Global Projection versus Local Perturbation Solutions for Macro-Finance Models with Nonlinear Dynamics*

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We provide a general solution method for macroeconomic models featuring nonlinear dynamics. This class of models include macroeconomic models for unconventional monetary policies with financial intermediaries. Within this framework, we highlight several important limitations of current models and methods, including the fact that local-linearization approximations omit important nonlinear dynamics, yielding biased impulse-response analysis and parameter estimates.

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1 Introduction

The recent financial crisis and the Great Recession of 2007–2009 revealed serious gaps in commonly used approaches to define, measure, and manage financial sector activities that pose risks to the macroeconomy as a whole.

One emerging narrative is that macroeconomic models commonly employed at policy institutions for evaluating monetary policy lack the analytical specificity to account for important financial sector influences on the aggregate economy. A new generation of enhanced models and advanced empirical and quantitative methodologies are needed by policymakers and need to be provided by researchers to better study the impact of shocks that are initially large or build endogenously over time.

This paper provides nonlinear global solution methods and estimation approaches are necessary, if one wishes to guarantee that key nonlinear dynamics in the financial market and the macroeconomy are eventually captured in quantitative analysis.

To illustrate the general solution method and algorithm, we present a fully specified canonical example of New Keynesian DSGE model with financial sectors in Section 2 and Appendix E that readers can work with immediately (an open-source software implementation is provided at MFMWEBSITE). The model in Section 2 is solved globally. We hope the contribution of our code and global solution method to this review may be of general interest to a broader group of researchers in the macrofinancial and monetary economics community. We calibrate and estimate this model using historical data in Section F, and explore the empirical implications of our canonical model. These implications will uncover clear weaknesses of the local linear approximation solution methods.

2 A Canonical Macro-Finance Model

The purpose of this section is to provide a benchmark DSGE model for unconventional monetary policy analysis. This model will incorporate two defining features: the nonlinear dynamics of risk premia and the endogenous financial risks originating from imperfect intermediation. The financial crisis of 2008 and the accompanying Great

Recession have highlighted the need for such models. Monetary authorities have become particularly aware of nonlinear risk premia and the real investment dynamics caused by dysfunctional financial intermediaries. As a result, unconventional monetary policies have been brought into the limelight by the monetary authorities following the financial crisis, and their role has fast become a focus of academic research.

Our model is a simplified version of the New Keynesian DSGE model proposed by Gertler and Kiyotaki (2010), yet extending it with regard to asset pricing dynamics. Importantly, instead of determining local solutions, we solve our model globally, emphasizing that the nonlinear features of the system are vital for optimal policy design, rather than focusing on a linear approximation around the small perturbations of the deterministic steady state. We believe the global method developed here is of general interest as well as an useful asset for the macro finance and monetary economics communities. In fact, one of our purposes is to show that global solutions are necessary; they capture the crucial nonlinear features designed into models, which are usually missed by local-linearization approximations. Nonlinear dynamics, especially those due to the instability of financial intermediation, require an unconventional monetary policy whose intensity directly depends on the observable indicators of financial intermediaries to stabilize the capital markets and the greater economy.

We also extend the simple model into a full-fledged standard New Keynesian DSGE model, incorporating a New Keynesian component similar to Christiano et al. (2005) and Smets and Wouters (2003). We include the full New Keynesian DSGE model in Appendix E; there we introduce the model details, estimate the parameters, and analyze the impact of shocks and implications of policies numerically.

2.1 Households

We begin with a description of households in the model, and then turn to firms and intermediaries. There is a continuum of households of unit mass. The members of each household are either workers or bankers. Although there are two types of household members, and certain portfolio constraints among them, we assume the representative household framework following Gertler and Kiyotaki (2010) and Gertler and Karadi

(2011) by assuming the household members to be part of a large family, sharing everything or, equivalently, assuming that the full set of Arrow-Debreu securities are available to the members within each household (but not across households), so that the idiosyncratic consumption risks can be fully insured, and all household members have identical preferences. A fraction ϖ of the members of the household are bankers. At any time, a fraction $1-\theta$ of randomly selected existing bankers exit and become workers, and return their net worth to their household. At the same time, an equal number of workers become bankers within each household, so the proportion of workers and bankers remains fixed. The new bankers receive some start-up funds from their household, which we describe below. The "perpetual youth" assumption in our model is purely technical, with the purpose of guaranteeing the survivorship of workers and preventing the economy from evolving into a degenerate situation.[1] It can be seen analytically from the condition (40).

Each banker within a household manages a financial intermediary. Workers deposit funds into these financial intermediaries. Household members do not hold capital directly by themselves. Instead, these financial intermediaries hold equity claims on a firm's capital; their funding, in turn, comes partly from the deposits put down by household members. At the same time, all household members provide labor to the firm for production. The firm and intermediaries will be described in details in Sections 2.2 and 2.4, respectively.

Since all household members have identical preferences, there are no incentives for bankers to pay dividends from their financial intermediaries. Rather, bankers would choose to accumulate the net worth of the financial intermediary up to a critical level from which the financial intermediation will be out of the credit constraints and stay there forever. If the critical value is the total value of all assets, the workers will be eliminated from the economy in the long run. As mentioned previously, to avoid this outcome, we assume that bankers and workers switch roles with probabilities $1-\theta$ and $(1-\theta)\frac{\varpi}{1-\varpi}$, respectively. When a banker switches roles, she pays all the accumulated net worth to her household. On the other hand, when a worker becomes

[1] Another way to prevent the over-accumulation of intermediary net worth is to assume efficiency losses, as in Bolton et al. (2011).

a banker, she needs funds to operate. To be precise, the start-up fund is transferred from the household, and it is equal to a fraction $\frac{\aleph}{(1-\theta)\varpi}$ of the aggregate asset value for each new banker. The parameter $\aleph > 0$ characterizes the intensity of funding transfers from workers to bankers. We denote Π_t the net transfer from bankers to workers, which will be defined later.

There are two points worth mentioning. First, the members of each household in this economy are divided into bankers and workers, both of whom supply labor, but only bankers own capital. In this way, bankers decide how much capital to accumulate given the capital adjustment costs. The heterogeneity of agents, however, mainly serves as an interpretational device.[2] Second, a potentially important deficiency is that the role of bankers is hardwired into the model: there is no other way to provide capital finance in financial markets. This sidesteps potentially important opportunities for flexibility in funding sources, an issue discussed more substantially in de Fiore and Uhlig (2011) and de Fiore and Uhlig (2015), for example.

The preferences of the household are given by

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau} \frac{C_{t+\tau}^{1-\gamma}}{1-\gamma} \right], \tag{1}$$

where C_t is the consumption and L_t is the labor supply at time t.

We denote by $R_{f,t}$ the real interest rate. Let B_t denote the quantity of the risk-free bank debt held by the household at the end of period t, and $B_{g,t}$ denote the quantity of the risk-free government debt held by households at the end of period t. Bank debt and government are both risk free, so they are perfect substitutes to the households, but bank debt enables banks to lever up and invest in capital privately, while government debt finances government purchase of capital. The household then

[2]For example, the equilibrium and its implications should not be affected if all households are homogeneous. Each household manages an intermediary. Households can invest in a firm's equity only through their intermediaries. Each household randomly terminates its intermediary and transfers its net worth to all households. Afterwards, they immediately start new intermediaries with funds collected from the households.

faces a state-by-state aggregate budget constraint

$$C_t = W_t L_t + \Pi_t - T_t + (1 + R_{f,t-1})(B_{t-1} + B_{g,t-1}) - (B_t + B_{g,t}), \tag{2}$$

where W_t is the real wage, Π_t is the profit from exiting intermediaries. T_t is the real lump sum taxes, and $R_{f,t-1}$ is the net real risk-free rate from the end of period $t-1$ to the end of period t. We assume each household provides one unit of labor inelastically, and thus the total labor supply remains $L_t \equiv 1$. Implicitly, we assume that households (workers and bankers) can trade a full set of Arrow-Debreu securities so that consumption risk is perfectly insured. The total payoffs of Arrow-Debreu securities are zero in aggregate. The intertemporal Euler equation for risk-free bond holding is

$$1 = \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} (1 + R_{f,t}) \right], \quad \text{where } \Lambda_t \equiv \beta^t (C_t)^{-\gamma}. \tag{3}$$

Here, Λ_t is the marginal utility of consumption C_t at date t.

2.2 Consumption Goods Sector

There is a continuum of firms of mass unity in the consumption goods sector. Each firm produces its output using an identical Cobb-Douglas production function with capital and labor as its input. The labor market is perfectly competitive, and labor is perfectly mobile across firms. As a result, there exists a representative firm with the same Cobb-Douglas production function:

$$Y_t = A_{c,t} K_t^{\alpha} L_{c,t}^{1-\alpha}, \quad 0 < \alpha < 1, \tag{4}$$

where $A_{c,t}$ is an exogenous and stochastic total factor productivity (TFP) parameter for consumption goods production, K_t is aggregate capital installed at the end of period $t-1$, and $L_{c,t}$ is aggregate labor demand in the consumption goods sector.

Denote $a_t \equiv \ln A_{c,t}$. The TFP and the capital quality evolve as

$$a_t = a_{t-1} + \sigma_a \epsilon_{a,t}, \tag{5}$$

where $\epsilon_{a,t}$ are i.i.d. standard normal variables.

Firms are owned by intermediaries. There is no friction between firms and intermediaries. A firm's investment is still subject to constraints, however, since intermediaries can be financially constrained. Firms always choose to pay their earnings to their intermediaries since the marginal value of cash for intermediaries is never less than that for firms. In addition, the capital structure of firms is irrelevant, since the firm's leverage may always be neutralized by the intermediary leverage, and it is the total leverage that is eventually subject to the credit constraint. Therefore, similar to most macroeconomic and asset pricing models, it is assumed that firms are all-equity firms, and pay out all their earnings. Such a firm has no wealth of its own, i.e. retained earnings. In period t, it issues new equity to intermediaries and uses the proceeds to purchase capital I_t, to be used for production in the next period[3] $t + 1$. The number of shares issued by the firm is normalized to one, although equity issuances occur over time.

Let us describe more details about the firm's production, hiring, and investment decisions along the timeline. Shocks are realized at the very beginning of each period. Observing the shocks, the firm hires labor at a perfectly competitive wage W_t and uses the capital K_t chosen at the end of period $t - 1$ to produce the consumption goods using the production function specified in (4).

After production takes place, the firm makes its investment by converting investment goods into new capital, and trading with other firms in a capital spot market. Together with the newly created capital, the depreciated old capital is traded freely in the spot market, and the amount of capital stock is optimally chosen for the next

[3]Note that we adopt the more conventional timing assumption, and index capital with the date when it is used in production, not with the date of the decision. As is well known, one needs to be careful when implementing this in solution software such as Dynare or Uhlig's Toolkit. See Uhlig (1999).

period. We denote the aggregate investment as I_t and the depreciation rate as δ. The law of motion for aggregate capital stock is given by

$$K_{t+1} = I_t + (1-\delta)K_t. \tag{6}$$

There are convex adjustment costs for the rate of investment, I_t/K_t. We assume that the cost for creating I_t units of new capital in terms of investment goods is

$$\Upsilon_t = \Upsilon(I_t; K_t) \equiv I_t + g(I_t, K_t), \quad \text{where } g(I_t, K_t) \equiv \frac{\vartheta}{2}\left(\frac{I_t}{K_t}\right)^2 K_t, \tag{7}$$

where $\vartheta > 0$ is a constant. The investment goods are produced by investment good firms. In the simpler cases adopted by macroeconomic asset pricing models (e.g. Gomes et al., 2003; Uhlig, 2007; Guvenen, 2009), a firm's investment converts consumption goods directly into new capital. By introducing an investment goods sector which exogenously maintains a stable scale relative to the whole economy, we can show that there exists a competitive equilibrium fluctuating around the balanced growth path. Similar methods have been adopted by Kogan et al. (2015) and Dou (2016). Details about the investment goods sector are introduced in Section 2.3.

Let us introduce the payout and valuation of firms. Arriving in period t with capital stock K_t chosen in $t-1$, the firm will choose labor L_t and investment I_t to maximize the cash flow (net payout)

$$X_t = \underbrace{\left[A_{c,t}K_t^{\alpha}L_{c,t}^{1-\alpha} - W_t L_{c,t}\right]}_{\text{cashflow of assets in place}} + \underbrace{\left[Q_t I_t - P_t \Upsilon(I_t; K_t)\right]}_{\text{cashflow of growth options}}, \tag{8}$$

where Q_t and P_t are the equilibrium spot prices of capital and investment goods, respectively. Here, the modeling of a firm's investment and payout follows the standard macro view of measuring cash flow for investors who own the entire corporate sector (e.g., Larrain and Yogo, 2008). This assumption is commonly adopted in standard production-based asset pricing models for stock returns (e.g., Papanikolaou, 2011). From a macro point of view, net repurchases of equity and debt are cash outflows from the corporate sector. The net capital gain at the end of period t is $Q_t(1-\delta)K_t = Q_tK_{t+1} - Q_tI_t$, since all firms trade in a perfectly competitive spot market of capital with equilibrium spot price Q_t. In effect, there is zero net trade among firms in equilibrium, since all firms are assumed to be homogeneous, even in the ex post situation.

The value of capital after depreciation, $Q_t(1-\delta)K_t$, can be viewed as the net capital gain of holding the "corporate sector". We do not explicitly model assets in place and growth options separately on firm balance sheets. More precisely, the capital in stock K_t implicitly contains both assets in place and growth options, and thus the value Q_t contains two components: the value of assets in place and the value of growth opportunities. Thus, the stock return can be represented as follows:

$$\begin{aligned} 1+R_{k,t+1} &\equiv \underbrace{\frac{Y_{t+1}-W_{t+1}L_{c,t+1}+Q_{t+1}(1-\delta)K_{t+1}}{Q_tK_{t+1}}}_{\text{return due to assets in place}} + \underbrace{\frac{Q_{t+1}I_{t+1}-P_{t+1}\Upsilon(I_{t+1},K_{t+1})}{Q_tK_{t+1}}}_{\text{return due to growth options}} \\ &= \underbrace{\frac{X_{t+1}}{Q_tK_{t+1}}}_{\text{net payout return}} + \underbrace{\frac{Q_{t+1}(1-\delta)K_{t+1}}{Q_tK_{t+1}}}_{\text{net capital gain return}}, \end{aligned} \tag{9}$$

where Q_tK_{t+1} is the value of assets at the end of period t. Capital gains from holding corporate shares are included in the net payout X_t. To sort out the consumption component in the return, we introduce the "dividend" of firms:

$$D_t \equiv Y_t - W_tL_{c,t} - P_t\Upsilon(I_t,K_t). \tag{10}$$

The stock return can be rewritten in terms of dividends as

$$1+R_{k,t+1} = \underbrace{\frac{D_{t+1}}{Q_tK_{t+1}}}_{\text{total dividend return}} + \underbrace{\frac{Q_{t+1}K_{t+2}}{Q_tK_{t+1}}}_{\text{total capital gains return}}. \tag{11}$$

It is worth mentioning that the quantitative model in full (see the appendix) follows Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), and includes a separate capital goods sector, a sector which produces capital and sells to consumption good firms. Basically, the model, rather than taking the macro view of stock returns, takes the portfolio view, as defined in Larrain and Yogo (2008).[4] Here in this model, consumption good firms are assumed to be short-lived, and the value of their outstanding shares is assumed to be equal to the size of capital.

The adjustment cost function has no intertemporal feature in itself; as a result, the intertemporal and dynamic aspects of investment decisions are captured by the forward-looking capital price Q_t. The trading in the competitive spot market of capital breaks the direct link between the current investment I_t and the capital stock for next period's production K_{t+1} for each firm. Thus, the current decision I_t has no effect on the following decisions I_{t+1} through K_{t+1}. As a result, the investment decision is not dynamic, and the standard q theory of Hayashi (1982) holds. To see this more clearly, consider the firm's optimization problem at the end of period t:

$$
\begin{aligned}
&K_{t+1}Q_t \geq \max_{L_{t+1},I_{t+1}} \mathbb{E}_t \left\{\mathcal{M}^{\mathcal{I}}_{t,t+1}\left[D_{t+1} + Q_{t+1}(1-\delta)K_{t+1}\right]\right\} \qquad (12)\\
&\geq \max_{L_{c,t+1},I_{t+1}} \mathbb{E}_t \left\{\mathcal{M}^{\mathcal{I}}_{t,t+1}\left[Y_{t+1} - W_{t+1}L_{c,t+1} + Q_{t+1}\left(I_{t+1} + (1-\delta)K_{t+1}\right) - P_{t+1}\Upsilon(I_{t+1};K_{t+1})\right]\right\}
\end{aligned}
$$

where $\mathcal{M}^{\mathcal{I}}_{t,t+1}$ is the effective intertemporal marginal rate of substitution (IMRS) of financial intermediaries. The equilibrium asset pricing condition depends on inequality (i.e. the supermartingale condition), instead of equality (i.e. the martingale condition), due to the credit constraints. Furthermore, due to the intermediary's credit constraint, the intermediary's IMRS can be different from the household's actual IMRS $\mathcal{M}_{t,t+1} \equiv \Lambda_{t+1}/\Lambda_t$. We denote $\Omega^{\mathcal{I}}_{t,t+1}$ as the wedge between the intermediary's

[4] The difference between two views as regards stock returns is the growth option component. To be more precise, the stock return under the macro view is $1 + R^{\mathrm{m}}_{k,t+1} \equiv \frac{Y_{t+1} - W_{t+1}L_{c,t+1} + Q_{t+1}(1-\delta)K_{t+1}}{Q_tK_{t+1}} + \frac{Q_{t+1}I_{t+1} - \Upsilon(I_{t+1},K_{t+1})}{Q_tK_{t+1}}$. In Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) which takes the portfolio view, the stock return is $1 + R^{\mathrm{p}}_{k,t+1} \equiv \frac{Y_{t+1} - W_{t+1}L_{c,t+1} + Q_{t+1}(1-\delta)K_{t+1}}{Q_tK_{t+1}}$. Effectively, the gap between $R^{\mathrm{m}}_{k,t+1}$ and $R^{\mathrm{p}}_{k,t+1}$ is the net return due to growth options.

effective IMRS ($\mathcal{M}^{\mathcal{I}}_{t,t+1}$) and the household's IMRS ($\mathcal{M}_{t,t+1}$); more precisely,

$$\mathcal{M}^{\mathcal{I}}_{t,t+1} = \Omega^{\mathcal{I}}_{t,t+1}\mathcal{M}_{t,t+1}. \tag{13}$$

The wedge $\Omega^{\mathcal{I}}_{t,t+1}$ is derived and discussed in Section 2.4. Briefly, the wedge $\Omega^{\mathcal{I}}_{t,t+1}$ characterizes the economic tightness of the credit constraint. The wedge $\Omega^{\mathcal{I}}_{t,t+1}$ becomes larger when the credit constraint is binding more tightly for financial intermediaries. In addition, when the credit constraints are not binding for intermediaries, the equality holds in the asset pricing condition (12). In fact, the Hamilton-Jacob-Bellman (HJB) equation in (12) can be rewritten in terms of stock returns as

$$1 \geq \mathbb{E}_t\left[\mathcal{M}^{\mathcal{I}}_{t,t+1}(1 + R_{k,t+1})\right]. \tag{14}$$

Finally, let us characterize the equilibrium relationships between optimal investment, hiring, and consumption. Given the capital price Q_{t+1} and the price of investment goods P_{t+1}, the problem of optimal investment for firms can be decomposed into a sequence of state-by-state static (intratemporal) optimization problems:

$$\max_{I_{t+1}} Q_{t+1}I_{t+1} - P_{t+1}\Upsilon(I_{t+1}; K_{t+1}). \tag{15}$$

The state-by-state first-order condition with respect to I_{t+1} gives

$$Q_{t+1}/P_{t+1} = 1 + \vartheta i_{t+1}, \quad \text{where} \;\; i_{t+1} \equiv \frac{I_{t+1}}{K_{t+1}}. \tag{16}$$

This is the standard q theory of investment developed by Hayashi (1982), in which the investment decision I_{t+1}/K_{t+1} is directly linked to the marginal q (marginal value) of the capital.

Similarly, the optimal labor demand can also be derived from the state-by-state (static) optimization problem:

$$\max_{L_{c,t+1}} A_{t+1} K_{t+1}^{\alpha} L_{c,t+1}^{1-\alpha} - W_{t+1} L_{c,t+1} \tag{17}$$

The first-order condition with respect to $L_{c,t+1}$ gives

$$L_{c,t+1} = \left[(1-\alpha) \frac{A_{t+1}}{W_{t+1}} \right]^{1/\alpha} K_{t+1}. \tag{18}$$

The consumption goods are non-durable, and thus the market clearing condition implies the characterization for aggregation consumption goods:

$$Y_t = D_t + W_t L_{c,t} + W_t L_{\iota,t}. \tag{19}$$

2.3 Investment Goods Sector

There is a continuum of investment good firms which produce investment goods using labor. These firms are identical, and they have the same production function:

$$\Upsilon_t = A_{\iota,t} L_{\iota,t} \tag{20}$$

where $A_{\iota,t}$ is the productivity of investment goods production, and $L_{\iota,t}$ is the labor demand in the investment goods sector. We assume that the scale of the investment goods section is co-integrated with the scale of the consumption goods sector. More precisely, we simply assume that $A_{\iota,t} = Z_{\iota,t} K_t$ where $Z_{\iota,t}$ follows a stationary stochastic process. For simplicity, the process $Z_{\iota,t}$ is assumed to be constant. It is worth mentioning that K_t is the total physical capital stock that cannot be internalized by single investment good firms, and thus it is the exogenous scale of the investment goods sector. This guarantees that the balanced growth path is $A_{c,t} K_t^{\alpha}$.

This assumption makes the allocation of the model suboptimal even in the absence of financial frictions. For firms that make investment decisions, they do not internalize the enhancement of investment good sector TFP, so that capital accumulation is lower than the first best case. Appendix A.3 provides the details of the social planner's problem and optimality conditions. The difference can be clearly seen from the first order condition (79) in appendix A.3. When the social planner chooses optimal K_t^i for each firm, they consider its effect on investment good sector TFP, but individual firms do not. The externality works in the same way as Romer (1986).

The market clearing condition for labor market requires

$$L_{c,t} + L_{\iota,t} = L_t \ \text{ for all } t. \tag{21}$$

All the investment good firms produce and sell investment goods competitively. As a result, they are zero-profit firms. We denote the competitive price of investment goods as P_t.

