A Generalized Approach to Indeterminacy in Linear Rational Expectations Models

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Introduction

Modern economics is based on dynamic general equilibrium theory

$\Rightarrow$ Possibility of indeterminacy under realistic parameter values

How to think of indeterminacy?

- Consider an *infinite time horizon* economy with a *unique* steady state
- If <u>deterministic</u> $\rightarrow$ multiple equilibrium paths
- If <u>stochastic</u> $\rightarrow$ multiple adjustment paths in response to fundamental shocks

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Renewed interest of empirical and theoretical macro literature in testing for indeterminacy in rational expectations models.

$\Rightarrow$ *We propose a novel approach to deal with the problem of indeterminacy in Linear Rational Expectations models*

Testing for indeterminacy: Bayesian approach

Common practice to empirically test for determinacy:

1. Estimate model under determinacy and indeterminacy, separately;
2. Compare the corresponding marginal data densities;
3. Draw inference about (in)determinacy from the highest marginal data density.

Limitations:

→ The researcher estimates the *same* model in two regions of the parameter space (not two *structurally different* models);

→ The conventional approach is harder to implement if the region of determinacy is unknown (e.g. Taylor principle).

→ Coding on the side of the researcher is often required (estimation under indeterminacy is usually not part of standard packages).

Contribution

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- Estimates a LRE model over the *entire* parameter space;
  - $\rightarrow$ Posterior distributions could lie in both regions of the parameter space;
- Is applicable also when the region of determinacy is *unknown*;
- Is implementable in standard software packages.
- Delivers solutions equivalent to:
  - standard solutions under determinacy (Sims, 2002);
  - the solutions of Lubik and Schorfheide (2003) and Farmer et al. (2015) under indeterminacy;

Building the intuition

- Consider the univariate LRE model from Lubik and Schorfheide (2004)

$$y_t = \frac{1}{\theta} E_t(y_{t+1}) + \varepsilon_t, \qquad \varepsilon_t \sim iidN(0, \sigma^2), \qquad \theta \in [0, 2].$$

- Define $\xi_t \equiv E_t(y_{t+1})$ and $\eta_t \equiv y_t - \xi_{t-1}$.

- Rewrite LRE model as

$$\xi_t = \theta \xi_{t-1} - \theta \varepsilon_t + \theta \eta_t$$

BK condition in original model

Consider the LRE model

$$\xi_t = \theta \xi_{t-1} - \theta \varepsilon_t + \theta \eta_t \tag{1}$$

BK condition: determinacy if number of unstable roots equals number of expectational variables.

Let

- p be number of expectational variables $\rightarrow p = 1$
- n be number of unstable roots of the system

$\Rightarrow$ Degrees of indeterminacy: $m \equiv p - n$

Solution:

- Determinacy if $\theta > 1$: $\Rightarrow$ $m = 0$ $\Rightarrow$ $\{\eta_t = \varepsilon_t, \ \xi_0 = 0, \ y_t = \varepsilon_t\}$
- Indeterminacy if $\theta < 1$: $\Rightarrow$ $m = 1$ $\Rightarrow$ $\{y_t = \theta y_{t-1} - \theta \varepsilon_t + \eta_t\}$

BK condition in augmented model

Our approach proposes to solve the augmented system:

$$\begin{aligned} \xi_t &= \theta\xi_{t-1} - \theta\varepsilon_t + \theta\eta_t, \\ \omega_t &= \alpha\omega_{t-1} - \nu_t + \eta_t. \end{aligned} \quad (2)$$

Table: Blanchard-Kahn condition in the augmented representation

α	Unstable Roots	BK condition in augmented model (2)	Solution
Determinacy $\theta > 1$ in original model (1)			
< 1	1	Satisfied	$\{y_t = \varepsilon_t, \, \omega_t = \alpha\omega_{t-1} - \nu_t + \varepsilon_t\}$
> 1	2	Not satisfied	-
Indeterminacy $\theta < 1$ in original model (1)			
< 1	0	Not satisfied	-
> 1	1	Satisfied	$\{y_t = \theta y_{t-1} - \theta\varepsilon_t + \eta_t, \, \omega_t = 0\}$

Applying our method

1. Apply our methodology to New-Keynesian model in Lubik and Schorfheide (2004).

2. We run two simulations of the model, under determinacy and indeterminacy respectively.

3. We estimate the augmented representation when the region of determinacy is assumed to be *unknown*.

4. We show that the posterior estimates recover the true parameters used for the simulations.

The New-Keynesian model in LS

- Dynamic IS curve

$$x_t = E_t(x_{t+1}) - \tau(R_t - E_t(\pi_{t+1})) + g_t$$

- NKPC

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa(x_t - z_t)$$

- Monetary policy

$$R_t = \rho_R R_{t-1} + (1-\rho_R)[\psi_1 \pi_t + \psi_2(x_t - z_t)] + \varepsilon_{R,t}$$

- Rational expectation forecast errors

$$\eta_{1,t} = x_t - E_{t-1}(x_t) \qquad \eta_{2,t} = \pi_t - E_{t-1}(\pi_t)$$

Conditions for determinacy

Let

- p be number of expectational variables $\rightarrow p = 2$
- n be number of unstable roots of the system

$\Rightarrow$ Degrees of indeterminacy: $m \equiv p - n$

Define $\psi^* \equiv \psi_1 + \frac{(1-\beta)}{\kappa}\psi_2$.

