A Narrative Approach To A Fiscal DSGE Model

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New Methods for Macroeconomic Modelling,
Model Comparison and Policy Analysis
CEPR-IMFS – April 05, 2016

Why a narrative approach to a DSGE model?

1. Structural DSGE models are widely used in business cycle analysis.
2. Narrative studies also purport to identify structural shocks.
   Is the DSGE model misspecified compared to narrative studies?

Idea

> *It may be an advantage of VAR's that they restrain our impulse to take our storyspinning too seriously. If Bayesian DSGE's displace methods that try to get by with weak identification and in the process reinforce the excess weight we give to story-spinning, they may set us back.*

Chris Sims (p. 2, 2005)

> *Before we use these [DSGE] models in the Solow-style mode of helping to organize our thinking and refine our trained intuitions, it seems only sensible that we check first where the models reflect and where they contradict common understanding.*

Jon Faust (p. 63, 2009)

This paper

- Do narrative and structural methods agree?
  1. Theoretically: Yes. For DSGE models with Taylor-type policy rules identifying shocks with narrative variables as instruments is correct.
     Stock & Watson (2012), Mertens & Ravn (2013)
  2. Empirically: No. The data likes DSGE model dynamics, but not the covariance structure implied by the identification scheme.
     Del Negro & Schorfheide (2004)
- Implications:
  - Different policy rules?
  - Policy foresight?
    Ramey (2011), Campbell et al. (2012), Leeper et al (2013)
  - Question narrative policy measures, e.g. the monetary policy shock.
    Caldara & Herbst (2015), Ramey (2015)

DSGE-VAR framework

- State space representation of DSGE model:

$$Y_t = B^* X^*_{t-1} + A^* \epsilon_t \quad \text{(DSGE-Y)}$$

$$X^*_t = D^* X^*_{t-1} + C^* \epsilon_t. \quad \text{(DSGE-X)}$$

- VAR(p) approximation:

$$Y_t = B X_{t-1} + A \epsilon_t \quad \text{(VAR-Y)}$$

$$X_t = [Y_t, \ldots, Y_{t-(p-1)}]. \quad \text{(VAR-X)}$$

- VAR problem: Observe $A^*(A^*)' \approx AA'$, but not A.
- Use DSGE model rotation to identify shocks.

Narrative DSGE-VAR framework

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$$X^*_t = D^* X^*_{t-1} + C^* \epsilon_t. \tag{DSGE-X}$$

- VAR(p) approximation:

$$Y_t = B X_{t-1} + A \epsilon_t \tag{VAR-Y}$$
$$X_t = [Y_t, \dots, Y_{t-(p-1)}]. \tag{VAR-X}$$

- VAR problem: Observe $A^*(A^*)' \approx AA'$, but not A.
- ~~Use DSGE model rotation to identify shocks.~~
- Use "narrative" instruments Z to recover 1st columns of A:

$$Z_t = F X^z_{t-1} + G \epsilon_{1,t} + \mathbf{0} \times \epsilon_{2,t} + \text{noise}_t. \tag{VAR-Z}$$

What I will show today

1. Narrative BVAR
   1.1 Shock identification conditional on parameters.
   1.2 SUR-type inference over parameters.
2. Narrative DSGE-VAR
   2.1 Taylor-type policy rules allow partial identification.
   2.2 Implement DSGE-VAR prior.
3. Application: Fiscal and monetary policy.
   3.1 Vary weight on DSGE models and compare via marginal likelihoods.
   3.2 Additional moments: IRFs, historical shock correlations.

Narrative BVAR: Setup and assumptions

- Consider VAR(1) in Y_t, m_z instruments Z_t:

$$\begin{bmatrix} Y_t \\ Z_t \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} B_y Y_{t-1} \\ B_z X^z_{t-1} \end{bmatrix}, \begin{bmatrix} \Sigma & \Gamma' \\ \Gamma & \Omega \end{bmatrix} \right)$$

$$\Sigma \equiv \mathsf{Var}_{t-1}[Y_t] = AA'$$

$$\Gamma \equiv \mathsf{Cov}_{t-1}[Z_t, Y_t]$$

$$A \equiv \begin{bmatrix} A_{IV} & A_{\text{other}} \end{bmatrix}$$

- Standard SUR Gibbs sampler for reduced form parameters:
  1. Vector of coefficients B_y, B_z is conditionally Normal distributed given covariance.
  2. Inverse covariance matrix is conditionally Wishart given coefficients.

Identification given parameters

Assumption

Assume that for some invertible matrix G:

$$\Gamma \equiv \begin{bmatrix} G & \mathbf{0} \end{bmatrix} \begin{bmatrix} A_{IV} & A_{\text{other}} \end{bmatrix} = G A_{IV}'$$

Proposition (Mertens & Ravn, 2013)

Given Γ, Σ, A_{IV} is identified up to a factorization of $S_1 S_1'$.