2.4 Financial Intermediaries

Financial intermediaries borrow funds from households at a risk-free rate, pool the funds with their own net worth and invest the sum in the equity of the representative consumption good firm. The balance sheet of intermediary j at the end of time t is given by

$$Q_t K_{t+1} S_{j,t} = N_{j,t} + B_{j,t}, \tag{22}$$

where Q_t is the price of the firm's equity, $S_{j,t}$ is the quantity of equity held by the intermediary, $N_{j,t}$ is the net worth, and $B_{j,t}$ is the deposits raised from households. The intermediary earns a return $R_{k,t+1}$ from the equity investment at time $t+1$, and must pay the interest, R_t, on the deposit. The net worth of the intermediary, therefore,

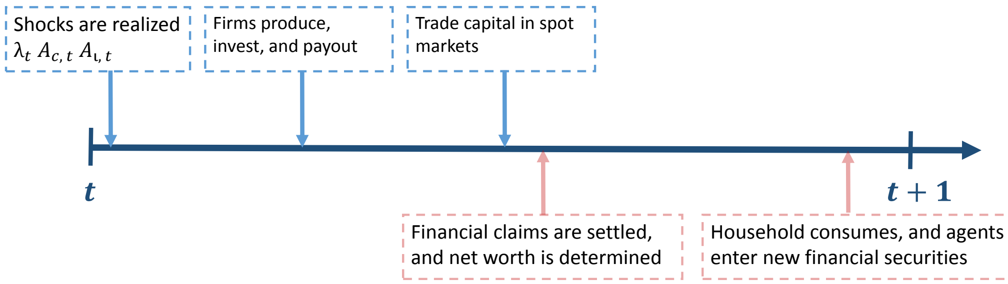


Figure 1: Timeline convention.

evolves as

$$\begin{aligned} N_{j,t+1} &= (1+R_{k,t+1})Q_t K_{t+1} S_{j,t} - (1+R_t)B_{j,t} \\ &= (R_{k,t+1}-R_t)Q_t K_{t+1} S_{j,t} + (1+R_t)N_{j,t}. \end{aligned} \quad (23)$$

where $N_{j,t+1}$ is intermediary j's net worth at the end of period $t+1$.

The intermediaries face a constraint on raising deposits from households. They cannot raise deposits beyond a certain level, which is determined endogenously in the equilibrium. We shall describe this constraint in more detail below. Since bankers own the intermediaries, we use the bankers' IMRS, which coincides with the IMRS of the representative household, $\mathcal{M}_{t,t+1} \equiv \Lambda_{t+1}/\Lambda_t$, to determine the value of assets held by an intermediary according to the cash flows received by the bankers.

The following schematic (Figure 1) is the timing convention for financial intermediaries and firms, to help explain the ordering of these events within the model.

Since an intermediary exits exogenously in each period with probability $1-\theta$, the value of intermediary j's terminal wealth to its household is given by

$$V_{j,t} = \max_{\{S_{j,t+\tau},B_{j,t+\tau}\}_{\tau\geq 1}} \mathbb{E}_t \left[\frac{\Lambda_{t+\tilde{\tau}_j}}{\Lambda_t} N_{j,t+\tilde{\tau}_j} \right], \quad (24)$$

where $\tilde{\tau}_j$ is the stochastic stopping time for the financial intermediary j to exit and pay out the net worth $N_{\tilde{\tau}_j}$ to its banker. Thus, the value of the financial intermediary j can be expressed as weighted average of discounted possible "payouts" (net worth $N_{t+\tau}$):

$$
\begin{aligned}
V_{j,t} &= \max_{\{S_{j,t+\tau},B_{j,t+\tau}\}_{\tau\geq 1}} \sum_{\tau=1}^{+\infty} \mathbb{P}(\tilde{\tau}_j = \tau)\mathbb{E}_t \left[\frac{\Lambda_{t+\tau}}{\Lambda_t} N_{j,t+\tau}\right] \\
&= \max_{\{S_{j,t+\tau},B_{j,t+\tau}\}_{\tau\geq 0}} \sum_{\tau=1}^{+\infty} (1-\theta)\theta^{\tau-1}\mathbb{E}_t \left[\frac{\Lambda_{t+\tau}}{\Lambda_t} N_{j,t+\tau}\right] \\
&= \max_{S_{j,t+1},B_{j,t+1}} \mathbb{E}_t \left\{\frac{\Lambda_{t+1}}{\Lambda_t} \left[(1-\theta)N_{j,t+1} + \theta V_{j,t+1}\right]\right\}. \qquad (25)
\end{aligned}
$$

The equation (25) is the HJB equation for the value of financial intermediary j.

In order to motivate the borrowing constraint faced by financial intermediaries, we introduce a simple moral hazard/costly enforcement problem following Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). We assume that the banker can choose to liquidate the financial intermediation and divert the fraction of available funds, λ_t, from the value of the financial intermediation.

The borrowing constraint is modeled as follows. At any time t, the banker of the intermediary can divert a fraction λ_t of the intermediary's assets for his own benefit, where λ_t is an exogenous parameter. The log of margin parameter $\ln \lambda_t$ follows a first-order Markov chain with long-term mean $\overline{\lambda}$, autocorrelation ρ_λ, and long-term variance $\overline{\sigma}^2_\lambda$. The quantity $1-\overline{\lambda}$ measures the steady-state pledgeability of the intermediary's asset. If the value of the intermediary falls below $\lambda_t Q_t S_{j,t}$, the banker will simply divert the assets and terminate the intermediary. In such a case, households will get a zero gross return from their deposits. In order for the households to have an incentive to deposit cash with the intermediary, the following condition must hold:

$$
V_{j,t} \geq \lambda_t Q_t K_{t+1} S_{j,t}. \qquad (26)
$$

Because utility functions are homothetic, the optimal portfolios are linear in terms of the net worth. Thus, the value of financial intermediaries is also linear in the net worth and therefore can be characterized as follows:

$$V_{j,t} = \Omega_t N_{j,t} \tag{27}$$

where Ω_t is the marginal value of net worth for financial intermediaries. Since each financial intermediary is atomistic, it does not affect the equilibrium. Furthermore, the cross-sectional distribution of the net worths of the intermediaries does not affect the equilibrium either, due to the linearity of the optimal portfolio holdings of intermediaries guaranteed by the homothetic utilities. Instead, the total net worth share $n_t \equiv \dfrac{N_t}{Q_t K_{t+1}}$ is the only endogenous state variable needed for characterizing the equilibrium, where $N_t \equiv \int_j N_{j,t} \mathrm{d}j$ is the aggregate net worth in all intermediaries. Let $S_{p,t} \equiv \int_j S_{j,t} \mathrm{d}j$ be the aggregate outstanding shares of firms held by private financial intermediaries. The total supply of $S_{p,t}$ is determined by the credit policy of the government.

We conjecture that Ω_t only depends on the aggregate exogenous state $z_t \equiv (\xi_t, \lambda_t)$ and the aggregate endogenous state variable x_t. The aggregate net worth characterizes the average leverage of financial intermediaries, and thus the incentives for bankers to walk away from their financial intermediaries. As a consequence, it also determines intuitively the tightness of the credit constraint, and in turn, the expected returns to financial holdings. The multiplier Ω_t can be interpreted as the marginal value ("marginal q") of the net worth held by intermediaries.

Following (25) and (26), the portfolio problem of the financial intermediary with credit constraints can be written as,

$$\Lambda_t \Omega_t N_{j,t} = \max_{S_{j,t+1}, B_{j,t+1}} \mathbb{E}_t \left[\Lambda_{t+1} \left(1 - \theta + \theta \Omega_{t+1}\right) N_{j,t+1} \right] + \mu_{j,t} \Lambda_t \left(\Omega_t N_{j,t} - \lambda_t S_{j,t} Q_t K_{t+1} \right)$$

subject to

$$N_{j,t} = S_{j,t}Q_tK_{t+1} - B_{j,t}, \quad \text{and} \tag{28}$$

$$N_{j,t+1} = S_{j,t}Q_tK_{t+1}(1 + R_{k,t+1}) - B_{j,t}(1 + R_{f,t}) \tag{29}$$

and $\mu_{j,t} \geq 0$ and $\Omega_t N_{j,t} \geq \lambda_t S_{j,t}Q_tK_{t+1}$. Here, $\mu_{j,t}$ is the Lagrangian multiplier normalized by Λ_t; it is non-negative, and becomes positive if and only if the credit constraint becomes binding for intermediary j. This is the HJB equation which formulates the optimization problem of the intermediaries. Because the intermediaries are the only channel to hold risky assets of the economy, and they face a leverage constraint, the marginal value of cash of the intermediaries $\Lambda_t(1 - \theta + \theta\Omega_t)$ is always larger than the marginal value of cash outside the intermediaries Λ_t. As a result, the intermediaries are the natural borrowers in the economy. That is, $B_{j,t} \geq 0$ for each intermediary j. In aggregate, it holds that $0 < N_t \leq S_{p,t}Q_tK_{t+1}$ and thus $0 < n_t \leq 1$.

The first-order condition of substituting between $S_{j,t}$ and $B_{j,t}$ gives

$$0 \leq \mu_{j,t}\lambda_t\Omega_t^{-1} = \mathbb{E}_t\left[\mathcal{M}^{\mathcal{I}}_{t,t+1}(R_{k,t+1} - R_{f,t})\right], \tag{30}$$

where $\Omega^{\mathcal{I}}_{t,t+1} \equiv \dfrac{1 - \theta + \theta\Omega_{t+1}}{\Omega_t}$. The wedge $\Omega^{\mathcal{I}}_{t,t+1}$ is the core component of the so-called "intermediary asset pricing theory" and the effective IMRS of intermediaries is $\mathcal{M}^{\mathcal{I}}_{t,t+1} \equiv \mathcal{M}_{t,t+1}\Omega^{\mathcal{I}}_{t,t+1}$. The condition shows that $\mathbb{E}_t\left[\mathcal{M}^{\mathcal{I}}_{t,t+1}(R_{k,t+1} - R_{f,t})\right] > 0$ when the credit constraint is binding. This condition by itself appears to violate the supermartingale condition of self-financed cash flows. Thus, there appears to be a possible arbitrage opportunity by going long on equity and going short on risk-free bonds. The absence of arbitrage still holds since the intermediary cannot further increase its leverage when its credit constraint is binding.

Plugging (30) into the HJB equation for intermediaries leads to pricing rules for risk-free bonds and firm equity, respectively:

$$1 \geq 1 - \mu_{j,t} = \mathbb{E}_t \left[\mathcal{M}^{\mathcal{I}}_{t,t+1}(1 + R_{f,t}) \right], \quad \text{and} \tag{31}$$

$$1 \geq 1 - \mu_{j,t}(1 - \lambda_t \Omega_t^{-1}) = \mathbb{E}_t \left[\mathcal{M}^{\mathcal{I}}_{t,t+1}(1 + R_{k,t+1}) \right]. \tag{32}$$

The inequality in (32) holds because λ_t is between 0 and 1. The supermartingale conditions hold for equity returns and risk-free rates separately. From (31) and (32), we can see that the Lagrangian multipliers $\mu_{j,t}$ for intermediaries should be the same. Upon reflection, we turn our focus to a symmetric equilibrium. Denote $\mu_t \equiv \mu_{j,t}$ for all j. In the symmetric equilibrium, each intermediary chooses the shares of equity to hold proportionally to its net worth in the following sense:

$$S_{j,t} Q_t K_{t+1} = s_t N_{j,t}, \tag{33}$$

for all j and s_t only depending on aggregate state variables. Given equilibrium asset prices and returns, the optimal holding s_t can be characterized by only the market-clearing condition in the equity market. The supply to private financial intermediaries is $S_{p,t} = 1 - S_{g,t}$ and the market-clearing condition is

$$S_{p,t} Q_t K_{t+1} = s_t N_t. \tag{34}$$

Thus, the optimal holding increases in the total supply of equity ($S_{p,t}$), and decreases in the total net worth of intermediaries (x_t):

$$s_t = \frac{S_{p,t}}{n_t}. \tag{35}$$

2.5 Net Worth Evolution

After integrating the dynamic equations in (23) over all intermediaries and accounting for the net fund transfer (**??**), the aggregate net worth evolves as

$$N_{t+1} = \tilde{N}_{t+1} - \Pi_{t+1}$$

where:

$$\Pi_{t+1} = (1-\theta)\widetilde{N}_{t+1} - \aleph Q_{t+1} K_{t+2} \tag{36}$$

$$\widetilde{N}_{t+1} = (R_{k,t+1} - R_{f,t}) Q_t K_{t+1} S_{p,t} + (1 + R_{f,t}) N_t. \tag{37}$$

Here, N_t is the (end-of-period) aggregate net worth of of intermediaries in time t after intermediary payout. The quantity $\widetilde{N}_{t+1} = (R_{k,t+1} - R_{f,t}) Q_t K_{t+1} S_{p,t} + (1 + R_{f,t}) N_t$ is the aggregate net worth before intermediary payout, and a fraction $1 - \theta$ exits the market and pay out their net worth. In the meantime, $\aleph Q_{t+1} K_{t+2}$ of net startup fund are given to newly entered intermediaries in time $t + 1$.
Plug in the expression for payout, we have:

$$N_{t+1} = \theta[(R_{k,t+1} - R_{f,t}) Q_t K_{t+1} S_{p,t} + (1 + R_{f,t}) N_t] + \aleph Q_{t+1} K_{t+2} \tag{38}$$

Thus, the net worth share of intermediaries evolves as

$$n_{t+1} = \theta \left[(R_{k,t+1} - R_{f,t}) S_{p,t} + (1 + R_{f,t}) n_t \right] / G_{k,t+1} + \aleph \tag{39}$$

where $G_{k,t+1} \equiv \dfrac{Q_{t+1} K_{t+2}}{Q_t K_{t+1}}$ is the total capital gain of equity.

Let μ^* be the upper bound of the dividend-price ratio of stocks in the frictionless economy. Then the net worth share n_t is always less than 1 when

$$\mu^* < (1-\theta) - \aleph. \tag{40}$$

The specifications of μ^* can be found in Appendix A. In the case when (40) holds, there exists $\underline{n}_t \in (0, \lambda_t S_{p,t})$ characterizing the constraint-binding boundary such that

$$\Omega_t = \begin{cases} \dfrac{\lambda_t S_{p,t}}{n_t}, & \text{when} \quad n_t \in (0, \underline{n}_t]; \\ \Omega(n_t, \lambda_t) > \max\left\{\dfrac{\lambda_t S_{p,t}}{n_t}, 1\right\}, & \text{when} \quad n_t \in (\underline{n}_t, 1). \end{cases}$$

The net worth share n_t never reaches the limit 1 since there is no efficiency loss attached to the intermediary net worth. Solving the equilibrium is effectively the functional form of $\Omega(n_t, \lambda_t)$.

2.6 Government Policies

The ultimate goal of our model is to analyze the effectiveness of unconventional monetary policies in fighting financial crises and their destructive impact on the macroeconomy as a whole. Importantly, we demonstrate that the global solution of our model, which captures its nonlinear features, is vital to guarantee the proper and useful analysis of its policy implications.

According to Section 13.3 of the Federal Reserve Act, the Fed is allowed to take risky positions through making loans in the private sector (provided that they are not unduly so), under "unusual and exigent circumstances." This legislation basically makes the Fed the lender of last resort of the economy. Meanwhile, the Treasury, the Fed, the FDIC, and the bailout bills passed by Congress together took unconventional policy measures, including equity injection into the private sector, asset purchases from distressed banks, the lifting of caps on deposit insurance for certain bank accounts, and lending guarantees for certain types of bank loans. All these policies and interventions were structured to encourage firms to bring in private capital. For instance, it was intentionally designed that firms returning capital to the government by certain dates would get better terms for the government's stake. The central plank of all these unconventional measures was to attract private capital.

These different measures work together in practice, and were intentionally designed to complement each other. As a result, it is unrealistic to discuss them individually in a unified framework. Given their primary goal and common ideas, however, our model adopts a single abstract unconventional policy as a modeling device, yet one still relevant enough to serve an illustrative purpose. We assume the government is willing to buy the shares of the firm directly to facilitate lending. Such policies were studied by Gertler and Kiyotaki (2010), and Gertler and Karadi (2011). This captures the unconventional policy of purchasing risky, privately managed, non-government assets, implemented in the U.S., the U.K. and the eurozone in the wake of the financial crisis. The U.S. Federal Reserve's program of buying \$600 million of mortgage-backed securities in 2008-09 (QE-1) and the European Central Bank's Covered Bond Purchase Programs (CBPPs) for buying private sector debt are examples of such policies. Our intention of appealing to such a simple form of unconventional monetary policy (or a credit policy) is to develop a baseline model for analysis.

More precisely, in our model, the government buys a fraction $S_{g,t}$ of the total outstanding shares of firms (normalized to one), so that

$$Q_t K_{t+1} = S_{p,t} Q_t K_{t+1} + S_{g,t} Q_t K_{t+1}, \tag{41}$$

where $S_{p,t} \equiv \int S_{j,t} \mathrm{d}j$ is the total share of equity held privately, and the share of government-held equity is $S_{g,t} = 1 - S_{p,t}$. To conduct the credit policy, the government issues government debt to households that pay the risk-free rate $R_{f,t}$ and then lends the funds to firms or purchases the equity stakes of firms with returns $R_{k,t+1}$. The government credit has an efficiency cost of $\tau > 0$ units per unit of credit supplied. This deadweight loss may reflect the government's fundraising costs or its investment search costs.

We then introduce the key assumption which makes the government's balance sheet, and thus the credit policy, non-neutral as regards its macroeconomic implications. This is the only special feature of government intermediation in our model. A general discussion about the special characteristics that make a government's balance sheet relevant can be found in the companion review paper Dou et al. (2017). More precisely, government intermediation is not financially constrained in our model, unlike private financial intermediation. This can be justified by the assumption that the government always honors its debt, and thus incurs no agency problems between it and its household creditors.

We define the total leverage ratio $\phi_{c,t}$ as follows

$$Q_t K_{t+1} = \phi_{c,t} N_t. \tag{42}$$

The leverage ratio, $\phi_{c,t}$, is the leverage ratio for total intermediated funds, public as well as private, and has the following relation with the private leverage ratio, $\phi_t \equiv S_{p,t} Q_t K_{t+1}/N_t$, and the intensity of government credit intervention, $S_{g,t}$,

$$\phi_{c,t} = \frac{\phi_t}{1 - S_{g,t}}. \tag{43}$$

The government issues government bonds $B_{g,t} = S_{g,t} Q_t K_{t+1}$ to fund the purchase of these shares. From this activity, the government thus earns an amount $(R_{k,t+1} - R_{f,t})B_{g,t}$ every period.

We assume that at the onset of a crisis, which is defined loosely to mean a period when the log risk premium $\Xi_t \equiv \mathbb{E}_t [\ln(1 + R_{k,t+1})] - \ln(1 + R_{f,t})$ rises sharply and becomes much higher than the frictionless benchmark $\Xi^* \equiv \mathbb{E}_t [\ln(1 + R_k^*)] - \ln\left(1 + R_f^*\right)$, the government injects credit in response to movements in risk premia. Similar to a standard Lucas-tree economy, the log risk premium is

$$\Xi^* \approx \gamma \sigma_a^2 - \frac{1}{2}\sigma_a^2,$$

where $\frac{1}{2}\sigma_a^2$ is Jensen's term for the log return. The frictionless benchmark is described in Appendix A. We consider the credit policy that follows the rule for $S_{g,t} = 1 - S_{p,t}$:

$$S_{p,t} = \frac{1}{1 + \nu_g \times (\Xi_t - \Xi^*)}, \tag{44}$$

where the sensitivity parameter, ν_g, is positive. According to (44), the government expands credit as the risk premium gap increases. Our specification is a global version of the credit policy considered by Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). In the local-linear approximation when $\Xi_t - \Xi^*$ is small,

$$S_{g,t} = 1 - S_{p,t} \approx \nu_g \times (\Xi_t - \Xi^*) \,. \tag{45}$$

The rationale behind this policy specification is as follows. In the absence of financial friction that prevents the financial intermediaries from leveraging too much, the equilibrium outcome is efficient. The inefficiency arises due to the inability of households to buy the risky assets directly, and to the limit on the leverage of their financial managers. This inefficiency manifests itself in the form of large risk premia, since the financial intermediaries must be compensated adequately in the absence of high leverage. The government does not intervene when the risk premium is at its steady-state level, but it does intervene when the premium rises to increasingly inefficient levels beyond it.

We shall show that the global solution of this nonlinear system allows for a state-dependent sensitivity coefficient. For example, we can specify a policy rule as follows:

$$\nu_g = \nu_{g,0} + \nu_{g,1} \times \left(\frac{1}{n_t} - 1 \right), \tag{46}$$

with $\nu_{g,0} \geq 0$ and $\nu_{g,1} \geq 0$. The idea is that it should be better for the government to conduct more aggressive credit policy (i.e., the sensitivity ν_g is larger) when the financial system is already more fragile (i.e., n_t is smaller).

From (43), it is clear that when the private leverage ratio ϕ_t is kept fixed, the expanding credit policy $S_{g,t}$ increases the total leverage of intermediation, i.e., $\phi_{c,t}$ rises. This captures the idea that the government's balance sheet acts as an intermediary to channel household funds to the asset market when the financial intermediaries are constrained. The government's intermediation prevents asset prices from becoming overly distressed when this is caused by the inefficiency of financial intermediaries after a sequence of negative shocks.

