Are Nonlinear Methods Necessary at the Zero Lower Bound?

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Introduction

- What is the best way to deal with the zero lower bound (ZLB) constraint on the short-term nominal interest rate?
- People use a variety of solution methods and estimation procedures that differ in their treatment of the constraint
- We examine the importance of the ZLB constraint by estimating three versions of a New Keynesian model with a particle filter:
  1. Nonlinear: occasionally binding ZLB constraint (solved globally)
  2. Constrained linear: only imposes the constraint in the filter
  3. Unconstrained linear: never imposes the constraint
- We compare the global nonlinear solution to the one from `OccBin`
- We extend the nonlinear model with a banking sector that includes additional states, shocks, and observables (e.g., interest rate spread)

New Keynesian Model

- A representative household chooses $\{c_t, n_t, b_t\}_{t=0}^{\infty}$ to maximize

$$E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t [\log(c_t - h c^a_{t-1}) - \chi n_t^{1+\eta}/(1+\eta)],$$

where $\tilde{\beta}_0 \equiv 1$ and $\tilde{\beta}_t = \prod_{j=1}^{t} \beta_j$ for $t > 0$ subject to $c_t + b_t = w_t n_t + i_{t-1} b_{t-1}/\pi_t + d_t$. Optimality implies $w_t = \chi n_t^{\eta}(c_t - h c^a_{t-1})$ and $1 = i_t E_t[q_{t,t+1}/\pi_{t+1}]$ where $q_{t,t+1} \equiv \beta_{t+1}(c_t - h c^a_{t-1})/(c_{t+1} - h c^a_t)$.

- Firm optimality condition:

$$\varphi\left(\frac{\pi_t}{\bar{\pi}} - 1\right)\frac{\pi_t}{\bar{\pi}} = 1 - \theta + \theta\frac{w_t}{z_t} + \varphi E_t\left[q_{t,t+1}\left(\frac{\pi_{t+1}}{\bar{\pi}} - 1\right)\frac{\pi_{t+1}}{\bar{\pi}}\frac{y_{t+1}}{y_t}\right]$$

- Production Function: $y_t = z_t n_t$
- Monetary policy rule: $i_t = \max\{\underline{\imath}, i_t^*\}$ where i^* is the notional rate

$$i_t^* = (i_{t-1}^*)^{\rho_i}(\bar{\imath}(\pi_t/\bar{\pi})^{\phi_\pi}(c_t/(\bar{g} c_{t-1}))^{\phi_c})^{1-\rho_i}\exp(\sigma_\nu \nu_t),$$

- Resource constraint: $c_t = [1 - \varphi(\pi_t/\bar{\pi} - 1)^2/2] y_t$
- Discount factor (β): $\beta_t = \bar{\beta}(\beta_{t-1}/\bar{\beta})^{\rho_\beta}\exp(\sigma_\upsilon \upsilon_t)$
- Technology (z) follows a random walk:

$$z_t = z_{t-1} g_t \quad \text{where} \quad g_t = \bar{g}(g_{t-1}/\bar{g})^{\rho_g}\exp(\sigma_\varepsilon \varepsilon_t)$$