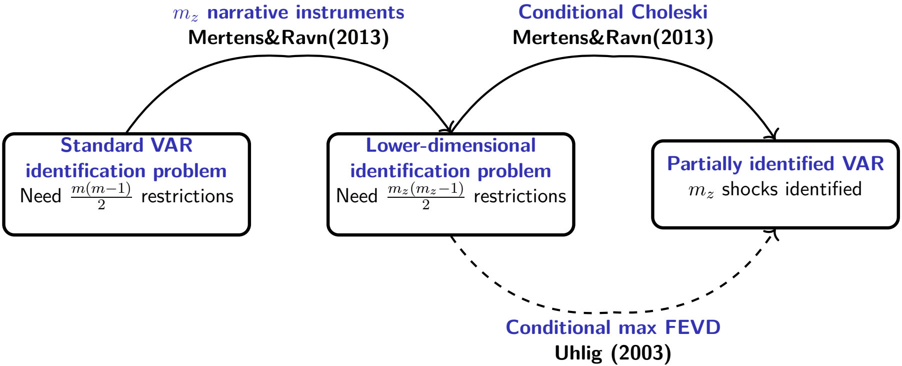
$$A_{IV} = \begin{bmatrix} (I - \eta\kappa)^{-1} \\ (I - \kappa\eta)^{-1}\kappa \end{bmatrix} S_1$$

where

Sketch of proof

- η, κ are functions of $\Sigma \equiv AA', \Gamma$,
- $S_1 S_1'$ is the residual variance of $v_{1,t}$ unexplained by $v_{2,t}$.

Identification: Overview



Narrative DSGE-VAR

1. Provide conditions when VAR and DSGE model agree on $A_{IV} = A^*_{IV}$.
2. Incorporate DSGE model prior.

Identifiable policy rules

Definition

A *simple Taylor-type rule* in the (DSGE) economy for $y_{p,t}$ is of the form:

$$y_{p,t} = \sum_{i=m_p+1}^{m} \psi_{p,i} y_{i,t} + \lambda_p X_{t-1} + \sigma_p \epsilon_{p,t}, \quad \epsilon_{p,t} \subset \epsilon_t \sim iid, y_{i,t} \subset Y_t, i > m_p.$$

Example

Consider the following Taylor-rules for interest rates r_t:

$$r_t = (1-\rho_r)\left(\gamma_\pi \pi_t + \gamma_y (y_t - y_{t-1})\right) + \rho_r r_{t-1} + \omega_r \epsilon_t^r. \qquad \text{(a)}$$

$$r_t = (1-\rho_r)\left(\gamma_\pi \pi_t + \gamma_y (y_t - y_t^f)\right) + \rho_r r_{t-1} + \omega_r \epsilon_t^r. \qquad \text{(b)}$$

$$r_t = (1-\rho_r)\left(\gamma_\pi \pi_t + \gamma_g g_t\right) + \rho_r r_{t-1} + \omega_r \epsilon_t^r. \qquad \text{(c)}$$

If $[y_t, \pi_t] \subset Y_t$ and $y_f^f \not\subset Y_t$: (a) is a simple Taylor rule, but not (b) or (c).

Narrative VAR consistent with DSGE models

Proposition

If $A^*(A^*)' = AA'$ then (up to sign) $S_1 = \text{chol}(S_1 S_1')$ if:

(a) (DSGE) has $m_p = m_z$ simple Taylor rules with m_z instruments, or

(b) (DSGE) has $m_p = m_z - 1$ simple Taylor rules ordered first and $\psi_{p,m_z} = 0, p = 1, \ldots, m_p$ with m_z instruments.

Corollary

If $A^*(A^*)' = AA', \Gamma = GA_{IV}'$, G^{-1} exists, and (a) or (b) hold:

$$A_{IV} = \begin{bmatrix} (I - \eta\kappa)^{-1} \\ (I - \kappa\eta)^{-1}\kappa \end{bmatrix} S_1 = A_{IV}^*$$

up to sign normalization.

Sketch of proof

DSGE-VAR

Statistical background

- Prior for VAR via artificial observations from DSGE model (Del Negro & Schorfheide, 2004):
  - Priors over DSGE model parameters θ standard to elicit.
  - Approximate DSGE model with VAR based on model moments given θ.
  - Parameterize prior precision via number of dummy observations.
- Measure fit via marginal likelihood $p(Y, Z|T_0^B, T_0^V)$
  - If $p(Y, Z|T_0^B, T_0^V)$ increases in T_0^V, evidence in favor of DSGE model identification.
  - If $p(Y, Z|T_0^B, T_0^V)$ increases in T_0^B, evidence in favor of DSGE model dynamics.

