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#### R-Star:

The long-run equilibrium rate has not declined by much



seit 1558

## $r^*$ : 3 concepts

- Short run: Flexible-price real rate in New Keynesian
   DSGE models
- Medium run: Equilibrium real rate in AD-Phillips curve model (Laubach-Williams).
- 3. Long run: Steady state equilibrium rate.

One-sided estimates Percent 4 3 Current estimates 2 estimates 2004 2007 2010 2013 1980 1983 1986 1989 1992 1995 1998 2001

Figure 5: Laubach-Williams model estimates of the natural rate of interest

Source: Laubach and Williams (2015)

### Huge policy impact

- L. Summers: 2014, "The Laubach/Williams methodology demonstrates a very substantial and continuing decline in the (equilibrium) real rate of interest."
- P. Krugman: 2015, NYT, "the low natural rate is as solid a result as anything in real time can be" referring to LW.
- J. Yellen, 2015, "Under assumptions that I consider more realistic under present circumstances, the Taylor rule calls for the federal funds rate to be close to zero."

## $r^*$ in the Taylor rule

$$r = p + .5y + .5(p - 2) + (2) \tag{1}$$

#### where

- r is the federal funds rate,
- p is the rate of inflation over the previous four quarters
- y is the percent deviation of real GDP from a target.

The 2-percent "equilibrium" real rate is close to the assumed steady-state growth rate of 2.2 percent.

- $r^*$  in the Taylor rule refers to the long-run equilibrium rate, not a medium term concept.
- Also, the average real federal funds rate was about 2%.
- Now (1965Q1-2017Q1) the average real federal funds rate is 1.82%.

# We use New Keynesian DSGE models to estimate long-run equilibrium rates

- DSGE models distinguish structural factors not considered by simple state space models like Laubach-Williams: demand compoments, technology shocks, investment shocks, risk premia, monetary policy rule and deviations.
- Models let us evaluate sources of deviations of real rates from equilibrium
- Allow some time variation by using 20-year moving window samples to account for possible breaks/trends not captured by model.
- Real-time data.
- Builds on Wieland (2016, Annual Report of the German Council of Economic Experts).

Steady state equilibrium rate in New Keynesian models (notation as in Smets and Wouters, 2007):

Euler equation: 
$$\Xi_t = \beta \varepsilon_t^b R_t E_t \left[ \frac{\Xi_{t+1}}{\Pi_{t+1}} \right]$$

 $\Xi_t$ : marginal utility of consumption,  $\beta$ : discount factor,  $\varepsilon_t^b$  risk premium shock,  $R_t$ : nominal gross interest rate,  $\Pi_{t+1}$ : inflation.

$$\Xi_t = (C_t - \lambda C_{t-1})^{-\sigma_c} exp\left(\frac{\sigma_c - 1}{1 + \sigma_l} L_t(j)^{1 + \sigma_l}\right)$$

 $C_t$ : Consumption,  $\lambda$ : habit formation,  $\sigma_c$  inv. int. elast. of substitution,  $\sigma_l$  Frisch elasticity of labor supply.

Detrending with  $\xi_t = \Xi_t/\gamma^{-\sigma_c t}$  (output grows with a deterministic trend  $\gamma^t$ , i.e.  $\Xi_t$  with  $\gamma^{-\sigma_c t}$ )

Euler equation:

$$\xi_t = \frac{\beta}{\gamma^{\sigma_c}} \varepsilon_t^b R_t E_t \left[ \frac{\xi_{t+1}}{\Pi_{t+1}} \right]$$

Steady state:

$$R^* = \frac{\gamma^{\sigma_c}}{\beta} \Pi^*$$

Long –run real equilibrium interest rate

$$r^* = \frac{\gamma^{\sigma_c}}{\beta}$$

 $\rightarrow$  Estimates mainly depend on priors and posteriors of  $\gamma$ ,  $\sigma_c$  and  $\beta$ 

### Estimates based on the Smets and Wouters model.

### Original sample 1966-2004:

	Prior distribution		Posterior distribution			n	
Parameter	Distr.	Mean	Std.	Mode	Mean	5 %	95 %
$\sigma_c$	Normal	1.5	0.37	1.39	1.38	1.16	1.59
$rac{\sigma_c}{ar{\gamma}}$	Normal	0.4	0.10	0.43	0.43	0.40	0.45
$100(\beta^{-1} - 1)$	Gamma	0.25	0.1	0.16	0.16	0.07	0.27
implied parameters							
eta		0.9975		0.9984	0.9984	0.9993	0.9974
$r^* = \frac{\gamma^{\sigma_c}}{\beta}$		1.0085		1.0075	1.0075	1.0053	1.0099
$\gamma_{ann}$		1.60		1.72	1.72	1.60	1.80
$r_{ann}^*$		3.4436		3.0339	3.0339	2.1369	4.0192