Calibrated Parameters

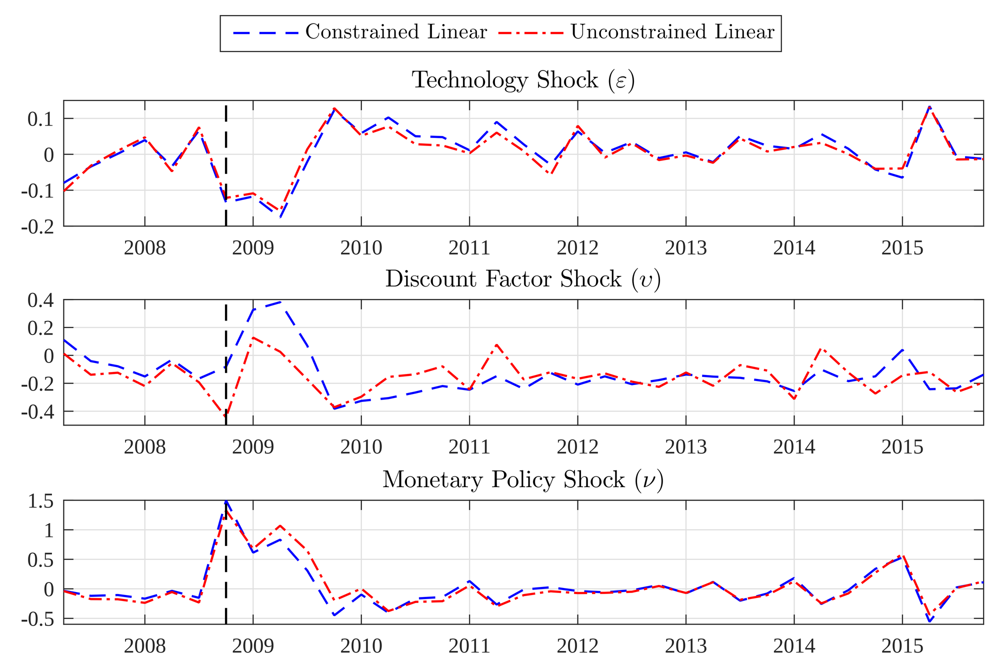
Steady-State Discount Factor	$\bar{\beta}$	0.9987
Frisch Elasticity of Labor Supply	$1/\eta$	3
Elasticity of Substitution between Goods	θ	6
Steady-State Labor	$\bar{n}$	0.33
Nominal Interest Rate Lower Bound	$\underline{\imath}$	1.00035

Estimated Parameters

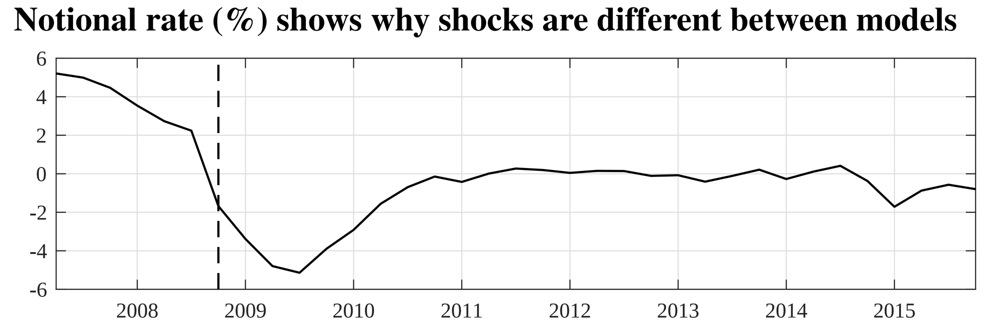
	Prior		Posterior Mean $(5\%, 95\%)$		
			Nonlinear	Linear	
Parameter	Dist	Mean (SD)	Constrained	Constrained	Unconstrained
φ	Gam	80.000 (20.000)	96.46409 (65.7392,130.3952)	90.61107 (59.9691,124.0710)	89.75551 (60.0572,124.9987)
h	Beta	0.500 (0.200)	0.46334 (0.33583,0.58604)	0.44752 (0.30823,0.58102)	0.43851 (0.29796,0.56827)
ϕ_π	Norm	2.500 (1.000)	4.07825 (3.32372,4.85669)	4.12383 (3.30709,5.01204)	3.74194 (3.02972,4.53073)
ϕ_c	Norm	1.000 (0.400)	1.46414 (1.10608,1.85105)	1.37493 (1.01308,1.79434)	1.24805 (0.90772,1.63401)
ρ_i	Beta	0.500 (0.200)	0.81158 (0.75375,0.86060)	0.83541 (0.78091,0.87712)	0.83846 (0.78707,0.88103)
ρ_g	Beta	0.500 (0.200)	0.20064 (0.06547,0.37805)	0.19063 (0.06085,0.36443)	0.18582 (0.05543,0.36851)
ρ_β	Beta	0.500 (0.200)	**0.90245** (0.87001,0.92958)	0.92920 (0.88436,0.96588)	0.92326 (0.88096,0.96010)
σ_ε	IGam	0.010 (0.010)	0.00968 (0.00738,0.01241)	0.00981 (0.00743,0.01274)	0.00975 (0.00752,0.01239)
σ_υ	IGam	0.010 (0.010)	0.00215 (0.00159,0.00286)	0.00216 (0.00161,0.00290)	0.00197 (0.00147,0.00261)
σ_ν	IGam	0.010 (0.010)	0.00199 (0.00148,0.00261)	0.00187 (0.00139,0.00242)	0.00181 (0.00137,0.00232)
$\log(ML)$			**1586.01**	**1586.25**	1578.04

