Optimal Macroprudential Rules*

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Abstract

This paper presents a model to study macroprudential rules for bank capital requirements. It features two financial frictions: entrepreneurs' borrowing from bankers is constrained by the expected future value of their capital stock, while bankers' borrowing from households is limited by the regulatory bank capital requirement. Binding borrowing constraints create inefficient distribution and variance of consumption. I consider macroprudential rules based on the dynamics of several macroeconomic variables. Entrepreneurs benefit significantly from countercyclical macroprudential policy as it helps to reduce the variance and smooth the path of their consumption. If the Pareto weight in the social welfare function attached to entrepreneurs is not too low, all the optimal rules are countercyclical, and the welfare maximizing one-variable rule is based on loans. An optimal rule based on the loans-to-output ratio, as designed in the Basel III framework, achieves the lowest welfare gain.

1 Introduction

The importance of macroprudential dimension of financial regulation[1] was emphasized as early as by Crockett (2000).[2] But it was not until the recent global financial crisis that the problem started receiving substantial attention from both policymakers and academic economists. It is a general consensus now that addressing systemic risk is crucial, both taking into account the interlinkages between financial institutions, a potential for contagion and the role of especially big institutions on one hand; and time variations in aggregate risk on the other hand.

One of the most appealing and widely discussed macroprudential instruments is dynamic bank capital requirements. This can be viewed as an extension of a more standard microprudential requirement for bank capital. A real-world example is the countercyclical capital buffer—part of the Basel III framework, which is now gradually implemented from January 1, 2016 to January 1, 2019. The buffer must be built up by banks in "good times" and can be used as a safety net in "bad times". One problem with the capital regulation model of Basel III is that minimum bank capital requirements still remain procyclical because in recessions default probabilities rise increasing the estimated values of risk-weighted assets, and hence capital requirements per unit of loans increase. Higher capital losses in recessions further exacerbate the situation. Moreover, the choice of the credit-to-GDP ratio as an indicator of a build-up of systemic risk has also been questioned.[3]

Motivated by these problems, I study the welfare implications of simple macroprudential policy rules for minimum bank capital requirements based on the dynamics of several macroeconomic variables.[4] These rules in essence mimic the standard Taylor rules for policy interest rates. As indicators of systemic risk I consider loans, output, loans-to-output ratio, as well as loans and output jointly[5]. The one-variable rules have

[1]The objective of macroprudential regulation is to reduce the risk of failure of financial system as a whole. The focus of microprudential regulation is to limit the risks of individual institutions.

[2]See also Borio (2003).

[3]See Repullo and Saurina (2012) and Repullo and Suarez (2013) for the discussion of these issues.

[4]Notice that I focus on cyclical fluctuations of capital requirements about the (deterministic) steady state level which I calibrate. On the other hand, the optimal constant (steady state) level could be analyzed jointly with optimal cyclical response parameters. The optimal constant level in the baseline model happens to be significantly higher than values used in practice (see subsection 4.3). I, therefore, calibrate the steady state level and focus on cyclical fluctuations.

[5]There are many other possible indicators. See Drehmann et al. (2010) and (BCBS, 2010b).

an advantage of being more intuitive and easy to explain to the general public which is beneficial for the practical implementation of macroprudential policy. Moreover, a theoretical evaluation of a policy with the same objectives and the same indicator variable as in the countercyclical capital buffer of Basel III can be performed. The optimal two-variable rule, being a generalization of one-variable rules will necessarily attain at least as high welfare. However, it may be more difficult to implement this rule in practice.

The model that I employ contains some standard ingredients of dynamic general equilibrium models used for policy analysis, while at the same time including two key financial frictions. On one hand, risk averse entrepreneurs' borrowing from bankers is limited by the expected future value of their capital stock as in Kiyotaki and Moore (1997). On the other hand, risk averse bankers' borrowing from households is constrained by the regulatory requirement on the minimum amount of bank capital as in Iacoviello (2015). As a result, consumption smoothing of entrepreneurs and bankers is distorted when the corresponding borrowing constraints are binding. In this case, entrepreneurs' consumption is volatile also due to the dependence of the value of collateral on the price of capital and the resulting financial accelerator effect, while bankers' consumption is highly dependent on their net worth, which is affected by the regulatory capital requirements. Under the baseline calibration, the variance of entrepreneurs' consumption is inefficiently high, while the variance of bankers' consumption is inefficiently low.

First, I analyze rules responding to one target variable. I obtain parameter values that maximize the social welfare function defined as an expected value of a weighted sum of bankers', entrepreneurs' and households' value functions. In the baseline case, all the agents are given equal Pareto weights. For all the rules that I consider, optimal parameter values are positive, which means that optimal rules are countercyclical: capital requirements are raised when a target variable is above the steady state level. The intuition for this is simple. In the baseline case, the efficient equilibrium implies equal variance of consumption for utility maximizing agents at the beginning. This does not hold in the decentralized equilibrium. Countercyclical capital requirements help reduce the variance of entrepreneurs' consumption, relaxing the collateral constraint in recessions and tightening it in expansions by affecting the supply of loans, loan interest rate, investment and the price of physical capital. Bankers accumulate more net worth in expansions that can be used in recessions. With dynamic capital requirements, the variance of their consumption naturally increases. The equilibrium is thus closer to the efficient one. The welfare maximizing one-variable rule is based on loans as a target variable. At the same time, the rule based on the loans-to-output ratio achieves the lowest aggregate welfare gain. This corresponds to the criticism of using this indicator mentioned before.

Second, I consider a macroprudential rule responding to both loans and output. The optimal rule in the range of nonnegative parameter values is described by a strong response to loans and a mild response to output. Although, such a rule improves aggregate welfare a bit comparing to one-variable rules, it does not Pareto dominate any of the optimal one-variable rules. Moreover, the implementation of a two-variable rule in practice would be more difficult for several reasons. Hence, optimal one-variable rules seem to be working good enough, especially the rule based on loans as a target variable.

I explore how robust are the results to alternative Pareto weights in the social welfare criterion, alternative calibrations of the steady state loan-to-value (LTV) ratio, and alternative monetary policy rule. Countercyclical macroprudential policy is optimal if the Pareto weight for entrepreneurs is not too low. Entrepreneurs are subject to financial frictions and benefit from countercyclical policy. Households, however, would prefer procyclical policy. Countercyclical macroprudential policy is optimal for every rule considered if the LTV ratio is high enough. If entrepreneurs' borrowing is constrained too much, they benefit from countercyclical policy less and the optimal rule based on output is close to being constant or slightly procyclical. Rules based on other variables are still countercyclical. Finally, under flexible inflation targeting, all the optimal rules remain countercyclical as in the baseline case of strict inflation targeting.

There is no clear consensus on what the optimal macroprudential policy should be like. One set of papers study macroprudential subsidies or taxes. Bean et al. (2010) analyze the interaction between optimal monetary policy operated through a Taylor rule and macroprudential policy affecting bank capital through a levy/subsidy. They find that macroprudential policy is more effective than monetary policy in affecting credit supply, while they admit that the type of macroprudential policy they consider may be not realistic. They use a reduced-form welfare loss function in the analysis.

Perotti and Suarez (2011) study short-term funding decisions of heterogeneous banks in the presence of systemic externalities. In their model, decentralized equilibrium is associated with excessive systemic risk, and the distorition can be eliminated through a Pigovian tax on short-term borrowing. Gertler, Kiyotaki and Queralto (2012) in a framework of Gertler and Karadi (2011) analyze how a subsidy to equity issuance with a tax on total assets affects banks' decisions to take short-term debt. They find that this type of macroprudential policy generates a significant welfare gain. Bianchi and Mendoza (2013) study how state-contingent tax on debt can prevent overborrowing in decentralized equilibrium. This tax is welfare improving and significantly reduces the probability of crises. They also show that constant tax is much less effective, while dynamic tax based on the dynamics of debt is more effective than constant tax, but still significantly less effective than the optimal state-contingent tax.

Other authors focus on dynamic LTV ratios using models with collateral constrained borrowers. Funke and Paetz (2012) analyze linear and non-linear (threshold) dynamic LTV ratios. They conclude that both are effective in lowering the magnitude of housing price cycles, and the non-linear rule is more effective in doing so. They calibrate response parameters and do not conduct welfare analysis. Rubio and Carrasco-Gallego (2014) do conduct welfare analysis of an LTV rule responding to credit growth jointly with monetary policy rule. They conjecture a welfare loss function which includes the variances of inflation, output and borrowing and optimize it under coordination and no coordination between monetary and macroprudential authorities. In both cases there is a welfare gain for borrowers and a welfare loss for savers, while in aggregate there is a welfare gain, and it is higher under no coordination.

Finally, the most relevant for this paper is the literature studying bank capital requirements. Angeloni and Faia (2013) find that countercyclical capital requirements perform better than procyclical and constant requirements, though they do not optimize over the parameters of macroprudential rules. Repullo (2013) shows that welfare maximizing capital requirements are countercyclical. Angelini, Neri and Panetta (2014) also find that countercyclical macroprudential policy is welfare improving, but results depend on types of shocks hitting the economy, and whether monetary and macroprudential policy work in coordination or not. Martinez-Miera and Suarez (2014), on the contrary, find that countercyclical capital requirements are welfare reducing due to the increased systemic risk taking by banks. Mendicino et al. (2015) have a rich model with defaulting agents and find that countercyclical macroprudential policy is welfare improving, but welfare gains are small. Collard et al. (2016) study optimal Ramsey policy and state-contingent capital requirements. The optimal procyclicality or countercyclicality of capital requirements depend on the particular version of the model that they consider.

My contribution to this literature results from the unique (to my knowledge) combination of the following. First, I have a general equilibrium model where bankers, entrepreneurs and households are all risk-averse utility-maximizing agents. Second, both bankers and entrepreneurs are subject to financial frictions. Third, the model contains the standard features of monetary models with nominal rigidities. Fourth, the welfare analysis is conducted by constructing a social welfare function and considering what happens under a wide range of Pareto weights.

The paper is structured as follows. Section 2 describes the model. Section 3 outlines additional model properties, parameter values, and explains the model dynamics after shocks. Section 4 presents the welfare analysis. Section 5 contains robustness exercises. Section 6 concludes. The Appendix A contains the list of equilibrium conditions, characterization of the deterministic steady state of the model, as well as computational details.

2 Model

The model describes the behavior of bankers, entrepreneurs, households, capital good producers, retailers, final good producers and the central bank. There is also a macroprudential authority which conducts macroprudential policy as explained in section 4.

2.1 Bankers

Bankers are assumed to be utility maximizing agents. They consume the final good, while also playing the role of financial intermediaries—borrow from households and lend to entrepreneurs. Bankers operate subject to a regulatory constraint on the minimum amount of bank capital. The existence of such a constraint is taken as a fact, reflecting the real world practice. However, capital requirements can be rationalized in various ways.[6] This structure is thus similar to that in Iacoviello (2015).

The objective of the banker is to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_b^t U_b(C_t^b), \tag{1}$$

subject to

$$\frac{R_t^l}{1+\pi_t} L_{t-1} - \frac{R_{t-1}}{1+\pi_t} D_{t-1} + D_t = C_t^b + L_t, \tag{2}$$

$$(1-k_t^m) L_t \geq D_t, \tag{3}$$

where $\mathbb{E}_t(\cdot) \equiv \mathbb{E}(\cdot \mid \mathbb{I}_t)$ and $\mathbb{I}_t$ is the information set at t, $\beta_b \in (0,1)$ is the discount factor, $U_b(\cdot)$ is strictly concave and strictly increasing, satisfies Inada conditions, C_t^b is consumption[7], R_t^l is the gross nominal loan rate, π_t – inflation rate, L_t denotes loans, R_t – gross nominal deposit rate, D_t – deposits, k_t^m denotes the regulatory capital requirement.

The demand for deposits and supply of loans is described by the Euler equations

$$\frac{\gamma_t^b}{U_b'(C_t^b)} = 1 - \mathbb{E}_t \left\{ \beta_b \frac{U_b'(C_{t+1}^b)}{U_b'(C_t^b)} \frac{R_t}{1+\pi_{t+1}} \right\}, \tag{4}$$

$$\frac{\gamma_t^b}{U_b'(C_t^b)} (1-k_t^m) = 1 - \mathbb{E}_t \left\{ \beta_b \frac{U_b'(C_{t+1}^b)}{U_b'(C_t^b)} \frac{R_{t+1}^l}{1+\pi_{t+1}} \right\}, \tag{5}$$

where $\gamma_t^b \geq 0$ is the Lagrange multiplier on the regulatory constraint (3). It is evident that non-zero capital requirements create an equilibrium credit spread.

It can be shown that in the steady state, (4) is $\frac{\gamma^b}{U_b'(C^b)} = 1 - \frac{\beta_b}{\beta_h}$, where β_h is the household discount factor. Hence, the regulatory constraint is binding in the neighborhood of the steady state as long as $\beta_b < \beta_h$. In this case, bankers' consumption smoothing is distorted.