2.7 Resource and Government Budget Constraints

The resource constraint for the final good in our model is given by

$$Y_t = C_t + G_t + \tau S_{g,t} Q_t K_{t+1}. \tag{47}$$

The government spends a fraction $\overline{g}$ of output Y_t in period t, where $\overline{g}$ is an exogenously specified constant. That is,

$$G_t = \overline{g} Y_t. \tag{48}$$

In addition to funding government expenditure, the government also needs to fund the central bank's purchase of shares by issuing purchasing bonds worth $B_{g,t} = S_{g,t} Q_t K_{t+1}$ and the efficiency loss by taxes. Its revenues include taxes, T_t, and the government's income from intermediation, $S_{g,t-1} Q_{t-1} K_t (R_{k,t} - R_{f,t-1})$. Thus, the government budget constraint is

$$G_t + (1+\tau) S_{g,t} Q_t K_{t+1} = T_t + S_{g,t-1} Q_{t-1} K_t (R_{k,t} - R_{f,t-1}) + B_{g,t}. \tag{49}$$

Since the taxation T_t effectively takes up any slack that shows up on the government balance sheet, and given the existence of representative agents in the economy, the intertemporal budget constraint of the representative household and the intertemporal budget constraint of the government can be combined with taxes left out. Intuitively, then, by Walras' Law, both budget constraints are redundant in determining the equilibria. However, this is very different from saying that the size and composition of the government balance sheet are irrelevant for pinning down the equilibrium under efficient financial market conditions in the sense of Wallace (1981). This is simply because not all investors can purchase an arbitrary amount of the same assets at the same market prices as the government in this model. Put more precisely, unlike private financial intermediation, government intermediation is not constrained by the balance sheet.

2.8 Quantitative Analysis

Our model can be used to understand the response of the economy to various shocks. We highlight the role of nonlinear dynamics of risk premia in determining the intensity of the credit policy. Furthermore, we show that it requires global solutions to properly characterize these nonlinear dynamics and to analyze the effectiveness of policies.

We use a calibrated version of the model, basing our parameter choices mainly on those in Gertler and Karadi (2011) and Gourio (2012), and the estimated dynamic parameters in Smets and Wouters (2007). Further discussion on parameter calibration can be found in Appendix F. Using the dynamic parameters estimated in the full model in the appendix, we calibrate them here for illustrative purposes. In our numerical analysis, the exogenous autoregressive processes are discretized into homogeneous Markov chains according to the method proposed by Rouwenhorst (1995). The calibrated parameters are summarized in Table 1. The parameter choices are basically close to those picked in Gertler and Karadi (2011).

Equilibrium We first demonstrate the equilibrium policy functions of the endogenous variables. Here, we assume the baseline parameter specifications, except that of the margin of financial intermediaries, stay constant $\overline{\lambda}$. That is, the volatility of margin is zero: $\sigma_\lambda = 0$. The nonlinearity of this equilibrium is severe, even having kinks over the endogenous state space. We compare the equilibrium outcomes with the frictionless economy equilibrium.

In Figure 2, we show the equilibrium financial variables. The red dashed curves in Panels A – E are endogenous variables in equilibrium in a frictionless economy, whereas the blue solid curves characterize the equilibrium of an economy with financial friction. The only deviation of the economy with financial friction from the frictionless economy is the leverage constraint faced by intermediaries. The equilibrium of the frictionless economy does not depend on the net worth of financial intermediaries, as

Table 1: Baseline Parameters (Quarterly)

Parameter	Symbol	Value	Source
Household preference			
Discount rate	β	$0.999^{\frac{1}{4}}$	Standard
Relative risk aversion	γ	6	Standard
Total labor supply	$\bar{L}$	1	Standard
Financial intermediaries			
Steady-state fraction of divertible capital	$\overline{\lambda}$	0.381	Gertler and Karadi (2011)
Proportional transfer to new bankers	$\aleph$	$0.002 \times \frac{1}{4}$	Gertler and Karadi (2011)
Survival rate of bankers	θ	0.8	Non-degenerate condition
Consumption-good firms			
Effective capital share	α	0.33	Standard
Depreciation rate	δ	$0.06 \times \frac{1}{4}$	Standard
Adjustment cost coefficient	ϑ	6	Standard
Investment-good firms			
TFP of investment good production	Z_ι	1	Standard
Government policies			
Government expenditure ratio	$\overline{g}$	20%	Standard
Government efficiency loss	τ	10%	Calibration
Sensitivity coefficient	$\nu_{g,0}$	5	Calibration
Sensitivity coefficient	$\nu_{g,1}$	0	Calibration
Dynamic			
Volatility of TFP	σ_a	$0.04 \times \frac{1}{2}$	Standard
Persistence of margin	ρ_λ	0.6	Gertler and Karadi (2011)
Volatility of margin	$\overline{\sigma}_\lambda$	0.267	Gertler and Karadi (2011)

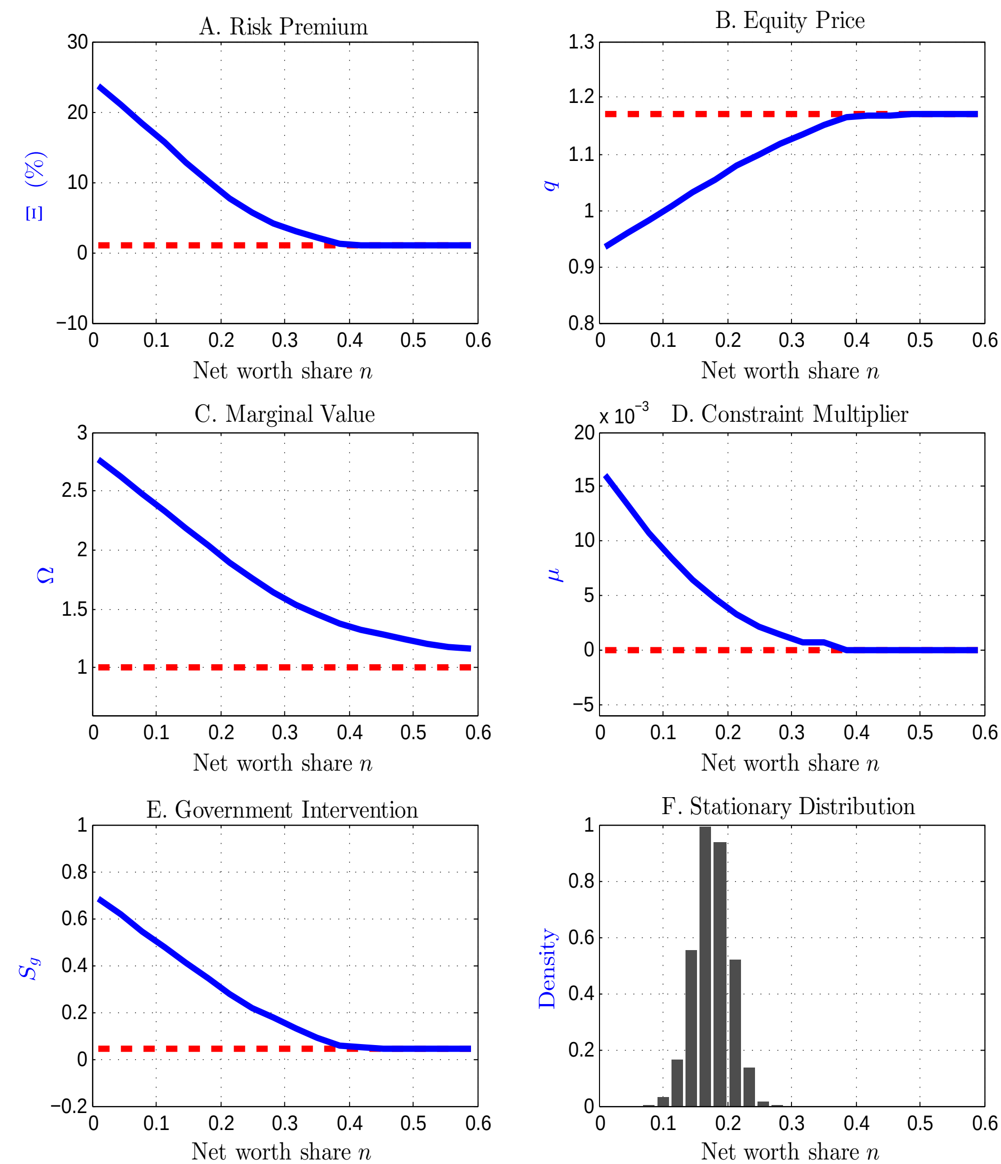
Note: All parameters are standard except the credit-policy related parameters τ, $\nu_{g,0}$, and $\nu_{g,1}$. We pick $\nu_{g,0}$ and τ here to provide reasonable private holding shares of risky assets and average risk premia. The parameter $\nu_{g,1}$ is chosen to be zero in the baseline calibration, though we emphasize that the non-zero $\nu_{g,1}$ is important as part of the optimal policy given the nonlinear dynamic of risk premia and the economy. The closest comparison in the literature is Gertler and Karadi (2011); however, their model is solved using log-linearization techniques. The local approximation significantly understates the magnitude and the volatility of the conditional risk premium, even though the model is capable of generating both quantitatively. The implication of biased asset pricing makes quantitative discussion of unconventional monetary policies itself biased. For example, the suppressed risk premium and its nonlinear dynamics require extremely sensitive unconventional monetary policy in order to have a quantitatively significant stabilization effect on the aggregate quantities. In Gertler and Karadi (2011), the credit policy sensitivity parameter $\nu_{g,0}$ is chosen about 100 and the cost parameter of the intervention τ is chosen at an extremely small value 10%. Parameters on the intermediary side θ, ρ_λ, $\bar{\lambda}$, and $\bar{\sigma}_\lambda$ are calibrated quarterly.

the equilibrium policy functions are all constant across different net worth share n_t. In contrast, the equilibrium of the economy with financial friction relies heavily upon intermediary net worth, i.e. the condition of how well the financial intermediaries are capitalized. Importantly, the dependence is largely nonlinear. The intermediary net worth, as an endogenous state variable, is endogenously driven by exogenous shocks hitting the economy, and in turn, it poses endogenous risks to the economy.

Panel D of Figure 2 shows the Lagrangian multiplier of the leverage constraints faced by intermediaries. In equilibrium, the constraint becomes binding if and only if the intermediary net worth becomes less than a critical value $\underline{n}$ that is about 0.4 under this calibration. Panel F displays the stationary distribution of intermediary net worth. The economy fluctuates mainly between 0.1 and 0.3. The median of the distribution is about 0.19. Comparing Panels D and F, we can see that the leverage constraint is almost always binding in equilibrium. Comparing Panels D and C, we can see that the marginal value of intermediary net worth Ω_t is higher than the frictionless benchmark $\Omega^* = 1$, even when n_t is far into the region where the leverage constraint is not binding for intermediaries. This is the expectation effect of financial constraints. From Panels A – E, it can be seen that the equilibrium of the frictional economy converges to that of the frictionless economy as the intermediary net worth approaches 1.

The equilibrium risk premium Ξ_t is shown in Panel A of Figure 2. Its unconditional expectation is about 10% each year, and it skyrockets as the intermediary net worth declines. Panel B shows the equity price q_t, which decreases rapidly as the intermediary net worth declines. The marginal value Ω_t, as the asset pricing wedge between intermediaries and households, increases quickly as n_t declines. The constraint of the Lagrangian multiplier μ_t shown in Panel D is intuitive: when the net worth declines, the leverage constraint becomes more binding, and thus the Lagrangian multiplier becomes larger. We can see, in Panel E, that the equilibrium government credit intervention is significant, even the intensity parameter $\nu_{g,0}$ is 5, much less than the choice of Gertler and Karadi (2011). Again, it shows the importance of nonlinear global solutions for the quantitative discussion on models of unconventional monetary policies. On average, the government holds about 30% of the risky assets in the economy.

Figure 2: Financial Variables at Equilibrium.



NOTE: Panels A – E show the equilibrium variables and Panel F displays the stationary distribution at equilibrium. The red dashed curves characterize the equilibrium policy functions for the frictionless economy. The blue solid curves are equilibrium policy functions of the frictional economy with $\overline{\lambda} = 0.381$. The economy mainly fluctuates between 0.1 and 0.3 in terms of net worth share n_t at equilibrium.

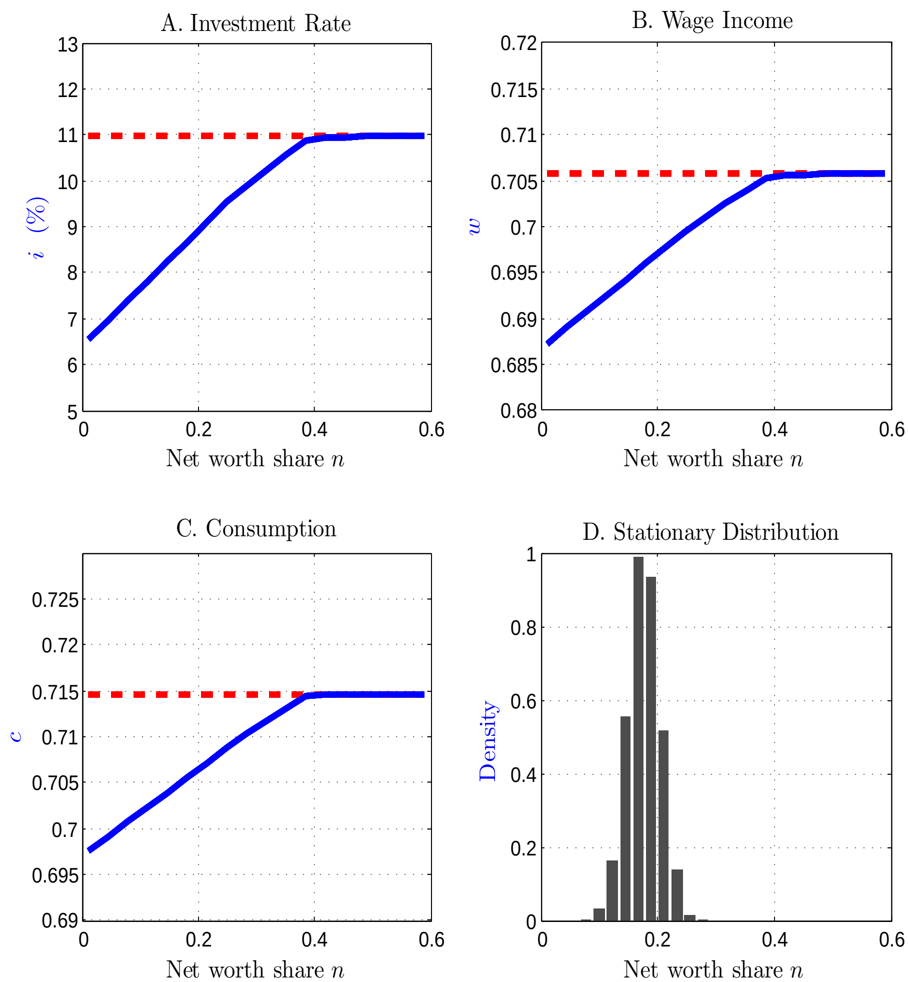
We now turn to the equilibrium macroeconomic quantities demonstrated in Figure 3. Panel A illustrates how the investment rate i_t varies when the intermediary net worth fluctuates. The investment rate decreases rapidly as n_t declines. The average investment rate at equilibrium is about 8.5%. Panels B and C shows wage income and consumption, respectively. Consistent with intuition, the dynamics of wage income and consumption are much more smooth relative to investment and other financial variables. More precisely, in this model, the wage income declines from about 0.7 to about 0.69, and the consumption declines from about 0.71 to about 0.7, as n_t declines from 0.3 to 0.2.

Financial Shocks We now describe the equilibrium of the economy with time-varying leverage constraints for intermediaries. Here, we assume that the log of margin parameter $\ln \lambda_t$ follows a first-order Markov chain with long-term mean $\overline{\lambda}$, autocorrelation ρ_λ, and long-term variance $\overline{\sigma}^2_\lambda$. Those parameters' values are chosen according to the baseline calibration in Table 1. We appeal to the discretization scheme proposed by Rouwenhorst (1995) to consider a two-state discretized process for λ_t. In particular, the margin parameter λ_t takes two values λ_H and λ_L with $\lambda_H > \lambda_L > 0$. The states of higher margin λ_H characterize the periods in which intermediaries are stressed by tightened leverage constraints. More precisely, according to the baseline calibration in Table 1 and the discretization method, we take λ_L to be 0.2917 and λ_H to be 0.4976. The annualized transition matrix of the two states is

$$\begin{array}{cc} & \begin{array}{cc} \lambda_L & \lambda_H \end{array} \\ \begin{array}{c} \lambda_L \\ \lambda_H \end{array} & \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix} \end{array}. \tag{50}$$

In Figure 4, we describe the equilibrium financial variables. The dashed blue curves are endogenous variables when $\lambda = \lambda_L$, whereas the solid blue curves characterize the equilibrium of the economy when $\lambda = \lambda_H$. The equilibrium of the economy relies upon the intermediary net worth. Moreover, the risk premium Ξ_t, the asset price q_t, the marginal value of net worth Ω_t, the constraint multiplier μ_t, and the government

Figure 3: Quantity Variables at Equilibrium.



NOTE: Panels A – C show the equilibrium variables and Panel F displays the stationary distribution of equilibrium. The red dashed curves characterize the equilibrium policy functions for the frictionless economy. The blue solid curves are equilibrium policy functions of the economy with financial friction and $\overline{\lambda} = 0.381$. The economy is mainly fluctuating between 0.1 and 0.3 in terms of net worth share n_t at equilibrium.

intervention scale $S_{g,t}$ are dramatically affected by financial shocks (i.e. shocks on λ_t). The effects on those financial variables become larger as the intermediary net worth declines. Intuitively speaking, when the financial system is already fragile (i.e., when intermediaries are not well capitalized), the adverse impact of financial shocks becomes especially substantial.

Panel D of Figure 4 shows the Lagrangian multiplier of the leverage constraints faced by intermediaries. The endogenous thresholds of binding leverage constraints are different in different states of λ_t. When $\lambda_t = \lambda_L$, the threshold of financial binding is about $\underline{n}_L = 0.14$, whereas when $\lambda_t = \lambda_H$, the threshold of financial bidding is $\underline{n}_H = 0.34$. It is intuitive that financial intermediaries become more fragile (i.e., leverage constraints become more binding) when the margin is higher at λ_H.

Panel F displays the stationary distribution of intermediary net worth. The economy fluctuates mainly between 0 and 0.3. Compared to Panel F of Figure 2, we can see that the financial shocks make the stationary distribution spread wider. The median of the distribution is about 0.13. From Panels D and F, we can see that the leverage constraint is almost always binding when $\lambda_t = \lambda_H$ in equilibrium, while interestingly enough, the leverage constraint occasionally becomes binding when $\lambda_t = \lambda_L$. From Panels D and C, we can see that the marginal value of intermediary net worth Ω_t is higher than the frictionless benchmark $\Omega^* = 1$, even when $\lambda_t = \lambda_L$ and the leverage constraint is not binding at all. This is a manifestation of the expectation effect of financial constraints. From Panels A – E, it can be seen that, no matter if λ_t is equal to λ_H or λ_L, the equilibrium of the frictional economy converges to that of the frictionless economy as the intermediary net worth approaches 1.

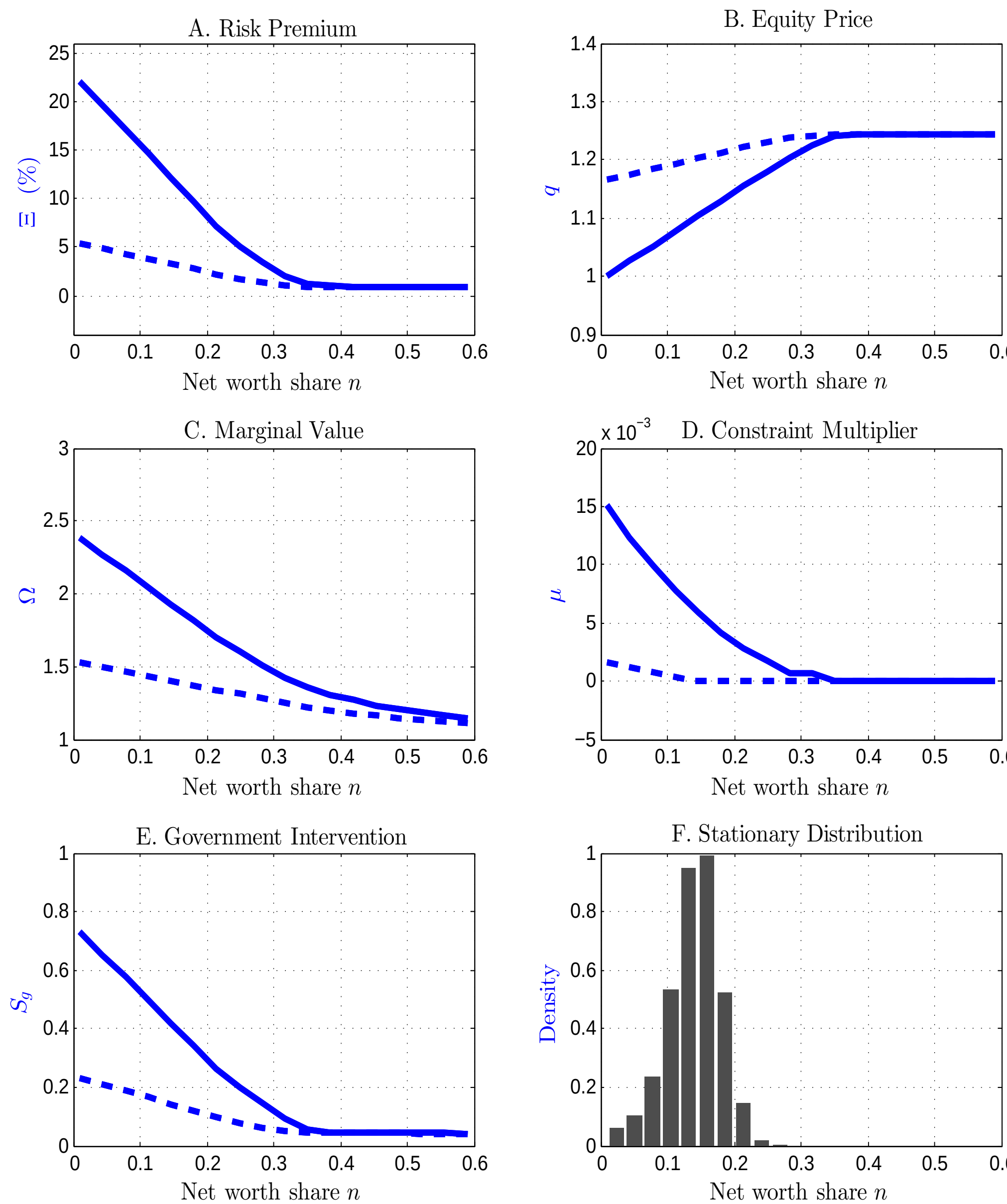
The equilibrium risk premium Ξ_t is shown in Panel A of Figure 4. The conditional average risk premium given $\lambda_t = \lambda_L$ is about 4% per year, and it skyrockets to about 13% per year as the economy is hit by an adverse financial shock (i.e. λ_t jumps to λ_H from λ_L). Conversely, Panel B shows that the equity price q_t suffers from a sizable drop as λ_t increases to λ_H. The marginal value Ω_t, shown in Panel C, also increases as λ_t jumps to λ_H. The Lagrangian multiplier constraint μ_t shown in Panel D indicates that the financial system becomes much more constrained when the margin λ_t rises. Finally, in Panel E, we can see that the equilibrium government credit intervention is very sensitive to the adverse financial shock. On average, the government holds about 17% of the risky assets in the economy when $\lambda_t = \lambda_L$, while it holds about 49% when $\lambda_t = \lambda_H$.