Results

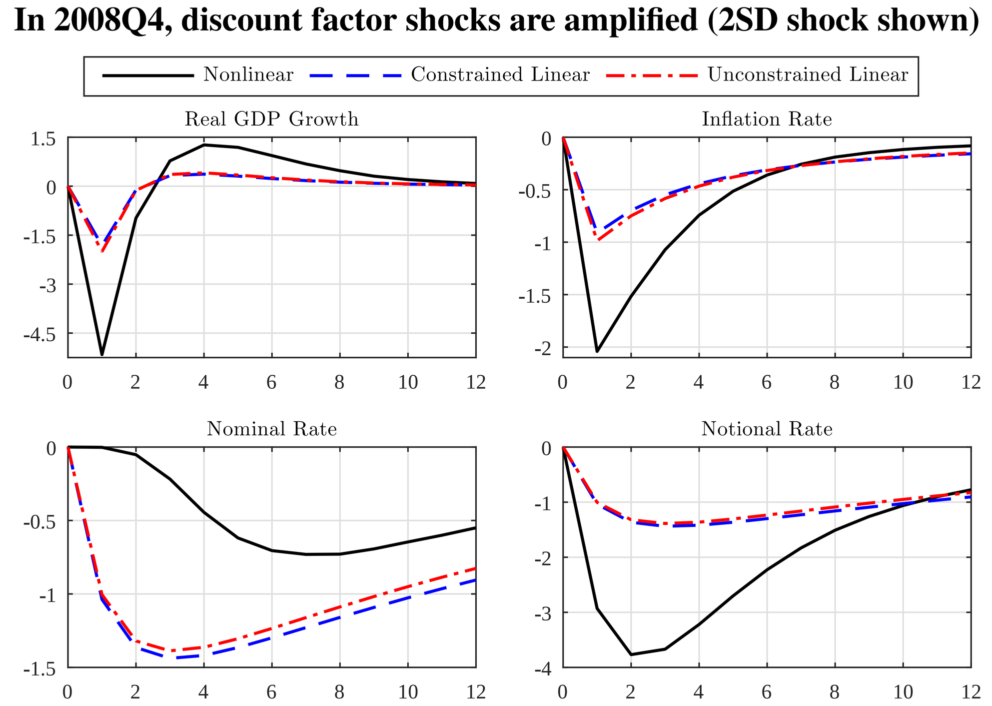
Differences between shocks from linear vs. nonlinear model in SDs



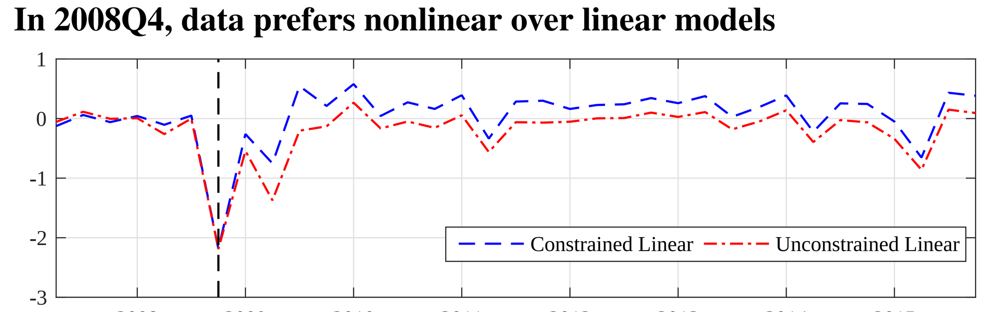
Notional rate (%) shows why shocks are different between models