One alternative approach to model the banking sector would be to follow the literature initiated by Gertler and Karadi (2011). For our needs, however, the incentive

[6] See, e.g. Holmstrom and Tirole (1997). In their model, a certain amount of intermediary capital is required for credible monitoring of entrepreneurs.

[7] The variables are in real terms (deflated by the aggregate price level), unless stated otherwise.

constraint for bankers have to be replaced with the regulatory capital requirement. It is interesting that such an approach leads to problems with existence of equilibria. The reason is that in the original framework the leverage ratio is an endogenous, forward-looking variable tightly interrelated with the bankers' franchise value. When the regulatory constraint is introduced, franchise value plays no role in the optimality conditions, other than defining whether the regulatory constraint is binding or not. There is one forward-looking variable less, which creates problems. Therefore, it was decided to introduce strictly concave utility and eliminate entry and exit of bankers (otherwise there would be problems with aggregation), which yields the framework outlined above.

2.2 Entrepreneurs

The entrepreneur uses capital and labor to produce the wholesale good which is sold to the continuum of retailers as in Bernanke, Gertler and Gilchrist (1999). The entrepreneur obtains utility from consuming the final good as in Iacoviello (2005) and can borrow from the bank subject to a collateral constraint, following Kiyotaki and Moore (1997). The objective is to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_e^t U_e(C_t^e), \tag{6}$$

subject to

$$\frac{Y_t^w}{X_t} + L_t = W_t N_t + Q_t I_t + \frac{R_t^l}{1+\pi_t} L_{t-1} + C_t^e, \tag{7}$$

$$m_t K_t \mathbb{E}_t \left\{ Q_{t+1}(1+\pi_{t+1}) \right\} \geq \mathbb{E}_t \{ R_{t+1}^l \} L_t, \tag{8}$$

where $\beta_e \in (0,1)$ is the discount factor, $U_e(\cdot)$ is strictly concave and strictly increasing, satisfies Inada conditions, C_t^e is consumption, $Y_t^w \equiv A_t K_{t-1}^{\alpha} N_t^{1-\alpha}$ is the wholesale output, A_t is the exogenous total factor productivity (TFP), K_t is physical capital and $\alpha \in (0,1)$ is the capital share, N_t is labor, X_t is the gross markup of the final good price over the wholesale good price, W_t is the real wage, Q_t is the real price of the capital good, $I_t \equiv K_t - (1-\delta)K_{t-1}$ is investment and $\delta \in [0,1]$ is the depreciation rate, m_t denotes the exogenous LTV ratio.

The following optimality conditions can then be derived.

$$W_t = (1-\alpha)\frac{Y_t^w}{N_t X_t}, \tag{9}$$

$$\begin{aligned} Q_t = \mathbb{E}_t \left\{ \beta_e \frac{U_e'(C_{t+1}^e)}{U_e'(C_t^e)} \left(\alpha \frac{Y_{t+1}^w}{K_t X_{t+1}} + (1-\delta) Q_{t+1} \right) \right\} \\ + \frac{\gamma_t^e}{U_e'(C_t^e)} m_t \mathbb{E}_t \left\{ Q_{t+1}(1+\pi_{t+1}) \right\}, \end{aligned} \tag{10}$$

$$\frac{\gamma_t^e}{U_e'(C_t^e)} \mathbb{E}_t\{R_{t+1}^l\} = 1 - \mathbb{E}_t \left\{ \beta_e \frac{U_e'(C_{t+1}^e)}{U_e'(C_t^e)} \frac{R_{t+1}^l}{1+\pi_{t+1}} \right\}, \tag{11}$$

where $\gamma_t^e \geq 0$ is the Lagrange multiplier on the collateral constraint (8). Equation (9) represents labor demand, (10) describes the demand for capital, while (11)—intertemporal consumption choice and demand for loans.

If $\gamma_t^e > 0$, the collateral constraint is binding. It can be shown that this is the case in the neighborhood of the steady state as long as $\beta_e < \widetilde{\beta}_e \equiv \frac{\beta_b \beta_h}{\beta_b + k^m(\beta_h - \beta_b)}$. If $\beta_b < \beta_h$ and $k^m \in (0,1)$, then $\widetilde{\beta}_e \in (\beta_b, \beta_h)$. When the collateral constraint is binding, the price of capital reflects not only its expected marginal product, but also its marginal value as collateral.

2.3 Households

The household chooses how much to consume, save through bank deposits, and how much labor to supply to the entrepreneur. The objective of the household is to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_h^t U_h(C_t^h, N_t), \tag{12}$$

subject to

$$W_t N_t + \frac{R_{t-1}}{1+\pi_t} D_{t-1} + T_t = C_t^h + D_t, \tag{13}$$

where $\beta_h \in (0,1)$ is the discount factor, $U_h(\cdot)$ is strictly concave, strictly increasing in the first argument and strictly decreasing in the second argument, satisfies Inada conditions, C_t^h is consumption, T_t – net transfers (profits of capital good producers and retailers).

The optimality conditions associated with this problem are

$$\frac{\partial U_h}{\partial N_t} + W_t \frac{\partial U_h}{\partial C_t^h} = 0, \tag{14}$$

$$\mathbb{E}_t \left\{ \beta_h \frac{\partial U_h / \partial C_{t+1}^h}{\partial U_h / \partial C_t^h} \frac{R_t}{1 + \pi_{t+1}} \right\} = 1, \tag{15}$$

where $\frac{\partial U_h}{\partial C_{t+j}^h}$ should be understood as $\frac{\partial U_h(C_{t+j}^h, N_{t+j})}{\partial C_{t+j}^h}$. Equation (14) describes labor supply, while (15)—consumption/saving choice.

2.4 Capital good producers

The capital good producer makes physical capital using the final good as an input. The real period profits are $\Pi_t^k \equiv Q_t I_t - C_t^k$, where C_t^k is consumption of the final good and $I_t \equiv C_t^k \left[1 - \frac{\phi_k}{2} \left(\frac{C_t^k}{C_{t-1}^k} - 1 \right)^2 \right]$, where $\phi_k \geq 0$ is the adjustment cost parameter. The functional form is analogous to the one used by Andrés, Arce and Thomas (2014). The capital good producer is owned by the household, so the objective is to maximize $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_{0,t}^h \Pi_t^k$, where $\beta_{0,t}^h \equiv \beta_h^t \frac{\partial U_h / \partial C_t^h}{\partial U_h / \partial C_0^h}$ is the relevant stochastic discount factor. The supply of capital is then described by

$$\begin{aligned} 1 = Q_t & \left[1 - \frac{\phi_k}{2} \left(\frac{C_t^k}{C_{t-1}^k} - 1 \right)^2 - \phi_k \left(\frac{C_t^k}{C_{t-1}^k} - 1 \right) \frac{C_t^k}{C_{t-1}^k} \right] \\ & + \mathbb{E}_t \left\{ \beta_{t,t+1}^h Q_{t+1} \phi_k \left(\frac{C_{t+1}^k}{C_t^k} - 1 \right) \left(\frac{C_{t+1}^k}{C_t^k} \right)^2 \right\}. \end{aligned} \tag{16}$$

Hence, the price of capital is unity in the steady state.

2.5 Final good producers

Final good producers use the CES production technology $Y_t \equiv \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$, where $Y_t(i)$ is the quantity of variety i used, and $\epsilon > 1$ is the elasticity of substitution between varieties. Minimizing costs $\int_0^1 P_t(i) Y_t(i) di$, where $P_t(i)$ is the price of variety i, subject to the production technology under perfect competition leads to the demand for retail good varieties of the form $Y_t(i) = Y_t \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}$, where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ is a variant of an aggregate price level.

2.6 Retailers

There is a measure one continuum of retailers who are owned by the household. Each retailer buys the wholesale good, differentiates it and sells its variety to the final good producer. Every period, the price of each variety "survives" to the next period with the probability $\theta \in [0,1]$ as in Calvo (1983). The real period profits in period $t+j$ under the price P_t^* set in the period t are $\Pi_{t+j|t} \equiv \left(\frac{P_t^*}{P_{t+j}} - \frac{1}{X_{t+j}}\right) Y_{t+j} \left(\frac{P_t^*}{P_{t+j}}\right)^{-\epsilon}$. Let $V_{j|k}$, where $j \geq k$, denote the value of a retailer in the period j under the price set in the period k. Then the objective of each retailer able to update its price in the period t is to choose P_t^* to maximize $V_{t|t} = \Pi_{t|t} + \mathbb{E}_t \left\{\beta_{t,t+1}^h \left(\theta V_{t+1|t} + (1-\theta) V_{t+1|t+1}\right)\right\}$. We then have an optimality condition $\frac{\partial V_{t|t}}{\partial P_t^*} = \frac{\partial \Pi_{t|t}}{\partial P_t^*} + \mathbb{E}_t \left\{\beta_{t,t+1}^h \theta \frac{\partial V_{t+1|t}}{\partial P_t^*}\right\} = 0$ or

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta_{t,t+j}^h \theta^j \frac{\partial \Pi_{t+j|t}}{\partial P_t^*} = 0. \tag{17}$$

After working with this equation, it can be represented as a system

$$1 + \pi_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\Pi_{1,t}}{\Pi_{2,t}}, \tag{18}$$

$$\Pi_{1,t} = (1+\pi_t)^{\epsilon} \left(\frac{\partial U_h}{\partial C_t^h} \frac{Y_t}{X_t} + \beta_h \theta \mathbb{E}_t \left\{\Pi_{1,t+1}\right\}\right), \tag{19}$$

$$\Pi_{2,t} = (1+\pi_t)^{\epsilon-1} \left(\frac{\partial U_h}{\partial C_t^h} Y_t + \beta_h \theta \mathbb{E}_t \left\{\Pi_{2,t+1}\right\}\right), \tag{20}$$

where $\pi_t^* \equiv \frac{P_t^*}{P_{t-1}} - 1$.

Finally, as the price survival probability is constant across retailers and across time and we have a continuum of retailers, $P_t = \left(\theta P_{t-1}^{1-\epsilon} + (1-\theta) P_t^{*1-\epsilon}\right)^{\frac{1}{1-\epsilon}}$ or

$$(1+\pi_t)^{1-\epsilon} = \theta + (1-\theta)(1+\pi_t^*)^{1-\epsilon}. \tag{21}$$

2.7 Monetary policy

The central bank is assumed to be following a simple interest rate rule of the form

$$R_t = R + \phi_\pi(\pi_t - \pi), \tag{22}$$

where variables without time indices are values in the steady state and $\phi_\pi \geq 0$, but in practice the requirement is more strict for the model to have a unique stable solution. Strict inflation targeting is assumed for simplicity, as monetary policy will be taken as given in the welfare analysis. In subsection 5.3, I explore how the results change under flexible inflation targeting.

2.8 Market clearing

First, the aggregate demand from retailers must be equal to the supply of the wholesale good: $\int_0^1 Y_t(i)di = Y_t^w$. Let $\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} di$. Using the demand equation for retailers' output, we get

$$Y_t \Delta_t = Y_t^w. \tag{23}$$

Taking into account the same considerations as for deriving equation (21), Δ_t can be rewritten recursively as

$$\Delta_t = \theta(1+\pi_t)^\epsilon \Delta_{t-1} + (1-\theta)\left(\frac{1+\pi_t}{1+\pi_t^*}\right)^\epsilon. \tag{24}$$

Finally, aggregate consumption must be equal to the output of the final good:

$$C_t^b + C_t^e + C_t^h + C_t^k = Y_t. \tag{25}$$

The list of all equilibrium conditions is provided in Appendix A.1 for convenience.

3 Model properties

3.1 Functional forms and exogenous processes

I specify the period utility from consumption as a constant relative risk aversion (CRRA) function for all agents. The household also obtains disutility from supplying labor, and I assume separability in the period utility function. Particularly, I define $U_h(C_t^h, N_t) \equiv \frac{{C_t^h}^{1-\sigma_h}-1}{1-\sigma_h} \cdot \mathbb{1}(\sigma_h \neq 1) + \ln C_t^h \cdot \mathbb{1}(\sigma_h = 1) - \frac{N_t^{1+\phi}}{1+\phi}$ and $U_i(C_t^i) \equiv \frac{{C_t^i}^{1-\sigma_i}-1}{1-\sigma_i} \cdot \mathbb{1}(\sigma_i \neq 1) + \ln C_t^i \cdot \mathbb{1}(\sigma_i = 1)$ for $i \in \{b, e\}$, where $\sigma_i > 0$ is the coefficient of relative risk aversion of agent $i \in \{b, e, h\}$, and $\phi \geq 0$ is the inverse Frisch elasticity of labor supply.[8]

The exogenous processes for logarithms[9] of TFP and LTV ratios are specified as AR(1):

$$\ln(A_t) = \rho_a \ln(A_{t-1}) + (1-\rho_a)\ln(A) + \epsilon_t^a, \tag{26}$$

$$\ln(m_t) = \rho_m \ln(m_{t-1}) + (1-\rho_m)\ln(m) + \epsilon_t^m, \tag{27}$$

where $\rho_a, \rho_m \in [0,1)$; $\epsilon_t^i \sim \mathcal{N}(0, \tau_i^2)$ are independent random variables in i and t, $\tau_i > 0$, $i \in \{a, m\}$. A and m are the steady state values of A_t, m_t, as well as their approximate expected values[10].