We now turn to the equilibrium macroeconomic quantity demonstrated in Figure 5. Panel A illustrates how investment rate i_t varies when financial margin λ_t and intermediary net worth n_t fluctuates. The investment rate drops rapidly as λ_t increases. The conditional average investment rate is about 11.6% in the state of $\lambda_t = \lambda_L$, and it becomes about 8.3% in the state of $\lambda_t = \lambda_H$. Panels B and C shows wage income and consumption, respectively. They are also negatively affected by the financial shock. However, their declines are much smaller than that of the investment rate.

Government Intervention and the Fragility of Intermediaries From the equilibrium cases in Figure 4 and Figure 5, showing the economy with time-varying leverage constraints for its financial intermediaries, we can see that (rare) adverse financial shocks can have dramatic and devastating effects on financial markets and the real economy. The negative effects become especially severe when intermediary net worth level n_t is already low. This is one of the key insights of this class of models: the fragility of financial intermediaries is largely captured by their level of capitalization. When the intermediaries are poorly capitalized (when net worth n_t is low in the model), the intermediaries are more fragile, and an exogenous financial shock can cause tragic financial turmoil and drag the economy into a recession. Therefore, government intervention should not only battle financial shocks, but also combat the fragility of the financial system. It is important when considering financial fragility that this model has global solutions, so it can be used as a laboratory for the analysis of simple nonlinear government intervention.

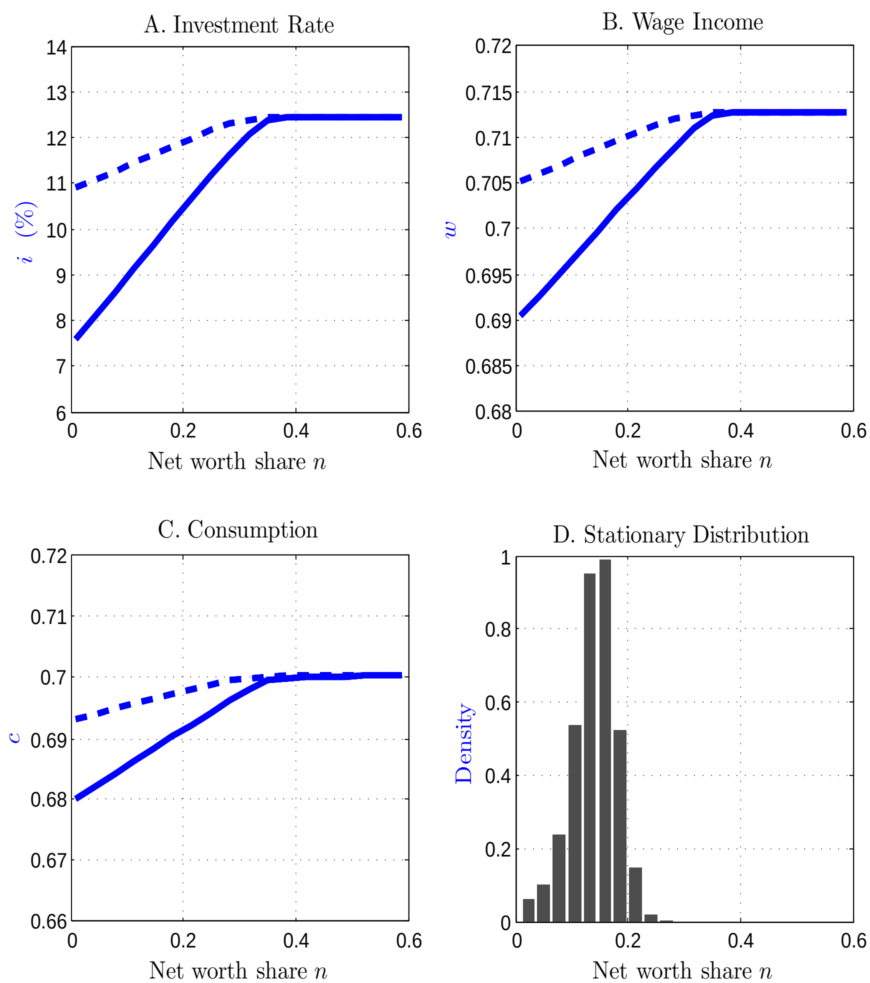
Here, we extend the credit policies considered in the previous analysis, where the intensity of intervention depends only on the observed risk premium, and the sensitivity ν_g is constant, no matter the current fragile state of the financial system. Here, we allow the sensitivity ν_g to be state-dependent, depending on the intermediary net worth level n_t as described in (46). Specifically, we choose $\nu_{g,0} = 5$ and $\nu_{g,1} = 2$ and keep the other parameter calibrations unchanged.

Figure 4: Financial Variables at Equilibrium of Economy with Financial Shocks.



NOTE: Panels A – E show the equilibrium variables and Panel F displays the stationary distribution at equilibrium. The dashed blue curves characterize the equilibrium policy functions of states with lower margin λ_L, and the solid blue curves are equilibrium policy functions of states with higher margin λ_H. The economy mainly fluctuates between 0 and 0.3 along the dimension of net worth share n_t at equilibrium.

Figure 5: Quantity Variables at Equilibrium of Economy with Financial Shocks.



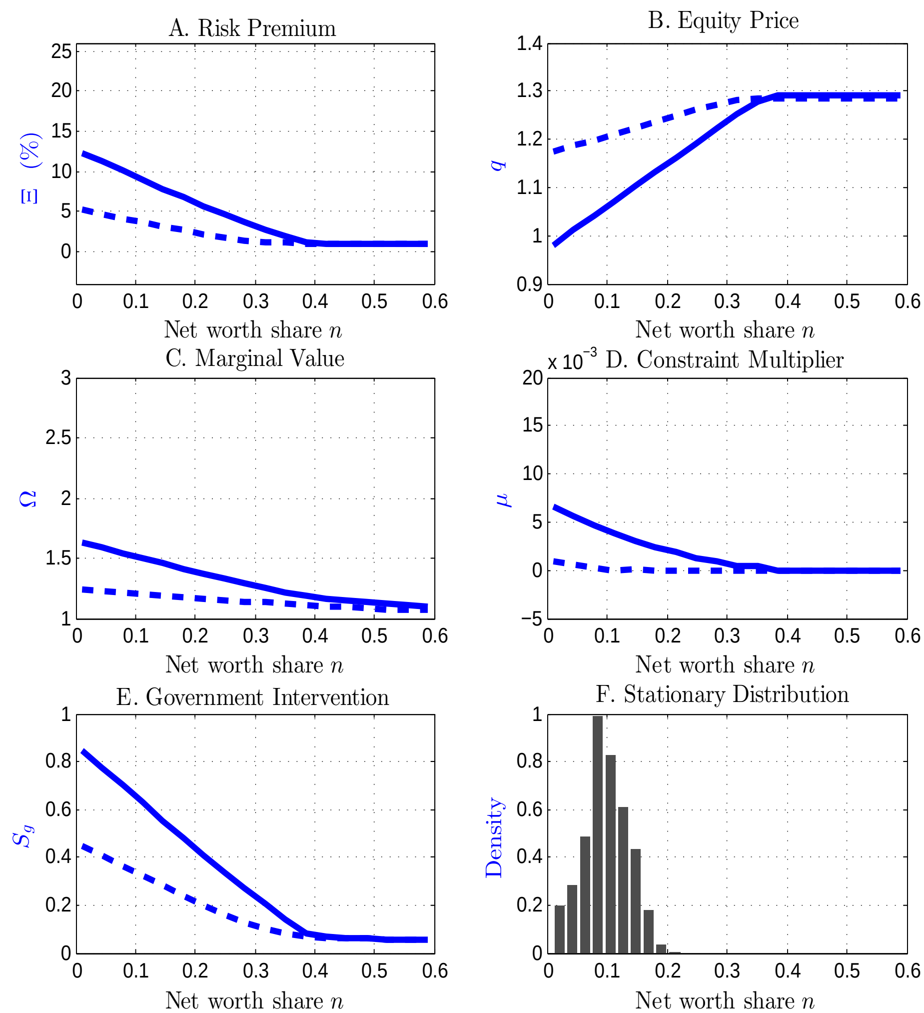
NOTE: Panels A – C show the equilibrium variables and Panel F displays the stationary distribution of equilibrium. The blue dashed curves characterize the equilibrium policy functions of states with lower margin λ_L, and the blue solid curves are equilibrium policy functions of states with higher margin λ_H. The economy is mainly fluctuating between 0 and 0.3 along the dimension of net worth share n_t in equilibrium.

In Figure 6, we describe the equilibrium financial variables. The dashed blue curves are endogenous variables when $\lambda = \lambda_L$, whereas the solid blue curves characterize the equilibrium of the economy when $\lambda = \lambda_H$.

Panel D of Figure 6 shows the Lagrangian multiplier of the leverage constraint faced by financial intermediaries. It is obvious that the leverage constraint binds much less compared to the case $\nu_{g,1} = 0$ in Figure 4. Interestingly, with nonlinear government intervention, financial intermediaries are almost unbound in the state $\lambda = \lambda_L$. This is because the intervention can also be intensive in the normal state $\lambda = \lambda_L$ when intermediaries are fragile (n_t is low), and it decreases the demand of risk sharing from intermediaries when n_t is low. As a result of the relaxed leverage constraints, we can see in Panel C of Figure 6 that the marginal value of the intermediary net worth becomes much smaller even when n_t is low, compared to Figure 4. The equilibrium government intervention can be seen clearly by comparing Panel E of Figure 6 to that of Figure 4: while the intensity of intervention in the state $\lambda = \lambda_H$ is similar, the government intervention in the state $\lambda = \lambda_L$ becomes more intense under the nonlinear credit policy. The nonlinear policy (the case of $\nu_{g,1} = 2$) makes the risk premium more stable relative to the case of linear policy (the case of $\nu_{g,0} = 0$). This can be seen by comparing Panel A of Figure 6 to that of Figure 4.

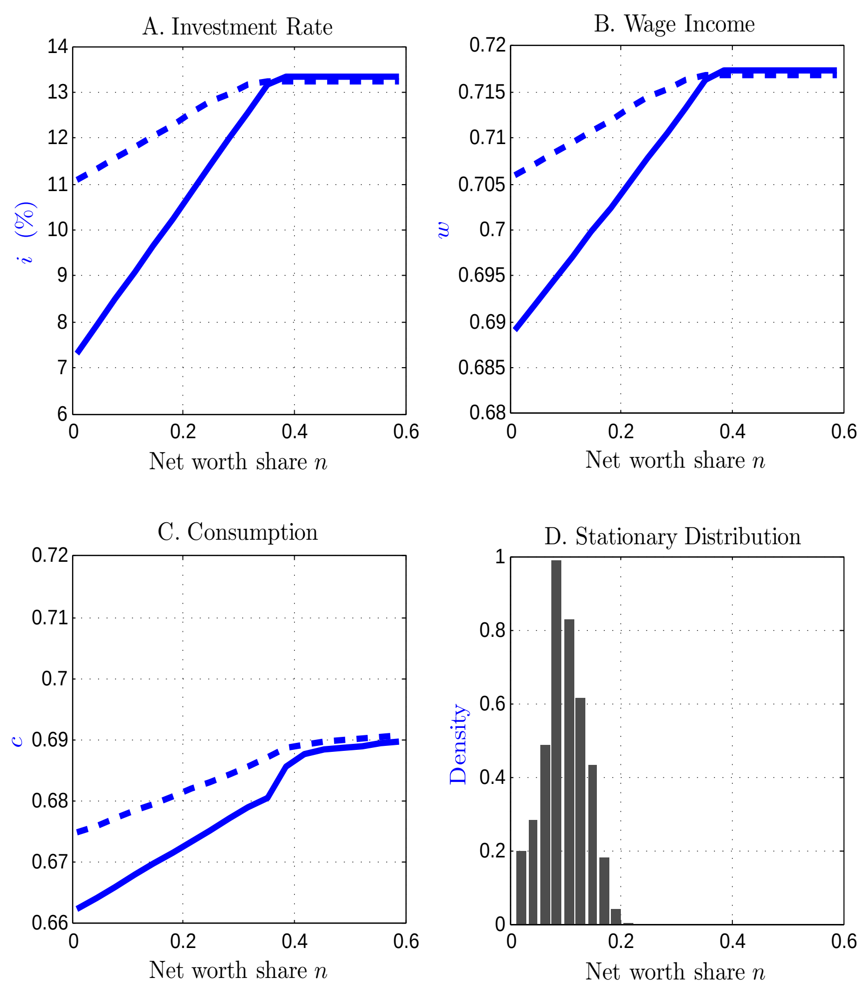
We now turn to the equilibrium macroeconomic quantities of Figure 7. Comparing Panels A and B of Figure 7 to those of Figure 5, it can be seen that the levels of investment and wages are higher under the nonlinear policy. This is due to the greater stability guaranteed by the nonlinear policy. However, Panel C of Figure 7 shows that the consumption level is lower than that of Figure 5. This is the result of the higher efficiency costs incurred by the more intensive government intervention suggested by the nonlinear policy. As can be seen clearly here, there is a crucial tradeoff between stability and efficiency in an optimal (nonlinear) government intervention – that is to say, through unconventional credit policies. To find more reliable quantitative answers to this important question will be a challenge. It will require a richer framework, and global solutions, to chart out a promising research agenda that will have a huge influence both in academia and among practicing monetary authorities.

Figure 6: Financial Variables at Equilibrium of Economy with Financial Shocks and Nonlinear Government Intervention.



NOTE: Panels A – E show the equilibrium variables and Panel F displays the stationary distribution at equilibrium. The dashed blue curves characterize the equilibrium policy functions of states with lower margin λ_L, and the solid blue curves are equilibrium policy functions of states with higher margin λ_H. The economy mainly fluctuates between 0 and 0.2 along the dimension of net worth share n_t at equilibrium. The y-axis scale of the plots is kept the same as the corresponding ones in Figure 4 for comparison purposes.

Figure 7: Quantity Variables at Equilibrium of Economy with Financial Shocks and Nonlinear Government Intervention.



NOTE: Panels A – C show the equilibrium variables and Panel F displays the stationary distribution at equilibrium. The dashed blue curves characterize the equilibrium policy functions of states with lower margin λ_L, and the solid blue curves are equilibrium policy functions of states with higher margin λ_H. The economy mainly fluctuates between 0 and 0.2 along the dimension of net worth share n_t at equilibrium. The y-axis scale of the plots is kept the same as the corresponding ones in Figure 4 for comparison purposes.

Appendix

A Frictionless Benchmark

A frictionless economy is used as a benchmark (1) for government policy in the larger model, (2) to calibrate the parameters, (3) as a starting point of the time-iteration algorithm, (4) as a check of the solution method for the full model, and (5) for economic analysis.

The equilibrium is the balanced growth path with stochastic trend $A_{c,t}K_t^{\alpha}$. Because $\tau > 0$, the government should not conduct credit policy in the frictionless economy, i.e. $\nu_g \equiv 0$.

The prices are

$$Q_t = qA_{c,t}K_t^{\alpha-1} \quad \text{and} \quad P_t = pA_{c,t}K_t^{\alpha-1}. \tag{51}$$

The optimal investment rate satisfies

$$q/p = 1 + \vartheta i. \tag{52}$$

The optimal wage is

$$W_t = (1-\alpha)A_{c,t}K_t^{\alpha}L_{c,t}^{-\alpha}. \tag{53}$$

Taking out the trend in W_t, the optimal normalized wage is

$$w = (1-\alpha)\ell_c^{-\alpha}, \tag{54}$$

where $\ell_c \equiv L_{c,t}$. Denote $\ell_\iota \equiv L_{\iota,t}$. The labor market clearing condition implies

$$\ell_c + \ell_\iota = 1. \tag{55}$$

The dividend is

$$D_t = \alpha A_{c,t}K_t^{\alpha}\ell_c^{1-\alpha} - P_t\left(iK_t + \frac{\vartheta}{2}i^2K_t\right). \tag{56}$$

We characterize the equilibrium dividend and consumption as respectively

$$D_t = dA_{c,t}K_t^{\alpha} \quad \text{and} \quad C_t = cA_{c,t}K_t^{\alpha}. \tag{57}$$

The market clearing condition for consumption goods implies

$$y = d + w\ell_c + w\ell_\iota. \tag{58}$$

And the relationship (56) can be rewritten as

$$d = \alpha\ell_c^{1-\alpha} - p\left(i + \frac{\vartheta}{2}i^2\right).$$

The equilibrium resource constraint implies that the consumption goods output y and the household consumption c are

$$y = \ell_c^{1-\alpha} \quad \text{and} \quad c = (1-\overline{g})\ell_c^{1-\alpha}. \tag{59}$$

The zero-profit condition in the investment goods sector is

$$W_t L_{\iota,t} = P_t K_t L_{\iota,t} \tag{60}$$

which implies that $w = p$, and thus (54) can be rewritten as

$$p = (1-\alpha)\ell_c^{-\alpha}. \tag{61}$$

The market clearing condition for the investment goods is

$$i + \frac{\vartheta}{2}i^2 = \ell_\iota. \tag{62}$$

The IMRS of household is

$$\mathcal{M}_{t,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} = \beta\left(\frac{A_{t+1}K_{t+1}^{\alpha}}{A_{c,t}K_t^{\alpha}}\right)^{-\gamma} = \beta e^{-\gamma\sigma_a\epsilon_{a,t+1}}\left(i+1-\delta\right)^{-\gamma\alpha} \tag{63}$$

The equilibrium interest rate satisfies

$$1 = \mathbb{E}_t \left[\mathcal{M}_{t,t+1} \right] (1 + R) \tag{64}$$

It implies that

$$\ln (1 + R) = -\ln \beta + \gamma\alpha \ln(i + 1 - \delta) - M(\gamma, \sigma_a), \tag{65}$$

where $M(\gamma, \sigma_a) \equiv \ln \left(\frac{1}{2} e^{-\gamma\sigma_a} + \frac{1}{2} e^{\gamma\sigma_a} \right)$. [We need to add a note, this is not Gaussian.]. As in the standard Lucas-tree economy, there are three components to the equilibrium interest rate. The first is the time value captured by the subjective discount rate $-\ln \beta$. The second is the sensitivity to the growth of consumption $\gamma \times \alpha \ln(i + 1 - \delta)$. The third component is the precautionary saving motive term $M(\gamma, \sigma_a)$. The interest rate depends heavily on the growth rate $i + 1 - \delta$, where the investment rate i is mainly governed by the adjustment cost coefficient ϑ.

The equity return satisfies the following Euler equation:

$$1 = \mathbb{E}_t [\mathcal{M}_{t,t+1}](1 + R_{k,t+1}) \tag{66}$$

The equilibrium stock return is

$$1 + R_{k,t+1} = \frac{D_{t+1} + Q_{t+1} K_{t+2}}{Q_t K_{t+1}} = e^{\sigma_a \epsilon_{a,t+1}} (i + 1 - \delta)^{\alpha - 1} \left(\frac{d}{q} + i + 1 - \delta \right), \tag{67}$$

with log return

$$\ln (1 + R_{k,t+1}) = (\alpha - 1) \ln(i + 1 - \delta) + \ln \left(\frac{d}{q} + i + 1 - \delta \right) + \sigma_a \epsilon_{a,t+1}. \tag{68}$$

Therefore, the conditional expected log return is

$$\mathbb{E}_t \left[\ln (1 + R_{k,t+1}) \right] = (\alpha - 1) \ln(i + 1 - \delta) + \ln \left(\frac{d}{q} + i + 1 - \delta \right). \tag{69}$$

The Euler equation for equity return can be rewritten as:

$$0 = \ln \beta + (\alpha - 1 - \gamma\alpha) \ln (i + 1 - \delta) + \ln \left(\frac{d}{q} + i + 1 - \delta \right) + M_k(\gamma, \sigma_a), \qquad (70)$$

where $M_k(\gamma, \sigma_a) \equiv \ln \left(\frac{1}{2} e^{(\gamma-1)\sigma_a} + \frac{1}{2} e^{-(\gamma-1)\sigma_a} \right)$. Combining (65), (68), and (70), the equilibrium risk premium can be derived without solving for the other equilibrium variables. The equilibrium risk premium is

$$\Xi^* \equiv \mathbb{E} \left[\ln (1 + R_k^*) \right] - \ln (1 + R^*) = M(\gamma, \sigma_a) - M_k(\gamma, \sigma_a) \approx \gamma \sigma_a^2 - \frac{1}{2} \sigma_a^2. \qquad (71)$$

This coincides with the equilibrium risk premium in the Lucas-tree economy with Jensen's term $\frac{1}{2}\sigma_a^2$. Similarly, the risk premium is independent of the growth rate of the economy.

Other equilibrium variables c^*, i^*, ℓ_c^*, ℓ_ι^*, p^*, q^*, and d^* can be solved from the system of equations including (52), (55), (58), (59), (61), (62), and (70).

A.1 Analytical Solution

Equation (70) can be solved analytically from a nonlinear equation. We conjecture i^* is a constant, and plug in d^* and q^*, we have derived a nonlinear equation for i^*:

$$0 = \ln \beta + (\alpha - 1 - \gamma\alpha) \ln(i^* + 1 - \delta) + \ln\left(\frac{\alpha(1 - i^* - \frac{\vartheta}{2}(i^*)^2) - (1 - \alpha)}{(1 - \alpha)(1 + \vartheta i^*)} + i^* + 1 - \delta \right) + M_k(\gamma, \sigma_a)$$

We solve this nonlinear equation with parameters shown in the main text. We compare the analytical result with what we obtain by solving the model with log-linearization.

