The Amplification of the New Keynesian Models and Robust Optimal Monetary Policy

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INTRODUCTION, CONCLUSION AND OUTLOOK

Questions

- Whether and how model uncertainty affects the amplification mechanism of the New Keynesian models
- Analyzing the optimal commitment policy, I compare the dynamics of the models under rational expectations and under model uncertainty.

Contribution

- The contribution is theoretical
- Makes the impact of model uncertainty comparable across encompassing models
  - As a cross-check with the literature

Conclusion and Outlook

- Model features plays a considerable role on the dynamics under model uncertainty.
- The impact of uncertainty on the optimal monetary policy conduct depends on the type of the shock.
  - Allowing for model uncertainty makes the optimal policy more *aggressive* in response to a demand shock.
  - Bringing additional persistence into the model deteriorates the effectiveness of monetary policy
  - Allowing for either habit formation or partial indexation of prices to lagged inflation rate requires a stronger response for the policy to a demand shock
  - Together with the specification doubts, in order to reassure the private sector and signal that it will stabilize the fluctuations in the output gap, the policymaker reacts more aggressively as persistence rises.
  - A supply shock is more persistent under model uncertainty. However, the initial response of the policymaker depends on the type of the model.
  - Habit formation in consumption eliminates -even reverses- the impact of uncertainty on the policy reaction to a supply shock.
- Policymaker always attributes less importance to nominal interest rate inertia with concerns about model uncertainty.

Comparison With The Literature

- Recent studies present mixed results whether robust control approach brings attenuation or aggressiveness to the optimal policy rule as in this study.
- Cateau (2006) argues that different New Keynesian models implies different monetary policy transmission mechanisms; hence, uncertainty reveals varying responses depending on the model.
- Leitemo and Soderstrom (2004) works in a discretionary solution for the monetary policy and present more aggressively responses to supply shock but unaltered responses to demand shocks. In a small open economy, their results indicate that the impact of robustness concerns depends on the type of shock and the source of misspecification.
- Dennis (2008) working on a hybrid New Keynesian model similar to the ones in this paper shows that a discretionary central bank stabilizes inflation and consumption more tightly than the optimal policy under rational expectations.
- A widely cited-work, Giordani and Soderlind (2004) stresses an aggressive behaviour for the central bank in different specifications of Keynesian models under uncertainty, which contradicts with my findings.

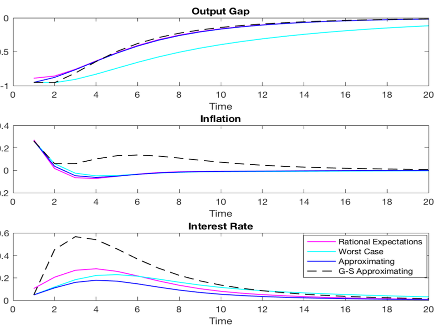


Fig. 1: Comparison with Giordani and Soderlind

METHODOLOGY

A New Keynesian Model With Price Stickiness

- Endogenous Persistence with
  - Habit Formation in Consumption

$$E_0\sum_{t=0}^{\infty}\beta^t\{\frac{(C_t-hC_{t-1})^{1-\sigma}}{1-\sigma}-\frac{L_t^{1+\eta}}{1+\eta}\}$$

  - Backward-Looking Firms

$$P_t=\theta P_{t-1}+(1-\theta)P_t^*$$
$$P_t^*=\omega P_t^b+(1-\omega)P_t^f$$
$$P_t^b\equiv P_{t-1}^*+\Pi_{t-1}$$

Model Summary

- **Benchmark Model** $(h=0,\omega=0)$
- **Habit Persistence** $(h>0,\omega=0)$
- **Inflation Persistence** $(h=0,\omega>0)$
- **Habit and Inflation Persistence** $(h>0,\omega>0)$

Hybrid IS Curve

$$y_t=\frac{h}{1+h}y_{t-1}+\frac{1}{1+h}E_t y_{t+1}-\frac{(1-h)}{\sigma(1+h)}(r_t-E_t\pi_{t+1})+u_t$$