Skip details

Generating DSGE-model instruments

$$Z_t = FX^z_{t-1} + G\,\epsilon_{1,t} + \mathbf{0} \times \epsilon_{2,t} + \text{chol}(\Omega)\,\mathcal{N}(\mathbf{0}, I_{m_z}) \quad \text{(VAR-Z)}$$

$$Z_t = \mathbf{0} + \text{diag}([\text{c}_i]_i)\,\epsilon_{1,t} + \mathbf{0} \times \epsilon_{2,t} + \text{diag}([\omega_i]_i)\,\mathcal{N}(\mathbf{0}, I_{m_z}) \quad \text{(DSGE-Z)}$$

Dummy variable DSGE prior

- Use (DSGE) economy to generate prior for B, Σ, Γ given (DSGE) parameters θ.

$$\begin{bmatrix} \bar{B}_0^y \\ \bar{B}_0^z \end{bmatrix} = \begin{bmatrix} \mathbb{E}[X_0' X_0]^{-1} \mathbb{E}[X_0' Y_0]^{-1} \\ \mathbf{0} \end{bmatrix} \quad \bar{V}_0 = \begin{bmatrix} (A^*)(A^*)' & \\ [I_{m_z}, \mathbf{0}](A^*)' & \bar{\Omega}_0(A_1^*) \end{bmatrix}$$

Vectorize as $\bar{\beta}_0$.

- Implement $\beta \sim \mathcal{N}(\bar{\beta}_0, N_{XX}(\bar{V}_0))$ via dummy observations:

$$\text{vec}([Y_0^B, Z_0^B]) = \bar{X}_{0,SUR}(\theta)\bar{\beta}_0(\theta) + \mathbf{0},$$

$$\text{vec}([Y_0^B, Z_0^B]) \sim \mathcal{N}(\bar{X}_{0,SUR}(\theta)\bar{\beta}_0(\theta), \bar{V}_0(\theta) \otimes I_{T_0^B})$$

- Implement $V^{-1} \sim \mathcal{W}(\bar{V}_0 T_0^V, T_0^V)$ via dummy observations:

$$[Y_0^V, Z_0^V] = \mathbf{0} \times \beta + \text{chol}(\bar{V}_0(\theta)) \otimes I_{T_0^V},$$

$$\text{vec}([Y_0^V, Z_0^V]) \sim \mathcal{N}(\mathbf{0}, V \otimes I_{T_0^V})$$

Inference with DSGE-VAR

Random-Blocking Metropolis-Hastings-within-Gibbs sampler (Chib & Ramamurthy, 2010).

1 Draw $B_{(i)}|V_{(i-1)}^{-1}, \theta_{(i-1)}$ from Normal distribution.

2 Draw $V_{(i)}^{-1}|B_{(i)}, \theta_{(i-1)}$ from Wishart distribution.

3 Draw $\theta_{(i)}|B_{(i)}, V_{(i)}^{-1}$ from $\pi(\theta|V^{-1}, B, Y, Z) \propto \pi(V^{-1}, B|\theta)\pi(\theta)$:

Inference with DSGE-VAR

Random-Blocking Metropolis-Hastings-within-Gibbs sampler (Chib & Ramamurthy, 2010).

1. Draw $B_{(i)}|V^{-1}_{(i-1)}, \theta_{(i-1)}$ from Normal distribution.
2. Draw $V^{-1}_{(i)}|B_{(i)}, \theta_{(i-1)}$ from Wishart distribution.
3. Draw $\theta_{(i)}|B_{(i)}, V^{-1}_{(i)}$ from $\pi(\theta|V^{-1}, B, Y, Z) \propto \pi(V^{-1}, B|\theta)\pi(\theta)$:
   - 3.1 Randomly permute parameter indices $j \to j'$.
     $\forall j' = 1, \ldots, \#(\theta)$: Assign $\theta_{j'}$ to new block s with iid probability.
   - 3.2 $\forall s = 1, \ldots, S_{(i)}$:
     Set $\theta^c_{(i-1),s} \equiv [\theta_{(i),1}, \ldots \theta_{(i),s-1}, \theta_{(i-1),s+1}, \ldots, \theta_{(i-1),S_{(i)}}]$.
     Draw ϑ_s from proposal density $q(\vartheta_s|\theta_{(i-1),s}, \theta^c_{(i-1),s})$.
     Compute $\alpha = \min\left\{1, \frac{\pi(\vartheta_s|V^{-1}_{(i)}, B_{(i)}, \theta^c_{(i-1),s})q(\theta_{(i-1),s}|\vartheta_s, \theta^c_{(i-1),s})}{\pi(\theta_{(i-1),s}|V^{-1}_{(i)}, B_{(i)}, \theta^c_{(i-1),s})q(\vartheta_s|\theta_{(i-1),s}, \theta^c_{(i-1)})}\right\}$
     Set $\theta_{(i),s} = \begin{cases} \vartheta_s & \Pr = \alpha \\ \theta_{(i-1),s} & \Pr = 1-\alpha. \end{cases}$

