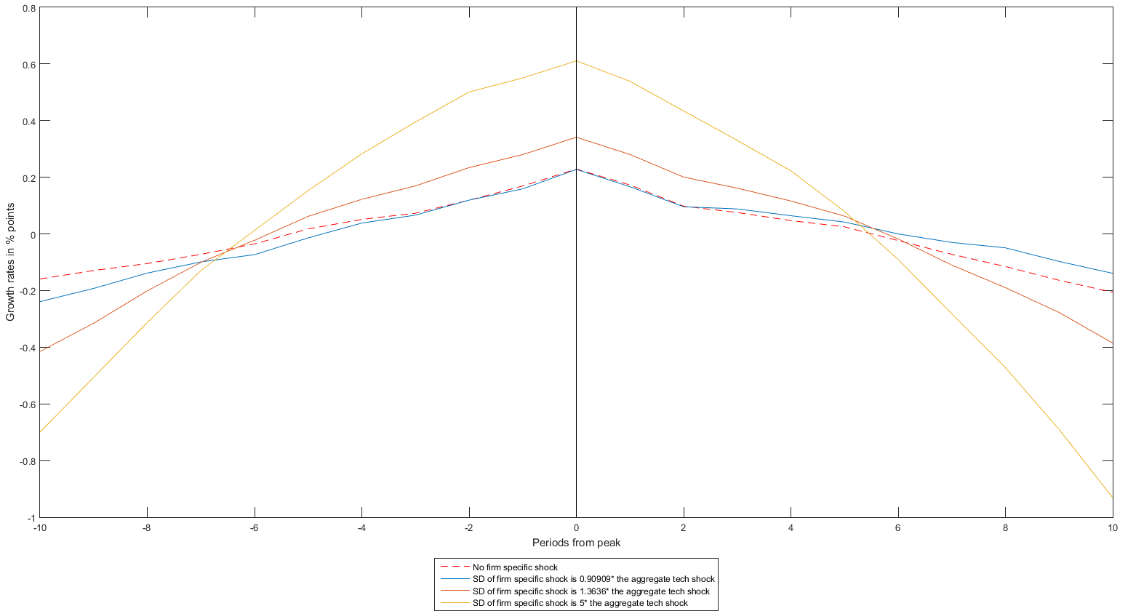
Note: This figure involves $N = 100$, $\alpha = 0,21$ and $\alpha_k = 0,03$.

Figure 3: Burns-Mitchell Diagram for Range of Profits



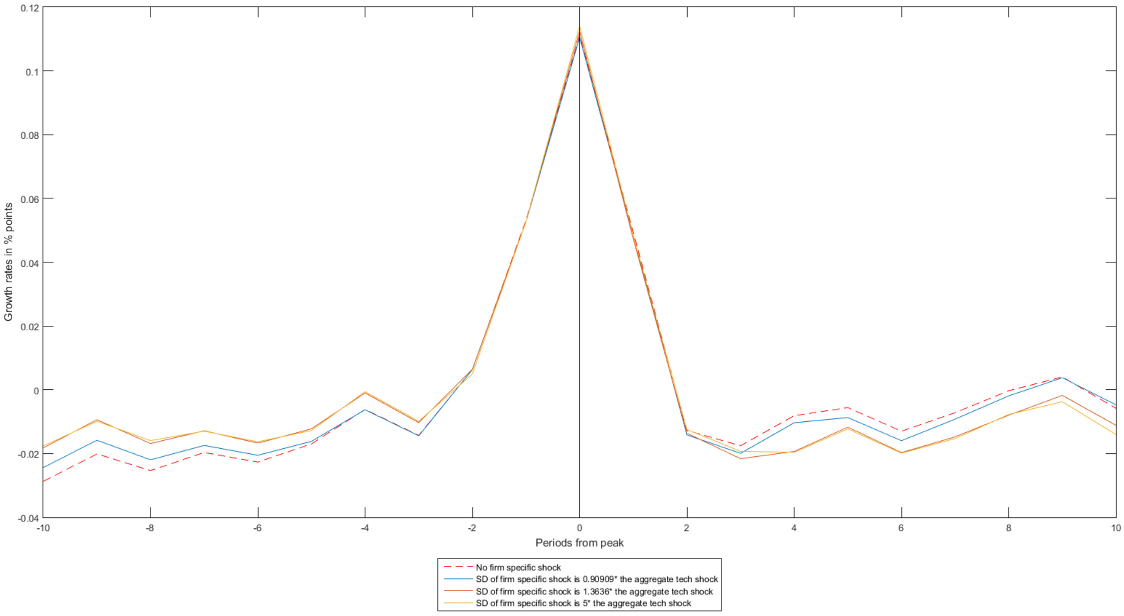
Note: This figure involves $N = 100$, $\alpha = 0,21$ and $\alpha_k = 0,03$.