The deterministic steady state of the model is described in Appendix A.2.

3.2 Calibration

The parameter values that I set are outlined in Table 1. The physical capital share (α) and depreciation rate (δ) take standard values used in the literature when the analysis is applied to the US economy using quarterly data. The household discount factor (β_h) implies an annualized steady state real deposit rate of 4.1%. The steady state minimum bank capital requirement (k^m) is close to the minimum total capital requirement recommended in the Basel III framework (8% plus 2.5% conservation

[8] If $\sigma_h = 1$, $U_h(\cdot)$ has a famous property of being consistent with the balanced growth path in the real business cycle model (King, Plosser and Rebelo, 1988).

[9] To ensure that $A_t, m_t > 0$.

[10] Since ϵ_t^a is normal and the process for $\ln(A_t)$ is stationary, A_t is log-normal with mean $Ae^{\frac{\tau_a^2}{2(1-\rho_a^2)}}$, which is very close to A for reasonable parameter values. Similar for m_t.

Table 1: Parameter values

Parameter	Value	Description
α	1/3	Physical capital share in wholesale good production
β_b	0.95	Bankers' discount factor
β_e	0.95	Entrepreneurs' discount factor
β_e	0.99	Households' discount factor
δ	0.025	Physical capital depreciation rate
ϵ	6	Elasticity of substitution between retail good varieties
k^m	0.1	Steady state value of bank capital requirement
m	0.9	Steady state value of LTV ratio
ω_b	1/3	Pareto weight for bankers
ω_e	1/3	Pareto weight for entrepreneurs
ω_h	1/3	Pareto weight for households
ϕ	1	Inverse of the Frisch elasticity of labor supply
ϕ_k	2.4	Investment adjustment cost parameter
ϕ_π	1.5	Central bank's response to inflation parameter
π	0.005	Steady state inflation rate
ρ_a	0.9	TFP AR(1) autocorrelation coefficient
ρ_m	0.9	LTV ratio AR(1) autocorrelation coefficient
σ_b	1	Bankers' utility RRA coefficient
σ_e	1	Entrepreneurs' utility RRA coefficient
σ_h	1	Households' utility RRA coefficient
τ_a	0.001	Standard deviation of the TFP shock
τ_m	0.001	Standard deviation of the LTV shock
θ	0.75	Retail price survival probability

buffer (BCBS, 2010a, p. 12 and 55)).[11] The bankers' discount factor then implies a steady state credit spread (difference between the loan and deposit rates) of about 1.8%. The entrepreneurs' discount factor (β_e) is set to a value similar to Iacoviello (2015). It implies that the collateral constraint is binding in the neighborhood of the steady state. The price survival probability (θ) takes a standard value which implies that on average prices of retail good varieties remain constant for a year. The value of the elasticity of substitution between wholesale good varieties (ϵ) then implies a steady state markup of final retail good price over wholesale good price (X) of 1.2 approximately, which is a standard target in the literature. The steady state LTV ratio (m) takes different values in the literature, e.g. 0.89 in Iacoviello (2005) and 0.9

[11]However, see the discussion in subsection 4.3.

in Iacoviello (2015), 0.64 in Andrés, Arce and Thomas (2014), 0.35 in Gerali et al. (2010).[12] I set m to 0.9 because this implies the optimal constant value of k^m of 0.23 which is the closest to the calibrated steady state value. The effect of different calibration of m on the results is explored in subsection 5.2. It is difficult to calibrate Pareto weights in the social welfare function ($\omega_i, i \in \{b, e, h\}$, see subsection 4.1), and I set them to be equal in the baseline case. This corresponds to the utilitarian measure of social welfare and to the fact that bankers, entrepreneurs and households are all representative agents from populations of measure one in the model. The effect of alternative Pareto weights is analyzed in subsection 5.1. The inverse Frisch elasticity of labor supply (ϕ) also varies substantially in the literature, the estimates depend on the data used (Chetty et al., 2011). I set the parameter to a "neutral" value, following (Galí, 2008, p. 52). The investment adjustment cost parameter (ϕ_k) is set as in Andrés, Arce and Thomas (2014). The response to deviations of inflation from the steady state in the interest rate rule (ϕ_π) takes a standard value, satisfying the Taylor principle. The steady state annual inflation rate (π) is about 2% which is consistent with the dynamics of advanced economies in non-crisis times.[13] Autocorrelation coefficients (ρ_a, ρ_m) are quite high, which is a standard finding for the TFP process. As for the LTV process, it is difficult to estimate it from the data, as also mention Andrés, Arce and Thomas (2013). I base the calibration of ρ_m on the posterior mean in Gerali et al. (2010). The standard deviations of shocks (τ_a, τ_m) are set to small values to have borrowing constraints mostly remain binding in the simulations. For simplicity, log utility from consumption is assumed for all agents ($\sigma_i = 1\ \forall i \in \{b, e, h\}$).

[12] In all these papers, however, the "value" is the value of a stock of housing, while in my case it is the value of the capital stock. However, the general idea is the same—a durable asset used as collateral, following Kiyotaki and Moore (1997).

[13] It also corresponds to the current IMF projections for advanced economies for 2018 and beyond.

3.3 Impulse responses

3.3.1 Productivity shock

The impulse responses[14] to a -0.05%[15] TFP shock ($\epsilon_t^a = -0.0005$) are shown in Figure 1. The shock leads to the decrease in the marginal product of labor and expected marginal product of capital. The demand for factors of production falls and their prices fall too. Entrepreneurs supply less wholesale output, marginal cost for retailers rises and inflation increases. The central bank responds increasing the deposit rate. The cost of borrowing for bankers rises, they decide to supply less loans, and the loan rate increases. Bankers' consumption reflects the dynamics of net worth. Lower wage income and higher deposit rate pushes household consumption downward. Lower expected value of collateral and rising loan rate curtail borrowing possibilities for entrepreneurs, and this negatively affects their consumption, which is volatile due to binding constraints and inhibited consumption smoothing.

3.3.2 LTV ratio shock

The impulse responses to an approximately -0.07% LTV ratio shock ($\epsilon_t^m = -0.0005$) are shown in Figure 2. The shock directly tightens the collateral constraint. On impact, borrowing decreases not very much, but the expected future decrease in borrowing possibilities immediately has a significant impact on entrepreneurs' consumption and demand for factors of production. The falling price of capital reinforces the tightening of the collateral constraint. Output decreases due to demand forces initiated by the entrepreneur, which pushes inflation down. The central bank reacts decreasing the deposit rate. Saving becomes less attractive for households and they decide to increase consumption, though after some periods it falls below the steady state value. The effective demand for loans and loan interest rate are falling which

[14] The impulse responses are obtained using the second order Taylor approximation of the model equations about the deterministic steady state, assuming that borrowing constraints are always binding.

[15] The value of the shock is small and reflects the standard deviation used in the welfare analysis section. Standard deviations of shocks have to be set small enough for borrowing constraints to remain mostly binding in the simulations. In the case of impulse responses to negative shocks, this issue is less relevant, however, because agents are more constrained after the shocks. But to be consistent with the welfare analysis part, the same values of standard deviations are set.

outweighs the lower cost of borrowing for bankers, so their net worth and consumption decreases.

4 Macroprudential policy and welfare

I consider the regulatory bank capital requirements (k_t^m) as a potential instrument of macroprudential policy, in line with the countercyclical capital buffer of the Basel III framework. I limit the analysis to continuous linear macroprudential rules.

4.1 Welfare criterion

As a welfare criterion, I consider the expected aggregate welfare of bankers, entrepreneurs and households—the utility maximizing agents in this model. The conditional aggregate welfare $\mathcal{W}_t$ is defined as

$$\mathcal{W}_t \equiv \omega_b \mathcal{W}_t^b + \omega_e \mathcal{W}_t^e + \omega_h \mathcal{W}_t^h, \tag{28}$$

where $\omega_b, \omega_e, \omega_h \geq 0, \omega_b + \omega_e + \omega_h = 1$ and

$$\mathcal{W}_t^b = U_b(C_t^b) + \beta_b \mathbb{E}_t \left\{ \mathcal{W}_{t+1}^b \right\}, \tag{29}$$

$$\mathcal{W}_t^e = U_e(C_t^e) + \beta_e \mathbb{E}_t \left\{ \mathcal{W}_{t+1}^e \right\}, \tag{30}$$

$$\mathcal{W}_t^h = U_h(C_t^h, N_t) + \beta_h \mathbb{E}_t \left\{ \mathcal{W}_{t+1}^h \right\} \tag{31}$$

are value functions of the banker, entrepreneur and household. The welfare criterion is then $\mathbb{E}(\mathcal{W}_t)$.

To ease the interpretation of results, it is convenient to report the implied permanent differences in consumption as proposed by Lucas (1987). Let $\widetilde{\mathcal{W}_t^h}$ be the conditional welfare of the household under the macroprudential policy in place. Notice that $\mathbb{E}(\mathcal{W}_t^h) = \mathbb{E}\left(\sum_{i=0}^{\infty} \beta_h^i U_h(C_{t+i}^h, N_{t+i})\right)$ by the law of iterated expectations. Then we seek for λ that satisfies $\mathbb{E}\left(\widetilde{\mathcal{W}_t^h}\right) = \mathbb{E}\left(\sum_{i=0}^{\infty} \beta_h^i U_h([1+\lambda^h] C_{t+i}^h, N_{t+i})\right)$. At this point, the assumption of log utility from consumption for households is useful because λ can be computed analytically.[16] Moreover, the expression is the same for

[16]For agents who do not supply labor, λ can be obtained analytically in the general case of

any log utility agent $i \in \{b, e, h\}$:

$$\lambda_i = \exp\left[(1-\beta_i)\mathbb{E}\left(\widetilde{\mathcal{W}}_t^i - \mathcal{W}_t^i\right)\right] - 1. \tag{32}$$

Similarly, the expression for λ applied to the aggregate welfare can be obtained, that is what are the implied permanent differences in consumption for all agents at once. In particular, λ satisfies

$$\mathbb{E}\left(\widetilde{\mathcal{W}}_t\right) = \mathbb{E}\left(\sum_{i=0}^{\infty}\Big[\omega_b\beta_b^i U_b([1+\lambda]C_{t+i}^b) + \omega_e\beta_e^i U_e([1+\lambda]C_{t+i}^e) + \omega_h\beta_h^i U_h([1+\lambda]C_{t+i}^h, N_{t+i})\Big]\right). \tag{33}$$

Hence,

$$\lambda = \exp\left[\frac{\mathbb{E}\left(\widetilde{\mathcal{W}}_t - \mathcal{W}_t\right)}{\frac{\omega_b}{1-\beta_b} + \frac{\omega_e}{1-\beta_e} + \frac{\omega_h}{1-\beta_h}}\right] - 1. \tag{34}$$

4.2 Efficient allocation

Let us consider the problem of a social planner, maximizing the conditional aggregate welfare (28) subject to the definition of production function $Y_t \equiv A_t K_{t-1}^{\alpha} N_t^{1-\alpha}$ (there are no nominal rigidities), market clearing condition (25), and the definition of production function for physical capital $K_t - (1-\delta)K_{t-1} \equiv C_t^k \left[1 - \frac{\phi_k}{2}\left(\frac{C_t^k}{C_{t-1}^k} - 1\right)^2\right]$.

The social planner's problem here cannot be specified as a dynamic programming problem due to differences in discount factors of utility maximizing agents. Therefore, we need to study the sequential problem. Let $A^t \equiv \{A_0, A_1, \ldots, A_t\}$ be the history of the TFP process. Equivalently, $A^t \equiv \{A^{t-1}, A_t\}$. Let $F_t(A^t)$ be the cumulative distribution function (cdf) of A^t and let $G_t(A_t|A^{t-1})$ be the conditional cdf of A_t given A^{t-1}. Denoting the corresponding probability density functions with lower case letters, we have $f_t(A^t) \equiv g_t(A_t|A^{t-1})f_{t-1}(A^{t-1})$. Then the Lagrangian of the

constant RRA utility. Namely, when $\sigma_i \neq 1$, then $\lambda_i = \left[\frac{1+(1-\beta_i)(1-\sigma_i)\mathbb{E}\left(\widetilde{\mathcal{W}}_t^i\right)}{1+(1-\beta_i)(1-\sigma_i)\mathbb{E}\left(\mathcal{W}_t^i\right)}\right]^{\frac{1}{1-\sigma_i}} - 1$.

planner's problem takes the form

$$\mathcal{L} = \sum_{t=0}^{\infty} \int \Bigg\{ \int \Bigg[\omega_b \beta_b^t \ln C_t^b(A^t) + \omega_e \beta_e^t \ln C_t^e(A^t) + \omega_h \beta_h^t \left(\ln C_t^h(A^t) - \frac{N_t^{1+\phi}(A^t)}{1+\phi} \right)$$
$$+ \lambda_t(A^t) \Big(A_t K_{t-1}^{\alpha}(A^{t-1}) N_t^{1-\alpha}(A^t) - C_t^b(A^t) - C_t^e(A^t) - C_t^h(A^t) - C_t^k(A^t) \Big)$$
$$+ \gamma_t(A^t) \left(C_t^k(A^t) \left[1 - \frac{\phi_k}{2} \left(\frac{C_t^k(A^t)}{C_{t-1}^k(A^{t-1})} - 1 \right)^2 \right] - K_t(A^t) + (1-\delta) K_{t-1}(A^{t-1}) \right) \Bigg]$$
$$dG_t \left(A_t | A^{t-1} \right) \Bigg\} dF_{t-1} \left(A^{t-1} \right), \tag{35}$$

where time $t-1$ variables are now indexed by a particular history A^{t-1}, and time t variables are indexed by history $A^t \equiv \{A^{t-1}, A_t\}$ which results from the particular A^{t-1} mentioned above.