Application: Fiscal and monetary policy rules

1. DSGE and VAR comparison by Bayes factors.
2. Informal model comparison: IRFs and historical shocks.

Empirical DSGE model

- Standard medium-scale New-Keynesian model + distortionary taxes.
  CEE (2005), Smets & Wouters (2007), CTW (2010)
  - Complete markets.
  - Habit formation in consumption.
  - Monopolistic labor and final goods markets.
  - Fixed cost in production.
  - Working capital.
  - Calvo-sticky prices and wages.
  - Investment adjustment costs. Variable capacity utilization.
  - Linear consumption, capital, and labor taxes.

  Estimate 40 parameters (+some calibrated values).
- Taylor-type policy rules.

Taylor-type policy rules

- Fiscal rules for government spending g_t and labor income taxes $d\tau_t^n$:
  (Leeper et al., 2010, and Fernandez-Villaverde et al., 2013)

$$\hat{g}_t = \mathsf{r}_g \hat{g}_{t-1} - (1 - \mathsf{r}_g)(\psi_{g,y}\hat{y}_t + \psi_{g,b}\frac{\bar{b}}{\gamma\bar{y}}\hat{b}_t) + \xi_t^g$$

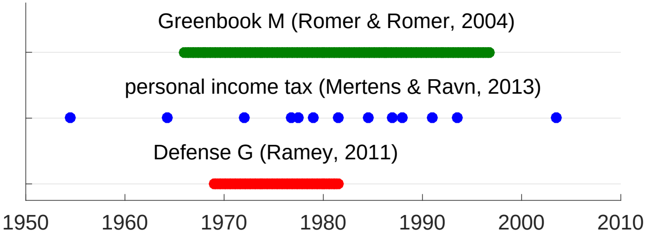
$$d\tau_t^n = \mathsf{r}_\tau d\tau_{t-1}^n + (1 - \mathsf{r}_\tau)(\psi_{\tau,y}\hat{y}_t + \psi_{\tau,b}\frac{\bar{b}}{\gamma\bar{y}}\hat{b}_t) + \xi_t^\tau$$

- Monetary policy rule for Federal Funds Rate:

$$\widehat{FFR}_t = \mathsf{r}_{FFR}\widehat{FFR}_{t-1} + (1 - \mathsf{r}_{FFR})(\psi_{FFR,\pi}\hat{\pi}_t + \psi_{FFR,y}\hat{y}_t) + \xi_t^{FFR}$$

Data: 1948:Q1 to 2007:Q4

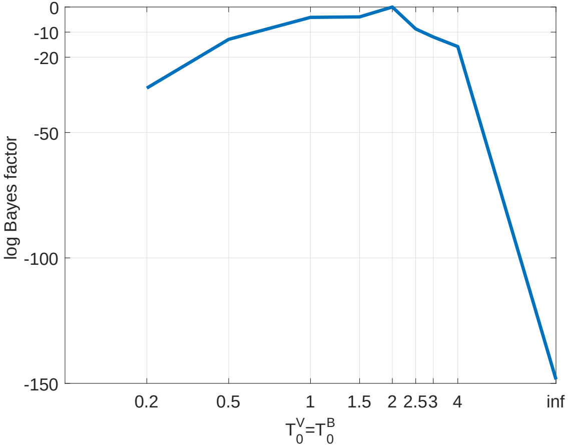
- Narrative signals Narrative example



  Set missing narrative data to zero.
- VAR: NIPA and FoF data
  G, personal income tax, output, investment, gov. debt to GDP, FFR, inflation.
- Quadratic detrending before estimation.

Bayes factors

- Bayes factors based on estimates of marginal likelihood (Chib (1995) + Geweke (1999)).



