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Monetary policy and PID control

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Abstract We show that many currently popular monetary policy rules fall structurally within a class of robust industrial control known as proportional-integral-differential, or PID, control. From this identification we propose a general class of PID-based monetary policy rules that include as limiting cases the original Taylor rule as well as lagged and forward-looking extensions of thereof. The effectiveness of parsimonious extensions of the Taylor rule are consistent with the well-known effectiveness and parsimony of PID control. We find that for the same reason encountered in other PID control applications—noisy data—most monetary policy rules fall in the proportional-integral subset of PID control known as PI control. We estimate both PID and PI monetary policy rules using the historical analysis approach of Taylor and compare the performance of our PI rule to other policy rules using a recently-developed macroeconomic-model comparison methodology. A key feature of PID control is its remarkable effectiveness for systems where the equations of motion are not known. Thus, PID-based rules both link monetary policy with a tradition of practical control in the absence of known dynamical equations and provide baseline rules for monetary policy in the face of macroeconomic model uncertainty.

Keywords Monetary-policy rules · Model uncertainty · Macroeconomic models · PID control

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1 Introduction

Although over a century has passed since their articulation by Wicksell (1898, 1907), monetary-policy rules and their role in monetary policy continue to engage the economics community.[1] Current expressions of these rules focus on interest-rate rules which gained heightened prominence following the publication of the parsimonious Taylor rule (Taylor 1993a). Since then a considerable literature has developed in which variations on the Taylor rule have been proposed in response to particular models of the macroeconomy.[2] This literature has been characterized generally by rules that are essentially variations of the Taylor rule and characterized recently by a view towards assessing rules less on their ability to perform for the macroeconomic model from which they originated and more on how they perform *across* macroeconomic models (Wieland et al. 2011; Taylor and Wieland 2012). These characteristics of monetary-policy rule development are strikingly similar to controller development in engineering (Bennett 1993; Åström and Murray 2008) and suggest that these fields share a common framework: a goal of this paper is to demonstrate that this is so.

When a central bank implements monetary policy by changing an interest rate in response to the deviation between desired and actual macroeconomic performance (e.g. inflation or output) the central bank is providing feedback into the economy designed to reduce, and hopefully eliminate, that deviation. This principle of feedback—"base correcting actions on the difference between desired and actual performance"—is found extensively in natural and technological systems (Åström and Murray 2008). Of the forms of feedback one can use, PID control is by far the most frequently encountered and also remarkably parsimonious: corrective action is based on (1) a proportion, P, of the deviation, (2) the history of the deviation represented by the integral, I, of the deviation over time and (3) the forecast or expectation of future deviations represented by the derivative, D, of the deviation with respect to time. Over 95 % of industrial applications and a wide range of biological systems employ elements of PID control (Åström and Murray 2008).[3]

In addition to this ubiquity and parsimony, decades of PID control across a wide range of applications have shown that "in the absence of any knowledge (in terms of a dynamical model) of the process to be controlled, the PID controller is the best form of controller" (Bennett 1993). This feature recommends PID control strongly as a way to address an issue that plagues the traditional approach to monetary-policy rule development: the intrinsic uncertainty in the equations of motion of the macroeconomy. Given a set of equations that describe a macroeconomy one can, in principle, derive a monetary-

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[1] For an engaging account of the history of monetary-policy rules see Woodford (2003), chapter 1.

[2] See, for example, Taylor (1999b), Woodford (2003), Taylor and Williams (2011), Taylor and Wieland (2012) and references therein.

[3] For an interesting and comprehensive account of the history, breadth, and current practice of feedback control in general and PID control in particular see Bennett (1993), Åström and Murray (2008). As discussed below, the vast majority of implementations of PID control in control engineering and, as we shall see in monetary policy, are in fact reduced forms of PID control such as PI control. In keeping with the tradition of control engineering, however, we use the term PID control as a general label for both PID and PI control.

policy rule (or controller) for that economy using well-known engineering techniques (Aoki 1967) that derive, ultimately, from the work of Maxwell (1867–1868), and this approach is used in monetary-policy rule development. As the complexity of a system increases, however, analytical tractability is often soon lost and system control can be realized only by using feedback control *tuned* to the system to be controlled, and the most popular approach to this problem in the field of control engineering is, as mentioned above, the proportional-integral-differential (PID) control (Bennett 1993; Åström and Murray 2008). Since recent work to assess monetary-policy rules *across* macroeconomic models (Wieland et al. 2011; Taylor and Wieland 2012) is a form of tuning, a natural question is whether PID control can be viewed as a monetary-policy rule. As we shall see, a subset of PID control—proportional-integral, or PI control—is nearly identical in mathematical form to the versions of the Taylor rule emerging from tuning work based on a global macroeconomic-model database (Wieland et al. 2011), and the PI-version of extended Taylor rules can be motivated on a purely economic basis. Thus, a contribution of this paper is a new PI-based monetary-policy rule that embraces the intrinsic uncertainty in our evolving understanding of macroeconomic dynamics.

