Testing DSGE models using indirect inference

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Introduction

- This paper addresses the issue of how best to test an already estimated DSGE model as judged by the power properties of the test.
- It can also be used to test (or estimate) a sub-set of equations in the DSGE model
- It is rare to test DSGE models estimated by Bayesian methods
- A possible reason is Sargent's comment that Lucas and Prescott thought too many good models were being rejected by classical methods such as ML
- This led to the use of calibration and then Bayesian estimation

- Current methodology developed in Cardiff over 15 years by Patrick Minford and myself helped by coauthors David Meenagh, Mai Le and recently Yongdeng Xu; also many former PhD students (citations in paper).
- Matlab Programme INDIRECT now available for download, plus supporting manual, papers etc.- www.patrickminford.net/INDIRECT

Bayesian estimation of DSGE models

- Bayesian estimates can be described as taking a weighted average of the prior information and the classical ML estimates with the weights inversely proportional to their variances
- The flatter the likelihood function, the more weight given to the prior information
- It is noticeable that many Bayesian estimates based on the posterior distribution are remarkably close to the prior information
- Another clue that something may be amiss is that many DSGE models is that they have extremely highly autocorrelated errors

How to test the model

- Traditionally tests of Bayesian estimated models are by the posterior odds ratio - comparing the probablity of the model on the null and alternative hypotheses
- We want a test which allows the model to have been already estimated perhaps by someone else
- Could just re-estimate the model but we want to avoid this too
- The early RBC calibrated models "matched the moments"
- This is an informal test and a form of Bayesian estimation with an extremely precise prior that leaves no role for the data

Indirect tests

- An extension of the idea of matching the moments
- Instead of comparing the moments of data simulated from the model with the moments of the observed data, the idea is to compare the estimates of an auxiliary model based on simulated and observed data
- For example, if the auxiliary model is a VAR, we could compare estimates of the VAR based on data simulated from the DSGE model and on the observed data
- Or we could compare their impulse response functions
- We form a test statistic by testing whether the two sets of estimates are significantly different

Previous papers

- In tests of the Smets-Wouters model we found that it didn't capture price-wage flexibility well
- A model that gave more flexibility performed better

This paper

- In this paper we investigate the power of two such tests for

- a 3-equation New Keynesian model with 5 structural parameters and 3 error parameters
- the 7-equation Smets-Wouters model with 12 structural parameters and 8 error parameters

- We use an LR ratio test in which the autoregressive processes generating the structural disturbances are re-estimated to maximise fit
- And a Wald test (IIW) in which the error autoregressive processes are not re-estimated
- In effect, due to the form of the solution to DSGE models, the LR test is based on the distance between the two models' one-period ahead forecasting errors.
- It depends on the falseness of the structural parameters
- It is also affected by re-estimating the AR parameters of the error processes which partly offsets the effect on the overall forecast error of the false parameters

Findings

- We find that the powers of the two tests turn on two factors
- First, whether the LR test is preceded by re-estimation of the model or of its error processes; if it is, the LR test's power is substantially weakened
- Second, the way the IIW test is implemented

- based on the variance matrix of the coefficients of the auxiliary VAR model estimated from the
observed data
- or based on data simulated from the DSGE model with its false structural parameters.

- Using the former, the powers of the LR and the indirect inference tests are roughly equivalent,
- Using the latter gives the IIW test more power

What does Bayesian estimation do?

Posterior distribution

$$p(\theta|x) = \frac{p(x|\theta)p(\theta)}{p(x)} = \frac{L(x|\theta)p(\theta)}{p(x)}$$

The mode of the posterior distribution

$$\arg\max_{\theta} \ln p(\theta/x) \equiv \arg\max_{\theta}[\ln L(x/\theta) + \ln p(\theta)]$$

Satisfies

$$[\frac{\partial \ln L(x/\theta)}{\partial\theta} + \frac{\partial \ln p(\theta)}{\partial\theta}]_{\theta=\widetilde{\theta}} = 0$$

If $\ln L(x/\theta)$ is flat then $\frac{\partial \ln L(x/\theta)}{\partial\theta} \simeq 0$ and the prior dominates