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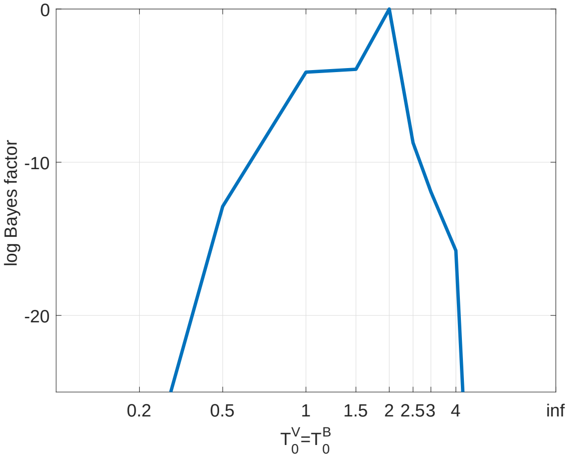
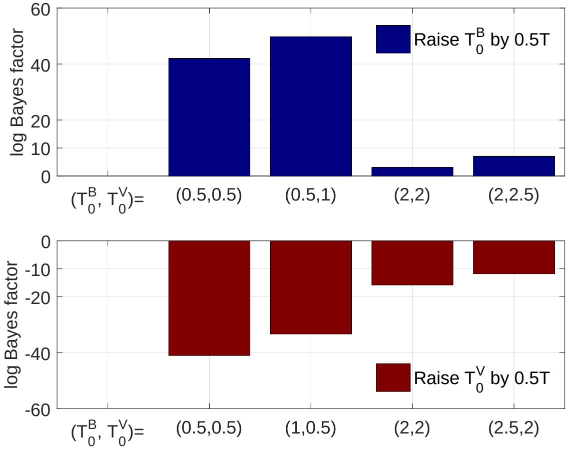


Bayes factors: Increasing weight on dynamics T_0^B vs covariance structure T_0^V



Developing intuition for model comparison

Sample moments matter for fit $\Rightarrow$ focus on results with weak prior.

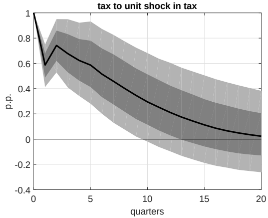
1. Flat prior narrative VAR.
   - Very wide confidence intervals; mostly qualitative results.
   - Similar results with conditional Choleski and max FEVD decompositions.
2. Narrative DSGE-VAR.
   - IRF comparisons: Inflation and tax dynamics not well captured.
   - Historical shocks: FFR shocks does best, tax shocks the noisiest. Plots
   - VAR Taylor rule estimates: Imprecise without prior information. Taylor rules
   - DSGE parameter estimates: Sensible Taylor rule estimates Parameters

