Cascades Along the Business Cycle

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Macroeconomic Modelling and Model Comparison

Goethe University Frankfurt

Does contagion matters?

"...network effects have important lessons for understanding the economic recession"

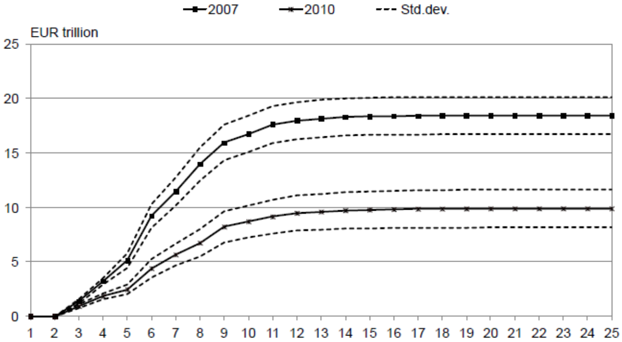


Figure: Cumulative sum of contagious European banks' total assets in 2007 and 2010. Source: Toivanen (2013).

Does contagion matters? (Cont)

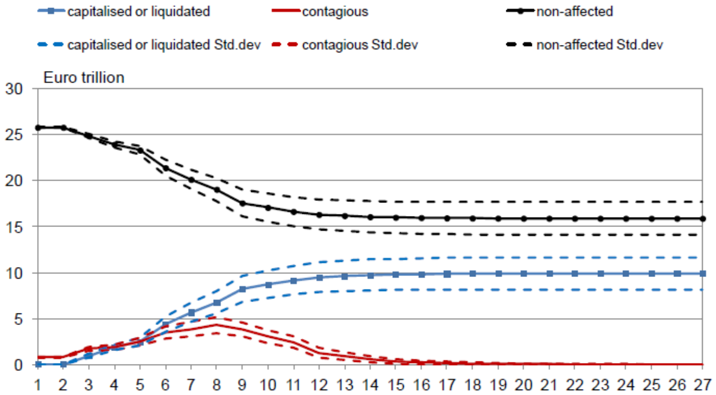


Figure: Average 2010 crisis propagation in European banking network. Source: Toivanen (2013).

Real banking network v.s. theoretical banking networks

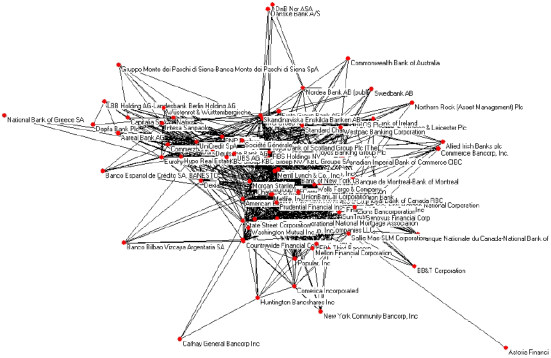


Figure: Global banking network in year 2006, source Houston 2015.

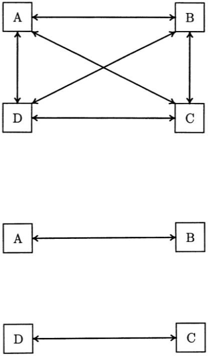


Figure: *Implied* banking networks in DSGE literature (source: Allena & Gale 2000).

Research question & selected literature

Research Question: how contagion spreads between banks? Can we model it? Can we integrate a contagion mechanism into a DSGE model and keep track of how default cascades evolve along the cycle?

Related Literature:

Gai and Kapadia (2010): agent-based simulated banking network. They keep track of the evolution of cascade size after different shocks.

Acemoglu et al. (2014): they built a micro model of banking with network. They compared ring and complete network finding evidences of phase transition.

Blasques et al. (2015): they simulated the interbank market, with banks as nodes of a network. Market activity is affected negatively by credit shocks and positively by larger rates corridors.

Capponi and Chen (2015): explore how to mitigate risk in a core-periphery and random (Poisson) banking network.

Coherent empirical literature on the structure of interbank network: Boss et al. (2004), Caldarelli et al. (2006), Somaraki (2007), Newman (2009), Cohen-Cole et al. (2011).