Mode is

$$\begin{aligned}\widetilde{\theta}-\theta_0 &= -[\frac{\partial \ln L(x/\theta)}{\partial\theta}\frac{\partial \ln L(x/\theta)}{\partial\theta'}+\frac{\partial \ln p(\theta)}{\partial\theta}\frac{\partial \ln p(\theta)}{\partial\theta'}]^{-1}\\ &\quad \times[\frac{\partial \ln L(x/\theta)}{\partial\theta}+\frac{\partial \ln p(\theta)}{\partial\theta}]_{\theta=\theta_0}\\ &= T^{-1}[Var(\widehat{\theta}_{MLE})^{-1}+Var(\theta)^{-1}]^{-1}[\frac{\partial \ln L(x/\theta)}{\partial\theta}+\frac{\partial \ln p(\theta)}{\partial\theta}]_{\theta=\theta_0}\end{aligned}$$

Choosing the auxiliary model

Starting with the general (linearised) DSGE model in the state-space form

$$\begin{bmatrix} x_{t+1} \\ E_t y_{t+1} \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} \\ A_{yx} & A_{yy} \end{bmatrix} \begin{bmatrix} x_t \\ y_t \end{bmatrix} + \begin{bmatrix} C_x \\ C_y \end{bmatrix} z_t$$

x_t are predetermined, y_t are not, z_t are exogenous (policy variables and disturbances)
The solution can be shown to take the form

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = M \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + N\Sigma_{s=0}^{\infty}\Gamma_{yy}^{-s} P_y E_t z_{t+s} + J z_{t-1} + K\xi_t$$

where ξ_t are innovations.

- This equation explains why DSGE models forecast no better than VARs

If the z_t are generated by the VAR

$$z_{t+1} = Rz_t + \varepsilon_{t+1}$$

then $E_t z_{t+s} = R^s z_t$, $(s > 0)$ and we can write the solution as the VARX

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = M \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + Hz_t + Jz_{t-1} + K\xi_t$$

The vector of observations on all of the variables can be written as the VAR

$$\begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix} = F \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ z_{t-1} \end{bmatrix} + G \begin{bmatrix} \xi_{xt} \\ \xi_{yt} \\ \varepsilon_t \end{bmatrix}.$$

where

$$F = \begin{bmatrix} M_{xx} & M_{xy} & J_x - H_x R \\ M_{yx} & M_{yy} & J_y - H_y R \\ 0 & 0 & R \end{bmatrix}$$

$$G = \begin{bmatrix} K_{xx} & K_{xy} & -H_x R \\ K_{yx} & K_{yy} & -H_y R \\ 0 & 0 & R \end{bmatrix}$$

New Keynesian model

$$
\begin{aligned}
\pi_t &= \omega E_t \pi_{t+1} + \lambda x_t + e_{\pi t}, \qquad \omega < 1 \\
x_t &= E_t y_{t+1} - \frac{1}{\sigma}(r_t - E_t \pi_{t+1}) + e_{xt} \\
r_t &= \gamma \pi_t + \eta x_t + e_{rt} \\
e_{it} &= \rho_i e_{i,t-1} + \varepsilon_{it} \qquad (i = \pi, x, r)
\end{aligned}
$$

The solution is

$$
\begin{bmatrix} \pi_t \\ x_t \\ r_t \end{bmatrix} = KH \begin{bmatrix} e_{\pi t} \\ e_{xt} \\ e_{rt} \end{bmatrix}
$$

The model is over-identified as the matrix $A = KH$ is restricted, having 9 elements but consists of only 5 structural coefficients (The ρ_i can be recovered directly from the error processes)

$$K=\begin{bmatrix} 1+\frac{\eta}{\sigma}-\rho_{\pi} & \lambda & -\frac{\lambda}{\sigma} \\ -\frac{1}{\sigma}(\gamma-\rho_{\pi}) & 1-\omega\rho_{y} & -\frac{1}{\sigma}(1-\omega\rho_{r}) \\ \gamma-(\gamma-\frac{\eta}{\sigma})\rho_{\pi} & \lambda\gamma+\eta-\eta\omega\rho_{y} & 1-(1+\omega+\frac{\lambda}{\sigma})\rho_{r}+\omega\rho_{r}^{2} \end{bmatrix},$$

$$H=\begin{bmatrix} H_{11} & 0 & 0 \\ 0 & H_{22} & 0 \\ 0 & 0 & H_{33} \end{bmatrix},$$