The efficient allocation is described by the following equations which hold for any history of the TFP process and $t \geq 0$.

$$\lambda_t = \frac{\omega_i \beta_i^t}{C_t^i} \quad \forall i \in \{b, e, h\}, \tag{36}$$

$$\frac{C_t^i}{C_t^j} = \frac{\omega_i}{\omega_j} \left(\frac{\beta_i}{\beta_j} \right)^t \quad \forall i, j \in \{b, e, h\}, \tag{37}$$

$$C_t^h N_t^{\phi} = (1-\alpha) A_t K_{t-1}^{\alpha} N_t^{-\alpha}, \tag{38}$$

$$\frac{\gamma_t}{\lambda_t} = \mathbb{E}_{A_{t+1}|A^t} \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(\alpha A_{t+1} K_t^{\alpha-1} N_{t+1}^{1-\alpha} + (1-\delta) \frac{\gamma_{t+1}}{\lambda_{t+1}} \right) \right\}, \tag{39}$$

$$1 = \frac{\gamma_t}{\lambda_t} \left[1 - \frac{\phi_k}{2} \left(\frac{C_t^k}{C_{t-1}^k} - 1 \right)^2 - \phi_k \left(\frac{C_t^k}{C_{t-1}^k} - 1 \right) \frac{C_t^k}{C_{t-1}^k} \right]$$
$$+ \mathbb{E}_{A_{t+1}|A^t} \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{\gamma_{t+1}}{\lambda_{t+1}} \phi_k \left(\frac{C_{t+1}^k}{C_t^k} - 1 \right) \left(\frac{C_{t+1}^k}{C_t^k} \right)^2 \right\}, \tag{40}$$

The condition (37) is of particular importance. If $\omega_i = \frac{1}{3} \; \forall i \in \{b, e, h\}$ as set in the baseline case in the welfare analysis, then in the initial period there should be equal variance of consumption for all utility maximizing agents. This is far from what is observed in the decentralized equilibrium under constant capital requirements: the variance of entrepreneurs' consumption is significantly higher than the variance of household consumption, while the variance of bankers' consumption is much lower. The condition also shows that the mean and variance of bankers' and entrepreneurs' consumption relative to household consumption is decreasing over time. This is because bankers and entrepreneurs are more impatient than households and value future consumption less, which is taken into account by the social planner. In terms of variance of consumption, this makes the difference between the decentralized and efficient allocation even bigger for entrepreneurs, while there is convergence for bankers up to some point, when the difference starts to build up again.

The model in section 2 is stationary, therefore no policy would be able to replicate the time patterns implied in the equation (37). If the social planner used β_h to discount future utility for all utility maximizing agents, then the situation would be simpler—it would just imply that relative levels, means and variances of consumption correspond to relative Pareto weights $\forall t \geq 0$. In particular, under the baseline Pareto weights, the above result of equal variance of consumption for the initial period would hold for all periods. In a sense, I focus on this simple case in what follows, or, equivalently, focus on the initial period differences. This could be rationalized based on the observation that if the planner solved the optimization problem in the period $t = 1$ again, then the new allocation for period $t = 1$ would imply the equal variance result as this period would be the initial period. Of course, this is exactly due to the fact that the optimization problem cannot be represented as a dynamic programming problem.

As for the other optimality conditions, equations (38)-(40) would hold in the decentralized equilibrium if $X_t = \Delta_t = 1$ (no monopolistic competition and nominal rigidities), households and entrepreneurs discount future utility equally ($\beta_h = \beta_e$), collateral constraint is not binding (implied by the previous condition). Then it follows that $Q_t = \frac{\gamma_t}{\lambda_t}$.

4.3 Optimal level of constant capital requirements

Consider the rule specifying that bank capital requirements are constant, that is $k_t^m = k^m \; \forall t$. What level of k^m is welfare maximizing? Figure 3 shows social welfare as a function of the level of constant capital requirements. Under the baseline calibration, the optimal k^m is 0.23[17]. Relative to the economy with $k^m = 0.1$ (calibrated value), the implied differences in permanent consumption are $\lambda_b = 94.33\%, \lambda_e = -8.12\%, \lambda_h = -6.90\%, \lambda = 3.23\%$.

When capital requirements are increased, households and entrepreneurs suffer from the lower scale of saving and borrowing possibilities. Bankers, however, benefit in this framework because their net worth and consumption increases. In the baseline model, the gains of bankers first dominate the losses of households and entrepreneurs up to a point when the situation is reversed, so that there is a unique maximizer. Obviously, this is a very simple framework for it to have a well-grounded optimal level of constant capital requirements (which set a limit on bank borrowing in the steady state). See Mendicino et al. (2015) for a macroeconomic model with defaulting agents, deposit insurance, and clear trade-offs from capital regulation. I, therefore, calibrate the level of capital requirements in the steady state to a value closer to the one used in practice, and focus on the analysis of dynamic capital requirements which fluctuate about the calibrated steady state value.

4.4 Macroprudential rules with one target variable

In this subsection, I present the welfare analysis of macroprudential rules of the form

$$k_t^m = k^m + \nu \left(\frac{x_t}{x} - 1 \right), \tag{41}$$

or alternatively

$$k_t^m = k^m + \nu \left(x_t - x \right), \tag{42}$$

where x_t is a target variable and x is its steady state value. I consider output, loans and the loans-to-output ratio as target variables[18]. The set of target variables is chosen based on considerations by policy institutions, such as Basel Committee on Banking Supervision[19]. The rules based on output or loans are specified according to (41), while for the case of the loans-to-output ratio it is more appropriate to use

[17] Computational details are explained in Appendix A.3.

[18] I also considered rules with the response to the growth rate (or first difference in the case of the loans-to-output ratio) of a variable instead of its percent deviation (or deviation) from the steady state. Such rules do not improve over the rules considered in this section.

[19] In particular, see Drehmann et al. (2010) and (BCBS, 2010b).

(42).

Table 2 reports parameter values which maximize aggregate welfare, as well as implied welfare gains in permanent consumption units. Figure 4 contains plots showing aggregate welfare as a function of the corresponding parameter in the macroprudential rule. The model has a stable solution when $\nu \in [-1.25, 2.73], [-0.59, 0.89], [-0.15, 0.13]$ for output, loans, and loans-to-output ratio rules respectively.

Table 2: Welfare gains under optimal one-variable rules

	Output	Loans	Loans/Output
Optimal ν	0.83	0.57	0.06
λ_b, %	0.45	-0.87	4.75
λ_e, %	6.45	11.62	-1.52
λ_h, %	-0.60	-0.97	-0.14
λ, %	0.53	0.75	0.34

Note: Welfare gains are in permanent consumption units, as defined in subsection 4.1, relative to the economy with constant capital requirements.

The optimal macroprudential rules are countercyclical with respect to the targets in all the cases. This means that capital requirements should be raised in "good times" corresponding to a target variable being above the steady state level. This way banks would accumulate more capital which then can be used in "bad times".

There is a notable difference in terms of welfare implications for output or loans rules and the loans-to-output ratio rule. In the case of output or loans as a target variable, there is a considerable welfare gain for entrepreneurs. Bankers gain a little under the output rule, while households lose a little under both rules.[20] The relative gain for entrepreneurs is much higher in both cases, and under the optimal loans rule their permanent consumption increases by 11.62%. This is a high number considering that standard deviations of shocks are small. In fact, these results may be considered as lower bounds for potential gains from the optimal macroprudential rule. At the same time, thinking about the results scaling is possible only under the assumption of always binding constraints. The overall gain in terms of λ is lower because

[20]Households could benefit in a model with default risks and deposit insurance as in Mendicino et al. (2015). With countercyclical macroprudential policy in place, the welfare gain for households could be achieved as a result of lower bank default probabilities and lower amount of lump-sum taxes associated with deposit insurance.

the steady state household consumption is considerably higher than consumption of entrepreneurs (or bankers).

The interpretation of the significant welfare gain for entrepreneurs in the case of the first two rules is as follows. After a negative TFP or LTV shock, the falling expected collateral value negatively affects borrowing and consumption of entrepreneurs. Falling investment demand pushes the price of capital down, further tightening the collateral constraint, which then curtails borrowing even more. Due to this accelerator effect and the presence of the binding collateral constraint by itself, consumption of the entrepreneur is volatile. When the countercyclical macroprudential policy is in place, the reduction in the capital requirements following the decrease in output or loans allows bankers to supply more loans which leads to a decrease in the equilibrium loan rate and a relaxation of the borrowing constraint for the entrepreneur, then the same accelerator effect works in the opposite direction and allows entrepreneurs to better smooth consumption. The variance of their consumption decreases and the decentralized equilibrium allocation becomes closer to the efficient one. Welfare increases.

The change in dynamics, explained above, is captured on Figures 5–6 which contain impulse responses under the optimal loans rule[21] comparing to constant capital requirements (the same impulse responses as on Figures 1–2). Furthermore, Table 3 reports standard deviations of consumption for bankers, entrepreneurs and households. Under constant capital requirements, the variance of consumption is much lower for bankers comparing to households, while the variance of entrepreneurs' consumption is significantly higher. Recall that under equivalent Pareto weights, (37) dictates the same variance of consumption for all agents in the initial period, and the same in all periods if the planner reoptimizes every period or applies household discount factor for all agents. Macroprudential rules push the variance of bankers' consumption upward, while optimal output or loans rules, especially the latter, decrease the variance of entrepreneurs' consumption. Under the optimal loans rule, the variance of entrepreneurs' consumption is very close to the variance of household consumption.

The case of the loans-to-output ratio rule is different. Although both loans and

[21] Similar figures under other optimal rules can be provided upon request.

Table 3: Standard deviations of consumption under alternative macroprudential rules

	Constant	Output	Loans	Loans/Output
Bankers	0.0002	0.0022	0.0012	0.0016
Entrepreneurs	0.0069	0.0050	0.0031	0.0076
Households	0.0039	0.0034	0.0034	0.0039

Note: Each row contains standard deviations of consumption of the corresponding agent under constant capital requirements, optimal output rule, loans rule, and the loans-to-output ratio rule.

output move in the same direction after shocks, the loans-to-output ratio moves in an opposite direction on impact. Therefore, capital requirements are raised after negative shocks, which makes the situation for entrepreneurs even worse. This is reflected in the increase of the variance of their consumption. At the same time, bankers gain the most under this rule. But in total, the welfare increase is the lowest. This finding corresponds to the criticism of the choice of the credit-to-GDP gap as a target variable in the Basel III framework. Repullo and Saurina (2012) find that this target empirically seems to be not a good indicator of the build-up of systemic risk because it is negatively correlated with the business cycle. They also find that credit growth seems to be a better target variable which is in a way consistent with the theoretical result of this paper—the loans rule is welfare maximizing among the one-variable rules considered.

As for the magnitude of the optimal parameter values, first, it should be noted that this is a simple model, and the obtained optimal rules should not be understood as a ready-made recipe for macroprudential policy. Taking this into account, the 0.57 response coefficient for loans, for example, means that if loans increase 1% above the steady state, then capital requirements increase from 10% to 10.57%. In practice, the loans rule could be specified in terms of deviations from trend, and the magnitude of the parameter intuitively seems reasonable.

4.5 Macroprudential rules with two target variables

A generalization of one-variable rules considered in the previous subsection is a rule based on several target variables. An optimal rule of this kind obviously would achieve at least as high welfare as any one-variable rule based on one of the components. However, a disadvantage of such a rule is that it may have not as clear interpretation and can be more difficult to explain to the general public in the case of its implementation in practice. Let us consider a two-variable rule responding to both output and loans:

$$k_t^m = k^m + \nu_y \left(\frac{Y_t}{Y} - 1 \right) + \nu_l \left(\frac{L_t}{L} - 1 \right). \tag{43}$$

Table 4 reports the unrestricted and restricted maximizers of the social welfare function, together with implied welfare differences. The unrestricted maximization is conducted on a $[-5, 5] \times [-5, 5]$ set of parameter values, while in the restricted case only nonnegative parameter values are considered, that is the grid is $[0, 5] \times [0, 5]$.