In 2008Q4, discount factor shocks are amplified (2SD shock shown)



In 2008Q4, data prefers nonlinear over linear models



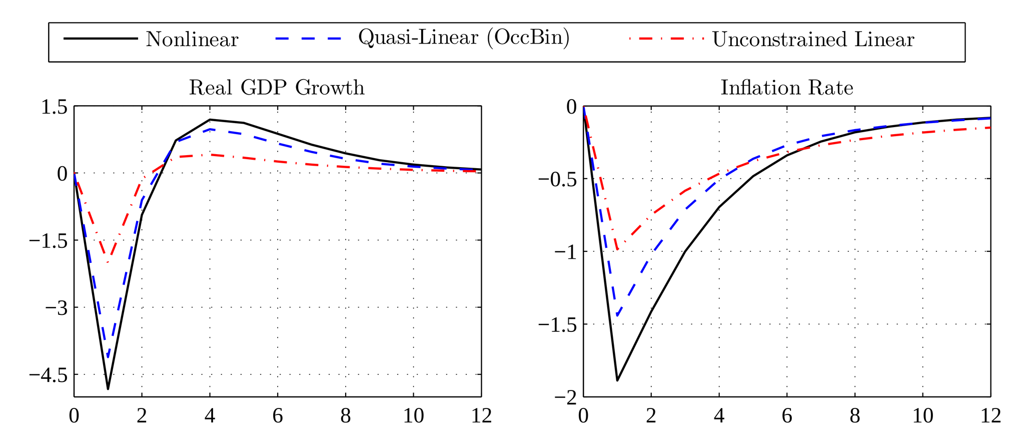
Real GDP Growth Moments: Data vs. Model

	Mean		SD		Skewness	
	Pre-ZLB	ZLB	Pre-ZLB	ZLB	Pre-ZLB	ZLB
Data	1.75	-0.75	2.21	3.96	-0.41	-1.35
Nonlinear	1.56 (0.70,2.43)	**0.02** (-2.10,2.04)	2.44 (1.99,2.98)	**4.60** (3.25,6.43)	0.08 (-0.32,0.49)	**-0.71** (-1.77,0.29)
Constrained Linear	1.50 (0.62,2.39)	1.02 (-1.08,3.04)	2.54 (2.09,3.07)	2.55 (1.54,3.83)	0.04 (-0.36,0.45)	-0.19 (-1.43,0.87)
Unconstrained Linear	1.51 (0.65,2.36)	1.07 (-1.01,3.12)	2.54 (2.10,3.05)	2.45 (1.48,3.62)	0.04 (-0.35,0.44)	-0.10 (-1.17,0.96)