$$\begin{aligned} H_{11} &= \frac{1}{1+\frac{\eta+\lambda\gamma}{\sigma}-[\frac{\lambda}{\sigma}+\omega(1+\frac{\eta}{\sigma})]\rho_{\pi}+\omega\rho_{\pi}^{2}} \\ H_{22} &= \frac{1}{1+\frac{\eta+\lambda\gamma}{\sigma}-[\frac{\lambda}{\sigma}+\omega(1+\frac{\eta}{\sigma})]\rho_{y}+\omega\rho_{y}^{2}} \\ H_{33} &= \frac{1}{1+\frac{\eta+\lambda\gamma}{\sigma}-[\frac{\lambda}{\sigma}+\omega(1+\frac{\eta}{\sigma})]\rho_{r}+\omega\rho_{r}^{2}}. \end{aligned}$$

Indirect inference tests

- Let the parameters of the DSGE model be θ and those of the auxiliary VAR be α

$$y_t = \alpha y_{t-1} + \eta_t$$

- If solved from the DSGE model we have $\alpha(\theta)$
- Calculate the structural errors from the estimated DSGE model with parameter estimates $\widehat{\theta}$ and, randomly drawing from these, generate simulated data from the model, $y_{s,t}(\widehat{\theta})$.

$$y_{s,t} = \alpha_s y_{s,t-1} + \eta_{s,t}$$

- Estimate the auxiliary model from the simulated data to obtain the estimates $\widehat{\alpha}_s(\widehat{\theta})$ and compute the test statistic
- Estimate the auxiliary model using the observed data y_t to give $\widehat{\alpha}$

- Both estimates of the auxiliary model are unrestricted VAR estimates, but those from the simulated data embody the parameter restrictions of the DSGE model
- Repeat with many simulated data generations (1000 times) to construct a numerical distribution of the test statistic
- The power characteristics of the tests are obtained using false values of the DSGE model parameters θ_F in the data simulations

Likelihood ratio test

- The LR test is based on comparing the log-likelihood function on the null and alternative hypotheses

$$
\begin{aligned}
LR &= 2(\ln L_U - \ln L_R) \\
&= T\left(\ln|\Sigma_R| - \ln\left|\widehat{\Sigma}\right|\right)
\end{aligned}
$$

- As the solution to a DSGE model is a VAR, the test compares different estimates of the residual covariance matrix of a restricted and an unrestricted VAR. In effect these are prediction errors.
- The estimates of $\ln L$ or $\ln|\Sigma|$ are from the estimated auxiliary model
- In order to give the DSGE model the best chance of passing the test, for the LR test only we re-estimate the AR error processes of the DSGE model for each simulated data set.
- In the power calculations we repeat this but with false parameter values for the DSGE model.

Wald test

- The Wald test (IIW) focuses on the VAR coefficients or functions of them such as the impulse response functions $g(\alpha)$

$$IIW = [\widehat{\alpha} - \alpha_S]' W_S^{-1} [\widehat{\alpha} - \alpha_S]$$

- W_S is the variance matrix of the limiting distribution of α_S
- Can also estimate W using the covariance matrix from the observed data

Auxiliary models used for the SW model

- The full model requires a 7-equation VAR with 3 lags - 7VAR3
- Can also consider sub-sets of variables
- The variables are added in the order:

- Output, Interest Rate, Inflation, Consumption, Investment, Wage, Return on Capital

- As the number of variables and lags increases so does the number of VAR coefficients
- The false parameter values are x% from the true values
- The IIW test is calculated using the unrestricted and the restricted error covariance matrix
- Unrestricted refers to calculating the covariance matrix from the observed data

Power of LR and Wald tests of SW model - full model and sub-sets of variables

VAR — no of coeffs	TRUE	1%	3%	5%	7%	10%	15
IIW TEST with unrestricted VAR							
2 variable VAR(1) — 4	5.0	6.2	20.3	69.6	61.0	99.8	100
3 variable VAR(1) — 9	5.0	3.4	7.5	30.7	75.0	97.4	100
3 variable VAR(2) — 18	5.0	3.8	5.2	19.1	57.5	84.3	98
3 variable VAR(3) — 27	5.0	3.9	6.4	21.6	54.5	84.0	97
5 variable VAR(1) — 25	5.0	2.8	3.2	2.6	5.4	6.2	4
7 variable VAR(3) — 147	5.0	5.1	3.4	1.4	0.9	0.2	0
IIW TEST with restricted VAR							
2 variable VAR(1) — 4	5.0	9.8	37.7	80.8	96.8	100.0	100
3 variable VAR(1) — 9	5.0	9.5	36.1	71.0	98.1	100.0	100
3 variable VAR(2) — 18	5.0	8.3	35.5	80.9	96.9	100.0	100
3 variable VAR(3) — 27	5.0	9.2	32.9	78.0	95.1	100.0	100
5 variable VAR(1) — 25	5.0	17.8	85.5	99.8	100.0	100.0	100
7 variable VAR(3) — 147	5.0	77.6	99.2	100.0	100.0	100.0	100