Table 4: Welfare gains under the optimal two-variable rule

	Unrestricted	Restricted
Optimal ν_y	3.86	0.10
Optimal ν_l	-1.86	0.86
λ_b, %	-85.63	0.86
λ_e, %	-24.90	9.50
λ_h, %	186.98	-0.89
λ, %	54.50	0.79

Note: Welfare gains are in permanent consumption units, as defined in subsection 4.1, relative to the economy with constant capital requirements.

The unrestricted maximization leads to magnitudes of welfare changes which are significantly larger than in the case of restricted maximization. This is due to the issue of model stability. When $\nu_y = 3.86$, the model is stable when $\nu_l \in [-1.87, -1.81] \cup [-1.56, -0.38]$. As explained in Appendix A.3, I exclude parameter values at the border of the stable regions. This means I exclude -1.87, -1.81, -1.56, and -0.38. However, $\nu_l = -1.86$ is still next to the border and social welfare changes significantly in this small stable region—falls sharply when ν_l is increased. When ν_l is equal to any of the parameter values from the second stable region ($[-1.56, -0.38]$), social welfare is less than the one obtained at the restricted maximum.

The optimal rule in the unrestricted case does not Pareto dominate any of the optimal one-variable rules: although the social welfare increases significantly due to a huge gain for households, bankers and entrepreneurs suffer from huge losses. It is difficult to consider such a rule as being implementable in practice. For this reason, and due to better stability properties in the neighborhood of the optimal rule, I focus on the optimal rule obtained in the case of restricted maximization. All the optimal one-variable rules are countercyclical—hence, it is reasonable to restrict the analysis of two-variable rules to nonnegative parameter values.

Under the optimal restricted rule, the aggregate welfare gain is not much higher than under the optimal loans rule. However, bankers now benefit from macroprudential policy together with entrepreneurs (who benefit less than under the optimal loans rule). The dynamics of the economy under the optimal two-variable rule is different in the first periods after the TFP shock (some variables respond in the opposite direction), but eventually similar. After the LTV shock, most variables respond less, so the economy returns to the steady state faster.

The optimal restricted two-variable rule does not Pareto dominate any of the optimal one-variable rules. The gains in terms of aggregate welfare are not significantly higher comparing to the welfare maximizing one-variable rule. A two-variable rule is also more complicated to follow in practice. Hence, it can be claimed that simpler one-variable rules are good enough in bringing the economy closer to the efficient state, especially the rule based on loans as a target variable.

5 Robustness

5.1 Alternative Pareto weights

Equal Pareto weights, corresponding to the utilitarian measure of social welfare, are consistent with the fact that bankers, entrepreneurs and households are all representative agents from populations of measure one in the model. However, it is true that in the real world the population of households-workers is bigger than the population of bankers or entrepreneurs, hence the Pareto weight for households may reflect this. Ideally, different population weights then must also be specified in the model directly.

On the other hand, the agents in this model have different discount factors, which automatically contributes to the fact that the steady state household welfare is considerably higher than welfare of bankers or entrepreneurs. Namely, $\mathcal{W}^b = -55.14, \mathcal{W}^e =$

$-50.35, \mathcal{W}^h = 14.24$ under the baseline calibration. This may also be taken into account. For example, the weights could be specified following the approach in Rubio and Carrasco-Gallego (2014), namely set $\omega_i = \frac{1-\beta_i}{3-\beta_b-\beta_e-\beta_h}$. If all the agents obtained utility only from consumption, this weighting would imply that agents would get the same welfare in the steady state. In the baseline model, however, households value leisure, so the meaning of such weighting is not the same. In any case, it is interesting to explore how the results change both when ω_h is increased or decreased. For simplicity, I keep $\omega_b = \omega_e$ in the analysis.

Table 5 reports the optimal parameter values for all types of rules considered under a wide range of Pareto weighting schemes[22].

If ω_h is decreased, the optimal level of constant capital requirements increases because the weight of bankers increases, and they are the ones who benefit from higher capital requirements. All the optimal one-variable rules remain countercyclical. The optimal output and loans-to-output ratio rules are characterized by similar parameter values as in the baseline case, while the optimal loans rule becomes more aggressive as it benefits entrepreneurs the most.

When ω_h is increased, the optimal constant capital requirements become lower by the reasoning symmetric to the above[23]. The output rule becomes less aggressive, and when the weight of households is more than 0.5, the rule becomes procyclical. The loans rule at first becomes more aggressive; however, the objective function is quite flat after some point as in the baseline case. When ω_h is increased, there is also a local maximum at a procyclical value, which becomes global when ω_h is high enough. The optimal loans-to-output ratio rule also becomes procyclical eventually. All this, of course, could be expected just from the fact that households lose from the countercyclical response.

The optimal unrestricted two-variable rule is similar to the baseline case for the wide range of Pareto weights. The optimal restricted rule changes similarly to one-variable rules. Mostly, it becomes less aggressive when ω_h increases, eventually hitting the nonnegativity bound by both parameters.

[22] Additional results can be provided upon request.

[23] For $\omega_h = 1$, the optimal k^m is zero in the limit. However, $k^m = 0$ cannot be formally checked because it implies that under the assumption of a binding capital constraint, bankers' consumption is zero in the steady state, and marginal utility of consumption is infinity. The reported optimal value of 0.01 is just the smallest value in the grid (it could be 0.0001 or smaller).

Table 5: Optimal rules under alternative Pareto weights

ω_h	Constant	Y	L	L/Y	L and Y (U)		L and Y (R)	
	k^m	ν	ν	ν	ν_y	ν_l	ν_y	ν_l
0.00	0.81	0.85	0.74	0.05	-0.15	0.87	0.00	0.74
0.05	0.66	0.85	0.73	0.05	-0.14	0.85	0.22	0.82
0.10	0.54	0.84	0.73	0.05	0.22	0.82	0.22	0.82
0.15	0.44	0.84	0.72	0.06	0.19	0.83	0.19	0.83
0.20	0.36	0.84	0.70	0.06	3.96	-1.89	0.19	0.83
0.25	0.30	0.84	0.66	0.06	3.96	-1.89	0.16	0.84
0.30	0.25	0.83	0.61	0.06	3.86	-1.86	0.13	0.85
0.33	**0.23**	**0.83**	**0.57**	**0.06**	**3.86**	**-1.86**	**0.10**	**0.86**
0.35	0.21	0.83	0.87	0.06	3.86	-1.86	0.07	0.87
0.40	0.18	0.82	0.88	0.07	3.86	-1.86	0.04	0.88
0.45	0.15	0.80	0.88	0.09	3.86	-1.86	0.04	0.88
0.50	0.13	0.72	0.88	0.12	3.86	-1.86	0.04	0.88
0.55	0.11	-0.01	0.88	0.12	3.86	-1.86	0.04	0.88
0.60	0.09	-0.23	-0.10	0.12	3.86	-1.86	0.04	0.88
0.65	0.07	-0.38	-0.26	0.12	3.86	-1.86	0.00	0.00
0.70	0.06	-0.50	-0.39	0.12	3.86	-1.86	0.00	0.00
0.75	0.05	-0.60	-0.45	-0.14	3.86	-1.86	0.00	0.00
0.80	0.04	-0.70	-0.49	-0.14	3.86	-1.86	0.00	0.00
0.85	0.03	-0.82	-0.52	-0.14	3.86	-1.86	0.00	0.00
0.90	0.02	-1.01	-0.54	-0.14	3.86	-1.86	0.00	0.00
0.95	0.01	-1.24	-0.55	-0.14	3.86	-1.86	0.00	0.00
1.00	0.01	-1.24	-0.57	-0.14	-1.63	0.76	2.70	0.00

Note: $\omega_b = \omega_e = 0.5(1 - \omega_h)$, Y – output, L – loans, U – unrestricted, R – restricted.

Entrepreneurs are the agents who are most severely affected by financial frictions in this model. The distortion is alleviated under the countercyclical macroprudential policy. When the Pareto weight of entrepreneurs is not too low (the weight of households is not too high), then countercyclical macroprudential policy is welfare maximizing. In the opposite case, procyclical policy is optimal.

5.2 Calibration of the steady state LTV ratio

Now let us analyze how the results are affected by the different calibration of the steady state LTV ratio. As mentioned in subsection 3.2, there is a considerable dispersion in the values of this parameter used in the literature.

Table 6 reports the optimal parameter values for environments with different values of the steady state LTV ratio. Pareto weights are equivalent as in the baseline economy.

Table 6: Optimal rules under alternative calibration of the steady state LTV ratio

m	Constant	Y	L	L/Y	L and Y (U)		L and Y (R)	
	k^m	ν	ν	ν	ν_y	ν_l	ν_y	ν_l
0.10	0.85	-0.08	-0.01	2.41	5.00	-0.71	0.00	0.00
0.20	0.91	-0.03	0.07	0.26	2.73	-0.42	0.00	0.07
0.30	0.94	0.00	0.09	0.18	-0.23	0.22	0.00	0.09
0.40	0.76	0.00	0.10	0.15	-0.28	0.24	0.00	0.10
0.50	0.56	-0.02	0.10	0.14	-0.37	0.29	0.00	0.10
0.60	0.43	-0.04	0.10	0.15	-0.49	0.37	0.00	0.10
0.70	0.33	-0.02	0.12	0.15	-0.62	0.50	0.00	0.12
0.80	0.27	0.83	0.18	0.14	4.72	-2.27	0.00	0.18
0.90	**0.23**	**0.83**	**0.57**	**0.06**	**3.86**	**-1.86**	**0.10**	**0.86**
1.00	0.78	2.47	0.77	0.04	-0.20	0.97	0.00	0.77

Note: Y – output, L – loans, U – unrestricted, R – restricted.

When m is increased, the optimal constant capital requirements in general decrease. As entrepreneurs become less credit constrained, it is optimal to increase the supply of loans available to them to maximize the social welfare.

The optimal output rule is close to a constant rule when entrepreneurs are very constrained because they cannot benefit from countercyclical macroprudential policy as much as in the baseline case, while households lose from the countercyclical response. The optimal loans rule is also less aggressive for small values of m, but mostly remains countercyclical because it is the most effective in helping entrepreneurs.

On the contrary, the optimal loans-to-output ratio rule is more aggressive when m is lower. Recall that this rule only helps bankers in the baseline case. When m is low, they can provide less loans to entrepreneurs, and a more aggressive macroprudential rule helps to create more variability in the supply of loans, bankers' net worth and consumption through varying the capital requirements more during the economic cycle (remember that the variance of bankers' consumption is inefficiently low in the decentralized equilibrium).

The optimal restricted two-variable rule inherits properties of the output rule and loans rule. When m is lower, it is less aggressive, and the output coefficient hits the non-negativity bound.

5.3 Monetary policy responding to output

I consider two alternative monetary policy rules:

$$R_t = R + \phi_\pi(\pi_t - \pi) + \phi_y\left(\frac{Y_t}{Y} - 1\right) \tag{44}$$

and

$$R_t = R + \phi_\pi(\pi_t - \pi) + \phi_y\left(\frac{Y_t}{Y_{t-1}} - 1\right), \tag{45}$$

where $\phi_y > 0$. Specifically, I set $\phi_y = 0.25$. Table 7 compares the optimal macroprudential rules under strict ($\phi_y = 0$) and two versions of flexible ($\phi_y = 0.25$) inflation targeting. Table 8 reports welfare gains under flexible rule (44), and Table 9 does the same for a flexible rule (45). First, the gains associated with moving from $\phi_y = 0$ to $\phi_y = 0.25$ under constant capital requirements (the same step for all rules) are reported. Second, the gains from optimal macroprudential rules under flexible targeting are shown (this is similar to what was reported in the baseline case). Finally, the total gains resulting from a transition from constant capital requirements and strict targeting to optimal rules under flexible targeting are reported. Pareto weights and the steady state LTV ratio are the same as in the baseline economy.

Table 7: Optimal rules under alternative monetary policy rules

Targeting	Constant	Y	L	L/Y	L and Y (U)		L and Y (R)	
	k^m	ν	ν	ν	ν_y	ν_l	ν_y	ν_l
Strict (22)	**0.23**	**0.83**	**0.57**	**0.06**	**3.86**	**-1.86**	**0.10**	**0.86**
Flexible (44)	0.22	1.31	0.84	0.13	-0.75	0.88	0.20	0.86
Flexible (45)	0.22	1.27	0.85	0.08	4.45	-2.03	0.30	0.86

Note: Y – output, L – loans, U – unrestricted, R – restricted.