A.2 Solution with Log-Linear Approximation

We can also solve the equilibrium of this economy by log-linearization approximation. The system of equations include static equations (52), (55), (56), (58), (59), (61), (62), and the two Euler equations:

$$E_t \underbrace{\beta e^{-\gamma g_{a,t+1}} (i_t + 1 - \delta)^{-\gamma\alpha}}_{\mathcal{M}_{t,t+1}} (1 + R_{f,t}) = 1 \qquad (72)$$

$$E_t \underbrace{\beta e^{-\gamma g_{a,t+1}}(i_t+1-\delta)^{-\gamma\alpha}}_{\mathcal{M}_{t,t+1}} \underbrace{e^{g_{a,t+1}}(i_t+1-\delta)^{\alpha-1}(\frac{d_{t+1}}{q_{t+1}}+i_{t+1}+1-\delta)}_{1+R_{k,t+1}} = 1 \tag{73}$$

where $g_{a,t+1} \equiv \sigma_a \epsilon_{t+1}$.

To solve the model with log-linearization approximation, we need to solve for the deterministic steady state first. The deterministic steady state for g_a is $g_{a,ss} = 0$. Plug it into equation (73), we have:

$$\beta(i+1-\delta)^{-\gamma\alpha}(i+1-\delta)^{\alpha-1}(\frac{d}{q}+i+1-\delta) = 1$$

Plug in d and q as a function of i, we can solve for the steady state value for investmet rate, i_{ss}. Hence, we can solve for $c_{ss}, \ell_{c,ss}, \ell_{\iota,ss}, p_{ss}, q_{ss}, d_{ss}$ in the same way as described above.

The equilibrium investment rate obtained from first order log-linear approximated solution is a constant, being smaller than the analytical solution. The difference is due to the omission of higher order term $\frac{1}{2}(\gamma-1)^2\sigma_a^2$ when expanding the Euler equation (73) around the steady state. This error can be used as a benchmark for solution comparison to assess the inaccuracy caused by omission of higher order term.

Furthermore, if we use second order approximation to solve the same system of equations, we will get exactly the same result as the analytical solution. In this economy, only a constant second order term $\frac{1}{2}(\gamma-1)^2\sigma_a^2$ shows up in the exact solution, so second order approximation suffices to characterize the behavior of the system.

A.3 Social Planner's Problem

We look at the social planner's problem in this economy. Suppose the economy consists of a continuum of households and firms (both consumption good firms and investment good firms), and the social planner can choose directly how much households consume, and how many consumption goods each consumption good producer produces, and how many investment goods the investment good producer produces. The social planner solves the following problem:

$$\max_{C_t, K_t^i, I_t^i, L_{c,t}^i} E \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} \tag{74}$$

subject to:

$$C_t \le \int_i [A_{c,t}(K_t^i)^\alpha (L_{c,t}^i)^{1-\alpha}]di \tag{75}$$

$$K_{t+1}^i = (1-\delta)K_t^i + I_t^i \tag{76}$$

$$\int_i I_t^i di + \int_i [\frac{\vartheta}{2}(\frac{I_t^i}{K_t^i})^2 K_t^i]di \le Z_\iota(\int_i K_t^i di)(1 - \int_i L_{c,t}^i di) \tag{77}$$

Assign $\beta^t\zeta_{1,t}, \beta^t\zeta_{2,t}^i, \beta^t\zeta_{3,t}$ the three Larangerian multipliers associated with the three constraints. Note that only $\beta^t\zeta_{2,t}^i$ is at firm level. The first order conditions are:

$$C_t^{-\gamma} = \zeta_{1,t} \tag{78}$$

$$\zeta_{2,t}^i = \beta E_t \alpha \frac{Y_{t+1}^i}{K_{t+1}^i}\zeta_{1,t+1} + \beta(1-\delta)E_t\zeta_{2,t+1}^i + \beta E_t \zeta_{3,t+1}[Z_\iota(1 - \int_i L_{c,t+1}^i di) + \frac{\vartheta}{2}(\frac{I_{t+1}^i}{K_{t+1}^i})^2] \tag{79}$$

$$\zeta_{2,t}^i = \zeta_{3,t}(1 + \vartheta\frac{I_t^i}{K_t^i}) \tag{80}$$

$$(1-\alpha)\zeta_{1,t}\frac{Y_{i,t}}{L_{c,t}^i} = \zeta_{3,t}Z_\iota K_t \tag{81}$$

All firms are identical, we can get rid of the superscript i. Further, we have:

$$Z_\iota L_{\iota,t+1} + \frac{\vartheta}{2}(\frac{I_{t+1}}{K_{t+1}})^2 = \frac{I_{t+1} + \vartheta(\frac{I_{t+1}}{K_{t+1}})^2 K_{t+1}}{K_{t+1}} = i_{t+1}(1 + \vartheta i_{t+1}) \tag{82}$$

The first order condition can be rewritten as:

$$E_t\beta(\frac{C_{t+1}}{C_t})^{-\gamma}\frac{\frac{\alpha Y_{t+1}}{K_{t+1}} + (1-\delta+i_{t+1})(1-\alpha)\frac{Y_{t+1}}{Z_\iota K_{t+1}L_{c,t+1}}(1+\vartheta\frac{I_{t+1}}{K_{t+1}})}{\frac{(1-\alpha)Y_t}{Z_\iota K_t L_{c,t}}(1+\vartheta\frac{I_t}{K_t})} = 1 \tag{83}$$

In the decentralized economy:

$$Q_t = \frac{(1-\alpha)Y_t}{Z_\iota K_t L_{c,t}}(1 + \vartheta i_t) \tag{84}$$

So we rewrite (83) as:

$$E_t\beta(\frac{C_{t+1}}{C_t})^{-\gamma}\frac{\alpha\frac{Y_{t+1}}{K_{t+1}}+(1-\delta)Q_{t+1}+i_{t+1}Q_{t+1}}{Q_t}=1 \tag{85}$$

Recall that in the decentralized economy, we have the Euler equation:

$$E_t\beta(\frac{C_{t+1}}{C_t})^{-\gamma}(1+R_{k,t+1})=1 \tag{86}$$

where:

$$1+R_{k,t+1}=\frac{D_{t+1}+Q_{t+1}K_{t+2}}{Q_tK_{t+1}}=\frac{\alpha Y_{t+1}}{Q_tK_{t+1}}-\frac{P_{t+1}(i_{t+1}+\frac{\vartheta i_{t+1}^2}{2})}{Q_t}+\frac{Q_{t+1}(1-\delta+i_{t+1})}{Q_t} \tag{87}$$

Compare (85) and (87), the differences reflect the externality created by K_t affecting the TFP of the investment good sector.

B Frictional Economy

B.1 Equilibrium Conditions

There is a leverage constraint for each intermediary. The Lagrangian multiplier is $\mu(\lambda,n)$ and is associated with the financial constraint. It is dual to the slackness in the balance sheet, and thus we define $\mu(\lambda,n)\equiv\max\{0,-\mu_b(\lambda,n)\}^2$. The slackness of the balance sheets of the intermediaries is

$$\Omega(\lambda,n)n-\lambda S_p(\lambda,n)=\max\{0,\mu_b(\lambda,n)\}^2 \tag{88}$$

These two conditions imply that

$$\mu(\lambda,n)\left[\Omega(\lambda,n)n-\lambda S_p(\lambda,n)\right]=0. \tag{89}$$

The resource constraint of the economy follows from (47) and (48) that

$$(1-\bar{g})y(\lambda,n)=c(\lambda,n)+\tau S_g(\lambda,n)q(\lambda,n). \tag{90}$$

Importantly, the expectation correspondence characterizes how the next period's aggregate normalized net worth n' as an endogenous state variable depends on the current states (λ, n) and exogenous state variables and shocks in the next period. The expectation correspondence is also an equilibrium to be solved:

$$n' = \Gamma(\lambda, n; \lambda', \epsilon_a'). \tag{91}$$

Following (39), the expectation correspondence can be expressed as:

$$\begin{aligned} n' &= \Gamma(\lambda, n; \lambda', \epsilon_a') \\ &= \frac{\theta \left\{ G_r(\lambda, n; \lambda', \epsilon_a') - [1 + R_f(\lambda, n)] \right\} S_p(\lambda, n) + \theta \left[1 + R_f(\lambda, n)\right] n}{G_k(\lambda, n; \lambda', \epsilon_a')} + \aleph. \end{aligned}$$

Here $G_r(\lambda, n; \lambda', \epsilon_a') \equiv 1 + R_k(\lambda, n; \lambda', \epsilon_a')$ is the total stock return, and $G_k(\lambda, n; \lambda', \epsilon_a') \equiv Q'K'/(QK)$ is the total return of capital gain on stocks whose expression can be found in (93). The stock return described in (11) can be rewritten as

$$1 + R_k(\lambda, n; \lambda', \epsilon_a') = \frac{d(\lambda', n')}{q(\lambda, n)} e^{\sigma_a \epsilon_a'} \left[i(\lambda, n) + 1 - \delta\right]^{\alpha - 1} + G_k(\lambda, n; \lambda', \epsilon_a') \tag{92}$$

where the capital gain return is

$$G_k(\lambda, n; \lambda', \epsilon_a') = \frac{q(\lambda', n')}{q(\lambda, n)} e^{\sigma_a \epsilon_a'} \left[i(\lambda, n) + 1 - \delta\right]^{\alpha - 1} \left[i(\lambda', n') + 1 - \delta\right] \tag{93}$$

According to (32), the Euler equation for stock returns is

$$\begin{aligned} &\Omega(\lambda, n) c(\lambda, n)^{-\gamma} - \mu(\lambda, n) c(\lambda, n)^{-\gamma} \left[\Omega(\lambda, n) - \lambda\right] \\ &\quad = \beta \left[i(\lambda, n) + 1 - \delta\right]^{-\gamma\alpha} \mathbb{E}\left\{ c(\lambda', n')^{-\gamma} e^{-\gamma \sigma_a \epsilon_a'} \left[1 - \theta + \theta \Omega(\lambda', n')\right] G_r(\lambda, n; \lambda', \epsilon_a') \middle| \lambda, n \right\} \end{aligned}$$

Define

$$\widetilde{G}_r(\lambda, n; \lambda', \epsilon_a') = G_r(\lambda, n; \lambda', \epsilon_a') / q(\lambda, n). \tag{94}$$

To obtain the Euler equation above, the key intermediate step is

$$\frac{\Lambda'}{\Lambda} = \beta \left[\frac{c(\lambda', n')}{c(\lambda, n)} \right]^{-\gamma} e^{-\gamma \sigma_a \epsilon_a'} \left[i(\lambda, n) + 1 - \delta \right]^{-\gamma\alpha}. \tag{95}$$

According to (31), the intermediary Euler equation for the risk-free rate is

$$\begin{aligned} &[1 - \mu(\lambda, n)] \, \Omega(\lambda, n) c(\lambda, n)^{-\gamma} \\ &\quad = \beta \left[i(\lambda, n) + 1 - \delta \right]^{-\gamma\alpha} \left[1 + R_f(\lambda, n) \right] \mathbb{E} \left\{ c(\lambda', n')^{-\gamma} e^{-\gamma \sigma_a \epsilon_a'} \left[1 - \theta + \theta \Omega(\lambda', n') \right] \middle| \lambda, n \right\} \end{aligned}$$

The household Euler equation for the risk-free rate is

$$1 = \beta \left[i(\lambda, n) + 1 - \delta \right]^{-\gamma\alpha} \left[1 + R_f(\lambda, n) \right] \mathbb{E}_t \left\{ e^{-\gamma \sigma_a \epsilon_a'} \middle| \lambda, n \right\} \tag{96}$$

The investment goods production is

$$u(\lambda, n) = \ell_\iota(\lambda, n). \tag{97}$$

The investment goods sector market clearing condition is

$$u(\lambda, n) = i(\lambda, n) + \frac{\vartheta}{2} i(\lambda, n)^2. \tag{98}$$

And the labor market clearing condition is

$$\ell_c(\lambda, n) + \ell_\iota(\lambda, n) = 1. \tag{99}$$

The zero-profit condition for investment good firms is

$$w(\lambda, n) = p(\lambda, n). \tag{100}$$

The optimal demand of labor in consumption goods sector is

$$w(\lambda, n) = (1 - \alpha) \ell_c(\lambda, n)^{-\alpha}. \tag{101}$$

The total dividend paid out from the consumption goods sector is

$$d(\lambda, n) = \alpha \ell_c(\lambda, n)^{1-\alpha} - p(\lambda, n) u(\lambda, n), \tag{102}$$

where $\alpha \ell_c(\lambda, n)^{1-\alpha}$ is the total consumption goods minus the labor cost and $p(\lambda, n)u(\lambda, n)$ is the expenditure of purchasing investment goods. The optimal investment decision is characterized by the traditional q theory relationship:

$$q(\lambda, n) = [1 + \vartheta i(\lambda, n)]\, p(\lambda, n). \tag{103}$$

The consumption goods production is

$$y(\lambda, n) = \ell_c(\lambda, n)^{1-\alpha}. \tag{104}$$

The log risk premium $\Xi(\lambda, n)$ is defined as

$$\Xi(\lambda, n) + \log\left[1 + R_f(\lambda, n)\right] = \mathbb{E}_t \left\{ G_r(\lambda, n; \lambda', \epsilon_a') | \lambda, n \right\}. \tag{105}$$

The credit policy can be written as

$$S_p(\lambda, n) \left[1 + \nu_g(\Xi(\lambda, n) - \Xi^*)\right] = 1, \tag{106}$$

where $\nu_g = \nu_{g,0} + \nu_{g,1}\left(\frac{1}{n_t} - 1\right)$ is state-dependent.

C Solution with Pertubation Method

This section provides the details of how to solve the model in the main text with log-linear approximation using Dynare 4.4.3. First, we list explicitly the system of equations we need to solve the model in dynare. Second, details of computing the deterministic steady state are provided. Third, we show how to convert the VAR(1)-from policy function reported by Dynare to functions of state variables $n, \log\lambda$ for comparison with global solution.

C.1 System of Equations

We assume that the constraint always binds, so we have:

$$\Omega_t n_t = \lambda_t S_{p,t} \tag{107}$$

The households' Euler equation for risk free rate is the same as (72). The two Euler equations for the intermediary are (31) and (32):

$$E_t \mathcal{M}^{\mathcal{I}}_{t,t+1}(1 + R_{f,t}) = 1 - \mu_t \tag{108}$$

$$E_t \mathcal{M}^{\mathcal{I}}_{t,t+1}(1 + R_{k,t+1}) = 1 - \mu_t + \frac{\lambda_t \mu_t}{\Omega_t} \tag{109}$$

where:

$$\mathcal{M}^{\mathcal{I}}_{t,t+1} = \beta(\frac{c_{t+1}}{c_t})^{-\gamma} e^{-\gamma g_{a,t+1}} (i_t + 1 - \delta)^{-\gamma\alpha} \frac{1 - \theta + \theta\Omega_{t+1}}{\Omega_t} \tag{110}$$

$g_{a,t+1} \equiv \sigma_a \epsilon_{a,t+1}$ as in the frictionless economy. The derivation of these two Euler equations (31) and (32) are in the main text. $1 + R_{k,t+1}$ is defined in (92) ane (93).

$$1+R_{k,t+1} = \frac{d_{t+1}}{q_t} \exp(g_{a,t+1})(i_t+1-\delta)^{\alpha-1} + \underbrace{\frac{q_{t+1}}{q_t} \exp(g_{a,t+1})(i_t + 1 - \delta)^{\alpha-1}(i_{t+1} + 1 - \delta)}_{G_{k,t+1}} \tag{111}$$

Aggregate net worth evolves as in equation (39):

$$n_{t+1} = \theta[(R_{k,t+1} - R_{f,t})S_{p,t} + n_t(1 + R_{f,t})]/G_{k,t+1} + \aleph \tag{112}$$

Risk premium is defined in (105) and government policy is characterized by (106).

The equilibrium of this economy is characterized by static equations (90), (97), (98), (99), (100), (101), (102), 103), (104), (106), two intermediary Euler equations (108),(109) and intermediary constraint (107), and net worth evolution (112).

C.2 Deterministic Steady State

The deterministic steady state of this economy is characterized by a set of nonlinear equations. We solve the deterministic steady state numerically using nonlinear solver as follows:

- Make a guess on i_{ss}, and solve for $c_{ss}, \ell_{c,ss}, \ell_{\iota,ss}, p_{ss}, q_{ss}, d_{ss}$ as in the frictionless economy.
- Solve for the deterministic steady state of $1 + R_{ss}$ and $1 + R_{k,ss}$:

$$R_{ss} = \frac{1}{\beta}(i_{ss} + 1 - \delta)^{\gamma\alpha} \tag{113}$$

$$R_{k,ss} = \frac{d_{ss}}{q_{ss}}(i_{ss} + 1 - \delta)^{\alpha-1} + (i_{ss} + 1 - \delta)^{\alpha} \tag{114}$$

- Solve for the steady state government policy from:

$$S_{p,ss}\left[1 + \nu_g(\ln(1 + R_{k,ss}) - \ln(1 + R_{ss}) - \Xi^*)\right] = 1$$

- The deterministic steady state of Ω_{ss}, μ_{ss} can be solved by (108) and (109) using a nonlinear solver.
- The steady state of net worth share is $n_{ss} = \frac{\lambda_{ss}}{\Omega_{ss}}$, according to (107).
- Check whether equation (112) holds. If not, iterate on i_{ss} until it holds.

C.3 Policy Function with First Order Approximation

Dynare delivers output in the form:

$$X_t = A_0 + A_1 X_{t-1} + A_2 \epsilon_t \tag{115}$$

where X_t includes all variables in the economy, and ε_t includes all the primitive shocks in the economy.

We need to convert the policy function in VAR(1) form into a function of n and λ. Suppose we have a variable $s(n, \lambda)$ (it can be any variable in the economy which only depends on the two state variables n, λ), we make a (log)-linear approximation [5] as:

$$s_t = C_{0,s} + C_{1,s} \log n_t + C_{2,s} \log \lambda_t \tag{116}$$

We extract the related rows in the VAR(1) form:

$$s_t = A_{0,s} + A_{1,s} X_{t-1} + A_{2,s} \epsilon_t \tag{117}$$

And the row for n and λ:

$$\log n_t = A_{0,n} + A_{1,n} X_{t-1} + A_{2,n} \epsilon_t \tag{118}$$

$$\log \lambda_t = A_{0,\lambda} + A_{1,\lambda} X_{t-1} + A_{2,\lambda} \epsilon_t \tag{119}$$

Plug (118) and (119) into (116), we have:

$$s_t = (C_{0,s} + C_{1,s} A_{0,n} + C_{2,s} A_{0,\lambda}) + (C_{1,s} A_{1,n} + C_{2,s} A_{1,\lambda}) X_{t-1} + (C_{1,s} A_{2,n} + C_{2,s} A_{2,\lambda}) \epsilon_t \tag{120}$$

We solve for $C_{1,s}$ and $C_{2,s}$ from:

$$C_{1,s} A_{1,n} + C_{2,s} A_{1,\lambda} = A_{1,s}, C_{1,s} A_{2,n} + C_{2,s} A_{2,\lambda} = A_{2,s} \tag{121}$$

The mean $C_{o,s}$ can be pinned down by:

$$C_{0,s} = \bar{s} - C_{1,s} \log \bar{n} - C_{2,s} \log \bar{\lambda} \tag{122}$$

where $\bar{s}, \log \bar{n}, \log \bar{\lambda}$ are the mean of respective variables.

D Literature Review

There are mainly two classes of macro finance models, from methodological perspective. The first class of models incorporate financial frictions into a macroeconomic model and study the economy with the constraint being always binding. This class of models includes Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Bernanke et al. (1999), Gertler and Karadi (2011), Jermann and Quadrini (2012), only to name a few.

[5] Variables of S_g, μ, and Ξ are in level, not in logs.

The second class of models feature nonlinear dynamics during the crisis (or sudden stops) in the economy: in normal times, the financial frictions do not matter while in crisis times, the tightening of the constraint will be greatly amplified. This class of models include Mendoza (2010), Bianchi (2011), Bianchi and Mendoza (2015), He and Krishnamurthy (2013), and Brunnermeier and Sannikov (2014).

The two classes of models differ in many aspects. For example, the first class of models focus on how financial sector accounts for business cycle fluctuations, while the second class of models focus on the nonlinear dynamics when the constraint binds. In terms of solution method, the first class of models can usually assume the constraint always binds and be solved using pertubation method. On the other hand, the second class of models must be solved using global method. Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2013) formulate the problem in continuous time, while other papers remain in the discrete setting and solve the model with value function iteration or policy function iteration.

There is a vast literature on numerically solving macroeconomic models locally and globally. Fernández-Villaverde et al. (2016) provides the most recent survey. Textbook treatment of numerical methods include Judd (1998), Miranda and Fackler (2004), Heer and Maussner (2009), Novales et al. (2008).

There are two broadly defined solution methods that are used to solve dynamic economic models: local solution (pertubation) and global solution. Perturbation method makes a Taylor expansion of policy functions around some deterministic steady state and transforms the nonlinear optimality conditions into linear systems. Judd and Guu (1993) showed how to apply perturbation methods in economic problems. Jin and Judd (2002) introduces higher-order pertubation in economic problems. Den Haan and De Wind (2012) improves the stability of high-order approximation. Judd (2002) compares two alternative ways of pertubation: using Taylor series in variable x and in variable $\log x$, and Fernández-Villaverde and Rubio-Ramírez (2006) explores how change of variables can reduce approximation error. Andreasen et al. (2013) improves the stability of economic system with higher-order approximations with a prunned state space system. In a word, this strand of literature explores how to compute the solutions to economic problems more accurately with higher-order and better approximation techniques. Schmitt-Grohé and Uribe (2004) provides a general introduction of using pertubation method.

The perturbation method is advantageous in its flexibility. It can handle an economic system with a large number of state variables with very nice global accuracy, as shown by Aruoba et al. (2006). The results are easy to interpret and computation is simple, especially with the usage of softwares such as dynare and dynare++. However, there are limitations to perturbation methods. First of all, it only provides accurate solutions around the deterministic steady state, around which we expand the policy function and cannot capture the highly nonlinear relationship between economic variables. An example is the net foreign asset dynamics in open economy models, as shown by Mendoza et al. (2016). Second, it is hard to handle problems in which policy function has kinks. These problems are very common in macroeconomic applications, including occasionally binding borrowing constraint, zero lower bound, corporate and sovereign default. Third, we cannot use perturbation methods to solve heterogenous agent models in which distribution of wealth is a key state variable. Lastly, it heavily relies on the existence of a deterministic steady state.