Table 1: Prior and Posterior Distribution of Parameters Relevant for  $r^*$ . Notes:  $\gamma_{ann}$  denotes the annualized trend growth rate  $(\gamma_{ann} = (\gamma^{*4} - 1) * 100)$ .  $r^*_{ann}$  denotes the annualized steady state interest rate  $(r^*_{ann} = (r^{*4} - 1) * 100)$ 

## Smets/Wouters: 20-year rolling window $r^*$ estimates

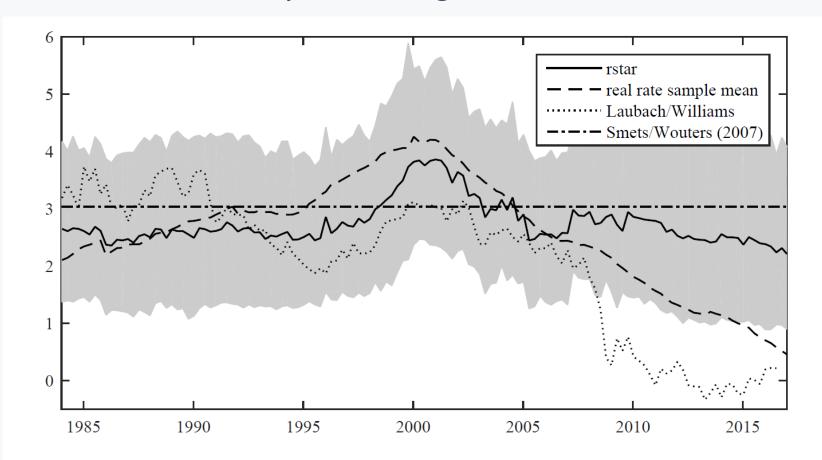


Figure 1: Realtime estimates of rstar with the Smets and Wouters model. Shaded areas show 95% probability bands. The dashed line shows the sample mean of the real interest rate.

## Structural parameters influence $r^*$

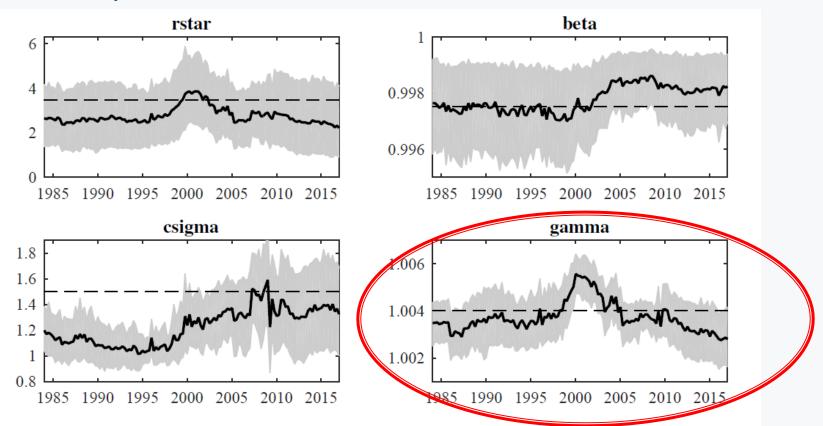


Figure 2: Structural parameters relevant for rstar. Shaded areas show 95% probability bands. The dashed line shows the implicit prior means (the parameters are not estimated directly, but functions of them). The dashed line for rstar shows the combination of prior means of  $\gamma^{\sigma_c}/\beta$ .

### Influence of Structural Parameters Decline of $r^*$

Holding fix one parameter at the 2000 subsample level (peak of  $r^*$ ) and vary the other two:

Specification	$r_{ann}$	$\gamma_{ann}$	eta	$\sigma_c$
$\gamma_{2000}, \beta_{2000}, \sigma_{c,2000}$	2.20%	2.18%	0.9974	1.27
$\gamma_{2017}, \beta_{2017}, \sigma_{c,2017}$	3.84%	1.12%	0.9982	1.32
$\gamma_{2017}, \beta_{2000}, \sigma_{c,2000}$	2.47%	1.12%	0.9974	1.27
$\gamma_{2000}, \beta_{2017}, \sigma_{c,2000}$	3.51%	2.18%	0.9982	1.27
$\gamma_{2000}, \beta_{2000}, \sigma_{c,2017}$	3.95%	2.18%	0.9974	1.32

Table 4: Influence of Structural Parameters on Decline of  $r^*$ .