Figure 4: Burns-Mitchell Diagram for Investments



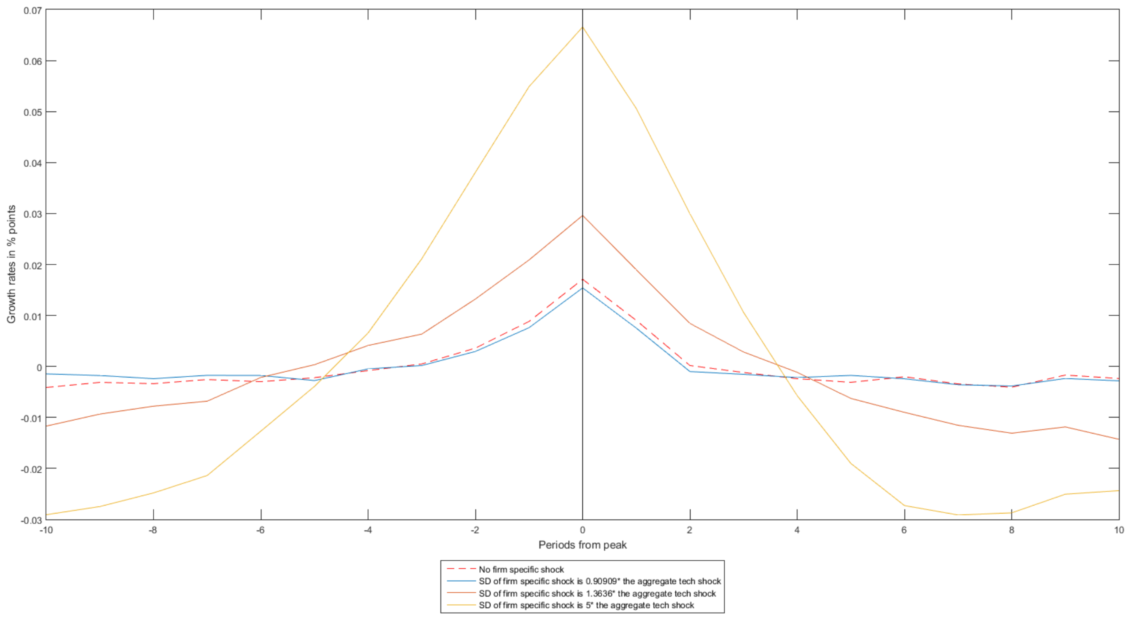
Note: This figure involves $N = 100$, $\alpha = 0,21$ and $\alpha_k = 0,03$.