Outline

- Construction of a model to describe diffusion in random networks
- Analyze the properties of the network described
- Integrate the network model into a standard DSGE model with frictions
- Analyze how cascades evolve along the economic cycle

Main results

- I have developed a general framework that can be applied to *any macro model*.
- With this framework it is possible to analyze the probability of a cascade and its size.
- I constructed the network using a power law distribution following empirical evidence, in contrast with previous attempts that use more simplistic assumptions (but the framework can be adapted to *any distribution*...).
- Robuts-yet-fragile properties that highlight a policy trade off during crisis.
- Probabilities and sizes of cascades evolve along the business cycle and are affected by real and financial shocks. Exists a "divine coincidence" with government spending shocks, while there is not for monetary policy.
- Next extension: estimate the key network parameters, without the need of restricted data.

What is a *cascade*?

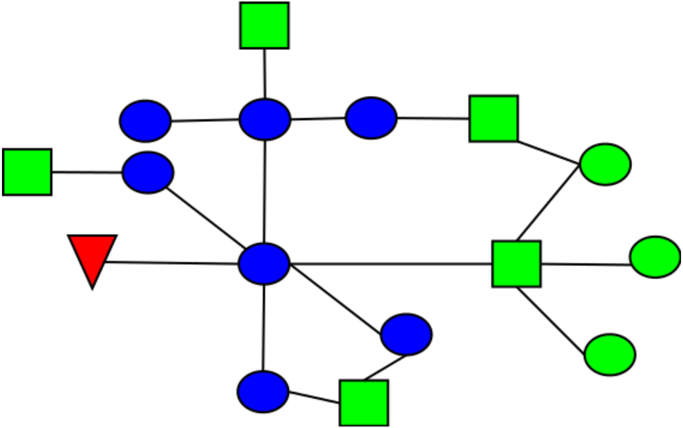


Figure: Diffusion on a network after an initial shock. The red triangular is the "starter" (called initial seed) of the cascade. Circles are vulnerable nodes, while squares are resilient nodes. Blue nodes are part of the cascade ("*infected*") while green nodes are not.

A model for cascades

Definition. Assume that nodes are in state 0 but can turn to state 1. We define a global cascade as a cascade that occupies a finite fraction of the network, with nodes shifted from state 0 to 1.

Global cascades are triggered by one node (called initial seed) that moves exogenously from state 0 to 1. The initial seed is able to spread to its neighbours that can shift as well or not. Only if at least one of the neighbours moves to state 1 the initial seed can spread.

Definition. A node is vulnerable if it turns to state 1 if at least a fraction ϕ of its k neighbours is in state 1.

If there are no short cycles, the initial seed can grow *if and only if* the initial seed is surrounded by at least one immediate neighbour with threshold $\phi \leq \frac{1}{k}$ or equivalently a degree $k = \frac{1}{\phi}$.

Moment generating functions approach

Borrowing from physics (Newman et al. (2001), Watts (2002)), we can describe a network trough a moment generating function (MGF) approach. Simply consider a random graph V with n nodes and d edges. We can define p_k as the probability of one node to have exactly k neighbours and assume its distribution to be correctly normalized.
We can than build a moment generating function for the entire graph as:

$$M_0(x) \equiv \sum_{k=0}^{\infty} p_k x^k \tag{1}$$

each moment of the distribution is simply the k^{th} derivative of $M_0(x) \mid x = 1$. With the average degree of the network z defined as:

$$M_0'(1) = z = \sum_{k=0}^{\infty} k p_k \tag{2}$$

in the case of a Poisson network $p_k = \frac{z^k e^{-z}}{k!}$.

The cascade into equations

Assume that each node has its how value of ϕ with ϕ drawn at random from a distribution such as $\int_0^1 f(\phi)\, d\phi = 1$.
We can define the moment generating function of vulnerable vertices from (1) as:

$$G_0(x) = \sum_{k=0}^{\infty} \rho_k p_k x^k \tag{3}$$

$$\rho_k = \begin{cases} 1 \ \text{if } k = 0 \\ P\left(k \leq \frac{1}{\phi}\right) \ \text{if } k \neq 0 \end{cases} \tag{4}$$

from this it is possible to characterize the distribution of vulnerable nodes and to compute probability and size of cascades.

For example we can compute: ▸ Math.

- The vulnerable fraction of the population: $P_v = G_0(1)$
- The average degree of vulnerable vertices: $z_v = G_0'(1)$
- The average degree of vertices v neighbours to an initial vertex u $G_1(x)$. This quantity is crucial in determining the spread of any initial seed.