Taylor principle and determinacy:

- If $|\psi^*| > 1 \quad \Rightarrow \quad m = 0 \quad \Rightarrow \quad$ Sims (2002)
- If $|\psi^*| < 1 \quad \Rightarrow \quad m = 1 \quad \Rightarrow \quad$ Farmer et al. (2015)

Our methodology

- We propose to append $m = 1$ autoregressive process

$$\omega_t = \alpha\omega_{t-1} + \nu_t - \eta_{2,t}$$

- For intuition, consider $\alpha \equiv 1/|\psi^*|$
- Under determinacy $|\psi^*| > 1$
  - $\rightarrow$ We introduce m stable roots ($\alpha < 1$) and BK conditions are satisfied
  - $\rightarrow$ Our solution equivalent to Sims (2002)
- Under indeterminacy $|\psi^*| < 1 \quad \rightarrow \quad m = p - n = 1$
  - $\rightarrow$ We introduce m unstable roots ($\alpha > 1$) and BK conditions are satisfied
  - $\rightarrow$ Our solution equivalent to Farmer et al. (2015)

Estimation for *unknown* region of determinacy

- Suppose the researcher *does not know* the region of determinacy.

- Our methodology still appends $m = 1$ autoregressive process,

$$\omega_t = \alpha\omega_{t-1} + \nu_t - \eta_{2,t}.$$

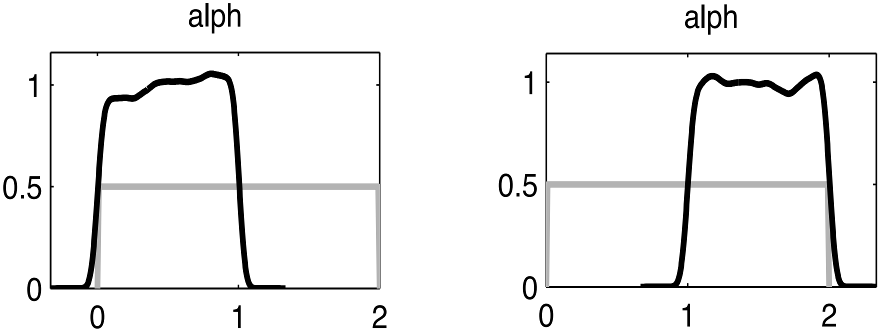
- However, we *cannot* set $\alpha \equiv 1/|\psi^*|$.

$\Rightarrow$ We assume uniform prior distribution: $\alpha \sim U[0, 2]$

$\rightarrow$ Equal probability of drawing α from $[0, 1]$ as well as from $(1, 2]$.

$\Rightarrow$ We estimate the augmented representation using both simulated time series and recover the true parameters.

Posterior distribution of α



(a) Determinacy (b) Indeterminacy

Note: the grey line represents the prior distribution of the parameter α. The black line is the posterior distribution.

Posterior estimates (determinacy)

	True values	Posterior estimates	
		Mean	90% probability interval
α	-	0.51	[0.11,0.99]
ψ_1	2.1	1.92	[1.63,2.20]
ψ_2	0.17	0.37	[0.06,0.67]
ρ_R	0.60	0.61	[0.57,0.65]
π^*	4.28	4.40	[4.25,4.56]
r^*	1.13	1.29	[1.08,1.50]
κ	0.77	0.73	[0.37,1.06]
τ^{-1}	1.45	1.36	[1.07,1.67]
ρ_g	0.68	0.68	[0.62,0.73]
ρ_z	0.82	0.80	[0.76,0.87]

Posterior estimates (determinacy)

	True values	Posterior estimates	
		Mean	90% probability interval
σ_R	0.23	0.22	[0.21,0.24]
σ_g	0.27	0.28	[0.23,0.33]
σ_z	1.13	1.12	[1.02,1.22]
ρ_{gz}	0.14	0.02	[-0.12,0.15]
ρ_{gR}	0	0.06	[-0.05,0.17]
ρ_{zR}	0	0.02	[-0.08,0.14]

Conclusions

- We provide a new method to deal with indeterminacy in LRE models.
- Our augmented representation ensures to estimate LRE models over the entire parameter space.
- Our method is applicable also when the region of determinacy is unknown.
- It is implementable in standard software packages.