Figure 5: Burns-Mitchell Diagram for Consumption



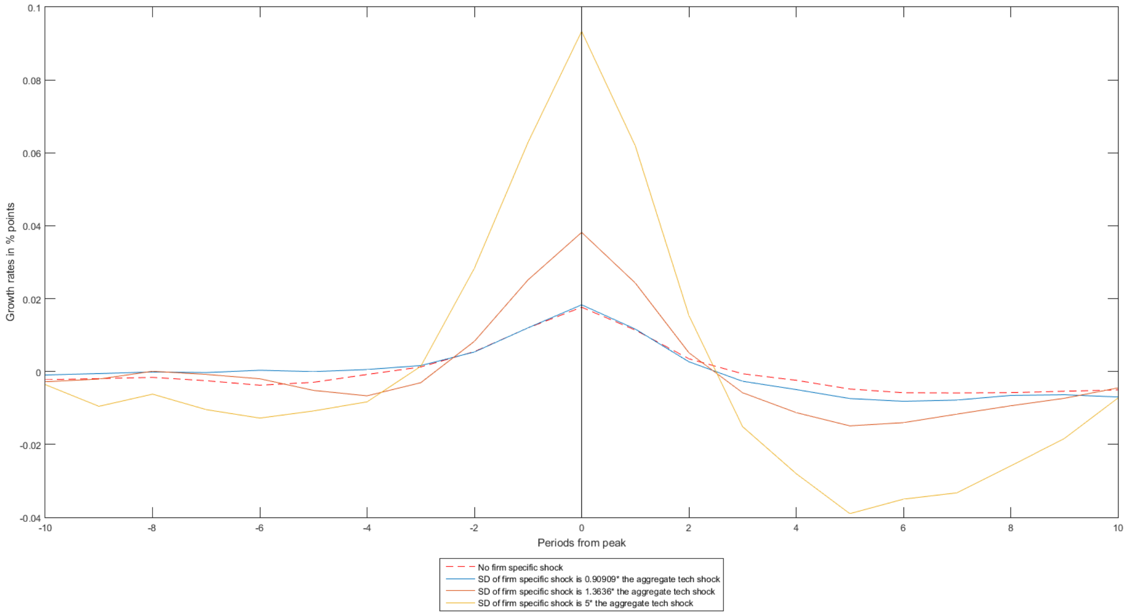
Note: This figure involves $N = 100$, $\alpha = 0,21$ and $\alpha_k = 0,03$.

Figure 6: Burns-Mitchell Diagram for Interest Rates



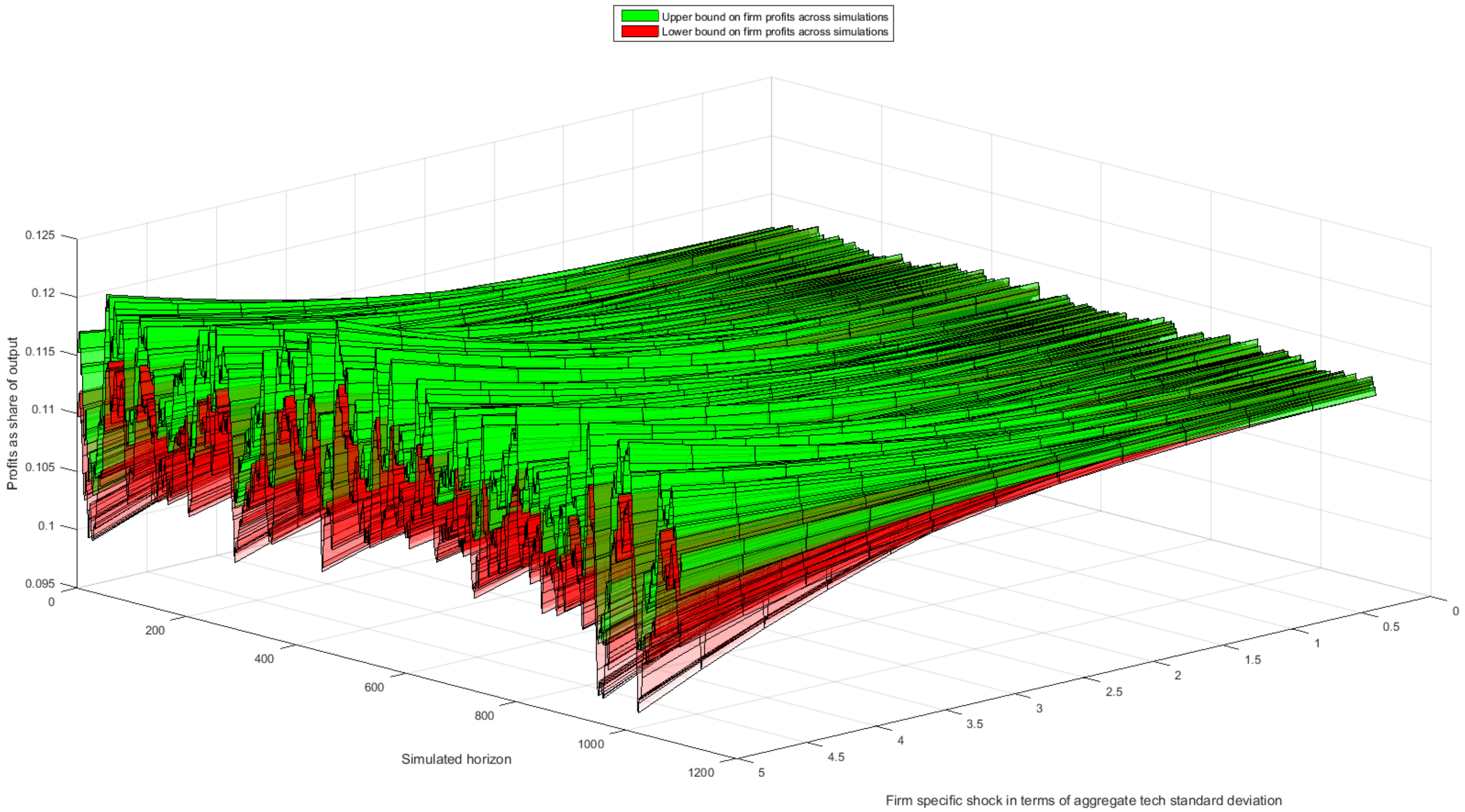
Note: This figure involves $N = 100$, $\alpha = 0,21$ and $\alpha_k = 0,03$.