Computing the cascade size

Define q_k the probability of a vertex being part of a vulnerable cluster and r_k the probability that one of its neighbours belongs to the cluster of vulnerable vertices, with the corresponding MGFs $H_0(x)$ and $H_1(x)$.
From the properties of generating functions follows that $H_0(x)$ and $H_1(x)$ satisfy the following self consistency conditions:

$$H_1(x) = [1 - G_1(1)] + xG_1(H_1(x)) \tag{5}$$

$$H_0(x) = [1 - G_0(1)] + xG_0(H_1(x)) \tag{6}$$

where the first term is the probability that the vertex chosen is not vulnerable and the second term the size distribution of vulnerable clusters attached to a vulnerable vertex.
With (some!) algebra, we can compute $H_0'(1) = \parallel n \parallel$ the average vulnerable cluster size as:

$$\parallel n \parallel = P_v + \frac{(z_v)^2}{z - G_0''(1)} \tag{7}$$

with $[z - G_0''(x)]$ the (to some extent transcendental) phase transition condition.

Some algebra. Self consistency.

The choice of p_k, ϕ and $f(\phi)$

The choice of the appropriate distribution is not trivial, because it captures some key characteristics of the underlying network. However this comes at the price of higher computational burden.

I will use a power law distribution of the form: $p_k = Ak^{-\gamma}$.
This distribution captures two key features of the interbank market: i) tales are not irrelevant; ii) there are signs of preferential attachment. Additionallly it is consistent with the empirical evidence.

ϕ is the number of counterparts that can default without leading to a default of the bank itself. It is possible to prove that $\phi = \frac{V^e}{I}$ with V^e the amount of dollars a bank can make fire-saling its assets and I the total interbank loans of each bank.

Probability and size of a cascade

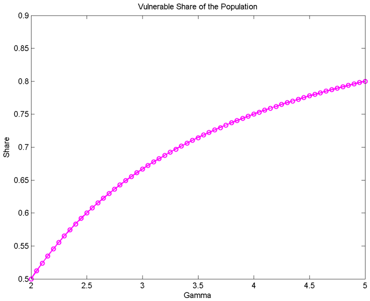


Figure: Cascade probability.

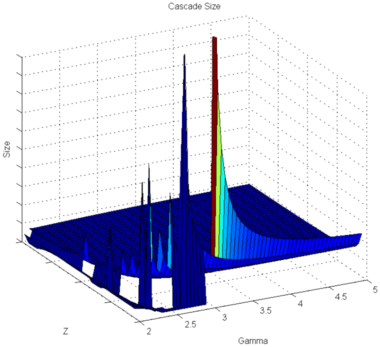


Figure: Cascade size.

Public policies: "immunization"

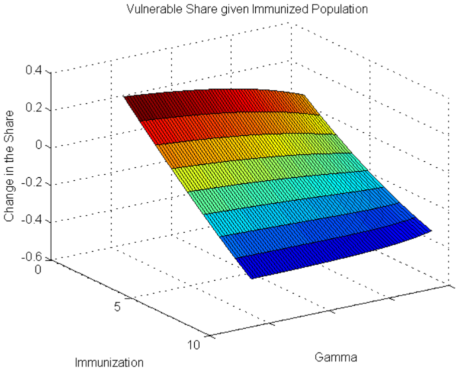


Figure: Cascade probability.

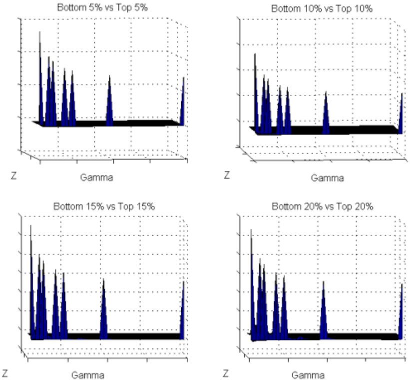
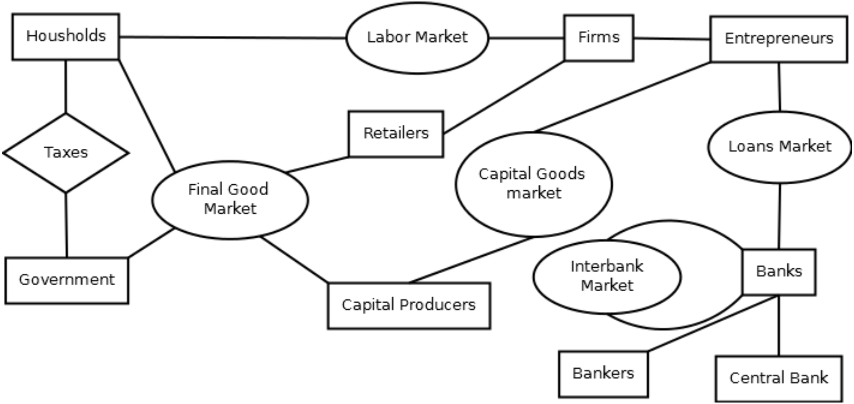


Figure: Cascade size.