Conclusion

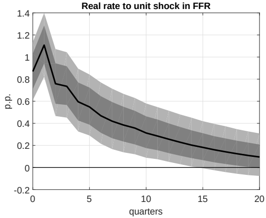
Effects on output

G to G shock

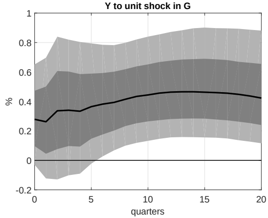
Tax rate to Tax



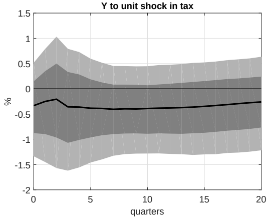
Real rate to FFR shock



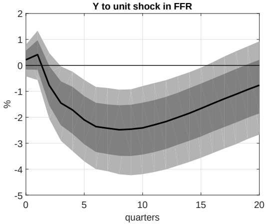
Y to G shock



Y to Tax



Y to FFR shock

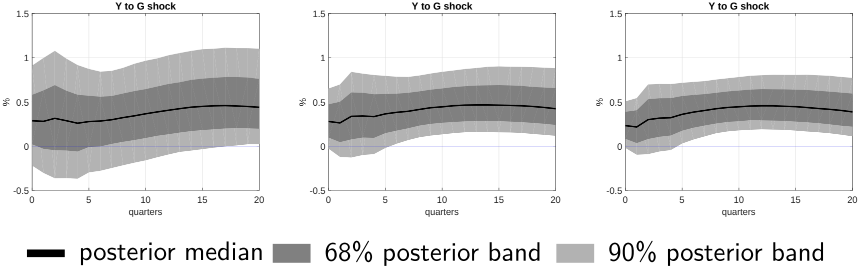


posterior median 68% posterior band 90% posterior band

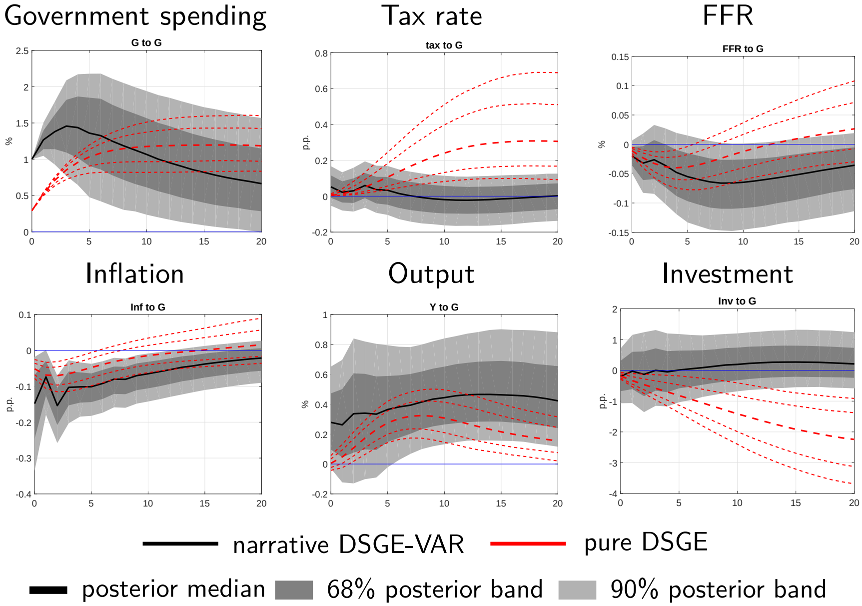
Moderate amounts of prior information go a long way

- Output response to government spending shock with prior information

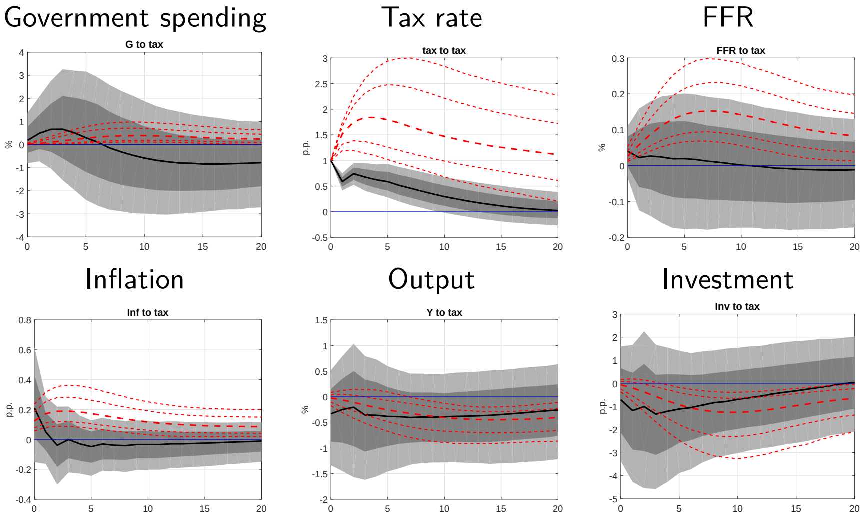
Flat prior | Benchmark weak prior $T_0 = \frac{1}{5}T$ | $T_0 = \frac{1}{2}T$