Figure 7: Burns-Mitchell Diagram for Inflation



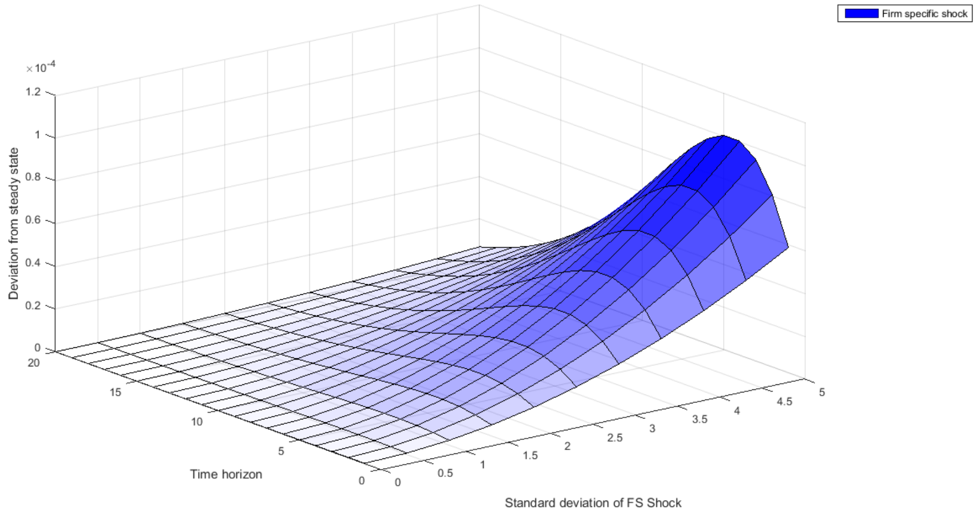
Note: This figure involves $N = 100$, $\alpha = 0,21$ and $\alpha_k = 0,03$.

Figure 8: Simulated Profits for 1200 Periods



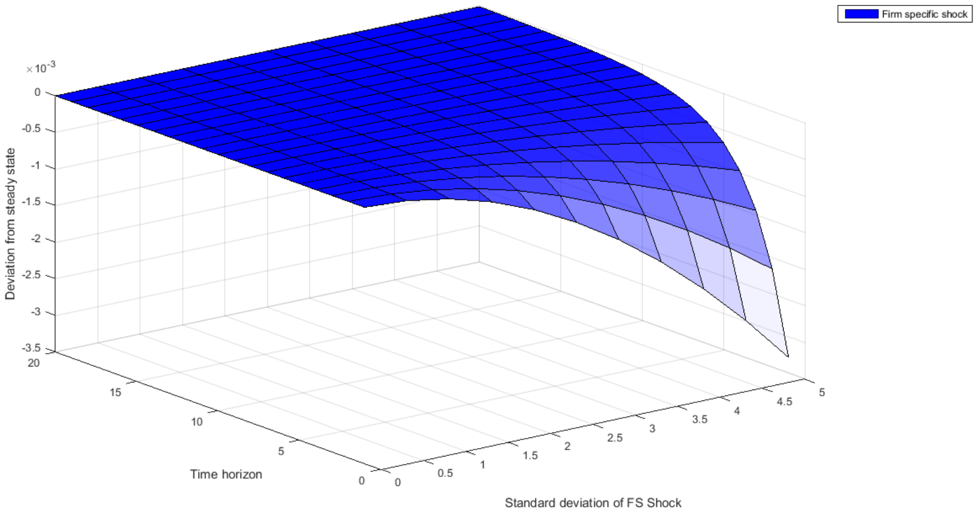
Note: This figure involves $N = 100$, $\alpha = 0.21$ and $\alpha_k = 0.03$. On the x-axis the firm specific shock is expressed in terms of the aggregate technology shock. On the y-axis the simulated periods are plotted. The z-axis represents the share of firm profits of output. The lower, blueish surface shows the share of profits for individual firm output for the worst off firm across all firms in a given period, the lower bound. The upper surface represents the best off firm for a given simulated period.