The model


Housholds
Labor Market
Firms
Entrepreneurs
Taxes
Retailers
Loans Market
Final Good Market
Capital Goods market
Government
Interbank Market
Banks
Capital Producers
Bankers
Central Bank

A complete DSGE model

The model I propose is an extension of Gertler & Karadi (2011), modified to incorporate banking sector on the line of Gerali et al. (2010) and Gambacorta & Signoretti (2013).

There are: households with habits, monopolistic competition and Calvo pricing. Firms finance each period's capital and investments with loans from the banking sector.
Following the assumption made by Acemoglu et al. (2014), each bank operates on the interbank market to loan and borrow funds using capital accumulated during previous periods. In each period banks are associated to a random number of counterparts, according to the network's laws.

Operatively, each bank is divided into a wholesale and a retail branch. Wholesale branches operate on the interbank market while retail branches extend loans to firms.

The bank's problem

The (consolidated) ballance sheet of a rapresentative bank is:

Assets	Liabilities
Loans to Firms (B)	Reserves (N)
Interbank Loans (I)	Deposits (D)
	Interbank Loans (I)

The aggregate bank's problem has the (well known) form:

$$E_t\left(V_t\right) = \sum_{i=0}^{\infty}\left(1-\theta\right)\theta^i \Lambda^C_{t,t+1+i} N_{t+1+i} \quad s.t \tag{8}$$

$$E\left(V_t\right) \geq \Theta B_t \tag{9}$$

the problem is solved considering also the peripheral decisions of the two branches.

Fire-sale threshold

The maximum amount of dollars that a bank can lose from its counterparts without defaulting is given by:

$$\eta_t V_t = -\frac{R_t^K B_t - R_t^D D_t - R_t^B Int_t}{R_t^B} = V_t^e \tag{10}$$

with $\eta_t \sim U \in [0,1]$ and being i.i.d. across banks and time. It is possible to show that $\phi = \frac{V_t}{Int_t}$ leading to:

$$P\left[\phi \leq \frac{1}{k}\right] = P\left[\frac{(V_t^e = \eta_t V_t)}{I_t} \leq \frac{1}{k}\right] = P\left[\eta_t \leq \frac{I_t}{kV_t}\right] \tag{11}$$

▸ Some algebra.

Nesting the network structure

Also in this relatively simple case, the computing equations (3)-(7) is far from trivial.
The power law exhibits fractal behavior and has not close form solution for a large set of parameter values. In fact, equation (3) can be solved as:

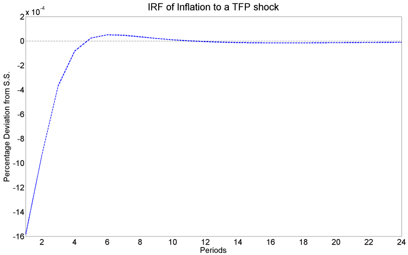
$$\frac{Li_{\gamma}(x)}{\zeta(\gamma)} \tag{12}$$

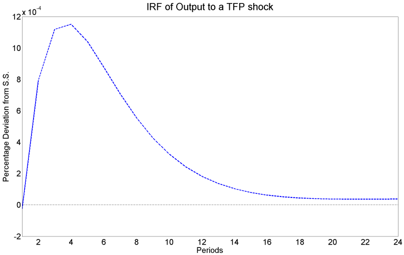
with $Li_{\gamma}(x)$ being the γ^{th} polylogarithm of x (a fractal function used in quantum statistics) and $\zeta(\gamma)$ the Riemann ζ function.
However, using the definition of integrals as limits of a Reinmann sum and relying on the properties of the network under consideration, after (some!) algebra, it is possible to redefine equation (3) as:

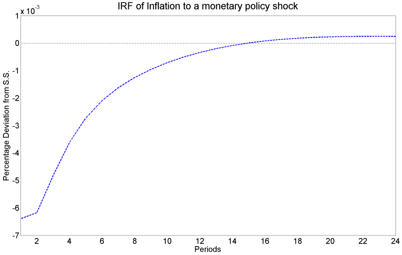
$$G_0(x) = \int_1^I \rho_k p_k x^k dk \mid x = 1 \tag{13}$$

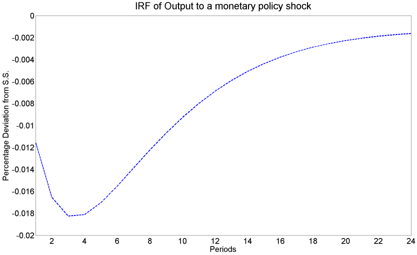
Some algebra. Network equations.

Standard results









Probability of a cascade after shocks

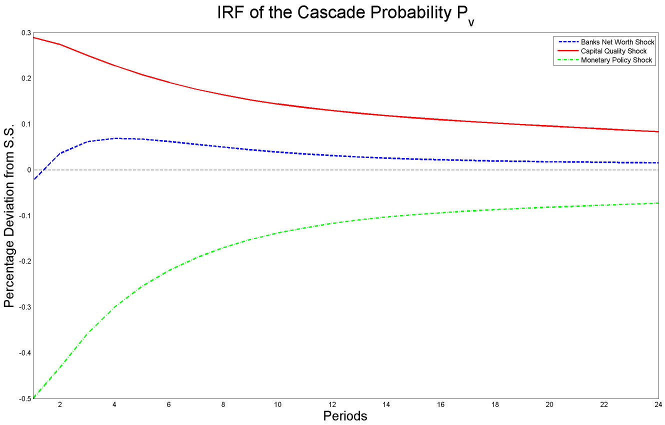


Figure: Cascade probability after monetary policy, capital quality and bank's networth shocks.

Probability of a cascade after shocks (cont)

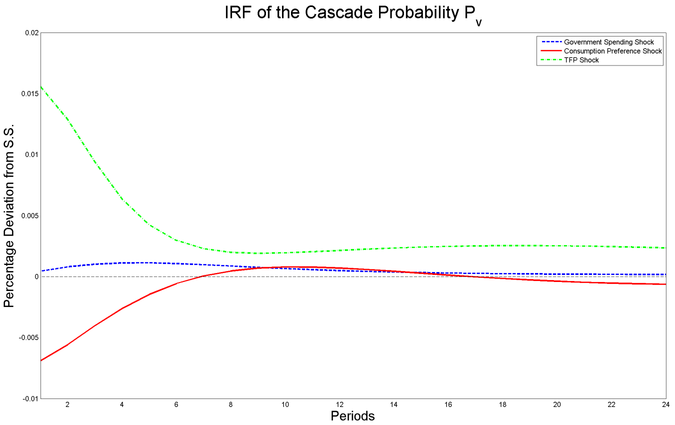


Figure: Cascade probability after TFP, consumption preference and government spending shocks.

Size of a cascade after shocks

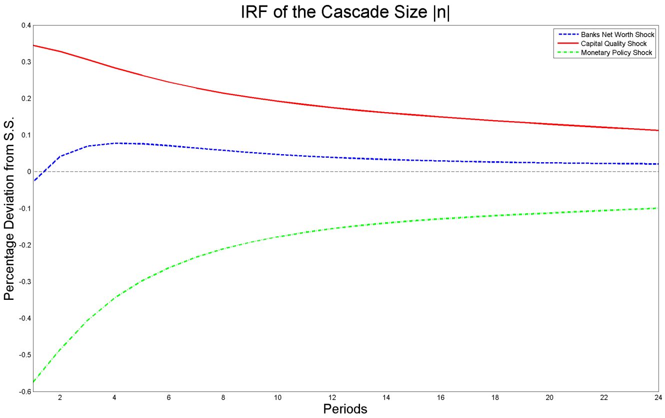


Figure: Cascade size after monetary policy, capital quality and bank's networth shocks.