Table 8: Welfare gains from macroprudential policy under flexible targeting (44)

	Step 1	Y	Total	L	Total	L/Y	Total	L and Y (R)	Total
λ_b, %	-1.91	3.02	1.04	8.13	6.06	0.91	-1.02	13.42	11.25
λ_e, %	10.69	-2.51	7.92	-7.85	2.00	-0.21	10.46	-12.10	-2.71
λ_h, %	-0.73	0.06	-0.67	0.47	-0.26	-0.02	-0.75	0.62	-0.12
λ, %	0.65	0.10	0.76	0.29	0.94	0.09	0.74	0.40	1.05

Note: Welfare gains are in permanent consumption units, as defined in subsection 4.1. Y – output, L – loans, R – restricted. "Step 1" is a change from $\phi_y = 0$ to $\phi_y = 0.25$ under constant capital requirements. "Total" is a change from strict targeting and constant capital requirements to a corresponding optimal rule under flexible targeting.

Table 9: Welfare gains from macroprudential policy under flexible targeting (45)

	Step 1	Y	Total	L	Total	L/Y	Total	L and Y (R)	Total
λ_b, %	-0.82	0.66	-0.17	1.71	0.87	2.81	1.96	4.05	3.19
λ_e, %	4.73	3.65	8.55	4.02	8.94	0.45	5.20	1.82	6.64
λ_h, %	-0.35	-0.40	-0.74	-0.47	-0.82	-0.22	-0.57	-0.42	-0.77
λ, %	0.30	0.32	0.62	0.47	0.76	0.30	0.60	0.52	0.82

Note: Welfare gains are in permanent consumption units, as defined in subsection 4.1. Y – output, L – loans, R – restricted. "Step 1" is a change from $\phi_y = 0$ to $\phi_y = 0.25$ under constant capital requirements. "Total" is a change from strict targeting and constant capital requirements to a corresponding optimal rule under flexible targeting.

The optimal constant capital requirements are almost unaffected. Moving from strict inflation targeting to flexible inflation targeting with constant capital requirements leads to a significant welfare gain for entrepreneurs, while bankers and households lose a bit. In terms of aggregate welfare, this transition is welfare improving, and the magnitudes are bigger in the case of rule (44).

Optimal one-variable rules and restricted two-variable rule are more aggressive under flexible targeting. But there is a notable difference between two types of monetary policy rules in terms of implied distribution and aggregate welfare gains from macroprudential policy.

In the case of flexible targeting responding to the relative deviation of output from the steady state (44), most of the gain is achieved just through a transition to flexible targeting. The gain is entirely due to a positive effect on entrepreneurs. Macroprudential policy is welfare improving, but the effect is weaker than in the baseline case. Entrepreneurs lose from macroprudential policy, while bankers and households mostly win. This is entirely due to the dynamics after TFP shocks (entrepreneurs benefit from macroprudential policy if there are only LTV shocks). Under the optimal macroprudential rule and flexible targeting, the combined policy response is such that output increases on impact after a negative shock, and recession starts only from the second period. There is additional variability in the price of capital, loan rate and entrepreneurs' consumption, which is the reason for the loss for entrepreneurs. At the same time, the variance of household consumption, output and inflation decreases, which contributes to the gain for households in the case of output and loans rules. Bankers benefit the most.

On the contrary, under flexible targeting responding to output growth (45), the positive effect of macroprudential policy is generally bigger than the effect of a change in the monetary policy rule. Similarly, to the baseline case, entrepreneurs are the ones who benefit the most under the output and loans rules. And the optimal loans rule is welfare maximizing among one-variable rules. A different effect under flexible targeting of the form (44) can be explained by the fact that monetary policy responds to the same variable as in the output rule for macroprudential policy, or to a related variable in the case of the loans rule. This leads to the positive effect of one policy replacing the positive effect of another policy.

In general, the joint analysis of optimal monetary and macroprudential policy is more complicated. The main point to be learned from this section, however, is that countercyclical macroprudential policy improves aggregate welfare both under strict and flexible inflation targeting.

6 Conclusion

The optimal macroprudential rules are countercyclical if the Pareto weight for entrepreneurs in the social welfare function is not too low. The result is generally robust under several model modifications. The welfare maximizing one-variable rule is based on loans as a target variable, while a rule based on the loans-to-output ratio leads to the least welfare gains. This is in line with the criticism of the countercyclical capital buffer of the Basel III framework. The buffer is based on the value of the credit-to-GDP gap. Repullo and Saurina (2012) find that this variable empirically is far from being a good indicator of a build-up of systemic risk, while loans perform much better.

The optimal loans rule generates a significant welfare gain for entrepreneurs who are subject to financial frictions in the model. The optimal rule helps to move the decentralized equilibrium allocation closer to the efficient one in terms of reducing the variance of entrepreneurs' consumption and increasing the variance of bankers' consumption. A more general two-variable rule leads to an additional aggregate welfare gain, but does not Pareto dominate any of the optimal one-variable rules.

It should be emphasized that the modeling of the banking sector is very simple in the model. There is no possibility for default, no deposit insurance. The introduction of these mechanisms would have created an additional role for capital requirements and macroprudential policy, as, for example, in the paper by Mendicino et al. (2015). At the same time, the model contains the standard and well-understood features of monetary models with nominal rigidities, while including financial frictions. It is reassuring that even this simple framework can rationalize countercyclical macroprudential policy.

One more direction for future research is the development of an alternative method for solving the model. All the results in this paper are fully valid only in the environment with always binding borrowing constraints, as if they were imposed as equalities from the beginning. It would be interesting to explore how the results change when constraints are occasionally binding. This, however, seems to be a computationally challenging task, at least for the current modeling framework.

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A Appendix

A.1 Equilibrium conditions

Let $\beta^i_{t,t+1} \equiv \beta_i \frac{\partial U_i / \partial C^i_{t+1}}{\partial U_i / \partial C^i_t}$, where $i \in \{b, e, h\}$.

Households

$$\frac{\partial U_h}{\partial N_t} + W_t \frac{\partial U_h}{\partial C^h_t} = 0, \tag{46}$$

$$\mathbb{E}_t \left\{ \beta^h_{t,t+1} \frac{R_t}{1 + \pi_{t+1}} \right\} = 1. \tag{47}$$

Entrepreneurs

$$\frac{Y^w_t}{X_t} + L_t = W_t N_t + Q_t[K_t - (1-\delta)K_{t-1}] + \frac{R^l_t}{1+\pi_t} L_{t-1} + C^e_t, \tag{48}$$

$$Y^w_t \equiv A_t K^\alpha_{t-1} N^{1-\alpha}_t, \tag{49}$$

$$\left[m_t K_t \mathbb{E}_t \left\{ Q_{t+1}(1+\pi_{t+1}) \right\} - \mathbb{E}_t\{R^l_{t+1}\} L_t \right] \cdot \mathbb{1}(\gamma^e_t > 0) + \gamma^e_t \cdot \mathbb{1}(\gamma^e_t \leq 0) = 0, \tag{50}$$

$$W_t = (1-\alpha) \frac{Y^w_t}{N_t X_t}, \tag{51}$$

$$Q_t = \mathbb{E}_t \left\{ \beta^e_{t,t+1} \left(\alpha \frac{Y^w_{t+1}}{K_t X_{t+1}} + (1-\delta) Q_{t+1} \right) \right\} + \frac{\gamma^e_t}{U'_e(C^e_t)} m_t \mathbb{E}_t \left\{ Q_{t+1}(1+\pi_{t+1}) \right\}, \tag{52}$$

$$\frac{\gamma^e_t}{U'_e(C^e_t)} \mathbb{E}_t\{R^l_{t+1}\} = 1 - \mathbb{E}_t \left\{ \beta^e_{t,t+1} \frac{R^l_{t+1}}{1+\pi_{t+1}} \right\}. \tag{53}$$

Bankers

$$\frac{R^l_t}{1+\pi_t} L_{t-1} - \frac{R_{t-1}}{1+\pi_t} D_{t-1} + D_t = C^b_t + L_t, \tag{54}$$

$$[(1-k^m_t)L_t - D_t] \cdot \mathbb{1}(\gamma^b_t > 0) + \gamma^b_t \cdot \mathbb{1}(\gamma^b_t \leq 0) = 0, \tag{55}$$

$$\frac{\gamma^b_t}{U'_b(C^b_t)} = 1 - \mathbb{E}_t \left\{ \beta^b_{t,t+1} \frac{R_t}{1+\pi_{t+1}} \right\}, \tag{56}$$

$$\frac{\gamma^b_t}{U'_b(C^b_t)}(1-k^m_t) = 1 - \mathbb{E}_t \left\{ \beta^b_{t,t+1} \frac{R^l_{t+1}}{1+\pi_{t+1}} \right\}. \tag{57}$$

Capital good producers

$$K_t - (1-\delta)K_{t-1} = C_t^k \left[1 - \frac{\phi_k}{2}\left(\frac{C_t^k}{C_{t-1}^k} - 1\right)^2\right], \tag{58}$$

$$1 = Q_t \left[1 - \frac{\phi_k}{2}\left(\frac{C_t^k}{C_{t-1}^k} - 1\right)^2 - \phi_k \left(\frac{C_t^k}{C_{t-1}^k} - 1\right)\frac{C_t^k}{C_{t-1}^k}\right]$$
$$+\mathbb{E}_t \left\{\beta_{t,t+1}^h Q_{t+1}\phi_k \left(\frac{C_{t+1}^k}{C_t^k} - 1\right)\left(\frac{C_{t+1}^k}{C_t^k}\right)^2\right\}. \tag{59}$$

Retailers

$$(1+\pi_t)^{1-\epsilon} = \theta + (1-\theta)\left(\frac{\epsilon}{\epsilon-1}\frac{\Pi_{1,t}}{\Pi_{2,t}}\right)^{1-\epsilon}, \tag{60}$$

$$\Pi_{1,t} = (1+\pi_t)^{\epsilon}\left(\frac{\partial U_h}{\partial C_t^h}\frac{Y_t}{X_t} + \beta_h \theta \mathbb{E}_t \left\{\Pi_{1,t+1}\right\}\right), \tag{61}$$

$$\Pi_{2,t} = (1+\pi_t)^{\epsilon-1}\left(\frac{\partial U_h}{\partial C_t^h} Y_t + \beta_h \theta \mathbb{E}_t \left\{\Pi_{2,t+1}\right\}\right). \tag{62}$$

Monetary policy

$$R_t = R + \phi_\pi(\pi_t - \pi). \tag{63}$$

Market clearing

$$Y_t^w = Y_t \Delta_t, \tag{64}$$

$$\Delta_t = (1+\pi_t)^{\epsilon}\left[\theta\Delta_{t-1} + (1-\theta)\left(\frac{\epsilon}{\epsilon-1}\frac{\Pi_{1,t}}{\Pi_{2,t}}\right)^{-\epsilon}\right], \tag{65}$$

$$Y_t = C_t^h + C_t^e + C_t^b + C_t^k. \tag{66}$$

Exogenous variables are A_t and m_t, while k_t^m is the macroprudential instrument, the rule for which is to be specified.

Equations (50) and (55) should be understood as follows. Suppose that the solution is such that the corresponding Lagrange multiplier is positive. Then the corresponding constraint is binding and becomes an equation, while the value of the multiplier is determined from the system. If this value turns out to be non-positive, then it must be equal to zero, and this equality is added to the system of equations instead of the binding constraint. In the latter case it must be verified that the constraint is satisfied. In the solution of the model it is assumed that constraints always bind.

A.2 Deterministic steady state

In the deterministic steady state, there are no shocks and all the variables are constant. In this model, it is possible to solve for the steady state analytically.