Hybrid Phillips Curve

$$\pi_t=\frac{\omega}{\theta+\omega[1-\theta(1-\beta)]}\pi_{t-1}+\frac{\beta\theta}{\theta+\omega[1-\theta(1-\beta)]}E_t\pi_{t+1}+\lambda(\frac{\alpha+\eta}{1-\alpha}+\frac{\sigma}{1-h})y_t-\lambda\frac{\sigma h}{1-h}y_{t-1}+e_t$$

Demand Shock

$$u_t=\rho_u u_{t-1}+\upsilon_t,\quad \upsilon_t\sim N(0,1),\quad |\rho_u|<1$$

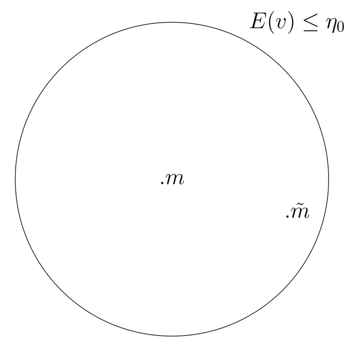
Supply Shock

$$e_t=\rho_e e_{t-1}+\epsilon_t,\quad \epsilon_t\sim N(0,1),\quad |\rho_e|<1$$

where $\lambda=\frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\theta+\omega[1-\theta(1-\beta)]}$

Introducing Model Uncertainty

- Robust Control to allow for unstructured uncertainty



- The decision-maker has a reference model (m) but suspects that the data are actually generated by a nearby model $(\tilde{m})$ which cannot be specified.
- Under model uncertainty, agents recognize that there are specification errors (v) and then seek for a decision rule that will work well, not only for the reference model but for a set of models in the neighbourhood of m.
- To express the idea that m is a good reference model, the neighbourhood of m for which the decision-maker wants a decision rule that works well is restricted to the set of models for which the size of the specification errors $(E(v))$ is bounded by a certain value, η_0.

- Following Hansen and Sargent (2008), to allow for model uncertainty, I introduce a second type of disturbances in the shock process which leads:

$$e_{t+1}=\rho_e e_t+\varepsilon_{t+1}+v_{t+1}^e$$

$$u_{t+1}=\rho_u u_t+\upsilon_{t+1}+v_{t+1}^u$$

- Hence, model uncertainty is described by the set $\{v_{t+1}^e,v_{t+1}^u\}$ satisfying the constraint:

$$E_0\sum_{\tau=0}^{\infty}\beta^t\{[v_{t+\tau}^e]^2+[v_{t+\tau}^u]^2\}\leq\eta_0,\quad \eta_0>0$$

Optimal Monetary Policy Under Uncertainty

- Reformulate the optimization problem to obtain a policy rule that performs well under the worst-case model
- Introduce a fictitious evil agent who shares the same reference model that the policymaker considers and tries to maximize the same objective function
- A two-person game → Each player simultaneously commits to sequences for $\{r_t\}$, and $\{v_{t+1}^e,v_{t+1}^u\}$ at time zero, taking the other player's moves as given.

Policymaker's Problem

$$\max_{\{v_{t+1}^e,v_{t+1}^u\}_{t=0}^{\infty}}\min_{\{r_t,\pi_t,y_t\}_{t=0}^{\infty}}(\frac{1}{2})E_0\sum_{t=0}^{\infty}\beta^t\{[\pi_t^2+\lambda_y y_t^2+\lambda_r(r_t-r_{t-1})^2]\}$$

subject to

$$\pi_t=\gamma_b\pi_{t-1}+\gamma_f E_t\pi_{t+1}+\lambda_1 y_t+\lambda_2 y_{t-1}+e_t$$
$$e_{t+1}=\rho_e e_t+[\epsilon_{t+1}+v_{t+1}^e]$$
$$y_t=\gamma_8 y_{t-1}+\gamma_9 E_t y_{t+1}+\gamma_{10}(r_t-E_t\pi_{t+1})+u_t$$
$$u_{t+1}=\rho_u u_t+[\upsilon_{t+1}+v_{t+1}^u]$$
$$E_0\sum_{t=0}^{\infty}\beta^t\{[v_{t+1}^e]^2+[v_{t+1}^u]^2\}\leq\eta_0$$