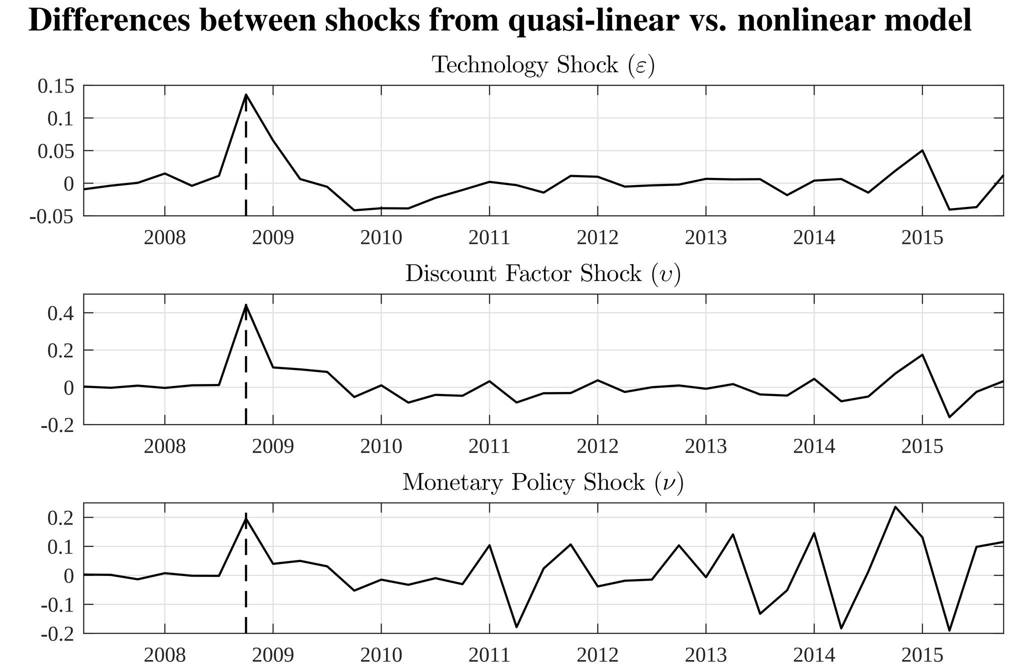
Quasi-Linear Solution

- Global solution to nonlinear model is the most accurate but also the most computationally burdensome to solve
- An alternative is to solve the quasi-linear model with `OccBin` [Guerrieri and Iacoviello (JME, 2015)]
- Solution is much faster, but has two drawbacks:
  1. Household do not form expectations about going to the ZLB
     - Decisions are unaffected by the proximity of the ZLB
     - Lower frequency of going to the ZLB
     - ZLB will bind later in the state space
  2. Must return to the regime where the ZLB does not bind
     - Costly to use filters based on repeated model simulations

In 2008Q4, IRF of 2SD discount factor shock closer to nonlinear



Differences between shocks from quasi-linear vs. nonlinear model



Filter Comparison

	Linear Unconstrained Model Posterior Mean $(5\%, 95\%)$			
Parameter	Particle	Kalman ME	Kalman No ME	Fair & Taylor
φ	89.75551 (60.0572, 124.9987)	88.32964 (58.9214, 122.5572)	79.63849 (52.7309, 110.7193)	115.27029 (81.4358, 157.3756)
h	0.43851 (0.29796, 0.56827)	0.43056 (0.29310, 0.56472)	0.43644 (0.31774, 0.55465)	0.54844 (0.43274, 0.65539)
ϕ_π	3.74194 (3.02972, 4.53073)	3.73074 (3.01440, 4.54203)	3.75178 (3.07512, 4.52112)	**1.99119** (1.79762, 2.21518)
ϕ_c	1.24805 (0.90772, 1.63401)	1.25149 (0.88588, 1.64862)	1.35632 (1.00980, 1.73330)	**0.73057** (0.51654, 0.94298)
ρ_i	0.83983 (0.78855, 0.88403)	0.84123 (0.79044, 0.88300)	0.89440 (0.87116, 0.91642)	0.85140 (0.82349, 0.87751)
ρ_g	0.19183 (0.05145, 0.37168)	0.19402 (0.05692, 0.38591)	0.12382 (0.03770, 0.25779)	0.22251 (0.06236, 0.43357)
ρ_β	0.92928 (0.88839, 0.96493)	0.93240 (0.89520, 0.96652)	0.94715 (0.91585, 0.97348)	**0.97460** (0.96040, 0.98621)
σ_ε	0.00938 (0.00729, 0.01189)	0.00929 (0.00718, 0.01178)	0.01122 (0.00911, 0.01359)	0.01356 (0.01025, 0.01710)
σ_υ	0.00191 (0.00143, 0.00257)	0.00186 (0.00140, 0.00244)	0.00160 (0.00124, 0.00206)	0.00109 (0.00087, 0.00134)
σ_ν	0.00177 (0.00133, 0.00227)	0.00174 (0.00133, 0.00226)	0.00123 (0.00106, 0.00141)	0.00125 (0.00108, 0.00144)