First, $A_t = A \equiv 1$ without loss of generality, $k_t^m = k^m$ (the corresponding macroprudential rule is defined in section 4), $m_t = m$, $\pi_t = \pi$ (the steady state inflation rate has to be imposed exogenously). From (15), $R = \frac{1+\pi}{\beta_h}$, while from (16), $Q = 1$. From (4) and (5), $R^l = R + k^m(1+\pi)\left(\frac{1}{\beta_b} - \frac{1}{\beta_h}\right)$. From (18)-(21) and (24),

$$X = \frac{\epsilon}{\epsilon - 1}(1+\pi)\frac{1-\beta_h\theta(1+\pi)^{\epsilon-1}}{1-\beta_h\theta(1+\pi)^{\epsilon}}\left(\frac{(1+\pi)^{1-\epsilon}-\theta}{1-\theta}\right)^{\frac{1}{\epsilon-1}}, \tag{67}$$

$$\Delta = \frac{(1-\theta)^{\frac{1}{1-\epsilon}}\left[(1+\pi)^{1-\epsilon}-\theta\right]^{\frac{\epsilon}{\epsilon-1}}}{(1+\pi)^{-\epsilon}-\theta}. \tag{68}$$

As mentioned before, as long as $\beta_b < \beta_h$, the regulatory constraint for the banker binds in the steady state. The collateral constraint for the entrepreneur binds as long as $\beta_e < \widetilde{\beta}_e \equiv \frac{\beta_b\beta_h}{\beta_b + k^m(\beta_h - \beta_b)}$. If $\beta_b < \beta_h$, then $\widetilde{\beta}_e \in (\beta_b, \beta_h)$ if $k^m \in (0,1)$. Assuming that these restrictions hold, which is ensured in the calibration, solving the rest of the system of static equations reduces to solving one equation in one unknown, N. Let

$$\Gamma \equiv \frac{X}{\alpha}\left[\frac{1}{\beta_e}\left(1 + \beta_e m - m\frac{1+\pi}{R^l}\right) - 1 + \delta\right], \tag{69}$$

$$\Psi \equiv \left[\Gamma\left(\frac{1}{\Delta} - \frac{\alpha}{X}\right) - m\frac{1+\pi}{R^l}(1-k^m)\left(1 - \frac{1}{\beta_h}\right)\right]^{\sigma_h}. \tag{70}$$

Then it can be shown that

$$N = \left(\frac{1-\alpha}{\Psi X}\Gamma^{\frac{\sigma_h - \alpha}{1-\alpha}}\right)^{\frac{1}{\sigma_h + \phi}}, \tag{71}$$

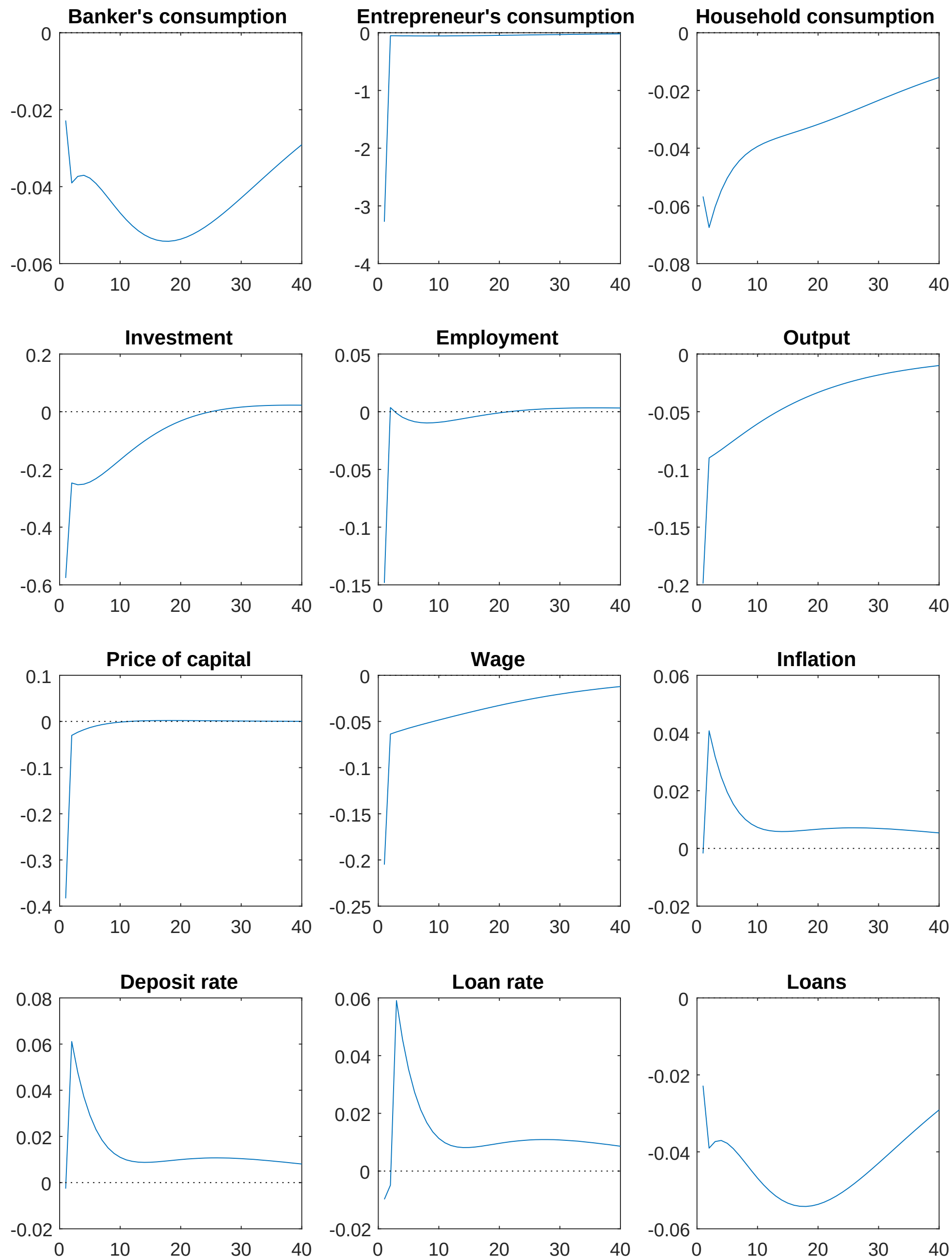
$$K = N\Gamma^{\frac{1}{\alpha - 1}}. \tag{72}$$

All the remaining steady state values are then easily obtained directly from the static equations of the model.

A.3 Computational details

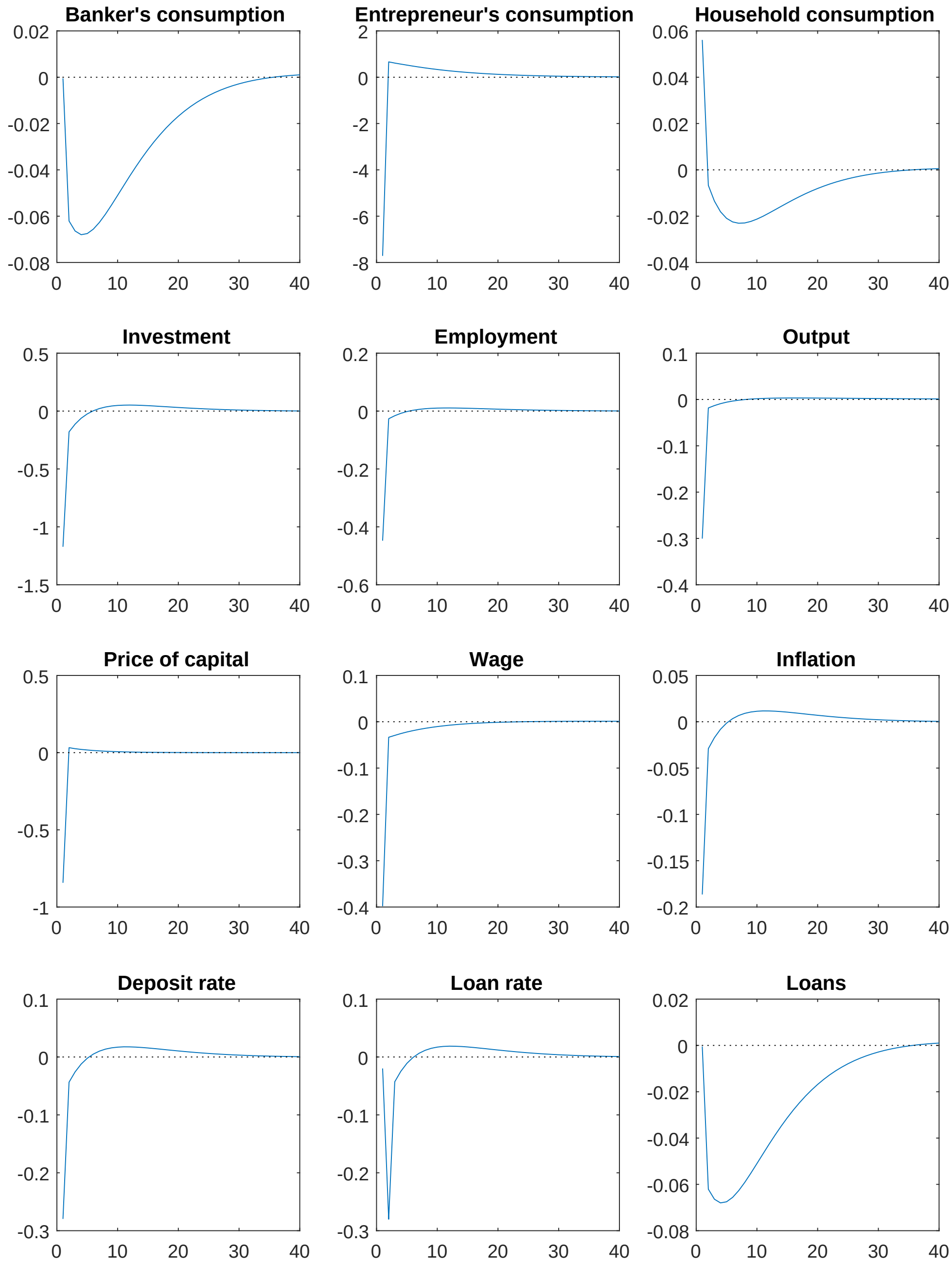
Quantitative analysis is conducted as follows. A set of shocks $\{\epsilon_t^a, \epsilon_t^m\}_{t=1}^T$ is drawn from $\mathcal{N}(0, \tau_i^2), i \in \{a, m\}$. TFP and LTV shocks are independent. I set $T = 10000, \tau_a = \tau_m = 0.001$. The standard deviations of shocks are small to have the collateral and minimum amount of bank capital constraints mostly remain binding in the simulations. A second order Taylor expansion of the equilibrium conditions around the steady state is taken (assuming that the borrowing constraints are binding). Approximate policy functions are computed, and then the model is simulated using the shocks mentioned above. This is done in software Dynare for each value of k^m or ν, ν_l, ν_y from a grid with a step 0.01. The average welfare ($\mathbb{E}(\mathcal{W}_t^i)$) and permanent consumption differences (λ_i) for each agent $i \in \{b, e, h\}$ are computed dropping 100 initial periods. Then the aggregate welfare ($\mathbb{E}(\mathcal{W}_t)$) and implied permanent consumption differences (λ) are obtained. Parameter values at the border of the parameter space yielding the stable solution to the model are not considered. The behavior of the welfare measure is usually unstable at the border with significant jumps or falls in welfare, which are mostly not local extrema.

Figure 1: Impulse responses to a TFP shock ($\epsilon_1^a = -0.001$)



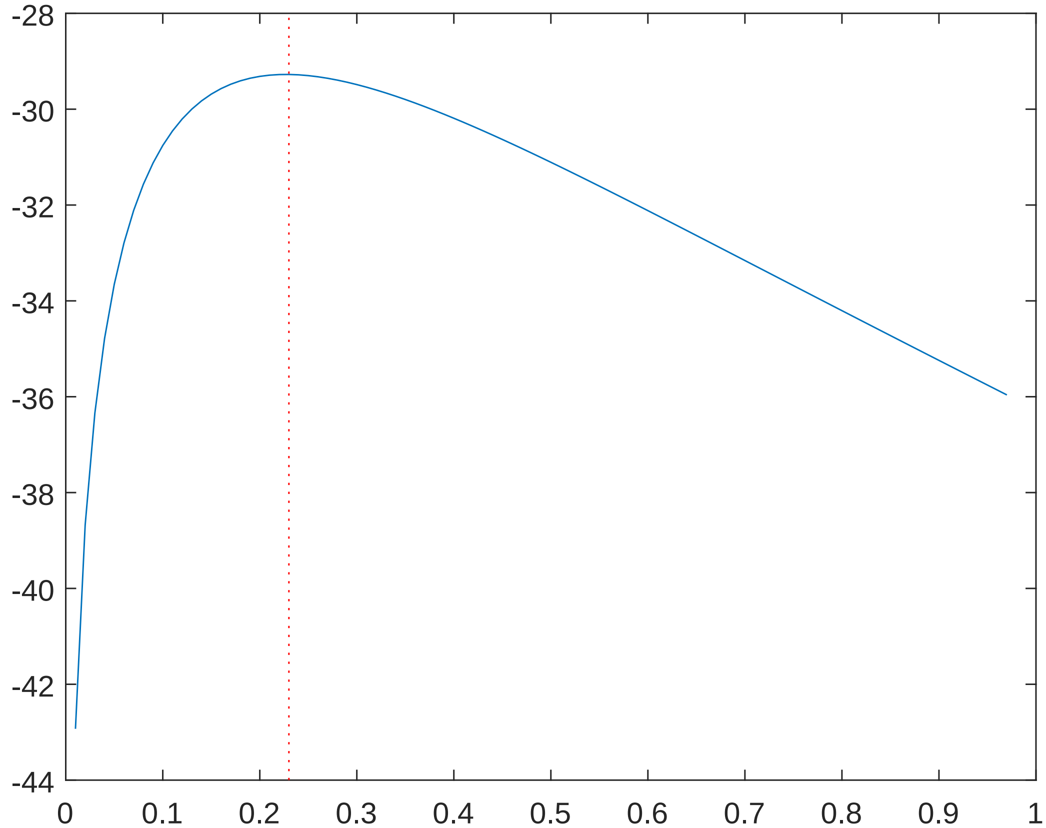
Note: Inflation and interest rates are in annualized percentage point deviations from the steady state. The rest of variables are in percent deviations from the steady state. Investment is defined as $Q_t I_t$. Horizontal axes are number of periods after the shock. Period 1 is the period of the shock, and this is the first period plotted.