Effects of G shocks: Comparison with DSGE model



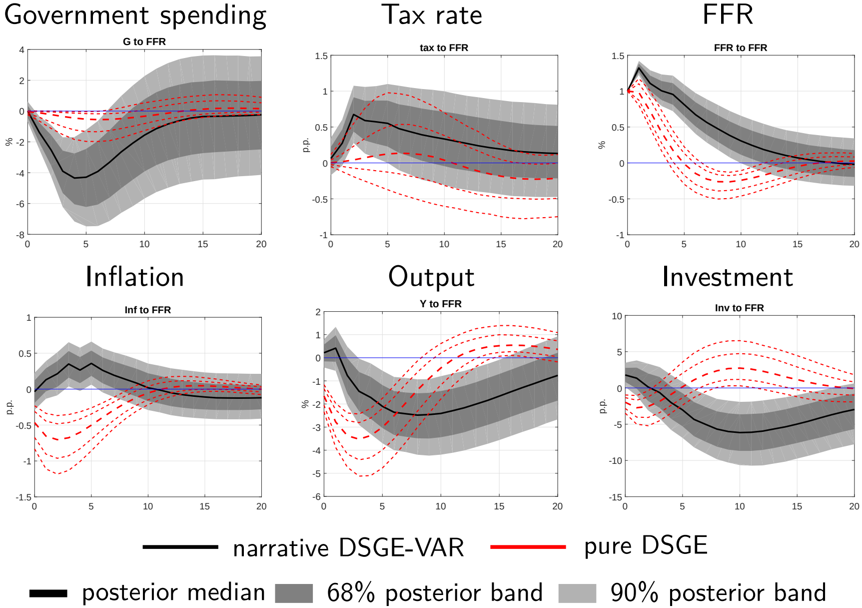
Effects of tax shocks



narrative DSGE-VAR pure DSGE

posterior median 68% posterior band 90% posterior band

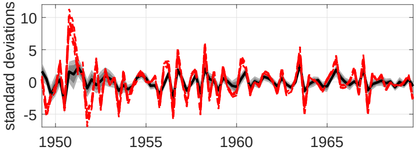
Effects of monetary policy shocks



Implied shocks: G

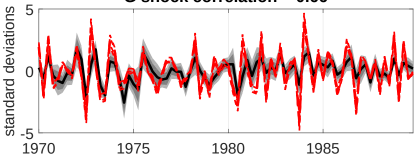
1949Q2 – 1969Q4

G shock correlation = 0.74



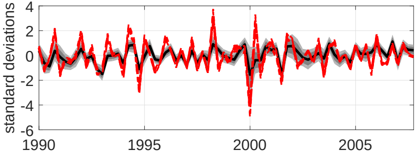
1970Q1 – 1989Q4

G shock correlation = 0.66



1990Q1 – 2007Q4

G shock correlation = 0.69



Overall correlation

Shock	Posterior median	(90% band)
G	0.62	(0.43, 0.73)
Tax	0.55	(0.37, 0.69)
FFR	0.82	(0.74, 0.88)

DSGE-VAR   pure DSGE

Detail Tax Detail FFR Overview Conclusion

Instruments and historical shocks

(a) Weak prior: $T_0 = \frac{1}{5}T$

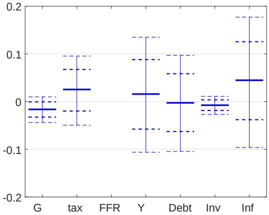
	DSGE-VAR vs DSGE		DSGE-VAR vs IV		DSGE vs IV	
Shock	Median	(90% band)	Median	(90% band)	Median	(90% band)
G	0.62	(0.43, 0.73)	0.50	(0.37, 0.59)	0.40	(0.37, 0.43)
Tax	0.55	(0.37, 0.69)	0.56	(0.39, 0.69)	0.30	(0.19, 0.40)
FFR	0.82	(0.74, 0.88)	0.65	(0.63, 0.67)	0.58	(0.55, 0.60)

(b) DSGE-VAR vs DSGE

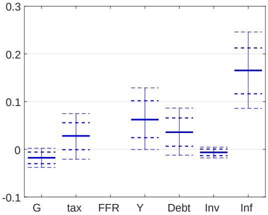
	$T_0 = \frac{1}{5}T$		$T_0 = 2 \times T$		$T_0 = 4 \times T$	
Shock	Median	(90% band)	Median	(90% band)	Median	(90% band)
G	0.62	(0.43, 0.73))	0.77	(0.70, 0.83)	0.81	(0.75, 0.86)
Tax	0.55	(0.37, 0.69)	0.73	(0.60, 0.82)	0.79	(0.67, 0.86)
FFR	0.82	(0.74, 0.88)	0.83	(0.75, 0.88)	0.87	(0.82, 0.91)

Monetary Taylor rule: Estimated loadings

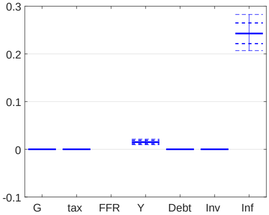
Weak prior:
$T_0 = \frac{1}{5}T$



Best fitting model:
$T_0 = 2 \times T$

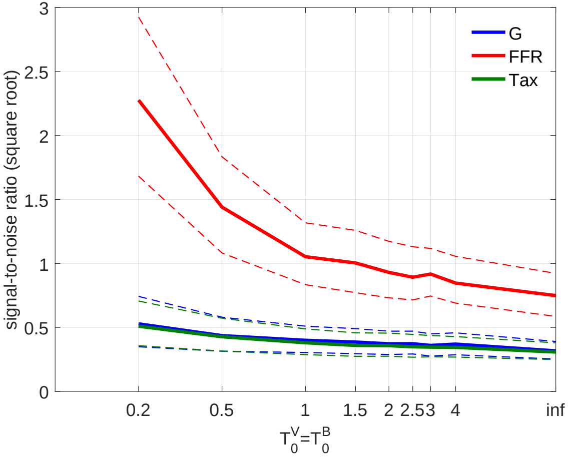


Dogmatic prior:
$T_0 = \infty$



Conclusion Overview

Estimated signal to noise ratios



This paper

- Method
  (1) Bayesian narrative VAR via SUR.
  (2) Narrative DSGE-VAR in SUR framework:
- Theoretical result:
  Narrative identification valid for DSGE model with class of policy rules.
- Empirical results:
  (1) Overall, intermediate weight on DSGE model optimal.
  (2) Evidence against DSGE identification – but even weak prior sharpens inference.
- Empirical caveats:
  (1) Narrative information is noisy.
  (2) Potentially problematic FFR instrument: Policy foresight?

What's next?