Size of a cascade after shocks (cont)

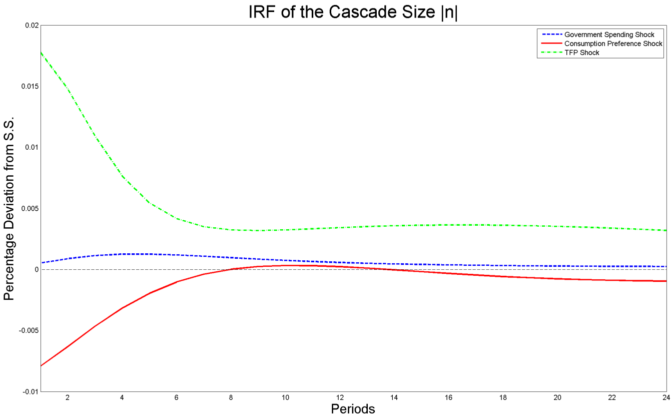


Figure: Cascade size after TFP, consumption preference and government spending shocks.

Leaning against the network

IRFs show that tightening can improve the system stability, so why the central bank does not *respond to financial variables*? Assume the augmented Taylor rule:

$$R_t^n = (1-\rho)\left[\overline{R^n} + \psi_\pi \pi_t + \psi_y (Y_t - Y^{ss}) + k_{pv}(P_{v,t} - P_v^{ss})\right] + \rho R_{t-1}^n + e_t^R \tag{14}$$

k_{pv}	Int. Rate		TFP		Inflation		Gov. Spending		Preference		Cap. Quality		Banks' Net Worth	
	Cummulative change in the contagion probability P_v in differences from the baseline													
0.05	2.34		-0.07		0.15		0.00		0.04		-1.33		-0.11	
0.10	2.91		-0.08		0.18		-0.01		0.04		-1.68		-0.20	
0.15	3.18		-0.08		0.20		-0.01		0.04		-1.87		-0.29	
0.20	3.34		-0.08		0.21		-0.01		0.04		-2.02		-0.41	
0.25	3.45		-0.07		0.22		-0.01		0.03		-2.17		-0.55	
	Ratio between the variances of Y and π under a specific rule and the baseline													
	Y	π	Y	π	Y	π	Y	π	Y	π	Y	π	Y	π
0.05	0.11	0.06	0.61	1.08	0.11	0.06	1.02	0.74	1.47	2.68	0.07	6.03	0.09	46.13
0.10	0.05	0.18	0.64	1.02	0.05	0.18	1.04	0.91	1.80	6.24	0.19	18.34	0.89	231.03
0.15	0.04	0.36	0.78	1.01	0.04	0.36	1.07	1.67	2.18	19.42	0.64	40.69	3.59	650.46
0.20	0.04	0.60	1.04	1.18	0.04	0.60	1.12	3.49	2.69	55.59	1.85	86.92	9.43	1509.48
0.25	0.06	1.00	1.51	1.76	0.06	1.00	1.18	7.61	3.49	151.39	4.97	196.47	22.26	3316.88

Conclusions

- This framework is flexible to different model specifications, additional frictions, different distributional assumptions. Can be extended to model other areas of the economy with network effects (i.e. technology, news, innovation).
- Real and financial shocks have a significant impact on the probability of a cascade and on its size.
- Financial shocks, as expected, have larger conseguences on the structure of the interbank market than real shocks.
- There is a mild "divine coincidence" between public spending and financial stability.
- However, the same is not true for expansionary monetary policies. Central banks can reduce volatility *weakly targeting* financial variables after real shocks, with limited losses in financial stability. This does not hold for financial shocks
- Policy makers should try to avoid the "chaotic area" of the network distribution, the most effective policy varies with the characteristics of the interbank market. Generally, targeting the most connected institutions is preferable.
- Extensions: estimate the model to indentify γ. This will allow researcher to indentify that parameter without the need of CB restricted data.

Questions?

Thank you!

Cascade equations

The first derivative of $G_0(x)$ is immediate to compute as:

$$G_0'(x) = \sum_k k\rho_k p_k x^{k-1}$$

$G_1(x)$ is the probability that a vertex a is second neighbour of a vulnerable vertex. The probability of choosing a is proportional to kp_k, therefore the corresponding generating function is:

$$G_1(x) = \frac{\sum_k k\rho_k p_k x^{k-1}}{\sum_k kp_k}$$

It is immediate that $\sum_k k\rho_k p_k x^{k-1} = G_0'(x)$. Recalling equation (2) it is possible to simplify:

$$G_1(x) = \frac{G_0'(x)}{z}$$

Go back.