Figure 9: Impulse Response for Output Respect to Firm Specific Shocks



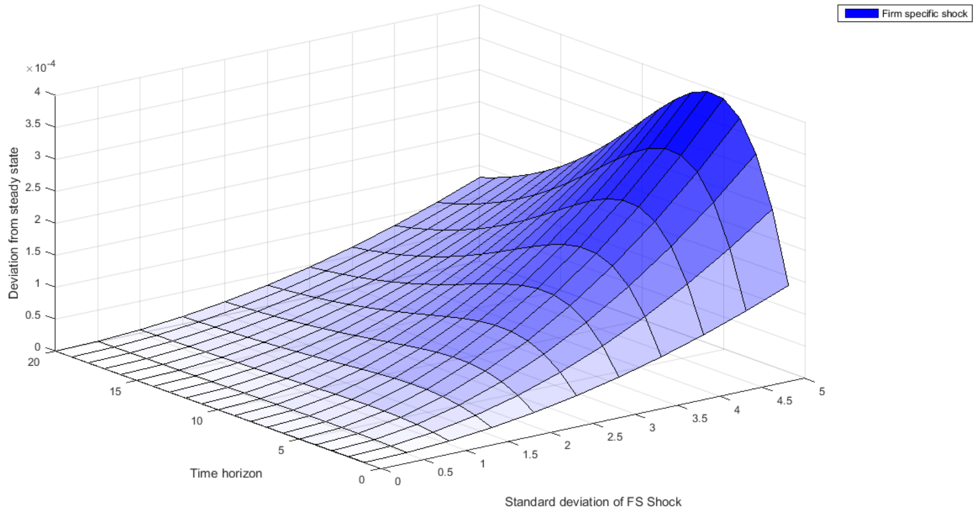
Note: This figure involves $N = 100$, $\alpha = 0,21$ and $\alpha_k = 0,03$.

Figure 10: Impulse Response for Average Profits Respect to Firm Specific Shocks



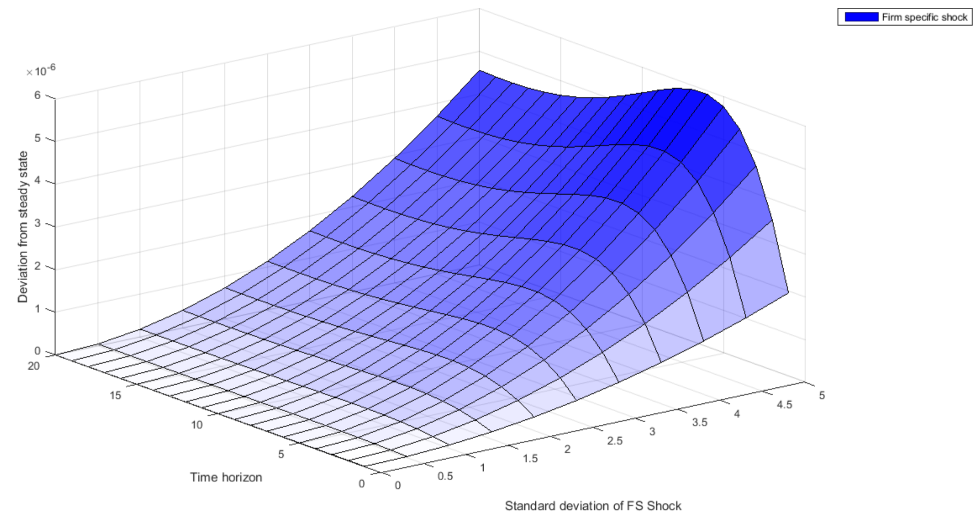
Note: This figure involves $N = 100$, $\alpha = 0,21$ and $\alpha_k = 0,03$.