Multiplier Version of the Stackelberg Problem

$$\min_{\{v_{t+1}^e,v_{t+1}^u\}_{t=0}^{\infty}}\max_{\{\pi_t,y_t,r_t\}_{t=0}^{\infty}}E_t\sum_{i=0}^{\infty}\beta^i\{[(\frac{1}{2})\pi_{t+i}^2+(\frac{1}{2})\lambda_y y_{t+i}^2+(\frac{1}{2})\lambda_r(r_{t+i}-r_{t+i-1})^2$$
$$-(\frac{1}{2})\beta\Theta([v_{t+i+1}^e]^2+[v_{t+i+1}^u]^2)]$$

$$+s_{1t+i}(\pi_{t+i}-\gamma_b\pi_{t+i-1}-\gamma_f\pi_{t+i+1}-\lambda_1 y_{t+i}-\lambda_2 y_{t+i-1}-e_{t+i})$$
$$+s_{2t+i}(\rho_e e_{t+i}+[\epsilon_{t+i+1}+v_{t+i+1}^e]-e_{t+i+1})$$
$$+s_{3t+i}(y_{t+i}-\gamma_8 y_{t+i-1}-\gamma_9 y_{t+i+1}-\gamma_{10}(r_{t+i}-\pi_{t+i+1})-u_{t+i})$$
$$+s_{4t+i}(\rho_u u_{t+i}+[\upsilon_{t+i+1}+v_{t+i+1}^u]-e_{t+i+1})\}$$
$$s_{1,t-1},s_{2,t-1}=0t=0$$

Calibration

Parameter	Description	Values
Parameters for the baseline model		
β	Discount Factor	0.99
σ	Inverse of the Elasticity of Intertemporal Substitution	2.75
η	Inverse of the Frisch Labor Supply Elasticity	5
θ	Calvo Price Stickiness Parameter	0.75
$(1-\alpha)$	Labor Share in the Production Function	0.75
λ_y	Weight of the output gap in loss function	0.25
λ_r	Weight of the interest rate in loss function	0.3
Θ	Robustness Parameter	12.5
Parameters specific to the model with habit		
h	Habit Persistence	0.6
Θ	Robustness Parameter	91
Parameters specific to the model with inflation persistence		
ω	the Fraction of Backward-looking Firms	0.25
Θ	Robustness Parameter	38
Parameters specific to the model with habit and inflation persistence		
Θ	Robustness Parameter	100

The Degree of Robustness

- As in Hansen and Sargent (2008) and Giordani and Soderlind (2004), I use likelihood ratio test to calculate the detection error probabilities.
- A detection-error probability is the probability that an econometrician observing equilibrium outcomes would make a wrong deduction about whether two competing models generate the data.
- The idea is to connect the value of the robustness parameter to the probability of making the incorrect choice of model between the reference model and the worst-case model.
- The probability of making this mistake is computed by simulations with

$$p(\Theta)=\frac{1}{2}[Pr(L^R>L^W|W)+Pr(L^W>L^R|R)]$$

- where R and W denotes for the reference model and the worst-case model respectively.

RESULTS

Robust Policy Rule

	e_{t-1}	u_{t-1}	s_{1t-1}	s_{3t-1}	r_{t-1}	ε_t	υ_t	y_{t-1}	π_{t-1}
Benchmark Model									
Rational Expectations Rule	0.084124	0.759322	-0.221848	-0.674672	0.553161	0.105155	0.949153	–	–
Robust Rule	0.038375	0.818540	-0.229571	-0.690094	0.522201	0.047968	1.023175	–	–
Sign of the Change	-	+	+	+	-	-	+		
Habit Persistence									
Rational Expectations Rule	0.247372	2.399231	-0.249915	-0.215174	0.616679	0.309215	2.999039	0.187211	–
Robust Rule	0.278031	2.820718	-0.258947	-0.212336	0.556326	0.347538	3.525898	0.203994	–
Sign of the Change	+	+	+	-	-	+	+	+	
Inflation Persistence									
Rational Expectations Rule	0.097847	0.717522	-0.165090	-0.758090	0.569876	0.122309	0.896903	–	0.064793
Robust Rule	0.011272	0.746094	-0.169593	-0.776358	0.555826	0.014090	0.932617	–	0.066296
Sign of the Change	-	+	+	+	-	-	+		+
Habit and Inflation Persistence									
Rational Expectations Rule	0.368632	2.280198	-0.174542	-0.234084	0.628676	0.460790	2.850248	0.172773	0.079458
Robust Rule	0.407979	2.680289	-0.180170	-0.231233	0.571973	0.509974	3.350362	0.189307	0.084361
Sign of the Change	+	+	+	-	-	+	+	+	+