We continue in Sect. 2 with the derivation of our monetary policy rule and a comparison of its formal structure with that of policy rules currently in use. The tuning of our rule and its performance are discussed in Sect. 3. In assessing rule performance we compare the performance of our policy rule against that of the original Taylor rule (Taylor 1993b) and of a rule recently proposed by Taylor and Wieland (2012) across 37 current-generation models of the macroeconomy. We close with a discussion and summary in Sect. 4.

2 Monetary policy rules and PID control

In his seminal work Taylor (1993a) expressed the policy rate in the United States, the Fed Funds rate r, in terms of the inflation gap π and output gap y as[4]

$$r(t) = K_0 + K_P^{(\pi)}\pi(t) + K_P^{(y)}y(t), \tag{1}$$

where the K_i are constants,[5] π is the difference between the average inflation over the 4 prior quarters and a target level of inflation, and "y is real [gross domestic product] (GDP) measured as a deviation from potential GDP" (Taylor 1993a). The idea here is that the control error represented by the inflation and GDP gaps (deviations from target or potential) can be closed (set to zero) by a proportional change in the actuation command represented by the interest rate: if the net gap is positive the rate should be raised, if negative lowered, and if zero left unchanged.

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[4] Taylor (1993a) originally presented his rule in terms of average inflation and of output gap as $r(t) = \pi(t) + 0.5y(t) + 0.5(\pi(t) - 2) + 2$ which can be easily rewritten in "gapped" form as $r(t) = 4 + 0.5y(t) + 1.5(\pi(t) - 2)$.

[5] K_0 is the nominal Fed funds rate associated with inflation gap and GDP gap being equal to zero. $K_P^{(\pi)}$ and $K_P^{(y)}$ are the proportionality constants for the inflation gap and output gap respectively.

The parsimony of the Taylor rule inspired a considerable literature in which one finds that with three slight econometrically-inspired modifications—the introduction of other gapped variables, the addition of lagged variables and of forward-looking estimates of the gaps—Taylor rules are able to describe the evolution of policy rates for a large number of economies.[6] The parsimony and ubiquity of these rules and a shared functional form suggests that they are related to a similarly parsimonious and ubiquitous solution in control theory: the proportional-integral-derivative (PID) controller.

The economic motivation for this paper is our observation that the original Taylor rule shown in Eq. (1) is a proportional controller—the level of the actuator (the Fed Funds rate) is *proportional to* the gap (or monetary policy ineffectiveness) in the system—which we can write as

$$r(t) = K_0 + \boldsymbol{K}_P \cdot \boldsymbol{g}(t), \tag{2}$$

where the vector $\boldsymbol{K}_P$ is the collection of proportionality constants for the vector of gapped variables $\boldsymbol{g}(t)$ and the dot product creates the sum. In the original Taylor rule $\boldsymbol{K}_P = \{K_P^\pi, K_P^y\}$ and $\boldsymbol{g}(t) = \{\pi(t), y(t)\}$. As discussed in Åström and Murray (2008), proportional control is the first element of the PID controller.

An implication of this policy rule is that the central bank considers only the current gaps in their deliberations on the policy rate. While the current gaps are clearly a part of their deliberations, one expects that the central bank also considers the history of the gaps (i.e. the effectiveness of their actions to date) in their deliberations which can represented as

$$r(t) = K_0 + \boldsymbol{K}_P \cdot \boldsymbol{g}(t) + \boldsymbol{K}_I \cdot \int_0^t \boldsymbol{g}(\tau) d\tau, \tag{3}$$

a proportional-integral (PI) controller. In addition to considering the results of prior actions, the central bank may also consider expectations of the gaps $E_{t+\Delta t}[\boldsymbol{g}(t)]$ which one can represent in a simple manner by the linear extrapolation $E_{t+\Delta t}[\boldsymbol{g}(t)] = \boldsymbol{g}(t + \Delta t) \approx \boldsymbol{g}(t) + \Delta t d\boldsymbol{g}(t)/dt$, whence

$$r(t) = K_0 + \boldsymbol{K}_P \cdot \boldsymbol{g}(t) + \boldsymbol{K}_I \cdot \int_0^t \boldsymbol{g}(\tau) d\tau + \boldsymbol{K}_D \cdot \frac{d\boldsymbol{g}(t)}{dt}, \tag{4}$$

the proportional-integral-differential, or PID, controller.