LIKELIHOOD RATIO TEST								
2 variable VAR(1) — 4	5.0	12.0	28.3	45.9	63.4	83.2	97.0	99
3 variable VAR(1) — 9	5.0	9.4	21.8	37.5	58.9	84.0	99.0	100
3 variable VAR(2) — 18	5.0	8.9	20.7	36.8	57.6	82.9	98.7	100
3 variable VAR(3) — 27	5.0	8.9	20.4	36.7	56.7	82.2	98.7	100
5 variable VAR(1) — 25	5.0	8.9	22.4	44.3	68.6	89.6	99.6	100
7 variable VAR(3) — 147	5.0	5.7	10.6	23.6	46.3	83.2	99.6	100

- Asymptotically the tests are the same
- All three tests have the correct size
- Power increases with

- the degree of falseness
- fewer coefficients
- using the estimate of the covariance matrix based on the restricted data

- The IIW tests have substantially greater power than the LR test for moderate degrees of falseness and reach 100% faster
- The power of the IIW(R) test is a greater than the IIW(U) test as the restricted estimates are more precise

Why does the IIW(R) test have more power than the LR test?

a) Different procedures

- In the LR test the AR structure is re-estimated for each sample to 'bring the model back on track'
- Makes it less likely that a false model will be rejected and so reduces the power of the LR test
- Check rejection rates using the 3-equation NK model

	Re-estimated $\rho's$	Pre-specified $\rho's$
True	5.0	5.0
1%	5.0	5.0
3%	5.3	9.6
5%	6.1	20.2
7%	8.0	39.1
10%	15.4	63.7
15%	48.1	90.7
20%	75.6	98.9

b) Same procedures but more general auxiliary model

- Can power be increased by extending the features of the structural model that the auxiliary model seeks to match?

- include elements of the variance matrix of the coefficients of the auxiliary model
- include more of the structural model's variables in the VAR
- increase the order of the VAR

- There may be a limit to the number of features of the DSGE model that can be included in the test as we run into the objection raised by Lucas and Prescott against tests of DSGE models that "too many good models are being rejected by the data"
- Focusing on particular features is a major strength of the restricted IIW test.

- Extending the auxiliary model for the 3-equation NK model
  1. Higher order - get little difference

Rejection rates at 95% confidence: T=200

	3 variable VAR(1)	3 variable VAR(2)
True	5.0	5.0
1%	4.9	4.3
3%	7.3	7.1
5%	16.1	21.7
7%	37.0	40.3
10%	73.3	76.3
15%	99.4	99.8
20%	100.0	100.0

2. Include an indexing lag in the Phillips Curve - increases the structural parameters to 9 and the reduced-form solution is a VAR(2) - get small increase in power

Rejection rates at 95% confidence: T=200

	3 variable VAR(1)	3 variable VAR(2)
True	5.0	5.0
1%	10.6	6.0
3%	20.7	19.5
5%	47.5	57.9
7%	65.6	91.2
10%	89.6	100.0
15%	98.8	100.0
20%	99.9	100.0

Conclusions

- IIW method has good power for tests of previously estimated DSGE models
- IIW has greater power than the LR test
- This may be due to re-estimating the AR processes generating the structural errors
- The power of the IIW test is improved the more restricted the DSGE model
- It may be preferable to focus on particular features of the DSGE model rather than the full model
- Programmes available for download to implement in user-friendly way.

Philosophical points on methodology

- All macro models are false - deliberately so as they are just simplifications
- Can always reject them by including enough restrictions
- Related to the Lucas-Prescott position of rejecting too many good models
- If Bayesian differ from classical estimates does this mean that the Bayesian estimates are imposing false prior information?
- Need to think about why we are building the model and our judgement criteria