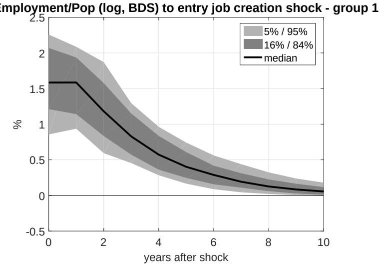
1. Fiscal policy application
    - Relax statistical assumptions (with P. Amir-Ahmadi and C. Matthes)
        1.1 Impute missing proxy variables
        1.2 Allow for time varying parameters
        1.3 Include forward-looking macroeconomic factors
    - Use Wieland et al. (2012) model data base to compare different DSGE models.
2. Entrepreneurship and local labor markets (with G. Carlino)
    - Systematically generate instruments using Bartik (1991) idea
    - Spatial Panel VAR
    - Compare effects of shocks to startups and established firms

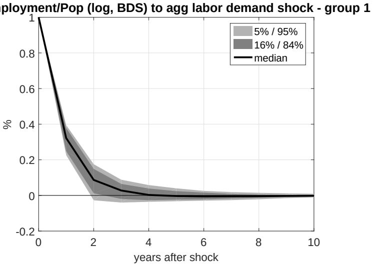
Labor demand shocks in low density areas

“Are Startups Special?” (with G. Carlino)

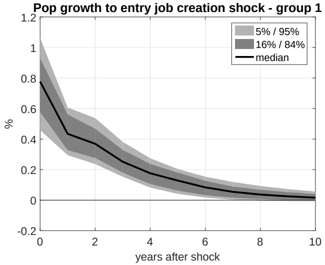
Startup labor demand | Overall labor demand

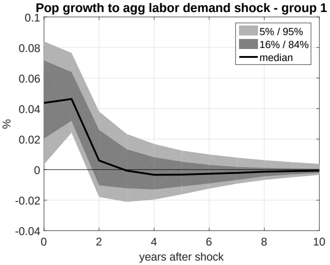
Emp / pop





Pop growth





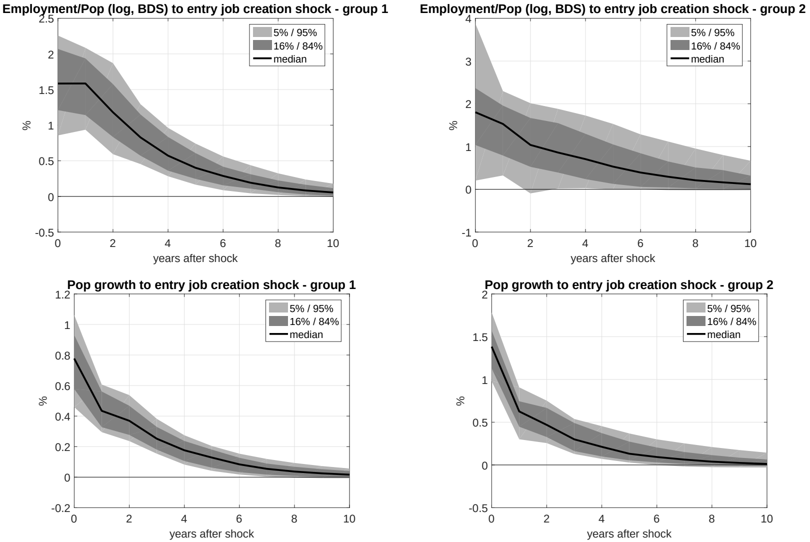
Startup labor demand shocks

“Are Startups Special?” (with G. Carlino)

Low density | Medium density

Emp / pop

Pop growth



Which structural shocks do narrative shocks reflect?

- Test identifying assumption within DSGE model.
- Compare flexible model with baseline: $\mathsf{c}_{ij} = 0 \forall i \neq j$?

$$z_t^i = \mathsf{c}_{ii}\epsilon_t^i + \sum_{\text{Shocks}_j, j\neq i} \mathsf{c}_{ij}\epsilon_t^j + \text{noise}_t$$

- Priors: $c_{ij} \sim \mathcal{N}(0,1), i \neq j$ and $c_{ii} \sim \mathcal{N}(1,1)$.

Bayes factor relative to best model

	Exclusion restriction is...		
Narrative shock	violated	satisfied	satisfied with $c_{ii} = 1$
Defense spending G	-10.6	0	-3.0
Personal income tax	-12.1	0	-2.8
Money shock, R&R	-5.6	0	-28.4
Money shock, R&R extended	0	-3.8	-68.2
Money shock, R&R CPI	-4.7	0	-79.5
Money shock, Kuttner	-7.3	0	-0.9
Money shock, TBill	-23.3	0	-191.4