As with most controller applications, monetary policy is based on measurements of the gap size at discrete points in time because the economic variables such as GDP and inflation that are used to form the gaps are measured on a periodic basis (e.g. quarterly). A discrete version of Eq. (4) can be obtained by discretizing time in units of Δt (the time between measurements) and employing the approximations

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[6] See, for example, Taylor (1999b), Woodford (2003) and references therein, especially Clarida et al. (1998).

$$\int_0^{t_k} \boldsymbol{g}(\tau)d\tau = \sum_{i=1}^{k} \boldsymbol{g}(t_i)\Delta t \ , \quad \text{and} \quad \frac{d\boldsymbol{g}(t_k)}{dt} = \frac{\boldsymbol{g}(t_k) - \boldsymbol{g}(t_{k-1})}{\Delta t}, \tag{5}$$

with which the continuous-time version of the PID controller given by Eq. (4) becomes

$$r(t_k) = K_0 + \boldsymbol{K}_P \cdot \boldsymbol{g}(t_k) + \Delta t \sum_{i=1}^{k} \boldsymbol{K}_I \cdot \boldsymbol{g}(t_i) + \boldsymbol{K}_D \cdot \left[\boldsymbol{g}(t_k) - \boldsymbol{g}(t_{k-1})\right] / \Delta t. \tag{6}$$

Taking the difference $r(t_k) - r(t_{k-1})$, setting $\Delta t = 1$ and collecting terms we obtain the general form of our monetary policy rule: the well-known velocity algorithm for the PID controller[7]

$$r_t = r_{t-1} + \boldsymbol{\beta}_0 \cdot \boldsymbol{g}_t + \boldsymbol{\beta}_1 \cdot \boldsymbol{g}_{t-1} + \boldsymbol{\beta}_2 \cdot \boldsymbol{g}_{t-2}, \tag{7}$$

where $r_t \equiv r(t_k)$, $\boldsymbol{\beta}_0 = [\boldsymbol{K}_P + \boldsymbol{K}_I + \boldsymbol{K}_D]$, $\boldsymbol{\beta}_1 = -[\boldsymbol{K}_P + 2\boldsymbol{K}_D]$ and $\boldsymbol{\beta}_2 = \boldsymbol{K}_D$. Written in terms of Taylor's original macroeconomic drivers of inflation and GDP our monetary policy equation becomes

$$r_t = r_{t-1} + \beta_0^{(\pi)}\pi_t + \beta_0^{(y)}y_t + \beta_1^{(\pi)}\pi_{t-1} + \beta_1^{(y)}y_{t-1} + \beta_2^{(\pi)}\pi_{t-2} + \beta_2^{(y)}y_{t-2} \tag{8}$$

for the full PID form and

$$r_t = r_{t-1} + \beta_0^{(\pi)}\pi_t + \beta_0^{(y)}y_t + \beta_1^{(\pi)}\pi_{t-1} + \beta_1^{(y)}y_{t-1} \tag{9}$$

for the PI form.[8]

Given the common stability and parsimony goals of system control engineering and central banking it is perhaps not surprising that there is structural similarity between PID control and monetary policy rules. Indeed, a particularly convenient feature of PID control is that it subsumes many well-known monetary policy rules as shown in Table 1. In the first row of Table 1 we see checks ($\surd$) in all of the columns for the

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[7] Alternatively, one could differentiate Eq. (4) with respect to time resulting in

$$\frac{dr(t_k)}{dt} = \frac{d\boldsymbol{K}_P \cdot \boldsymbol{g}(t_k)}{dt} + \boldsymbol{K}_I \cdot \boldsymbol{g}(t_k) + \frac{d^2\boldsymbol{K}_D \cdot \boldsymbol{g}(t_k)}{dt^2},$$

discretize the first-order derivatives with Eq (5) and the second-order derivatives with

$$\frac{d^2\boldsymbol{g}(t_k)}{dt^2} \approx \frac{\boldsymbol{g}(t_k) + \boldsymbol{g}(t_{k-2}) - 2\boldsymbol{g}(t_{k-1})}{(\Delta t)^2},$$

set $\Delta t = 1$ and collect terms to obtain Eq. (7).