Cascade size

By definition, the average vulnerable cluster size is the first derivative of the MGF of vulnerable vertices. Derive equation (5) for its argument:

$$H_1'(x) = G_1(H_1(x)) + xG_1'(H_1(x))H_1'(x) = \frac{G_1(H_1(x))}{1 - xG_1'(H_1(x))}$$

now derive equation (6):

$$H_0'(x) = G_0(H_1(x)) + xG_0'(H_1(x))H_1'(x)$$

combine the two to get:

$$H_0'(x) = G_0(H_1(x)) + xG_0'(H_1(x))\frac{G_1(H_1(x))}{1 - xG_1'(H_1(x))}$$

Go back.

Cascade size (cont)

Recall that:

$$G_1\left(x\right) = \frac{G_0'\left(x\right)}{z} \quad and \quad G_1'\left(x\right) = \frac{G_0''\left(x\right)}{z}$$

therefore it is possible to write H_0' as:

$$H_0' = G_0(H_1(x)) + \frac{x[G_0'(H_1(x))]^2}{z - G_0''(H_1(x))}$$

the average cluster size is given by $H_0' \mid x = 1$ so the previous equation becomes:

$$\| n \| = P_v + \frac{(z_v)^2}{z - G_0''(1)}$$

▸ Go back.

Self consistency

Equation (5) must fulfill so called self consistency conditions. Self-consistency (see Tarpey 1996) allows to approximate the distribution of a random vector X by a random vector Y whose structure is less complex without significant loss of information. In particular we can construct a self-consistent approximation of X dividing X into subsamples and defining Y as a random variable with values the means of each subset.

Definition

Consider two random vectors X and Y. Y is *self-consistent* for X if:

$$E(X \mid Y) = Y$$

Go back.

Self consistency (cont)

Lemma

Let X_n be a sequence of independent random variable with 0 mean and define $S_n = \sum_{i=0}^{n} X_i$. Then:

$$E(S_{n+k} \mid S_n) = S_n + E(X_{n+1} + ... + X_{n+k} \mid S_n)$$

$$E(S_{n+k} \mid S_n) = S_n + E(X_{n+1} + ... + X_{n+k})$$

$$E(S_{n+k} \mid S_n) = S_n$$

Thus S_n is self-consistent for S_{n+k}, $k > 0$. This holds more generally if $S_{n\,n>1}$ is a ***martingale process****.*

Notice that the exploration of a social network is a **martingale process** (i.e. the expectation of the next value in the sequence is equal to the present observed), in fact the after visiting node k, the exploration along any of its edges is ex-ante identical to the original situation (in expectations).

Go back.

Nesting the network structure

Recall the definition of integral as limit of a Riemann sum:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \frac{b-a}{n} \sum_{i=1}^{n} f(x_i)$$

with $x_i = a + i\frac{b-a}{n}$. With the MGF for the entire graph given by given by $\sum_k f(k)$.
With siple algebra it is possible to compute $a = 0$ and $b = n$, the maximum number of edges in the graph.
Given the system under consideration, therefore, one can easily obtain:

$$G_0(x) = \int_1^I \rho_k p_k x^k dk$$

with I defining the maximum number of edges that are obtainable as if each bank loans one unit of interbank funds to a differente counterpart. Clearly with $\rho_k = P\left[\phi \leq \frac{1}{k}\right]$ and ϕ a function of V_t, the capital buffer of banks.

Go back.

Full network system

From equation (3) we can have that:

$$G_0(1) = \int_1^{I_t} \Omega_t A k^{-\gamma} x^k dk$$

with $\Omega_t \equiv P\left[\eta_t \leq \frac{I_t}{kV_t}\right]$. It is now easy to derive the system to get:

$$z = \int_1^{I_t} L k^{-\gamma} x^k dk$$

$$G_0'(x) = \int_1^{I_t} \Omega_t L k^{-\gamma} k x^{k-1} dk$$

$$G_0''(x) = \int_1^{I_t} \Omega_t L k^{-\gamma} k(k-1) x^{k-2} dk$$

Go back.

Fire-sale threshold

Define ϕ as the fraction of neighbours of a bank that can default without leading to the default of the bank itself.

I follows that $i = \frac{I_t}{k}$ is the fraction of funds lend to each counterpart. The maximum number of clients that may default without threatening the bank itself is:

$$F = \frac{V}{i}$$

Recalling the definition of ϕ:

$$\phi = \frac{F}{k} \rightarrow \frac{V}{i}\frac{1}{k} \rightarrow \phi = \frac{V}{k}\frac{k}{Int} = \frac{V}{Int}$$

Go back.