Projection is an alternative way of solving macroeconomic models. We project policy function of the model onto some basis functions. Pertubation can be viewed as a special case of projection with linear basis. A well-chosen basis enables us to solve the model globally. Judd (1992) shows how to apply projection methods in solving macroeconomic models. A commonly used basis is Chebyshev polynomials, which can accurately approximate the nonlinear relationship in economic variables. In multidimensional settings, Krueger and Kubler (2004) and Malin et al. (2011) apply Smolyak collocation method to reduce the curse of dimensionality.

Value function iteration is another widely used global method to solve recursive dynamic models. Any recursive formulation that is a contraction mapping can be solved by value function iteration. Conditions for contraction mapping can be found in Stokey and Lucas (Stokey and Lucas). We can deal with kinks in policy function, heterogenous agents, and nonexistence of deterministic steady state. However, the method is subject to the curse of dimensionality. It is crucial to have a relatively small number of endogenous state variables, and computing time increases exponentially if more state variables are introduced. The choice of grids crucially determines the computational complexity. There are several methods of choosing grid points to simplify computation: quadrature method by Tauchen and Hussey (1991), randomized grid method by Rust (1997), endogenous grid by Carroll (2006) and Barillas and Fernández-Villaverde (2007). Value function iteration is especially useful in solving models with heterogenous agents. Aiyagari (1994) solves a heterogenous agent model without aggregate uncertainty, and Krusell and Smith (1998) develops an algorithm to solve heterogenous agent models with aggregate uncertainty. Reiter (2010) and Den Haan and Rendahl (2010) provide alternative algorithms. Den Haan (2010) compares the performance of these algorithms, and find these algorithms produce results that have similar implications on correlations, but differ in some other aspects.

E The Full Benchmark DSGE Model

The purpose of this section is to provide an extension of a benchmark New Keynesian DSGE model, incorporating financial intermediation, a a role for asset markets and unconventional monetary policy. In light of the financial crisis of 2008, such extensions appear to be in particularly high demand. The model here is kept simple on purpose, and can serve as a base for constructing more elaborate versions. We view this model as a multi-purpose tool rather than a model designed to answer a specific question at hand. Our model is a simplified version of Christiano et al. (2010) and Christiano et al. (2014), which are state-of-the-art New Keynesian DSGE models with nontrivial financial intermediation. This model is meant for illustrative purposes, and we have left out many exogenous shocks that would be studied in a full-scale DSGE model.

The New Keynesian component of the model is a simplified version based on Christiano et al. (2005) and Smets and Wouters (2003). A simple New Keynesian DSGE model featuring monopolistic competitive firms and rigid nominal prices without endogenous capital accumulation can be found in Galí (2008). Kollman (1997) and Erceg et al. (2000) both introduced nominal sticky wages which are adjusted according to the Calvo rule (see Calvo, 1983). In terms of endogenous capital accumulation, we follow Christiano et al. (2005), which incorporates endogenous capital accumulation with an adjustment cost characterized by the relative level of investment, rather than the investment-capital stock ratio, is commonly assumed in the RBC literature. We adopt their external habit formation in consumption, which helps generate persistence in the consumption process in the data.

The financial intermediation component in our model is based on Gertler and Karadi (2011) and Gertler and Kiyotaki (2010). We allow for time-varying rare disaster risk and its premia (see, e.g. Gourio, 2012). We also incorporate "capital quality shocks" similar to Smets and Wouters (2003) and Christiano et al. (2005) in a New Keynesian setting and Gourio (2012) in an asset pricing setting, inspired by Greenwood et al. (1988) and King and Rebelo (1999) in the RBC literature.

In this simplified but recognizable version of the real economy, households maximize their individual utility function with consumption and labor over an infinite horizon. The utility function is characterized by external habit formation. The habits depend on lagged aggregate consumption that is unaffected by any single household's decision. Abel (1990) calls this the "catching up with the Joneses" effect. For simplicity, we assume households face flexible nominal wages.

The members of each household are divided into bankers and workers. Bankers and workers can both supply labor, but only bankers own capital in this economy and rent capital to the intermediate goods firm to extract rents. In this way, bankers decide how much capital to accumulate given the capital adjustment costs. A potentially important deficiency of the model is that the role bankers is hard-wired into the model: there is no other way for provide capital finance. This side-steps potentially important opportunities for flexibility in funding sources, an issue discussed more substantially in de Fiore and Uhlig (2011) and de Fiore and Uhlig (2015), for example.

The representative intermediate goods firm produces one kind of intermediate good, which can be used for investing and creating capital goods, or sold wholesale to retailers who simply convert the intermediate goods into differentiated goods for consumption. The retailers produce differentiated goods, giving them some monopoly power over goods prices, with a downward sloping demand for goods from households. The intermediate goods firm decides on labor and capital inputs, and at the same time, the retailers re-optimize goods prices according to the Calvo rule. Eventually, under the assumption that the bankers and the workers can fully insure their idiosyncratic risks in consumption, the representative households have the full claim on the dividends paid out by the intermediate goods firm and the retailers.

It should be noted that in benchmark New Keynesian DSGE models such as

Christiano et al. (2005) and Smets and Wouters (2003), the financial and credit markets play no role in determining asset prices except for the term structure of real interest rates and expectations of future payouts, and have no impact on the real economy, beyond the repercussions of shocks added to the Euler equation. An equivalent statement is that these models adopt the assumptions underlying the Modigliani and Miller (1958) Theorem, which implies that financial structure is both indeterminate and irrelevant to real economic outcomes. In order to quantitatively study how credit market imperfections influence the transmission of monetary policy, Bernanke et al. (1999) incorporated a countercyclical credit-market friction into a dynamic model, which is endogenously generated from first principles (i.e., using agent optimization). This countercyclical credit-market friction, first emphasized in Kiyotaki and Moore (1997), is shown to amplify and propagate productivity shocks. In more recent work, Christiano et al. (2010) extend the model in Bernanke et al. (1999) in many dimensions, including financially-constrained intermediations, but the key financial and credit market imperfections are not far removed from the financial accelerator mechanism in Bernanke et al. (1999). The crucial feature of constrained financial intermediation has been formulated by Gertler and Karadi (2011) in a simple but transparent model, on which we build here. A drawback of this approach is that the particular friction is hard-wired into the model, with little scope for agents in the model to get around them. By contrast, real-life participants on financial markets often appear to be particularly creative in circumventing restrictions.

Our goal here is to understand and illustrate the effects of three shocks in this model economy: (1) a financial sector shock impairing the ability of banks to borrow and hold productive assets, (2) a technology shock that impairs the quality of the physical capital, and (3) a risk shock. We also desire to show how the economy would respond to various policy responses to these shocks.

E.1 Households

We begin with a description of households in our canonical model. There is a continuum of households of unit mass. The members of each household are either workers or bankers. Though there are two groups of agents, and certain portfolio constraints among them, we assume the representative framework following Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) by assuming the household members to be part of a large family, sharing everything or, equivalently, assuming that the full set of Arrow-Debreu securities are available to the members within each household (but not across households), so that the idiosyncratic consumption risks can be fully insured, and the agents in two groups have identical preferences. At any time, a fraction f of the members of the household are bankers. Bankers live for a finite number of periods with probability 1. At any time, a fraction $1 - \theta$ of randomly selected existing bankers exit and become workers, and return their net worth to their household. At the same time, an equal number of workers become bankers within each household, so the proportion of workers and bankers remains fixed. The new bankers receive some start-up funds from their household, which we describe below. The "perpetual youth" assumption in our model is purely technical, with the purpose of guaranteeing the survivorship of both groups of agents.

The preferences of the household are given by

$$\mathbb{E}_t\left[\sum_{\tau=0}^{\infty}\beta_{t+\tau}^{\tau}\left(\frac{(C_{t+\tau}-hC_{t+\tau-1})^{1-\gamma}}{1-\gamma}-\frac{\chi}{1+\varphi}L_{t+\tau}^{1+\varphi}\right)\right],$$

where C_t is the consumption and L_t is the labor supply at time t. φ is the Frisch elasticity of labor supply and it is positive. The subject discount rate $\beta \in (0,1)$ and habit parameter $h \in (0,1)$. Also, we assume that $\chi > 0$. The logarithm of the discount factor β_t follows an AR(1) process

$$\log\beta_t = (1-\rho_\beta)\log\beta_{ss} + \rho_\beta \log\beta_{t-1} + \sigma_\beta u_{\beta,t}, \tag{123}$$

where β_{ss} is the long term mean, and $u_{\beta,t} \sim$ i.i.d. $N(0,1)$.

Both bankers and workers within each household can hold nominally risk-free debt issued by the government, and can deposit its cash with a financial intermediary that pays a nominally risk-free rate. Assuming that both assets are perfect substitutes, we denote by $R_{f,t+1}$ the real gross interest rate paid by either of these assets. It should be noted that $R_{f,t+1}$ is possibly random up to the information set at time t because the debt contract is written on the nominal term and the inflation, Π_{t+1}, is random up to the information set at time t. Let B_{t+1} denote the quantity of this debt held by the household at the end of period t. The household then faces a state-by-state budget constraint

$$C_t = W_t L_t + \mathrm{Pr}_t + T_t + R_{f,t}B_t - B_{t+1},$$

where W_t is the real wage, Pr_t is the profits from the various firms the household owns (which we describe below), and T_t is the real lump-sum taxes. The first-order conditions to the household's utility maximization problem include the intertemporal Euler equation for working hours

$$\Lambda_t = \chi\frac{L_t^{\varphi}}{W_t}, \tag{124}$$

and the intertemporal Euler equation for risk-free bond holding

$$1 = \mathbb{E}_t \left[\beta \frac{\Lambda_{t+1}}{\Lambda_t} R_{f,t+1} \right], \tag{125}$$

where

$$\Lambda_t \equiv (C_t - hC_{t-1})^{-\gamma} - \beta h \mathbb{E}_t (C_{t+1} - hC_t)^{-\gamma} \ . \tag{126}$$

is the marginal utility of consumption C_t at date t.

E.2 Financial Intermediaries

Financial intermediaries borrow funds from households at a risk-free nominal rate, pool this with their own net worth or wealth and invest the sum in the equity of the representative intermediate goods firm. We describe the intermediary using real variables in what follows. The balance sheet of intermediary j at the end of time t is given by

$$Q_t S_{j,t} = N_{j,t} + B_{j,t+1}, \tag{127}$$

where Q_t is the price of the intermediate goods firm's equity, $S_{j,t}$ is the quantity of equity held by the intermediary, $N_{j,t}$ is the net worth, and $B_{j,t+1}$ is the deposits raised from households. The intermediary earns a gross return $R_{k,t+1}$ from the equity investment at time $t+1$, and must pay the gross interest, $R_{f,t+1}$, on the deposit. The net worth of the intermediary, therefore, evolves as

$$\begin{aligned} N_{j,t+1} &= R_{k,t+1} Q_t S_{j,t} - R_{f,t+1} B_{j,t+1} \\ &= (R_{k,t+1} - R_{f,t+1}) Q_t S_{j,t} + R_{f,t+1} N_{j,t}. \end{aligned}$$

The intermediaries face a constraint on raising deposits from households. They cannot raise deposits beyond a certain level, which is determined endogenously in the equilibrium. We shall describe this constraint in more detail below. Since the bankers own the intermediaries, we use the bankers' stochastic discount factor (SDF), which coincides with the SDF of the representative agent, $\beta^i \Lambda_{t+\tau}/\Lambda_t$, to compute the value of assets to the intermediary. The presence of the borrowing constraints implies

$$\mathbb{E}_t \left[\beta^\tau \frac{\Lambda_{t+\tau+1}}{\Lambda_t} \left(R_{k,t+\tau+1} - R_{f,t+\tau+1} \right) \right] \geq 0, \quad \forall \tau \geq 0, \tag{128}$$

with equality if and only if the intermediary faces no borrowing constraint. Note that, so far, the returns and SDF are all real.

Since the intermediary ceases being a banker each period with probability $1-\theta$, the value of intermediary j's terminal wealth to its household is given by

$$\begin{aligned} V_{j,t} &= \max_{\{S_{j,t+\tau}, B_{j,t+\tau+1}\}_{\tau \geq 0}} \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (1-\theta)\theta^\tau \beta^{\tau+1} \frac{\Lambda_{t+\tau+1}}{\Lambda_t} N_{j,t+\tau+1} \right] \\ &= \max_{\{S_{j,t+\tau}, B_{j,t+\tau+1}\}_{\tau \geq 0}} \mathbb{E}_t \Bigg[\sum_{i=0}^{\infty} (1-\theta)\theta^\tau \beta^{\tau+1} \frac{\Lambda_{t+\tau+1}}{\Lambda_t} \times \\ &\qquad [(R_{k,t+\tau+1} - R_{t+\tau+1}) Q_{t+\tau} S_{j,t+\tau} + R_{f,t+\tau+1} N_{j,t+\tau}] \Bigg]. \end{aligned} \tag{129}$$

In order to motivate the borrowing constraint faced by financial intermediaries and following Gertler and Karadi (2011) and Gertler and Kiyotaki (2010), we introduce a simple moral hazard/costly enforcement problem. We assume that the banker can choose to liquidate the financial intermediation and divert the fraction of available funds, λ_t, from the value of the financial intermediation.

The borrowing constraint is modeled as follows. At any time t, the manager of the intermediary can divert a fraction λ_t of the intermediary's assets to his household for his own benefit, where $\lambda_t \in [0,1]$ is an exogenous parameter. To make sure that λ_t is between zero and one, we use the transformation

$$\lambda_t = \frac{1}{1+\tilde{\lambda}_t}.$$

The logarithm of $\tilde{\lambda}_t$ follows an AR(1) process

$$\log \tilde{\lambda}_t = (1-\rho_{\tilde{\lambda}}) \log \tilde{\lambda}_{ss} + \rho_{\tilde{\lambda}} \log \tilde{\lambda}_{t-1} + \sigma_{\tilde{\lambda}} u_{\tilde{\lambda},t}, \tag{130}$$

where $\tilde{\lambda}_{ss}$ is the long term mean, and $u_{\tilde{\lambda},t} \sim$ i.i.d. $N(0,1)$. If the value of the intermediary falls below $\lambda_t Q_t S_{j,t}$, the intermediary will simply divert the assets and the households will get a zero gross return from their deposits. In order for the households to have an incentive to deposit cash with the intermediary, the following condition must hold:

$$V_{j,t} \geq \lambda_t Q_t S_{j,t}. \tag{131}$$

We conjecture that the value of the intermediary is linear in its net worth and the value of the assets it holds:

$$V_{j,t} = \nu_t Q_t S_{j,t} + \eta_t N_{j,t}. \tag{132}$$

for some ν_t and η_t. We see that the incentive constraint binds only if $0 < \nu_t < \lambda_t$, otherwise the marginal value to the intermediary of increasing the assets is larger than the marginal value of diverting them, and the intermediary has an incentive to increase its assets. As in the equilibrium in Gertler and Karadi (2011), we assume that the incentive constraint (131) always binds in the local region of the long run mean, λ_{ss}, of λ_t. When the constraint binds, we have the condition

$$\nu_t Q_t S_{j,t} + \eta_t N_{j,t} = \lambda_t Q_t S_{j,t}, \text{ or} \tag{133}$$

$$Q_t S_{j,t} = \frac{\eta_t}{\lambda_t - \nu_t} N_{j,t} = \phi_t N_{j,t}. \tag{134}$$

Using the definition of ϕ_t, we can rewrite the evolution of the intermediary's net worth as

$$N_{j,t+1} = N_{j,t} \left[(R_{k,t+1} - R_{f,t+1})\phi_t + R_{f,t+1} \right]. \tag{135}$$

This is the standard wealth or net worth law of motion with leverage, where ϕ_t can be viewed as the share of net worth invested in the risky asset (i.e., equity).

We verify the guess to the solution of the value function of the intermediary when the incentive constraint binds, and obtain

$$\nu_t = \mathbb{E}_t \left[(1-\theta)\beta \frac{\Lambda_{t+1}}{\Lambda_t} (R_{k,t+1} - R_{f,t+1}) + \theta\beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\phi_{t+1}}{\phi_t} \nu_{t+1} \left((R_{k,t+1} - R_{f,t+1})\phi_t + R_{t+1} \right) \right] \tag{136}$$

and

$$\eta_t = \mathbb{E}_t \left[1 - \theta + \theta\beta \frac{\Lambda_{t+1}}{\Lambda_t} \eta_{t+1} \left((R_{k,t+1} - R_{f,t+1})\phi_t + R_{f,t+1} \right) \right]. \tag{137}$$

Since $Q_t S_{j,t} = \phi_t N_{j,t}$, and since ϕ_t does not depend on intermediary-specific factors, we can aggregate over the equation to get

$$Q_t S_t = \phi_t N_t, \tag{138}$$

where S_t is the aggregate investment in the equity and N_t is the aggregate wealth of the intermediaries.

Finally, we determine the evolution of the aggregate net worth of the intermediaries. The aggregate net worth is the sum of the net worth of the existing intermediaries, $N_{n,t}$, and the net worth of the new entrants, $N_{e,t}$:

$$N_t = N_{e,t} + N_{n,t}. \tag{139}$$

Since a fraction θ of the bankers from $t-1$ survive up to t, we have

$$N_{e,t} = \theta N_{t-1} \left[(R_{k,t} - R_{f,t})\phi_{t-1} + R_{f,t} \right]. \tag{140}$$

The new entrants receive funds from the households to "start up". As in Gertler and Karadi (2011), we assume that each entering intermediary receives a fraction $\frac{\omega}{1-\theta}$ of the value of the final period assets of the exiting intermediaries, which is $(1-\theta)Q_t S_{t-1}$, the remainder being distributed to the households. This gives

$$N_{n,t} = \omega Q_t S_{t-1}.$$

Thus, we have the evolution of the aggregate net worth

$$N_t = \theta N_{t-1} \left[(R_{k,t} - R_{f,t})\phi_{t-1} + R_{f,t} \right] + \omega Q_t S_{t-1}. \tag{141}$$

E.3 Firms

There are three types of firms in our model: intermediate-goods, capital-producing, and retail firms.

Intermediate-Goods Firms

The financial intermediaries invest in the equity of the intermediate-goods firm. We assume that the representative intermediate-goods firm produces the intermediate goods used by the retail firms to produce differentiated goods. Such a firm has no wealth of its own. It is alive for two periods, $t-1$ and t, say. In period $t-1$, it issues shares to banks and uses the proceeds of the investment by the intermediaries to purchase capital K_t from the capital producing firm, to be used for production in the next period[6] t.. The number of shares issued by the intermediate goods firm is equal to the number of units of capital purchased, and we assume no frictions or wedges (such as those induced by an agency problem between the financial firms and the managers of the intermediate goods firms), so that

$$Q_{t-1}S_{t-1} = Q_{t-1}K_t. \tag{142}$$

or, stated as an equation for period t,

$$Q_tS_t = Q_tK_{t+1}. \tag{143}$$

The intermediate firm faces no informational or incentive problems. In period t, it hires labor at a wage W_t and uses the capital K_t chosen in period $t-1$ to produce the intermediate goods using the production function

$$Y_t^I = A_t(\xi_t K_t)^\alpha L_t^{1-\alpha}, \tag{144}$$

where A_t is an exogenous and stochastic total factor productivity (TFP) parameter, ξ_t is an exogenous and stochastic quality of capital parameter, and K_t is the capital

[6] Note that we adopt the more conventional timing assumption of indexing capital with the date when it is used in production, not with the date of the decision. As is well known, one needs to be careful with that, when implementing this in solution software such as Dynare or Uhlig's Toolkit, see Uhlig (1999).

stock determined at period $t-1$. The TFP and the capital quality evolve as

$$\log A_t = (1-\rho_A)\log A_{ss} + \rho_A \log A_{t-1} + \sigma_A u_{A,t}, \tag{145}$$
$$\log \xi_t = (1-\rho_\xi)\log \xi_{ss} + \rho_\xi \log \xi_{t-1} + \sigma_\xi u_{\xi,t}, \tag{146}$$

where the subscripts ss denote the long term steady states, and $u_{A,t}, u_{\xi,t} \sim$ i.i.d $N(0,1)$. After production takes place, the firm sells the quality-adjusted and depreciated capital back to the capital-producing firms at a price of Q_t per unit. Thus, the proceeds from the capital sale are $Q_t(1-\delta)\xi_t K_t$, where δ is the depreciation rate.