Figure 11: Impulse Response for Investment Respect to Firm Specific Shocks



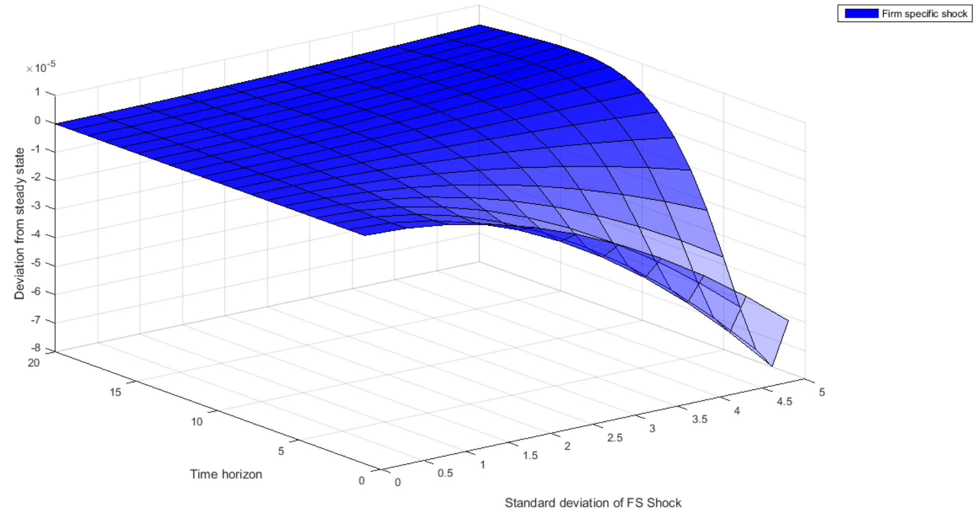
Note: This figure involves $N = 100$, $\alpha = 0,21$ and $\alpha_k = 0,03$.

Figure 12: Impulse Response for Consumption Respect to Firm Specific Shocks



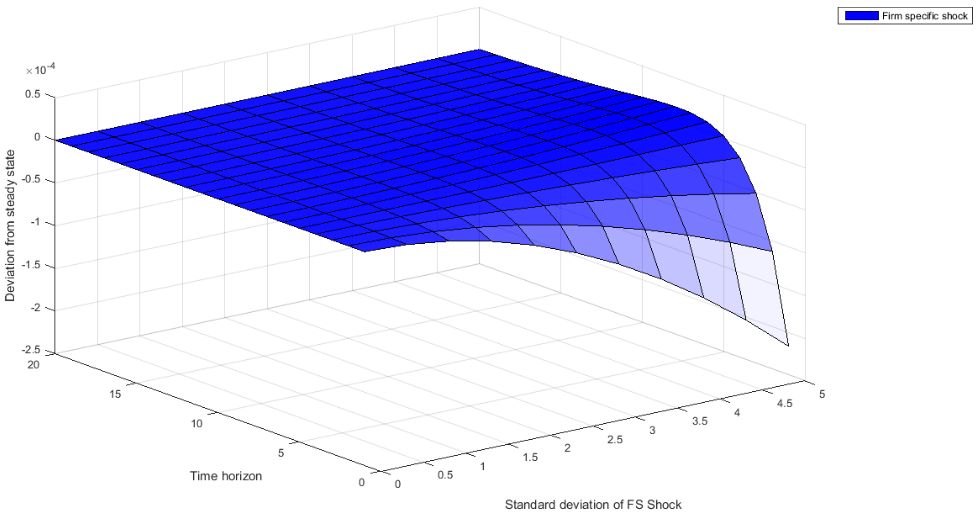
Note: This figure involves $N = 100$, $\alpha = 0,21$ and $\alpha_k = 0,03$.

