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Solving DSGE Portfolio Choice Models with Asymmetric Countries

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Motivation

- Large cross-country asset holdings $\rightarrow$ a general-equilibrium theory is needed to understand their drivers and macroeconomic implications.
- DSGE models with portfolio choice cannot be solved with standard perturbation methods due to indeterminacy in a certainty equivalent environment.
- The workhorse local approximation routine to overcome this problem is the technique developed by Devereux and Sutherland (2010, 2011), henceforth *DS* $\Rightarrow$ first-order approximation of international portfolios $\Rightarrow$ neglecting the direct effect of risk.
- Accounting for risk is especially important in a model with asymmetric countries because of existing heterogeneity in precautionary motives.

Goal: to improve upon the shortcomings of *DS*.

Highlights

The proposed method to solve DSGE portfolio choice models:

- corrects the approximate policy function for the presence of risk,
- accounts for asymmetries present in the model,
- and improves the quality of the approximation.

Related References

Devereux & Sutherland (2010, 2011), Guu & Judd (2001), Rabitsch et. al. (2015), Tille & van Wincoop (2010), Winant (2014)

Bifurcation Methods

- Bifurcation: varying the parameter governing the size of risk (σ) causes a change in the number of solutions to the portfolio choice problem:
  $-\sigma = 0$: infinitely many solutions.
  $-\sigma > 0$: a unique solution.
- Bifurcation theory: determines steady-state portfolio holdings and enables the use of the implicit function theorem.
- Implementation:
  1. Decompose the model into a portfolio-selecting equation and the macroeconomic part (consisting of the remaining equations).
  2. Solve the macroeconomic part conditional on some guess for the relevant component of the portfolio holdings.
  3. Use the result from (2) to update your guess according to the bifurcation theory. Iterate until a fixed point has been reached.
- I consider two types of bifurcation methods:
  1. *BIF*: the solution is expressed as a state space policy function, $y_t = g(\sigma, y_{t-1}^{state}, \epsilon_t)$.
  2. *BIFN*: the solution is expressed as a nonlinear moving average policy function (Lan and Meyer-Gohde, 2013), $y_t = y(\sigma, \epsilon_t, \epsilon_{t-1}, ...)$.

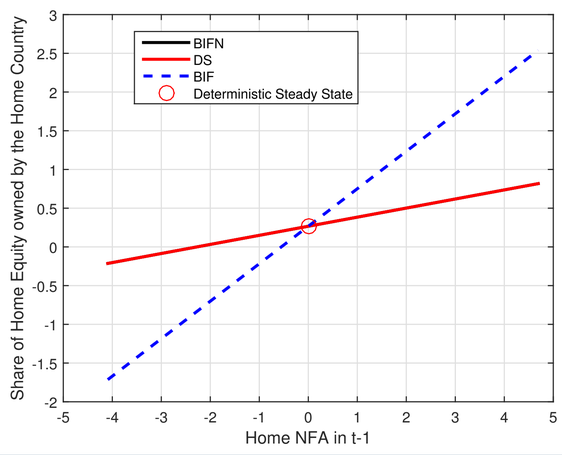
An Example Model

- The performance of three local solution methods, *DS*, *BIF* and *BIFN*, is assessed based on an endowment economy model with two countries: home (H) and foreign (F).
- Each country is populated by an infinitely-lived representative household whose preferences are given by the CRRA utility function.
- Two types of income: "capital income" Y_i^K and "labor income" Y_i^L with $i = H, F$. Income innovations follow a symmetric distribution.
- Asymmetric countries: the income stream in the foreign country is twice as volatile as the endowment in the home country.
- There are two assets that can be traded internationally. They represent claims on the capital income of the respective country and can thus be interpreted as equity.

Main Results

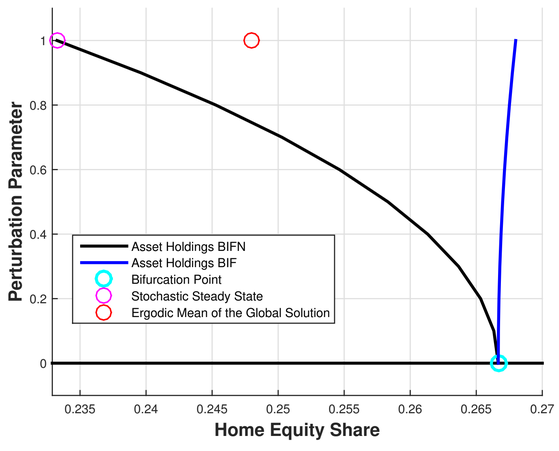
Up to first order of accuracy *DS* and *BIFN* yield the same results. On the other hand, *BIF* yields highly volatile portfolios.

First-Order Accurate Policy Function



By including second-order risk correction, *BIFN* accounts for cross-country differences in precautionary motives.

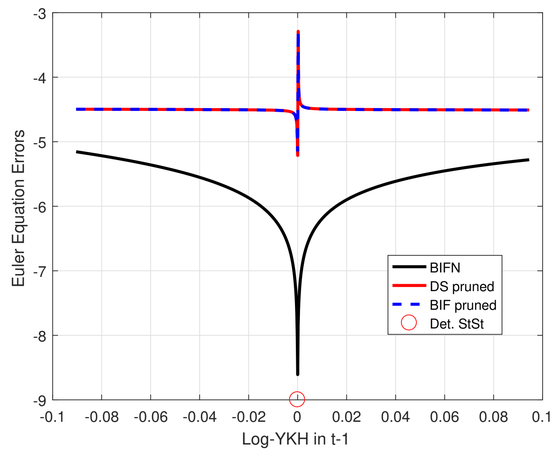
Risk-Adjusted Portfolio Holdings



The perturbation parameter (σ) governs the size of risk. $\sigma = 0$ corresponds to the deterministic steady state, whereas $\sigma = 1$ denotes the fully stochastic environment. The ergodic mean of the global solution is taken from Rabitsch et al. (2015).

BIFN improves the accuracy of the approximation.

Euler Equation Errors



Approximation errors are computed for portfolio Euler equations (Kazimov, 2012). They are reported for an interval based on the ergodic set of the home capital income under *DS*. All other state variables take their steady state values.

Ergodic Moments of Portfolio Holdings

	GS		BIFN		BIF		DS	
	mean	std	mean	std	mean	std	mean	std
θ_h^h	0.248	0.13	0.239	0.138	0.268	0.562	0.265	0.138
θ_h^f	0.723	0.066	0.728	0.07	0.735	0.36	0.736	0.07

θ_h^h and θ_h^f denote home household's holdings of home and foreign equity, respectively. Mean and standard deviation of the global solution are taken from Rabitsch et al. (2015).