Denote by $P_{m,t}$ the market price of the output of the intermediate goods, taken as given by the firm. Arriving in period t and given the amount of capital K_t chosen in $t-1$, the firm — or, perhaps better, the firm-owning banker — will choose labor L_t to maximize the cash flow

$$cf_t = P_{m,t} A_t (\xi_t K_t)^\alpha L_t^{1-\alpha} + Q_t(1-\delta)\xi_t K_t - W_t L_t \tag{147}$$

The first-order condition with respect to labor gives

$$P_{m,t}(1-\alpha)\frac{Y_t^I}{L_t} = W_t. \tag{148}$$

Since the intermediate firm has no wealth of its own and is owned entirely by its banks, it pays out the cash flow cf_t in proportion to the underlying shares or units of capital to the banks. The return per unit of resources for a bank investing at date t is therefore

$$R_{k,t+1} = \frac{P_{m,t+1}Y_{t+1}^I - W_{t+1}L_{t+1} + Q_{t+1}(1-\delta)\xi_{t+1}K_{t+1}}{Q_t K_{t+1}}, \tag{149}$$

where δ is the depreciation rate. Plugging in the first-order condition for labor, we get

$$R_{k,t+1} = \frac{\left(P_{m,t+1}\alpha \frac{Y_{t+1}^I}{\xi_{t+1}K_{t+1}} + Q_{t+1}(1-\delta)\right)\xi_{t+1}}{Q_t}. \tag{150}$$

Capital Producing Firms

We assume that the representative capital-producing firm purchases the depreciated capital $(1-\delta)\,\xi_t K_t$ from the intermediate goods firm at the end of every period. It also produces new capital or, possibly, turns existing old capital back into consumption goods, denoted as gross capital investment I_t. It then sells the total new capital to the new generation of intermediate goods firms at price Q_t per unit. The aggregate production function for new capital is

$$K_{t+1} = I_t + (1-\delta)\,\xi_t K_t. \tag{151}$$

We state the production function for new capital or gross capital investment as a resourcecost function per unit of investment,

$$x_t = z_t \left(I_t + \Phi\left(\frac{I_t}{\xi_t K_t}\right) \xi_t K_t \right) \tag{152}$$

where x_t is in units of the aggregate consumption good, z_t is an exogenous and stochastic investment cost parameter. and $\Phi(\cdot)$ is often called an adjustment cost function. The function Φ is assumed to satisfy $\Phi(\delta) = \Phi'(\delta) = 0$ and $\Phi''(\delta) > 0$. Specifically, we assume the quadratic adjustment cost function

$$\Phi(x) \equiv \frac{\vartheta}{2}(x-\delta)^2, \quad \text{with} \ \ \vartheta > 0. \tag{153}$$

The profits of the capital producing firms in period t are

$$Q_t(K_{t+1} - (1-\delta)\,\xi_t K_t) - x_t = Q_t I_t - z_t \left(I_t + \Phi\left(\frac{I_t}{\xi_t K_t}\right) \xi_t K_t \right) \tag{154}$$

We assume that the capital producing firms are directly owned by the households, and that there is free entry. Thus, the discounted value of the profits of the capital goods producer is

$$\max_{\{I_\tau\}_{\tau \geq 0}} \sum_{\tau=t}^{\infty} \mathbb{E}_t \left[\beta^{\tau-t} \frac{\Lambda_\tau}{\Lambda_t} \left(Q_\tau I_\tau - z_\tau I_\tau - z_\tau \Phi\left(\frac{I_\tau}{\xi_\tau K_\tau}\right) \xi_\tau K_\tau \right) \right]. \tag{155}$$

Maximising this gives the first-order condition

$$Q_t = z_t(1 + \Phi'(\cdot)). \tag{156}$$

$$\tag{157}$$

The exogenous and stochastic investment parameter evolves as

$$\log z_t = \rho_z \log z_{t-1} + \sigma_z u_{z,t}, \tag{158}$$

with $u_{z,t} \sim$ i.i.d. $N(0,1)$.

Retail Firms

Our model includes a unit-mass continuum of retail firms. Retail firms use the (single) intermediate good to produce differentiated goods. Each firm f can use one unit of the intermediate good to produce one unit of the differentiated good. If $Y_{f,t}$ denotes the final output of the retail firm, the final output composite good is the constant elasticity of substitution (CES) aggregator

$$Y_t = \left[\int_0^1 Y_{f,t}^{(\varepsilon_t - 1)/\varepsilon_t} \mathrm{d}f\right]^{\varepsilon_t/(\varepsilon_t - 1)}, \tag{159}$$

where $\varepsilon_t > 1$ is the elasticity of substitution. The steady state of the elasticity of substitution is ε_{ss}. Define $\tilde{\varepsilon} \equiv \varepsilon - 1$, which evolves as

$$\log \tilde{\varepsilon}_t = (1 - \rho_{\tilde{\varepsilon}}) \log \tilde{\varepsilon}_{ss} + \rho_{\tilde{\varepsilon}} \log \tilde{\varepsilon}_{t-1} + \sigma_{\tilde{\varepsilon}} u_{\tilde{\varepsilon},t}, \tag{160}$$

with $u_{\tilde{\varepsilon},t} \sim$ i.i.d. $N(0,1)$.

The retail firms are monopolistically competitive. Equation (159) and household optimization implies that retail firms face a downward sloping demand for their goods. More precisely, if $P_{f,t}$ is the nominal price that each retail firm charges for its good, then cost minimization for the users of the final good gives

$$Y_{f,t} = \left(\frac{P_{f,t}}{P_t}\right)^{-\varepsilon_t} Y_t, \tag{161}$$

where P_t is the ideal nominal price index

$$P_t = \left[\int_0^1 P_{f,t}^{1-\varepsilon_t} \mathrm{d}f \right]^{1/(1-\varepsilon_t)}. \tag{162}$$

We assume that, at each time t, a random fraction $1 - \varsigma$ of the retail firms can reset their price, while all other firms are stuck at the price from the previous period[7]. Retail firms are owned by the households, who instruct these firms to maximize the net present value of future cash flows, using the household discount factor. A retail firm which can reset its price at time t to $P_{f,t} = P_t*$ therefore sets it to maximize the discounted value of its future profits as long as it is stuck with that price, i.e.,

$$\max_{P_t^*} \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^\tau \varsigma^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t} \left(\frac{P_t^*}{P_{t+\tau}} Y_{f,t+\tau} - P_{m,t+\tau} Y_{f,t+\tau} \right) \right] \equiv \tag{163}$$

$$\max_{P_t^*} \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^\tau \varsigma^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t} \left(\frac{P_t^*}{P_{t+\tau}} \left(\frac{P_t^*}{P_{t+\tau}} \right)^{-\varepsilon_t} - P_{m,t+\tau} \left(\frac{P_t^*}{P_{t+\tau}} \right)^{-\varepsilon_t} \right) Y_{t+\tau} \right].$$

where it is useful to note that $P_{m,t}$ is the real price for the intermediate input good or real cost per unit of retail output, and where the second line follows, because $Y_{f,t+\tau}$ is a function of $P_{f,t+\tau} = P_t*$ per (161). The first-order condition with respect to P_t* is:

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^\tau \varsigma^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t} \left(\frac{P_t}{P_{t+\tau}} \right)^{1-\varepsilon_t} Y_{t+\tau} \right] \frac{P_t^*}{P_t} - \mathbb{E}_t \left[\beta^\tau \varsigma^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t} \frac{\varepsilon_t}{\varepsilon_t - 1} P_{m,t+\tau} \left(\frac{P_t}{P_{t+\tau}} \right)^{-\varepsilon_t} Y_{t+\tau} \right] = 0.$$

Defining

$$\begin{aligned} J_t &= \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^\tau \varsigma^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t} \left(\frac{P_t}{P_{t+\tau}} \right)^{1-\varepsilon_t} Y_{t+\tau} \right] \\ H_t &= \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^\tau \varsigma^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t} \frac{\varepsilon_t}{\varepsilon_t - 1} P_{m,t+\tau} \left(\frac{P_t}{P_{t+\tau}} \right)^{-\varepsilon_t} Y_{t+\tau} \right] \end{aligned}$$

[7]Often, this framework is extended to allow for price indexation for these firms, thereby largely reducing the effect of average inflation on the equilibrium. To keep matters simple, we do not include indexation here.

we have:

$$\frac{P_t^*}{P_t} = \frac{H_t}{J_t}.$$

At any time t a fraction ς of the retail firms will be unable to change their prices and their aggregate price index will be simply P_{t-1}. A fraction $1-\varsigma$ will be able to reset the prices, and they will all reset their price to P_t^*. Thus, by the law of large numbers, we have $P_t^{1-\varepsilon_t} = \varsigma P_{t-1}^{1-\varepsilon_t} + (1-\varsigma)P_t^{*1-\varepsilon_t}$. Rearranging, and substituting for P_t^* from the first-order condition, we get

$$\frac{1-\varsigma\Pi_t^{\varepsilon_t-1}}{1-\varsigma} = \left(\frac{H_t}{J_t}\right)^{1-\varepsilon_t}, \tag{164}$$

where $\Pi_{t+1} = P_{t+1}/P_t$ is the inflation. We can rewrite J_t and H_t recursively as

$$J_t = Y_t + \varsigma\beta\mathbb{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\Pi_{t+1}^{\varepsilon_t-1}J_{t+1}\right] \tag{165}$$

$$H_t = \frac{\varepsilon_t}{\varepsilon_t-1}P_{m,t}Y_t + \varsigma\beta\mathbb{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\Pi_{t+1}^{\varepsilon_t}H_{t+1}\right]. \tag{166}$$

The relationship between the intermediate goods output and the final goods output is:

$$\begin{aligned} Y_t^I &= \int_0^1 Y_{f,t}\mathrm{d}f = \frac{Y_t}{P_t^{-\varepsilon_t}}\int_0^1 P_{f,t}^{-\varepsilon_t}\mathrm{d}f = \frac{Y_t}{P_t^{-\varepsilon_t}}\left[(1-\varsigma)(P_t^*)^{-\varepsilon_t} + \varsigma P_{t-1}^{-\varepsilon_t}\right] \\ &= Y_t\left[(1-\varsigma)^{\frac{1}{1-\varepsilon_t}}(1-\varsigma\Pi_t^{\varepsilon_t-1}) + \varsigma\Pi_t^{\varepsilon_t}\right]. \end{aligned}$$

E.4 Government Policies

In our simple model, the central bank sets the short-term nominal risk-free interest rate, i_t, at time t. This gives the expression for the ex post real return on one period safe nominal bonds

$$R_{f,t+1} = \frac{1+i_t}{\Pi_{t+1}}. \tag{167}$$

The central bank uses a Taylor rule to set the short-term nominal interest rate:

$$i_t = \max\{(1-\rho_i)\,[i_{ss} + \kappa_\pi \log \Pi_t + \kappa_y(\log Y_t - \log Y_t^*)] + \rho_i i_{t-1} + m_t, 0\}, \qquad (168)$$

The interest rate is bounded below by zero. We assume that the central bank normally follows the Taylor rule to set the interest rate, but sets the interest rate to zero when the Taylor rule indicates a negative interest rate. This simple treatment of monetary policy abstracts from considerations of other policy tools such as forward guidance, as in Eggertsson and Woodford (2003) and Werning (2011).

In Equation (168), Y_t^* is the natural level of output that would hold in a flexible price equilibrium, i_{ss} is the steady-state nominal interest rate, the smoothing parameter, ρ_i, lies between zero and one, and κ_π and κ_y are constants which satisfy certain conditions, such as those in Woodford (2003), so that there is a unique equilibrium with non-exploding inflation. Here, m_t is an exogenous and stochastic component of the monetary policy rule. It evolves as

$$m_t = \rho_m m_{t-1} + \sigma_m u_{m,t}, \qquad (169)$$

with $u_{m,t} \sim$ i.i.d. $N(0,1)$.

Now we specify the credit policy. The central bank is also willing to buy the shares of the intermediate goods firm to facilitate lending. Such policies were studied by Gertler and Kiyotaki (2010), and Gertler and Karadi (2011). They capture the unconventional policies of purchasing risky, privately managed, non-government assets, implemented in the U.S., the U.K. and the Eurozone in the wake of the financial crisis. The U.S. Federal Reserve's program of buying $600 million of Mortgage Backed Securities in 2008-09 (QE-1), and the European Central Bank's Covered Bond Purchase Programs (CBPPs) for buying private sector debt, are examples of such policies.

The central bank buys a fraction ψ_t of the total outstanding shares of the intermediate goods firm, so that

$$Q_t S_t = \phi_t N_t + \psi_t Q_t S_t, \text{ or} \qquad (170)$$

where ϕ_t is the leverage ratio for the privately-held asset, i.e., $Q_t S_{p,t} \equiv \phi_t N_t$ and the government-held asset is $Q_t S_{g,t} = \psi_t Q_t S_t$.

We define the total leverage ratio $\phi_{c,t}$ as follows

$$Q_t S_t = \phi_{c,t} N_t. \qquad (171)$$

The leverage ratio, $\phi_{c,t}$, is the leverage ratio for total intermediated funds, public as well as private, and has the following relation with the private leverage ratio, ϕ_t, and the intensity of government credit intervention,

$$\phi_{c,t} = \frac{\phi_t}{1 - \psi_t}. \tag{172}$$

The central bank issues government bonds $B_{gt} = \psi_t Q_t S_t$ to fund the purchase of these shares. From this activity, the central bank thus earns an amount $(R_{k,t+1} - R_{f,t+1})B_{g,t}$ every period. The central bank credit has an efficiency cost of $\tau > 0$ units per unit of credit supplied.

We assume that at the onset of a crisis, which is defined loosely to mean a period when the credit spread rises sharply, the central bank injects credit in response to movements in credit spreads, according to the following rule for ψ_t:

$$\psi_t = \psi_{ss} + \nu \mathbb{E}_t \left[(\log R_{k,t+1} - \log R_{f,t+1}) - (\log R_{k,ss} - \log R_{f,ss}) \right], \tag{173}$$

where ψ_{ss} is the steady-state fraction of intermediation, $\log R_{k,ss} - \log R_{f,ss}$ is the steady-state risk premium, and the sensitivity parameter, ν, is positive. According to (173), the central bank expands credit as the credit spread increase relative to the the steady-state credit spread. The rationale behind this policy specification, used by Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), is as follows. In the absence of the financial friction that prevents the financial intermediaries from leveraging too much, the equilibrium outcome is efficient. The inefficiency arises due to the inability of the households to directly buy the risky assets, and the limit on their financial managers' leverage. The inefficiency manifests itself in the form of large risk premium, since the financial intermediaries must be compensated adequately in the absence of high leverage. The government does not intervene when the risk premium is at its steady state level, but does when it rises to increasingly inefficient levels beyond it.

From (172), it is clear that, when the private leverage ratio, ϕ_t, is kept fixed, the expanding credit policy increases the total leverage of financial intermediaries, i.e., $\phi_{c,t}$ rises.

E.5 Resource and Government Budget Constraints

The resource constraint for the final good in our model is given by

$$Y_t = C_t + I_t + \Phi\left(\frac{I_t}{\xi_t K_t}\right)\xi_t K_t + G_t + \tau \psi_t Q_t K_{t+1}.$$

The government spends a fraction g_t of output Y_t in period t, where g_t is an exogenous and stochastic parameter. That is,

$$G_t = g_t Y_t. \tag{174}$$

It also funds the central bank's purchase of shares by issuing purchasing bonds worth $\psi_t Q_t K_{t+1}$. Its revenues include taxes, T_t, and the central bank's income from intermediation, $\psi_{t-1} Q_{t-1} K_t (R_{k,t} - R_{f,t})$. Thus, the government budget constraint is

$$G_t + \tau \psi_t Q_t K_{t+1} = T_t + \psi_{t-1} Q_{t-1} K_t (R_{k,t} - R_{f,t}). \tag{175}$$

exogenously fixed at a constant fraction of output. We denote steady-state government spending share of output by g_{ss}. Government spending evolves as

$$\log g_t = (1 - \rho_g) \log g_{ss} + \rho_g \log g_{t-1} + \sigma_g u_{g,t}, \tag{176}$$

with $u_{g,t} \sim$ i.i.d $N(0, 1)$.

Since the taxation, T_t, effectively takes up any slack that shows up on the government balance sheet, and given the existence of representative agents in the economy, the intertemporal budget constraint of the representative household and the intertemporal budget constraint of the government can be combined with taxes left out. Intuitively, then, by Walras' Law, both budget constraints are redundant in determining the equilibria. However, this is very different from saying that the size and composition of the government balance sheet are irrelevant for pinning down the equilibrium under efficient financial market conditions, as was proposed by Wallace (1981). This is simply because not all investors can purchase an arbitrary amount of the same assets at the same market prices as the government in this model. Put more precisely, unlike private financial intermediation, government intermediation is not balance-sheet constrained.

F Calibration and Estimation

We use our model to understand the response of the economy to various shocks. We use a calibrated version of the model, basing our parameter choices mainly on those in Gertler and Karadi (2011) and Gourio (2012), and the estimated dynamic parameters in Smets and Wouters (2007). In particular, we set the steady-state government expenditure to be $g_{ss} = 20\%$ and the steady-state government credit intervention to be $\psi_{ss} = 0$. These values are close to the average government expenditure and investment in the U.S. over the time period 1934 to 2010. We note that credit intervention by the U.S. government has historically been negligible, only becoming substantial after the

Table 2: Static Parameter Calibration (Quarterly)

Parameter	Symbol	Value	Source
Household preference			
Discount rate	β	0.99	Standard
Relative risk aversion	γ	2	Standard
Habit parameter	h	0.815	Standard
Relative weight of labor	χ	3.409	Standard
Inverse Frisch elasticity of labor supply	ψ	0.276	Standard
Financial intermediaries			
Steady-state fraction of divertible capital	λ_{ss}	0.381	Gertler and Karadi (2011)
Proportional transfer to new bankers	ω	0.002	Gertler and Karadi (2011)
Survival rate of bankers	θ	0.972	Gertler and Karadi (2011)
Capital producing firms			
Depreciation rate	δ	0.025	Standard
Adjustment cost coefficient	ϑ	1	Standard
Retail firms			
Elasticity of substitution	ϵ	4.167	Standard
Probability of keeping prices fixed	ς	0.779	Standard
Price indexation	ς_p	0	Simplification
Government policies			
Inflation coefficients of Taylor rule	κ_π	1.5	Standard
Output coefficients of Taylor rule	κ_y	0.125	Standard
Persistence of interest rate	ρ_i	0.8	Standard
Government expenditure ratio	g_{ss}	20%	Standard
Steady-state government share of capital	ψ_{ss}	0	Standard

recent crisis. Nevertheless, we include such intervention in our analysis to understand the effects of modern policy responses. The parameter values are summarized in Tables 2 and 3. We use Dynare to perform the analysis. For the baseline analysis, we use the first-order approximation around the steady state.

We allow the nominal interest rate to have a lower bound at zero. The impulse response functions with the zero lower bound are computed using the algorithm from Holden (2011). (As demonstrated by Cochrane (2013), Duarte (2016), Holden (2016) etc. the zero lower bound leads to the existence of multiple equilibria, even with the appropriate Taylor coefficients, when the liquidity trap lasts for too long. However, in our simulations, this multiplicity does not arise.)

The presence of the financial friction implies that the steady state of the model features a non-zero risk premium. The low supply of risk-free assets causes the interest rate to be low. The inability of the financial intermediaries to finance the intermediate

Table 3: Dynamic Parameter Calibration (Quarterly)

Parameter	**Symbol**	**Value**	**Source**
Capital Quality			
Persistence	ρ_ξ	0.66	Gertler and Karadi (2011)
Volatility	σ_ξ	0.05	Gertler and Karadi (2011)
Margin			
Persistence	ρ_λ	0.66	Gertler and Karadi (2011)
Volatility	σ_λ	0.20	Gertler and Karadi (2011)
Monetary Policy			
Persistence	ρ_m	0.15	Smets and Wouters (2007)
Volatility	σ_m	0.24	Smets and Wouters (2007)
Government Spending			
Persistence	ρ_g	0.97	Smets and Wouters (2007)
Volatility	σ_g	0.53	Smets and Wouters (2007)
Markup			
Persistence	ρ_ε	0.89	Smets and Wouters (2007)
Volatility	σ_ε	0.14	Smets and Wouters (2007)
TFP			
Persistence	ρ_A	0.95	Smets and Wouters (2007)
Volatility	σ_A	0.45	Smets and Wouters (2007)
Investment			
Persistence	ρ_z	0.75	Smets and Wouters (2007)
Volatility	σ_z	0.45	Smets and Wouters (2007)

goods producers also lowers the price of capital, which increases the expected return on capital, further increasing the risk premium.

F.1 Calibration Experiments

We conduct two calibration experiments involving shocks to capital quality and margin. The capital quality shock allows us to examine the reaction of the economy to productivity shocks, whereas the margin shock allows us to examine the impact of financial market frictions on the real economy. A comparison of the two responses will provide insights into whether inclusion of the financial sector generates amplifications that do not appear in conventional DSGE models such as Smets and Wouters (2007).

Experiment 1: Capital Quality Shock

In our first experiment, we examine the effect of a capital quality shock on our model economy. We argue that an initial adverse disturbance of capital quality can approximately capture a decline in the quality of intermediary assets, leading to a severe decline in the net worth of the financial intermediaries. There are two major effects of the quality shock, one exogenous and one endogenous. The first effect is the exogenous impact of the destruction of capital on output and asset values. The second effect is endogenous. The balance sheet of intermediaries is weakened by the decline of asset values, and hence intermediaries reduce their demand for investment goods, which suppresses the price of capital, Q. The endogenous feedback effect of a decline in Q is to further weaken the balance sheet of the financial intermediaries.

We choose the shock size to be a 5% deviation from the steady-state ξ level (i.e., $\sigma_\xi = 0.05$). Conditional on occurring, the shock obeys an AR(1) decaying path with a persistent parameter $\rho_\xi = 0.66$. The shock path is displayed in the top-left corner of Figure 8. The real economy's responses to this capital quality shock are shown in Figure 8. The capital quality shock triggers dramatic drops in output, investment, labor, and capital stock. Also, the intermediate and capital goods prices decline due to the weakened demand for investment. Interestingly, however, we can see that credit policies do not show any significant power to combat the capital quality shock, which can be seen even more clearly in Figure 9. In Figure 9, we see that the change in credit policy lasts for a long period, although the total amount of credit intervention is low overall. The credit policy does bring down the leverage and the risk premium for the intermediaries, but only by a moderate amount.