Interest Rate Spread Model

- Extended model is based on Curdia and Woodford (JMCB, 2010)
- Two types of Households: Lending households, lh, and borrowing households, bh, that differ in their discount rates
- No habit persistence in consumption
- Banks that collect deposits, D, and issue loans, L, subject to a real $(\gamma_1 z (L/z)^{\gamma_0})$ and a financial cost (ψL) to lending
- Bank optimality condition: $i_t^\ell = i_t^d(1 + s_t)$ where i_t^ℓ is the loan rate, i_t^d is the deposit rate, and $s_t \equiv \gamma_0\gamma_1(L_t/z_t)^{\gamma_0 - 1} + \psi_t$ is the spread.
- The financial cost, ψ, follows a log-normal AR(1) process

$$\ln(\psi_t) = (1 - \rho_\psi)\ln(\bar{\psi}) + \rho_\psi \ln(\psi_{t-1}) + \sigma_\xi \xi_t$$

- Exogenous state variables: β_t, ψ_t, ν_t, g_t
- Endogenous state variables: c_{t-1}, i_{t-1}^{dn}, $i_{t-1}^{d}\ell_{t-1}$
- Policy functions: π_t, $c_{lh,t}$, $c_{bh,t}$, $n_{bh,t}$

Estimated Parameters

	Prior		Posterior Mean $(5\%, 95\%)$	
Parameter	Dist	Mean (SD)	Nonlinear	Constrained Linear
φ	Gam	80.000 (20.000)	77.86658 (53.20203,106.62357)	70.45353 (47.41162,99.07662)
ϕ_π	Norm	3.000 (0.750)	3.80130 (3.16238,4.50769)	3.86881 (3.16977,4.61810)
ϕ_y	Norm	1.000 (0.400)	1.79307 (1.41407,2.17524)	1.66688 (1.33652,2.04541)
ψ	Gam	0.006 (0.002)	0.00429 (0.00292,0.00549)	0.00427 (0.00273,0.00582)
s	Gam	0.008 (0.002)	0.00575 (0.00527,0.00629)	0.00603 (0.00543,0.00667)
γ_0	Gam	15.000 (5.000)	13.77215 (7.20076,21.14558)	16.67012 (9.08941,23.83093)
ρ_β	Gam	0.500 (0.200)	**0.90778** (0.87660,0.93162)	**0.94823** (0.91425,0.97638)
ρ_ψ	Norm	0.500 (0.200)	0.91655 (0.86779,0.95363)	0.92732 (0.86223,0.97353)
ρ_g	Norm	0.500 (0.200)	0.21388 (0.06651,0.41686)	0.2111 (0.06179,0.40569)
ρ_{id}	Norm	0.500 (0.200)	0.77221 (0.69787,0.82964)	0.79529 (0.73783,0.84627)
σ_ε	Norm	0.010 (0.010)	0.00157 (0.00124,0.00198)	0.00162 (0.00126,0.00203)
σ_υ	Gam	0.100 (0.100)	**0.14231** (0.10913,0.18669)	**0.17936** (0.11941,0.26234)
σ_ν	Gam	0.0100 (0.010)	0.00215 (0.00153,0.00296)	0.00204 (0.00147,0.00273)
σ_ξ	Gam	0.010 (0.010)	0.00675 (0.00535,0.00820)	0.00673 (0.00536,0.00808)
$\log(ML)$			**2263.35**	2245.68