Figure 13: Impulse Response for Interest Rate to Firm Specific Shocks



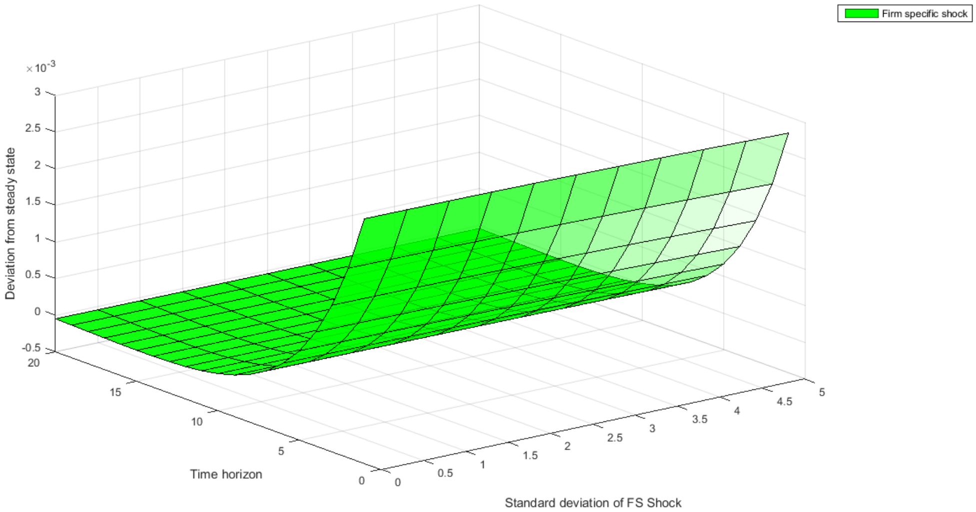
Note: This figure involves $N = 100$, $\alpha = 0,21$ and $\alpha_k = 0,03$.

Figure 14: Impulse Response for Inflation to Firm Specific Shocks



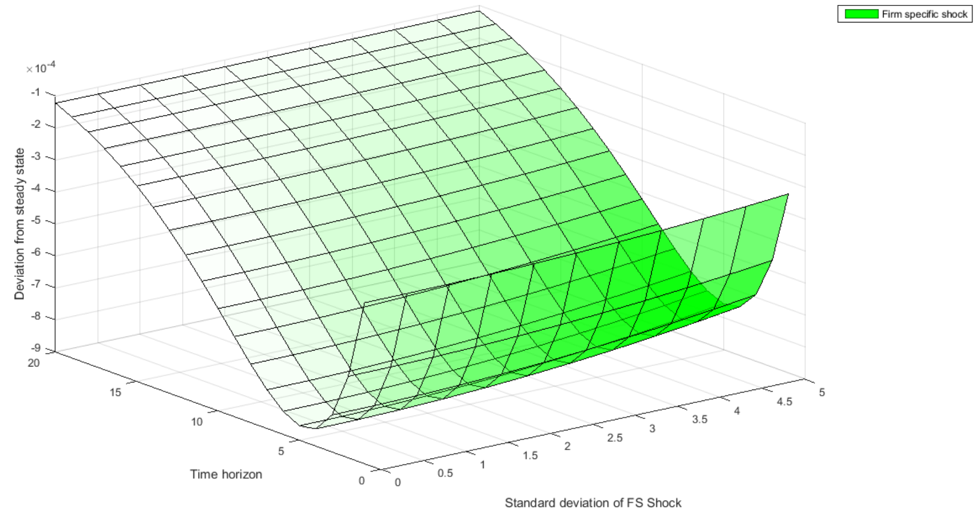
Note: This figure involves $N = 100$, $\alpha = 0,21$ and $\alpha_k = 0,03$.

Figure 15: Impulse Response for Interest Rate with Respect to a Monetary Policy Shock



Note: This figure involves $N = 100$, $\alpha = 0,21$ and $\alpha_k = 0,03$.

Figure 16: Impulse Response for Output with Respect to a Monetary Policy Shock



Note: This figure involves $N = 100$, $\alpha = 0,21$ and $\alpha_k = 0,03$.