Experiment 2: Margin Shock

In our second experiment, we examine the effect of a margin shock on our model economy. The crisis scenario of a sudden collapse of funding can be roughly captured by an adverse shock in λ_t, meaning the deterioration of the pledgeability of the

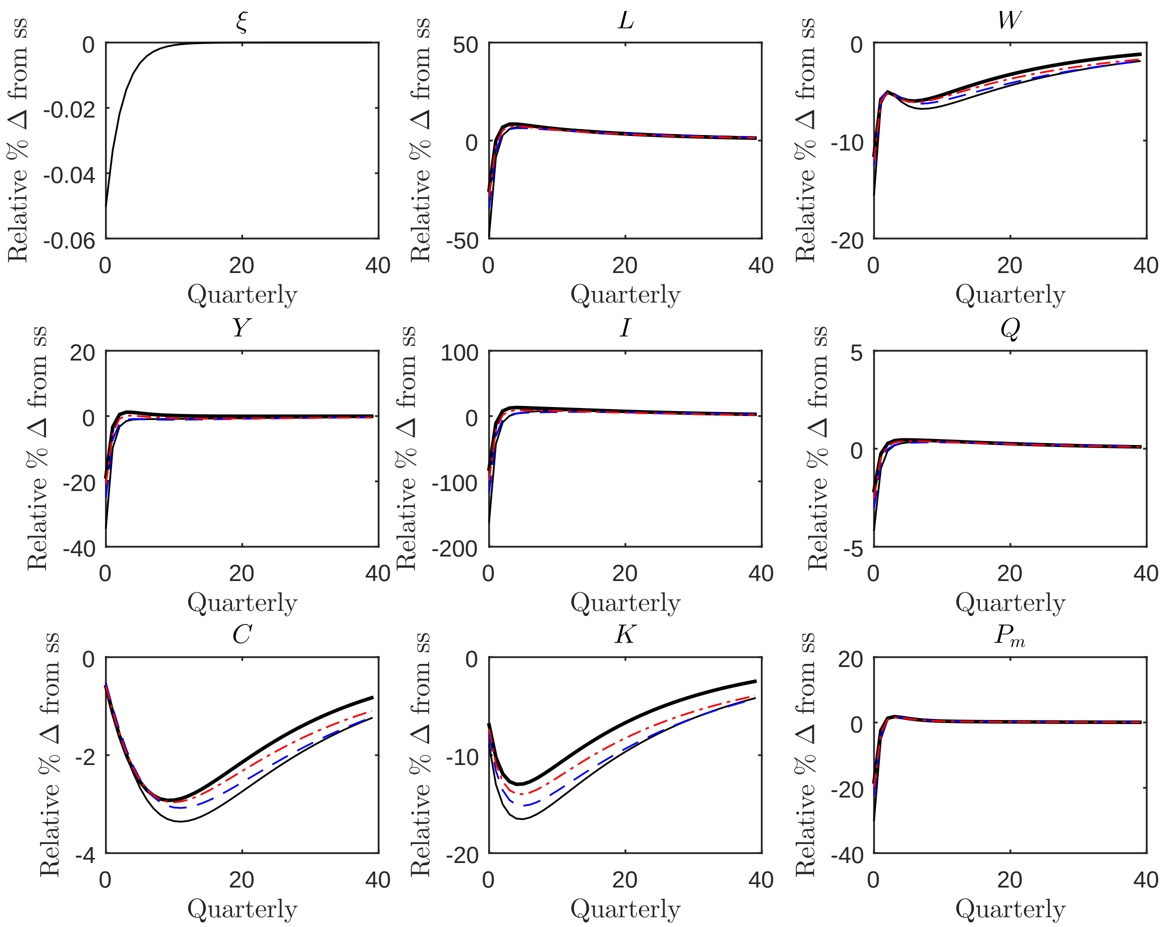


Figure 8: Real quantities' response to capital quality shock: 5% deviation from steady state. The thick solid curve represents the case in the absence of financial frictions. The solid curve is for the case of zero credit policy intervention ($\nu = 0$). The dashed curve is for the case of moderate credit policy intervention ($\nu = 10$). The dash-dotted curve is for the case of intensive credit policy intervention ($\nu = 100$). The nominal interest rate has a zero lower bound.

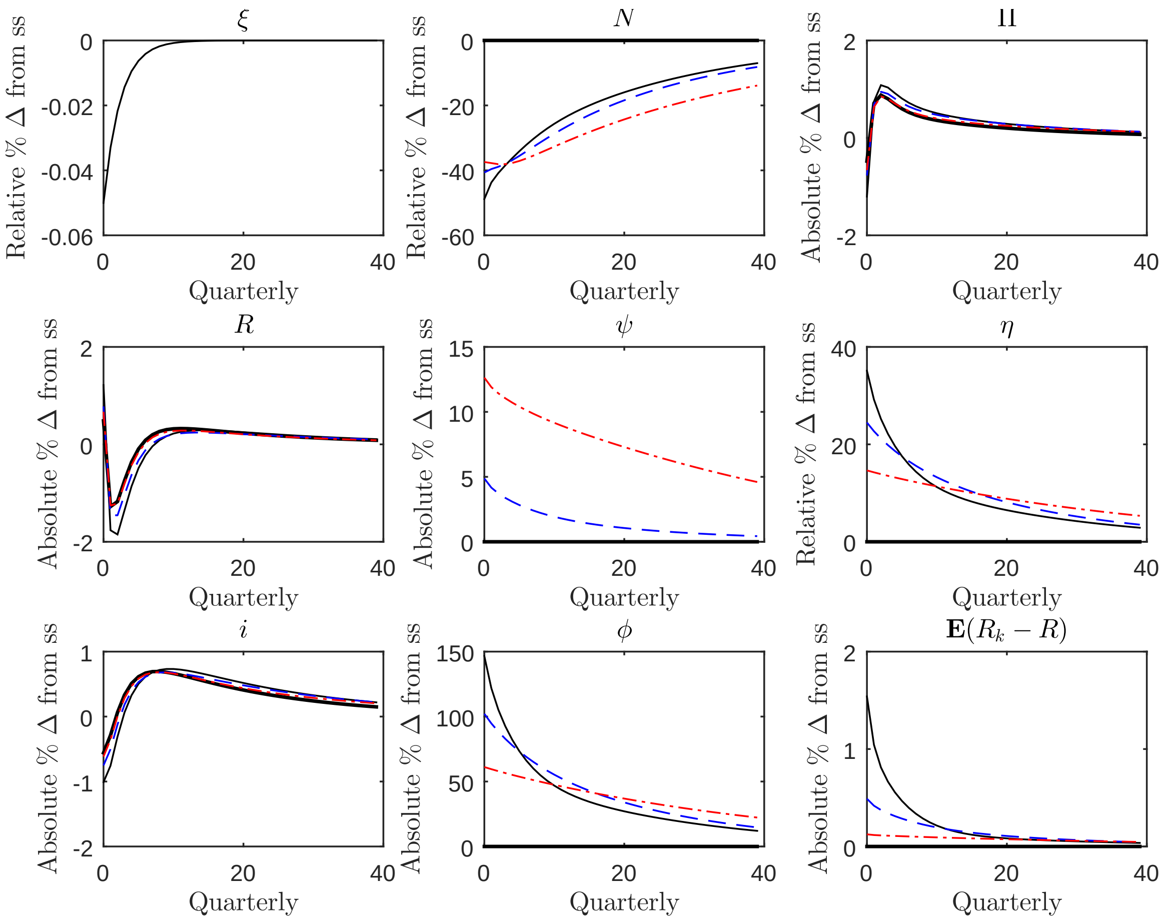


Figure 9: Financial variables' response to capital quality shock: 5% deviation from steady state. The solid curve is for the case of zero credit policy intervention ($\nu = 0$). The dashed curve is for the case of moderate credit policy intervention ($\nu = 10$). The dash-dotted curve is for the case of intensive credit policy intervention ($\nu = 100$). The nominal interest rate has a zero lower bound.

financial intermediaries' assets. This can also be interpreted as a margin shock, as is emphasized by Geanakoplos (2001, 2009), among others.

The adverse margin shock affects asset value only through the endogenous channel, by deteriorating the capacity of the intermediaries to take on leverage. Like the capital quality shock, it also triggers a feedback effect via the balance sheet of intermediaries.

We choose the shock size to be one standard deviation from the steady-state λ level (i.e., $\sigma_\lambda = 0.20$). We use such a large shock to get the impulse response to the real variables to be roughly the size they were during the recent financial crisis. Conditional on occurring, the shock obeys an AR(1) decaying path a with persistent parameter $\rho_\xi = 0.66$, the same as in experiment I. The shock path is displayed in the top-left corner of Figure 10. The real economy's responses to this margin shock are shown in Figure 10. The margin shock causes severe but very temporary drops in output, investment, and employment. However, it only generates a small decline in capital stock, different from the response of capital quality shock. Also, the prices of the intermediate and capital goods decline due to the weakened demand for investment. In contrast to the case of capital quality shock, credit policies are very efficient in alleviating the adverse impact of the margin shock, which can be seen even more clearly in Figure 11. In Figure 11, we find that the credit policy only lasts for about 10 quarters, with a level similar to that of the policy response to the 5% capital quality shock. From Figure 11, it is obvious that the credit policy is a powerful tool in maintaining the stability of the financial system when a sudden funding crisis occurs.

F.2 Estimation Analysis

In this section, we perform an estimation exercise to demonstrate the inadequacy of current estimation methods, which rely on linearized approximations and Kalman filtering, in incorporating the non-linear dynamics of asset pricing and risk premia. We estimate our model using macroeconomic and financial data. Since the static parameters of Table 2 are reasonably well understood and estimated, we fix them in place, and only estimate the dynamic parameters for exogenous shocks using a Bayesian method (see, e.g. Smets and Wouters, 2007). For simplicity, we assume that the nominal interest rate does not have a zero lower bound.

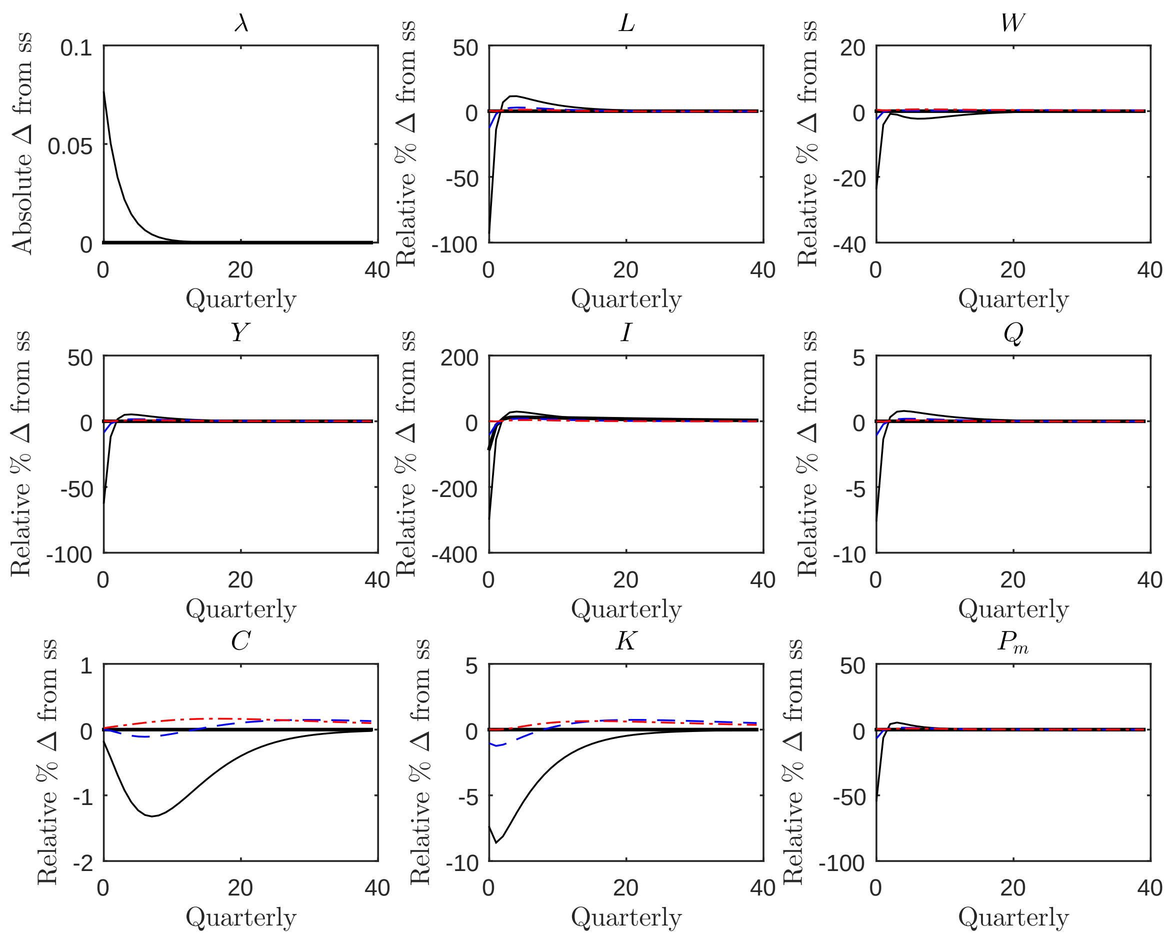


Figure 10: Real quantities' response to intermediary margin shock: one standard deviation from steady state. The thick solid curve represents the case in the absence of financial frictions. The solid curve is for the case of zero credit policy intervention ($\nu = 0$). The dashed curve is for the case of moderate credit policy intervention ($\nu = 10$). The dash-dotted curve is for the case of intensive credit policy intervention ($\nu = 100$). The nominal interest rate has a zero lower bound.

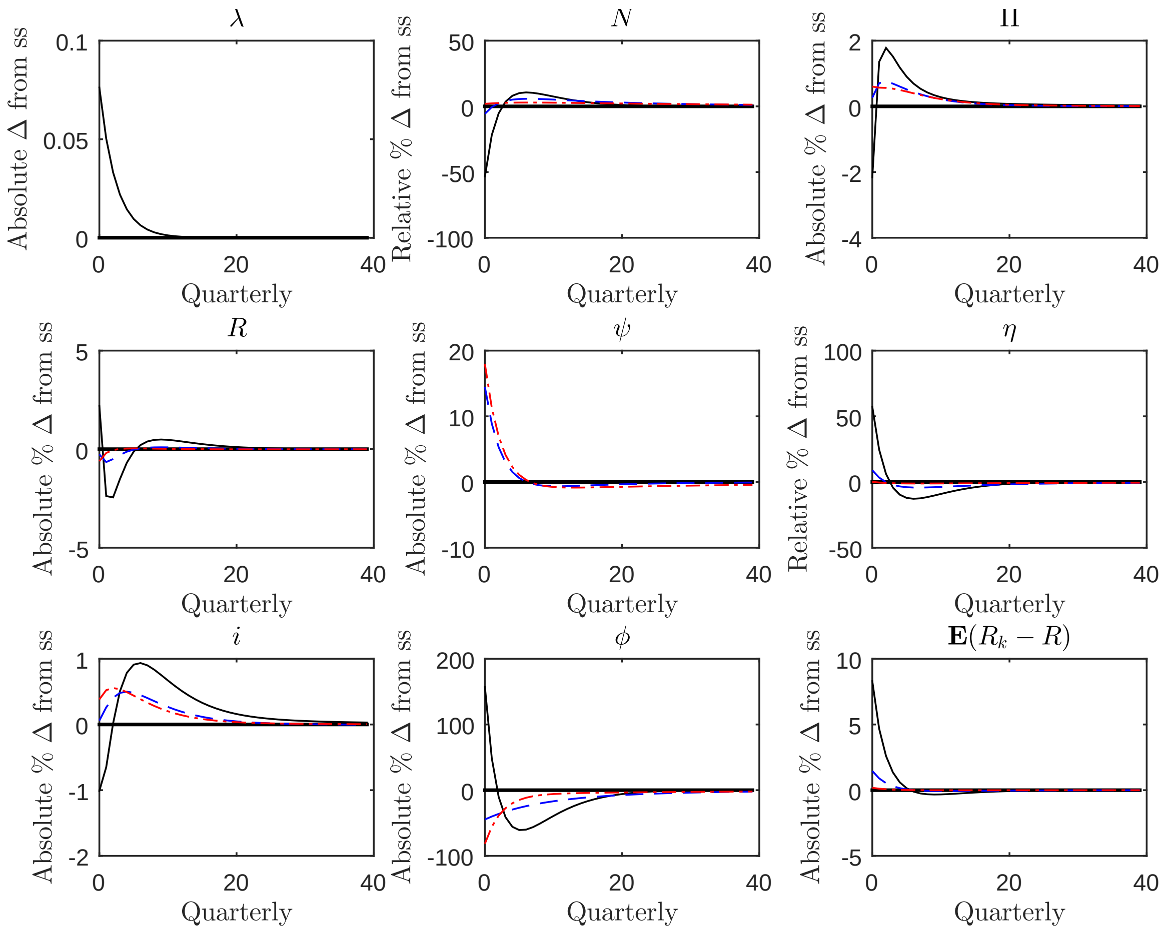


Figure 11: Financial variables' response to intermediary margin shock: one standard deviation from steady state. The solid curve is for the case of zero credit policy intervention ($\nu = 0$). The dashed curve is for the case of moderate credit policy intervention ($\nu = 10$). The dash-dotted curve is for the case of intensive credit policy intervention ($\nu = 100$). The nominal interest rate has a zero lower bound.

We use quarterly data on the data variables

$$\mathbf{Y}_t = \begin{pmatrix} \log(Y_t) - \log(Y_{t-1}) \\ \log(C_t) - \log(C_{t-1}) \\ \log(I_t) - \log(I_{t-1}) \\ \log(W_t) - \log(W_{t-1}) \\ \log(L_t) - \log(L_{t-1}) \\ \log \Pi_t \\ i_t \\ R_{k,t} \end{pmatrix}$$

from 1951Q4 to 2013Q4 for our estimation. For $R_{k,t}$ we use the quarterly return on the S&P 500 index, and unlever it using data on aggregate corporate leverage from the Flow of Funds tables. We ignore the zero lower bound on the nominal interst rate. We also add an AR(1) shock to the discount rate to identify the parameters (8 variables and 8 shocks). Formally, we calibrate the parameters Θ in Table 2 and estimate the dynamic parameters Ψ. The estimation is performed using Dynare. We use the Metropolis-Hasting algorithm to numerically compute the posterior distribution

$$f\left(\Psi|\mathbf{Y}_t, \Theta\right) \propto g\left(\mathbf{Y}_t|\Psi, \Theta\right) \cdot p(\Psi),$$

where $f\left(\Psi|\mathbf{Y}_t, \Theta\right)$ is the posterior distribution of the parameters, $g\left(\mathbf{Y}_t|\Psi, \Theta\right)$ is the likelihood function or the conditional distribution of the observables given the parameters, and $p(\Psi)$ is the prior distribution of the parameters. The prior means are based on the calibrations of Tables 2 and 3. We simulate the posterior using a sample of 20000 draws after dropping 40% of the draws. We report the priors and the estimated mean and the 90% HPD interval. In Table 4 we report the results of the estimation using all eight variables, and in Table 5 we report the results after dropping the variable for the return on assets $R_{k,t}$.

It is evident that the estimated shock sizes with asset return data are unreasonably large, even after imposing tight priors on the volatilities of the shocks. Particularly large is the volatility of the margin shock. This is because the likelihood function obtained using the Kalman filter in the Bayesian estimation is based on the first-order approximation around the deterministic steady state. The first-order approximated likelihood function suppresses the significant nonlinear structure of the model design. The nonlinear structural components are critical to capture the large fluctuations in risk premia and their important equilibrium feedback in the real economy. The Bayesian estimation based on first-order approximated likelihood functions performs successfully in DSGE models without financial intermediaries or risk premia data in

Smets and Wouters (2007). However, it fails in DSGE models with financial frictions, trying to capture the volatile dynamics of risk premia in the data by ignoring the nonlinear features in the model.

When we include the risk aversion and habit parameters in the estimation, the results are similar, with very large estimated values of the shock sizes.

Table 4: Estimated Parameters with R_k

Parameter	Description	Priors			Posteriors		
		Dist.	Mean	Stdev.	Mean	90% HPD interval	
Financial							
λ_{ss}^h	Steady state	Inv Gam	1.625	2	0.1523	0.1475	0.1571
λ_{ss}					0.8678	0.8642	0.8714
ρ_λ	Persistence	Beta	0.66	0.2	0.7190	0.7167	0.7213
σ_λ	Volatility	Inv Gam	0.5	0.2	0.6821	0.6192	0.7450
Capital Quality							
ρ_ξ	Persistence	Beta	0.66	0.2	0.6033	0.5996	0.6070
σ_ξ	Volatility	Inv Gam	0.5	0.2	1.3698	1.3020	1.4376
Discount Rate							
ρ_β	Persistence	Beta	0.66	0.2	0.9404	0.8929	0.9879
σ_β	Volatility	Inv Gam	0.5	0.2	1.4846	1.4778	1.4913
Monetary Policy							
ρ_{mp}	Persistence	Beta	0.66	0.2	0.5867	0.5860	0.5874
σ_{mp}	Volatility	Inv Gam	0.5	0.2	1.2175	1.2150	1.2199
Spending							
ρ_g	Persistence	Beta	0.66	0.2	0.3163	0.2877	0.3450
σ_g	Volatility	Inv Gam	0.5	0.2	0.2523	0.2476	0.2570
Markup							
ρ_{mk}	Persistence	Beta	0.66	0.2	0.9821	0.9720	0.9922
σ_{mk}	Volatility	Inv Gam	0.5	0.2	0.3807	0.3694	0.3921
TFP							
ρ_a	Persistence	Beta	0.66	0.2	0.6758	0.6645	0.6870
σ_a	Volatility	Inv Gam	0.5	0.2	1.0078	0.9667	1.0488
Investment							
ρ_z	Persistence	Beta	0.66	0.2	0.5070	0.4693	0.5466
σ_z	Volatility	Inv Gam	0.5	0.2	0.1461	0.1150	0.1171

Table 5: Estimated Parameters without R_k

Parameter	Description	Priors			Posteriors		
		Dist.	Mean	Stdev.	Mean	90% HPD interval	
Financial							
λ_{ss}^h	Steady state	Inv Gam	1.625	2	9.8212	9.3576	10.2124
λ_{ss}					0.0924	0.0892	0.0965
ρ_λ	Persistence	Beta	0.66	0.2	0.3864	0.3564	0.4226
σ_λ	Volatility	Inv Gam	0.5	0.2	0.6404	0.5850	0.6938
Capital Quality							
ρ_ξ	Persistence	Beta	0.66	0.2	0.5764	0.5481	0.6029
σ_ξ	Volatility	Inv Gam	0.5	0.2	0.1252	0.1135	0.1371
Discount Rate							
ρ_β	Persistence	Beta	0.66	0.2	0.0828	0.0274	0.1366
σ_β	Volatility	Inv Gam	0.5	0.2	0.0747	0.0693	0.0810
Monetary Policy							
ρ_{mp}	Persistence	Beta	0.66	0.2	0.6402	0.2994	0.9750
σ_{mp}	Volatility	Inv Gam	0.5	0.2	0.4773	0.2669	0.6937
Spending							
ρ_g	Persistence	Beta	0.66	0.2	0.9936	0.9883	0.9996
σ_g	Volatility	Inv Gam	0.5	0.2	0.0868	0.0799	0.0934
Markup							
ρ_{mk}	Persistence	Beta	0.66	0.2	0.9987	0.9975	0.9999
σ_{mk}	Volatility	Inv Gam	0.5	0.2	0.5112	0.4732	0.5495
TFP							
ρ_a	Persistence	Beta	0.66	0.2	0.9969	0.9938	0.9999
σ_a	Volatility	Inv Gam	0.5	0.2	0.0690	0.0633	0.0744
Investment							
ρ_z	Persistence	Beta	0.66	0.2	0.9204	0.9015	0.9385
σ_z	Volatility	Inv Gam	0.5	0.2	0.0888	0.0815	0.0964

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