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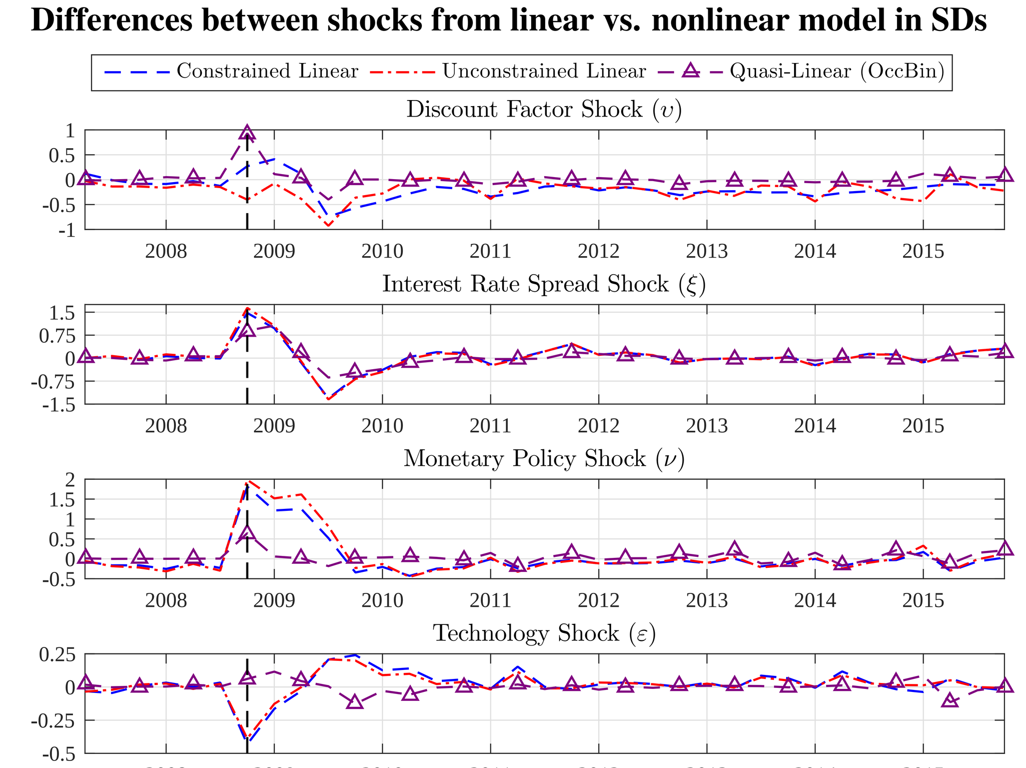
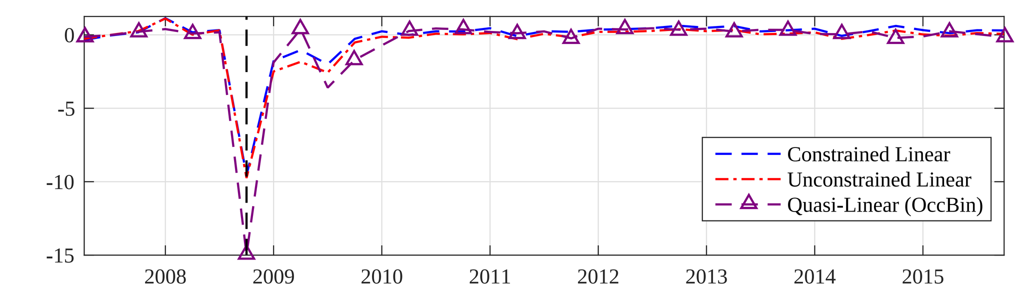
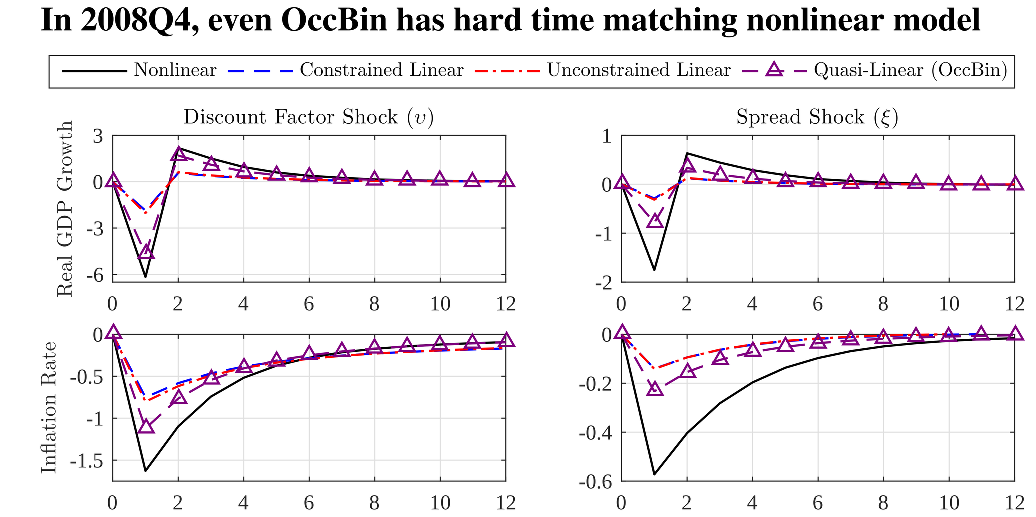