Note: The third row for each model demonstrates how doubts about model uncertainty changes the coefficients of the policy rule. A positive change reflects more aggressive response of the robust rule. The coefficients of the robust rule is calculated for a detection error probability of 20%

Figure 2: Impulse Responses for the Demand Shock

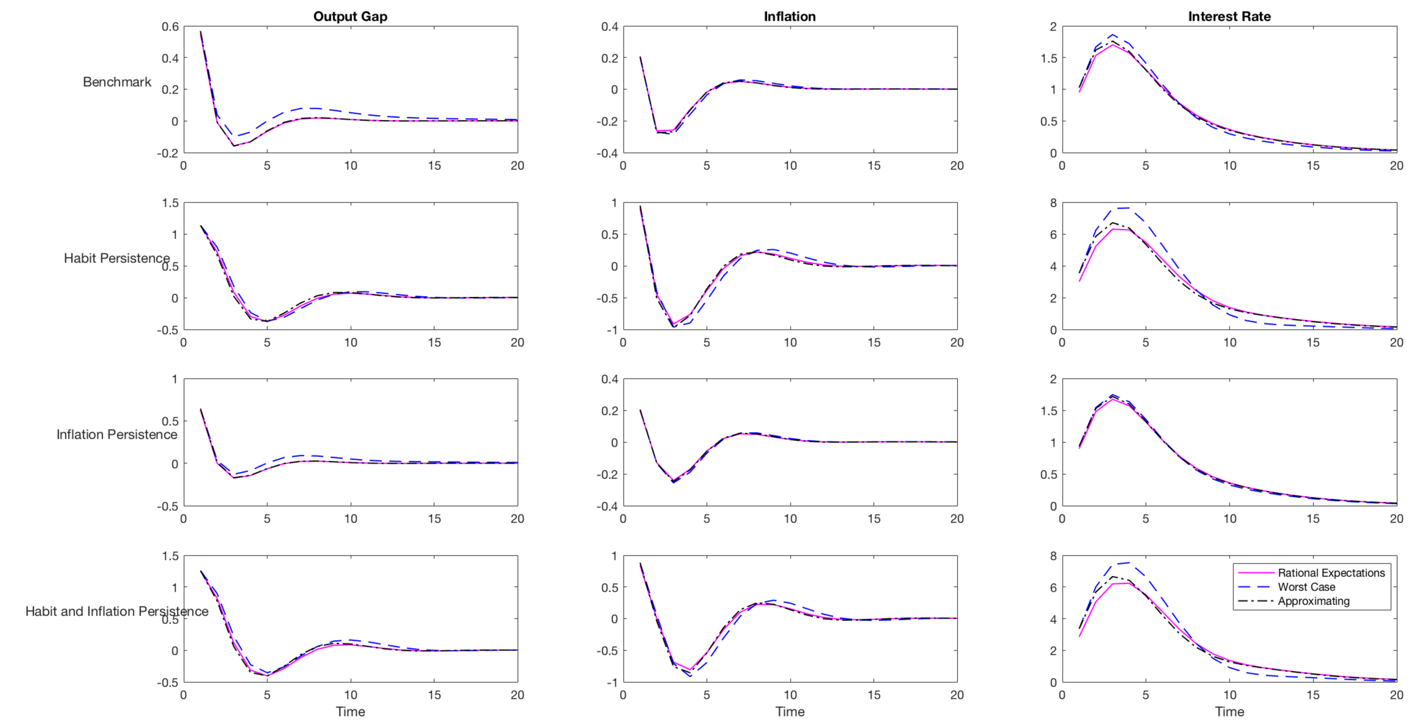


Figure 3: Impulse Responses for the Supply Shock