# Which factors are driving the real interest rate below $r^*$ (most recent sub-sample)?

Approach: Historical decomposition and then taking the mean over the sample

Shock	Contribution to $Mean(r_t) - r^*$ 0.45% - 2.20%	Share of Overall Difference
technology	-0.09%	5%
risk premium	-0.48%	27%
government spending	-0.04%	2%
investment spec. techn.	-0.24%	14%
monetary policy	-0.83%	47%
price markup	0.15%	-9%
wage markup	-0.01%	1%
initial values	-0.22%	13%

Table 5: Contribution of Shocks to Difference Between  $r^*$  and the Real Interest Rate.

## The role of priors (last subsample):

Use very wide priors for key parameters determining  $r^*$ :

		Poster	2.5%	97.5%		
Specification	$\sigma_c$	$\gamma_{ann}$	β	$r_{ann}^*$	$r_{ann}^*$	$r_{ann}^*$
Baseline	1.32	1.12	0.9982	2.20	0.89	4.06
Wide prior $\sigma_c$	1.30	1.12	0.9982	2.17	0.91	4.03
Wide prior $\bar{\gamma}$	1.30	0.88	0.9983	1.83	0.67	3.47
Wide prior $100(\beta^{-1}-1)$	1.37	1.12	0.9999	1.60	0.78	2.76
Wide prior $\sigma_c$ , $\bar{\gamma}$ , $100(\beta^{-1}-1)$	1.30	0.76	0.9999	1.03	0.46	2.02

# Robustness with respect to zero lower bound (last subsample):

	Posterior Mean		2.5%	97.5%		
Specification	$\sigma_c$	$\gamma_{ann}$	β	$r^*$	$r^*$	$r^*$
Baseline	1.32	1.12	0.9982	2.20	0.89	4.06
Shadow interest rate (Wu,Xia)	1.33	1.08	0.9982	2.20	0.82	4.03
Shadow int. + wide prior $\sigma_c$ , $\bar{\gamma}$ , $\beta$	1.37	0.96	0.9999	1.38	0.53	2.84
Shadow interest rate (Krippner)	1.30	1.21	0.9981	2.32	0.98	4.13
Shadow int. + wide prior $\sigma_c$ , $\bar{\gamma}$ , $\beta$	1.27	1.04	0.9999	1.35	0.52	2.65

Table 7: Robustness checks with shadow interest rate for data vintage 2017Q.

## Model uncertainty: 4 different variants

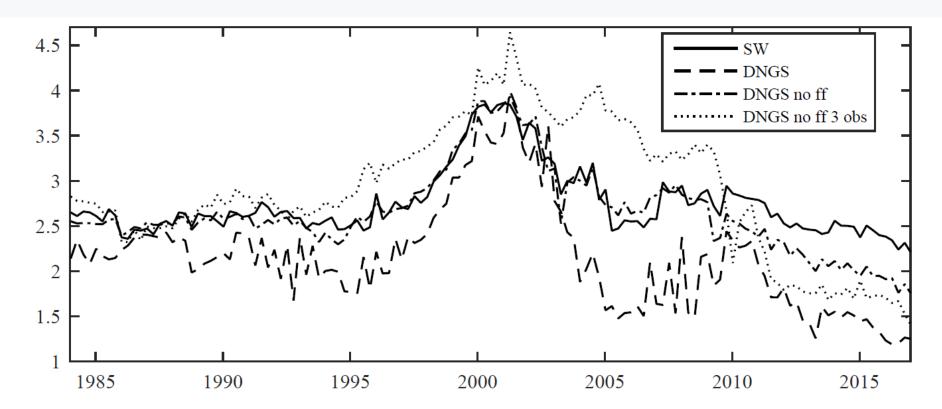


Figure 3: Posterior Mean Estimates of  $r^*$  from four different models. Notes: SW: Smets and Wouters, DNGS: Del Negro, Gianonni and Schorfheide, DNGS no ff: DNGS without financial frictions, DNGS no ff 3 obs: DNGS without financial frictions and three observables.

## Comparison to the literature:

Authors	$m{r}^*$ (recent years)	Change since late 90s
LW (2016)	0%	-3%
Kiley (2015)	1.25%	-0.6%
Lubik/Matthes (2015)	0.5%	-2.5%
Pescatori/Turunen (2015)	0.5-1%	-2.5%
Johannsen/Mertens (2016)	1.2%	-0.5%
Del Negro et al. (2017)	1-1.5%	-1 to -1.5%
Wieland/Wolters (2017)	1.2-2.2%	-1 to -2%