[8] The velocity algorithm is formally similar to the "first-difference rules" in the monetary policy literature (cf. e.g. Levin et al. 2003). Our derivation can be viewed as a formal representation of their observation that "[a] rule with a high value [of the lagged interest-rate parameter] implicitly makes the current interest rate depend on the complete history of output and inflation, albeit in a very restricted way." Levin et al. (2003), p. 287.

Table 1 A comparison of the elements of the PID and PI rules with those of other monetary policy rules

Rule	r_{t-1}	π_t	y_t	π_{t-1}	y_{t-1}	π_{t-2}	y_{t-2}
PID	√	√	√	√	√	√	√
PI	√	√	√	√	√	X	X
Taylor (1993a)	X	√	√	X	X	X	X
Gerdesmeier and Roffia (2005)	√	√	√	X	X	X	X
Levin et al. (1999, 2003)	√	√	√	X	√	X	X
Smets and Wouters (2007)	√	√	√	X	√	X	X
Taylor and Wieland (2012)	√	√	√	X	√	X	X
Christiano et al. (2005)	√	√	√	√	X	√	X

PID rule indicating that all of the corresponding elements are present as shown in Eq. (8). The PI rule shown in the second row does not contain the final two elements shown in the Table and this is indicated by the presence of an (“X”) in the right-most two columns. The entries of the third row indicate that the original Taylor rule is the simplest type of PID rule—a proportional, or P, rule—as discussed above.

The first of a series of very parsimonious rules begins in the fourth row of Table 1 with the rule of Gerdesmeier and Roffia (2005). The parsimony in gap drivers, however, is offset by the introduction of an extra variable in the treatment of the lagged rate: the parameter of r_{t-1} is not equal to unity. The rules of Levin et al. (1999, 2003) and of Smets and Wouters (2007) shown in rows five and six further this extension of the Taylor rule with the contribution from the singly-lagged output gap.

Finally, we round out our comparison with the rule of Christiano et al. (2005) which includes the expected inflation one period forward. In our PID rule expectations are modeled by a simple forward extrapolation in time using the derivative term introduced in Eq. (4) and appear as contributions to all gap parameters in general and the doubly-lagged gap parameter in particular. The implicit introduction of a basic expectation calculation though the introduction of lagged variables goes some way to explaining the observation[9] that the addition of explicit expectation calculations to a rule often provides less “pick up” in explanatory power in the presence of lagged variables. The lags effectively represent the first-order derivative term in an Taylor-series representation of the expectation which relegates any additional explanatory contribution of an explicit expectation calculation to higher-order terms: a contribution that the ubiquity of the PID controller would suggest is small.

Determining the parameters of any monetary policy rule is an exercise in tuning the rule to a given economy and it is to this exercise for the PI and PID rules that we now turn.

3 Tuning and performance of the PID and PI rules

The utility of PID control for complex systems comes at a cost: the parameters must be obtained by determining how a change in controller parameters changes the per-

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[9] See, for example Batini and Haldane (1999) and the discussion in Levin et al. (1999).

formance of the system: a process known as tuning. In the noneconomic context there are a number of approaches that can be used, all of which entail some sort of direct experimentation on the system to be controlled (Åström and Murray 2008; Bennett 1993). As experimentation on existing economies is currently not feasible, two alternate approaches to tuning economy controllers (monetary policy rules) have evolved: historical analysis and macroeconomic simulation.[10] Historical analysis consists of choosing a time period during which a given central bank is thought to have conducted monetary policy properly and fitting a policy rule to the historical time series of policy rates and economic drivers. Macroeconomic simulation involves embedding a monetary-policy rule within a model of a macroeconomy and varying the parameters of the monetary-policy rule to minimize some measure of performance of the macroeconomy.

3.1 Tuning

In keeping with both the historical analysis of Taylor and the considerable portion of the monetary-policy rule literature that employs econometric analysis for monetary-policy rule tuning,[11] we have tuned our rule using an econometric analysis of the actions of the FOMC on the Fed Funds rate from 1987 to 1992.[12] The tuned parameters the Taylor, PI and PID rules are shown in Table 2 together with their standard errors (below each parameter), the adjusted R-squared $\overline{R}^2$, sum of squared errors SSE and Akaike information criterion. In the first two rows we find inflation and output responses of 1.50 and 0.59 respectively which are remarkably close to Taylor's (non-econometric) estimates of 1.5 and 0.5 (Taylor 1993a) and consistent[13] with Orphanides' (econometric) parameters of 1.51 and 0.60 (Orphanides 2001). In the following four rows we see the results of this analysis for the PI and PID policy rules. For both policy rules we see a reduction in the strength of the contemporaneous response as explanatory power is distributed across the lagged terms. The drop in strength between the Taylor and PI rules is greater than that between the PI and PID rules and a similar diminishing return is seen also in the measures of rule fit associated with the move from PI to PID. As expected, the PI and PID rules both outperform the original Taylor rule, but that outperformance is almost certainly a function of the increasing number of rule parameters as one moves from Taylor to PI and PID. The PI and PID rules are preferred to the Taylor rule because the action of the Federal Open Market Committee (FOMC) implied by the PI and PID rules is more realistic: the FOMC almost certainly considers the history of the effectiveness of prior policy