In 2008Q4, data really prefers nonlinear over linear models



In 2008Q4, even OccBin has hard time matching nonlinear model



Conclusion

1. Over the entire sample, the mean posterior parameter estimates are strikingly similar across the three models
2. The nonlinear and constrained linear models have nearly identical marginal likelihoods over the whole sample, but the nonlinear model has a higher likelihood at the ZLB
3. In the linear models, larger discount factor and monetary policy shocks are necessary to explain the data in the ZLB period
4. `OccBin` provides a better approximation than a linear solution but filters used with this solution have drawbacks
5. Larger differences arise in models that match observables that had asymmetric and adverse effects during the crisis

Appendix

Solution Method

1. Solve the linear models using Sims's (Comp. Econ., 2002) algorithm
2. Solve the nonlinear models using policy function iteration:
   (a) Use linear solution as an initial conjecture: $\tilde{c}^A(\mathbf{z}_t)$, $\pi^A(\mathbf{z}_t)$
   (b) For all nodes $d \in D$, implement the following steps:
      i. Solve for $\{\tilde{w}_t, \tilde{y}_t, i_t^*, i_t\}$ given $\tilde{c}^A_{i-1}(\mathbf{z}_t^d)$ and $\pi^A_{i-1}(\mathbf{z}_t^d)$
      ii. Use piecewise linear interpolation to solve for updated values of consumption and inflation, $\{\tilde{c}_{t+1}, \pi_{t+1}\}_{m=1}^{M}$, given each realization of the updated state vector, $\mathbf{z}_{t+1}$
      iii. Given $\{\tilde{c}_{t+1}, \pi_{t+1}\}_{m=1}^{M}$, solve for future output, $\{\tilde{y}^m_{t+1}\}_{m=1}^{M}$, which enters expectations. Then numerically integrate.
      iv. Use `csolve` to determine the values of the policy functions that best satisfy the equilibrium system
   (c) On iteration i, $\text{maxdist}_i \equiv \max\{|\tilde{c}_i^A - \tilde{c}_{i-1}^A|, |\pi_i^A - \pi_{i-1}^A|\}$.
   Continue iterating until $\text{maxdist}_i < 10^{-7}$ for all d

Estimation Procedure

- Use quarterly data on per capita real GDP, the GDP price deflator, and the Fed Funds Rate from 1986Q1 to 2015Q4
- Use a Metropolis-Hastings algorithm with a particle filter to evaluate the likelihood of the posterior distribution
- Observation equation:

$$\begin{bmatrix} \log\left(\frac{RGDP_t/CNP_t}{RGDP_{t-1}/CNP_{t-1}}\right) \\ \log(DEF_t/DEF_{t-1}) \\ \log(1 + FFR_t)/4 \end{bmatrix} = \begin{bmatrix} \log(g_t c_t/c_{t-1}) \\ \log(\pi_t) \\ \log(i_t) \end{bmatrix} + \begin{bmatrix} \xi_{1t} \\ \xi_{2t} \\ \xi_{3t} \end{bmatrix},$$

where $\xi \sim \mathbb{N}(0, \Sigma)$ is a vector of measurement errors.

- We adapt the particle filter to incorporate the information contained in the current observation, which helps the model better match outliers in the data (e.g., 2008Q4).