#### Conclusions

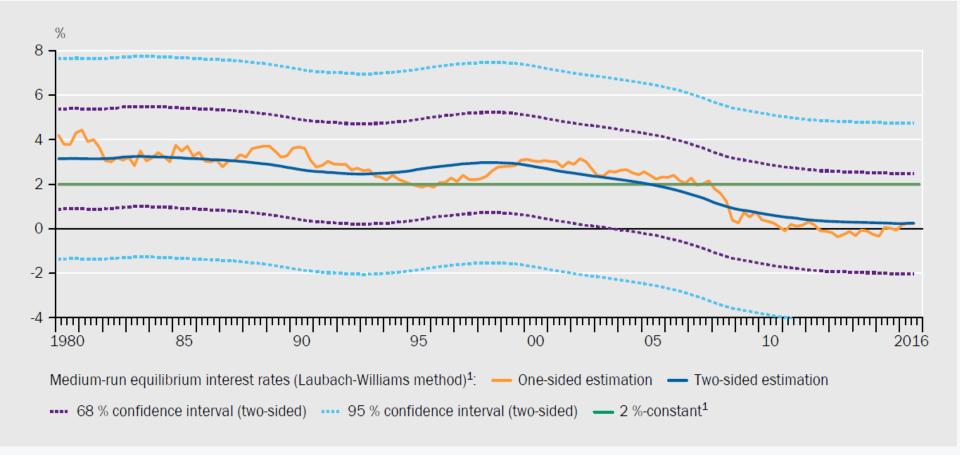
- Structural model estimates show that  $r^*$  has decreased from a peak of 3.5-4% around 2000 to a level of 1.2-2.2% in recent years. This is not much below the level that prevailed in the 1980s and early 1990s.
- $r^*$  is still significantly positive and above the decline in the mean real rate across several models.
- Decline in  $r^*$  mainly caused by productivity growth slowdown.
- Decline in the real rate below  $r^*$  caused by preference for liquid and safe assets (risk premium shock) and expansionary monetary policy.
- Discussions about the Taylor rule should focus on long-run equilibrium rate rather than short-run natural rate or medium-term concept.

## Thank you for your attention.



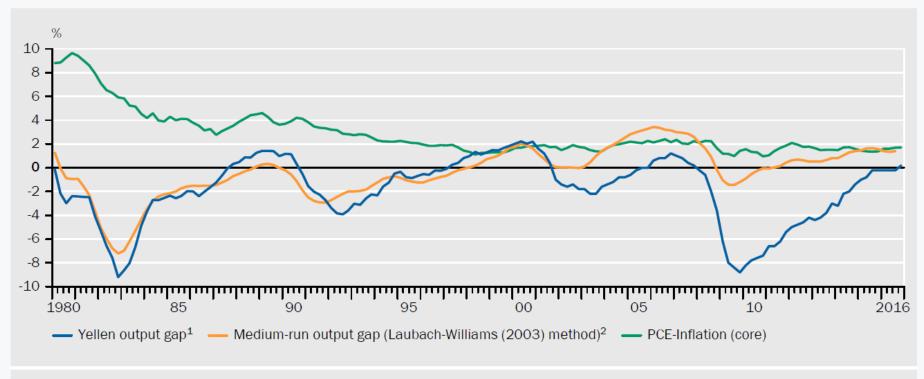
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## Laubach/Williams: High uncertainty around estimates



Source: Beyer and Wieland (2017)

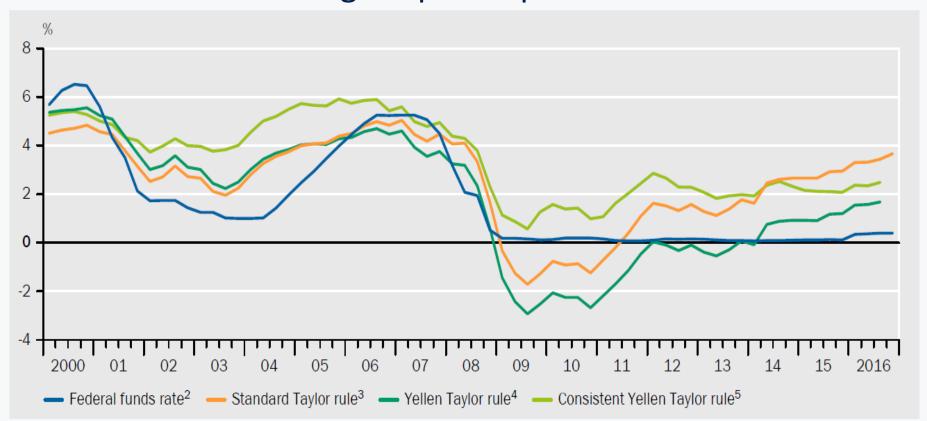
## Used with inconsistent output gap



1 - Based on the unemployment rate using Okun's law:  $Y_t = -2(U_t - U^*)$ , where U is the unemployment rate and  $U^*$  the natural rate of unemployment. 2 - Based on the two-sided estimation method.

Source: Beyer and Wieland (2017)

## Consistent use -> higher prescription



Source: Beyer and Wieland (2017)