Figure 2: Impulse responses to an LTV ratio shock ($\epsilon_1^m = -0.001$)



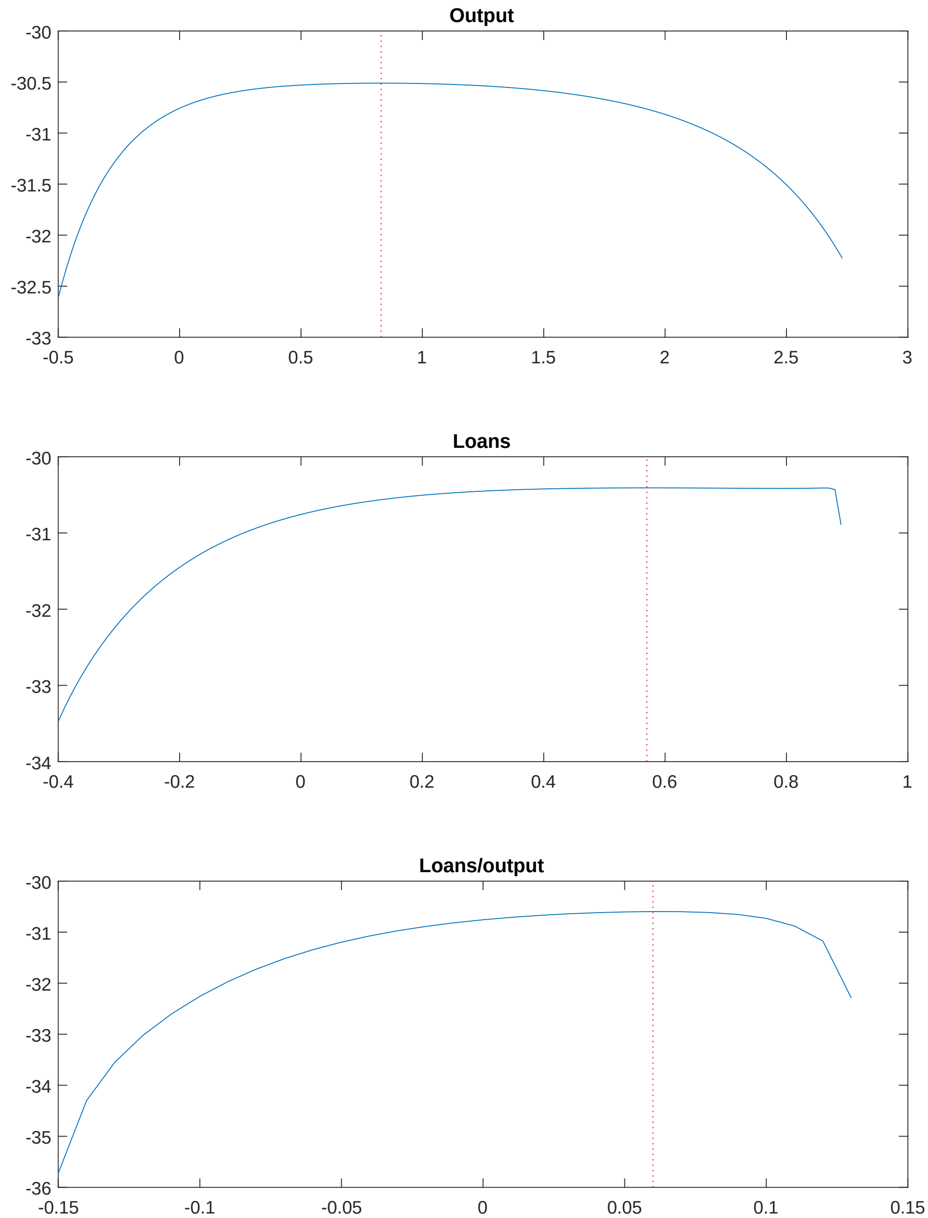
Note: Inflation and interest rates are in annualized percentage point deviations from the steady state. The rest of variables are in percent deviations from the steady state. Investment is defined as $Q_t I_t$. Horizontal axes are number of periods after the shock. Period 1 is the period of the shock, and this is the first period plotted.

Figure 3: Optimal level of constant capital requirements



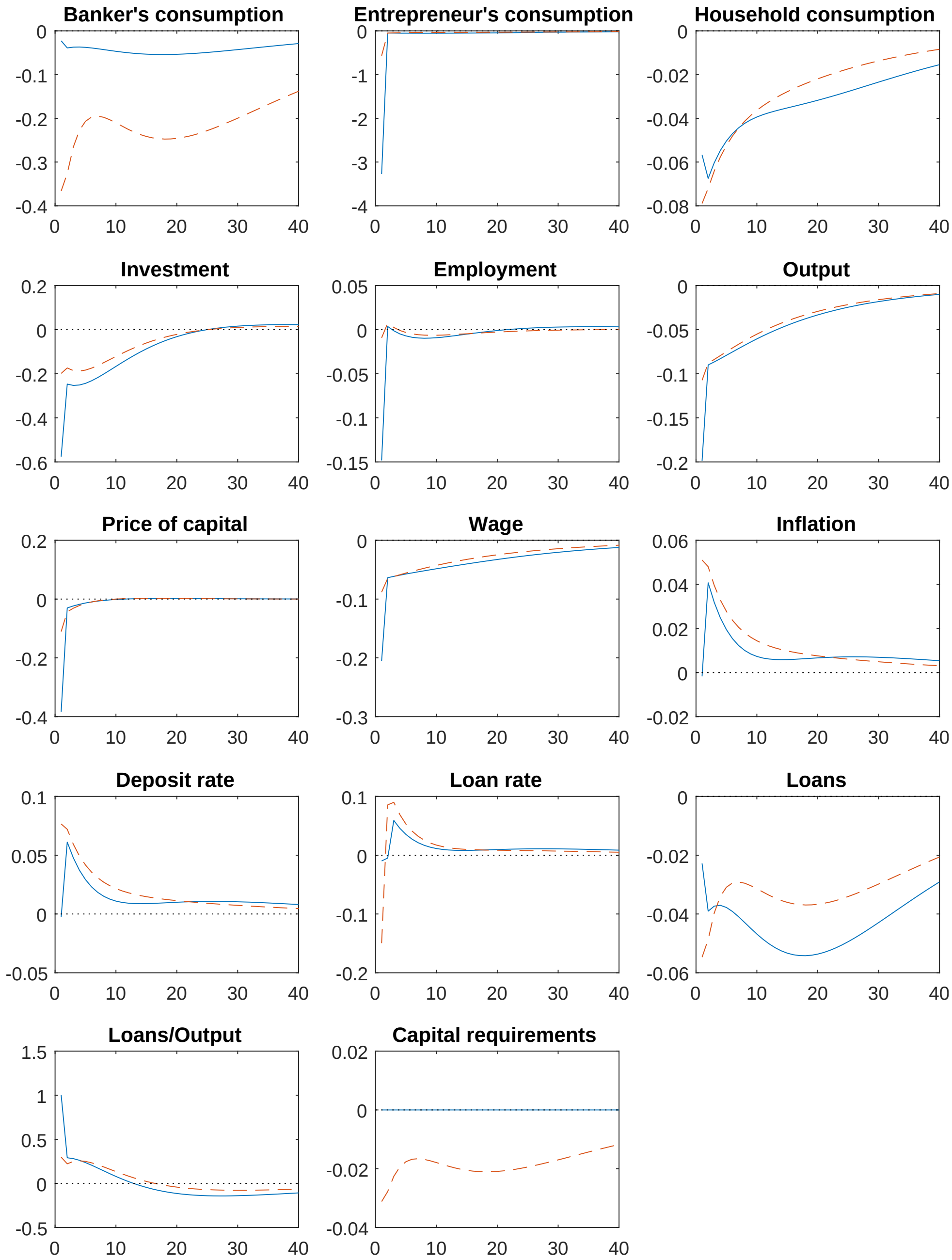
Note: The horizontal axis contains values of k^m, while the vertical axis is $\mathbb{E}(\mathcal{W}_t)$ as defined in subsection 4.1. The red dotted vertical line indicates the welfare maximizing value of k^m.

Figure 4: Optimal one-variable rules, $\omega_i = \frac{1}{3}$



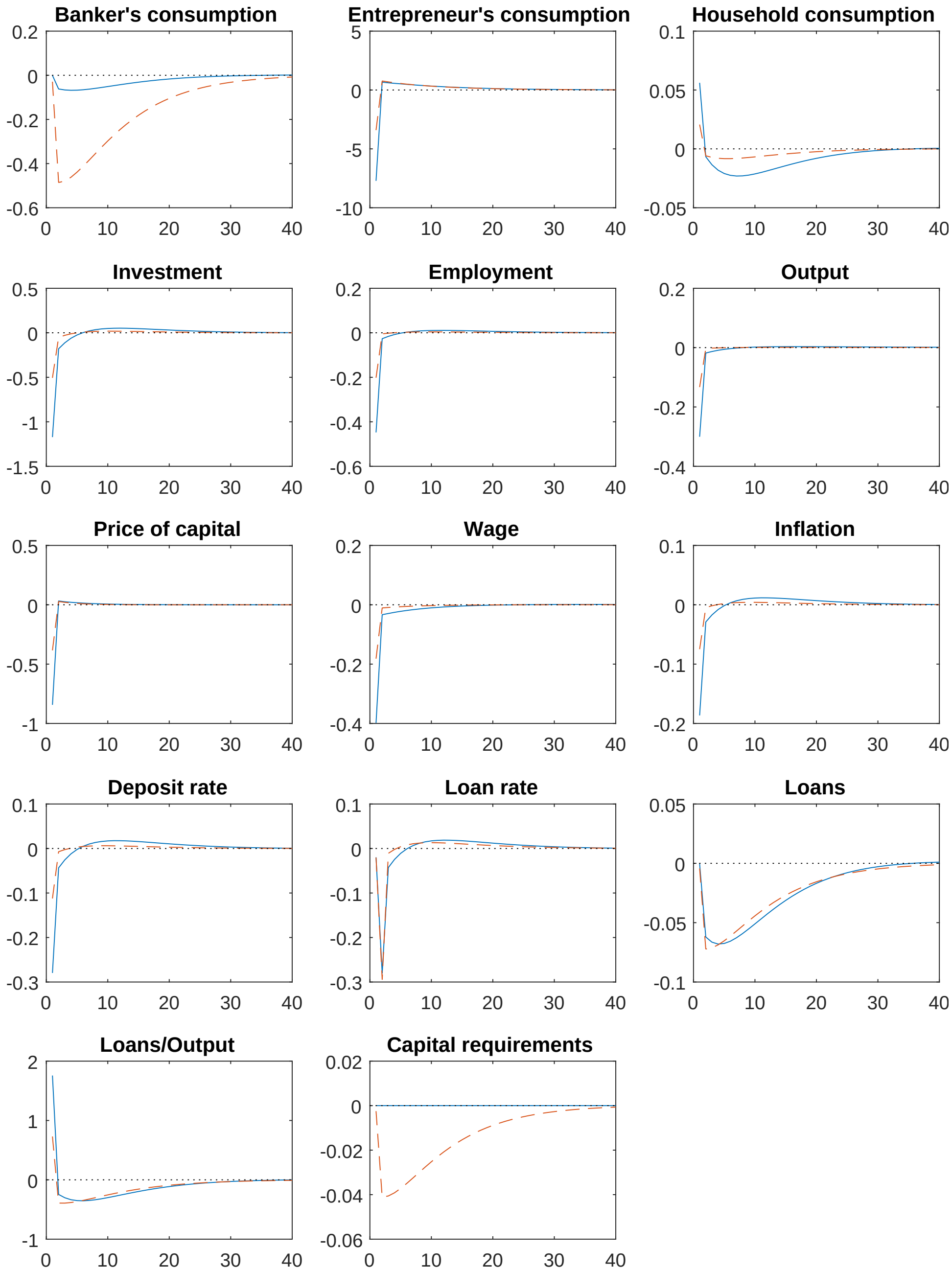
Note: Horizontal axes are values of ν, while vertical axes are $\mathbb{E}(\mathcal{W}_t)$ as defined in subsection 4.1. Red dotted vertical lines indicate welfare maximizing values of ν. The model has a stable solution when $\nu \in [-1.25, 2.73], [-0.59, 0.89], [-0.15, 0.13]$ for output, loans, and loans-to-output ratio rules respectively. For output and loans rules, not the entire stable region is plotted to show better how welfare changes around the maximum (in the regions not shown the shape is similar to the one near the left border).

Figure 5: TFP shock ($\epsilon_1^a = -0.001$), optimal rule responding to loans



Note: Red dashed lines are responses under the optimal rule. Blue lines correspond to constant capital requirements. Capital requirements are in percentage point deviations from the steady state.

Figure 6: LTV shock ($\epsilon_1^m = -0.001$), optimal rule responding to loans



Note: Red dashed lines are responses under the optimal rule. Blue lines correspond to constant capital requirements. Capital requirements are in percentage point deviations from the steady state.