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[10] For a discussion of and references to both approaches see Taylor (1999a).

[11] See, for example, Taylor (1999a), Orphanides (2001), Smets and Wouters (2007), and references therein.

[12] We employed ordinary least squares (OLS) analysis with Newey-West Heteroskedasticity and Autocorrelation Consistent (HAC) standard errors on quarterly (Q1 1987–Q4 1992) vintage data sourced from the ArchivaL Federal Reserve Economic Data (ALFRED) repository at the Federal Reserve Bank of St. Louis: http://alfred.stlouisfed.org/.

[13] This was expected given that we used similar data.

Table 2 Historical tuning of monetary policy rules

Model	$\beta_0^{(\pi)}$	$\beta_0^{(y)}$	$\beta_1^{(\pi)}$	$\beta_1^{(y)}$	$\beta_2^{(\pi)}$	$\beta_2^{(y)}$	$\overline{R}^2$	*SSE*	Akaike
Taylor	1.50	0.59	–	–	–	–	0.92	0.53	1.68
	(0.31)	(0.07)	–	–	–	–			
PI	1.08	0.37	−1.13	−0.28	–	–	0.98	0.30	0.57
	(0.29)	(0.13)	(0.29)	(0.11)	–	–			
PID	0.96	0.20	−0.70	0.12	−0.30	−0.28	0.98	0.28	0.52
	(0.28)	(0.11)	(0.26)	(0.24)	(0.23)	(0.19)			

actions in their deliberations.[14] The tradeoff between PI and PID rule is somewhat more subtle. The PID rule is favored somewhat based on the *SSE* and Akaike information criterion, but the modest improvement shown in Table 2 must be weighed against the challenges of real-time macroeconomic data (e.g. later restatement of the data): the well-known difficulty of estimating derivatives from any source of noisy empirical data in real time (Bennett 1993). Given these considerations we adopt the general practice in the engineering community of dropping the derivative term in favor of the more robust (and, consequently, dominant in engineering applications) PI policy rule.

3.2 Performance

A measure of the robustness of any monetary policy rule is the extent to which it can be used effectively in different macroeconomic models. Policy-rule reach across macroeconomic models is expected given that central bank deliberations share major concerns regardless of the country in which these deliberations take place. A comparison of monetary policy rules across macroeconomic models can be made using the recently developed macroeconomic model database of Wieland et al. (2011)[15] with which one can see how a monetary policy rule performs in terms of the unconditional variances of key macroeconomic variables (interest rate, inflation and output gap) across a large number of macroeconomic models. In Fig. 1 we present results from 37 macroeconomic models using this software to compare our PI rule with the original Taylor rule (see footnote 4) and a new rule by Taylor and Wieland (2012) which was tuned using macroeconomic simulation.[16] Included in Fig. 1 are results from macro-

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[14] A sense of the historical data that is reviewed in the FOMC meetings can be found in the material on the Board of Governors' website (http://www.federalreserve.gov/monetarypolicy/fomccalendars.htm).

[15] This reference and further information on The Macroeconomic Model Data Base (MMB) project can be found at http://www.macromodelbase.com. The macroeconomic models in this database are solved using Dynare (cf. Juillard 1996, 2001 and http://www.dynare.org).

[16] In a recent paper Taylor and Wieland (2012) present the results of a novel approach to monetary-policy rule tuning that used macroeconomic simulation to obtain rule parameters that minimized the variance of the inflation, output and interest rate across three current-generation macroeconomic models (Taylor 1993b; Altig et al. 2005; Smets and Wouters 2007). With this tuning they obtained the model-averaged policy rule $r_t = 1.06r_{t-1} + 0.19\pi_t + 0.67y_t - 0.59y_{t-1}$. Comparing the PI rule with their model-averaged rule we see first that the parameter for the lagged interest-rate (by definition unity in the PI rule) is also very close to

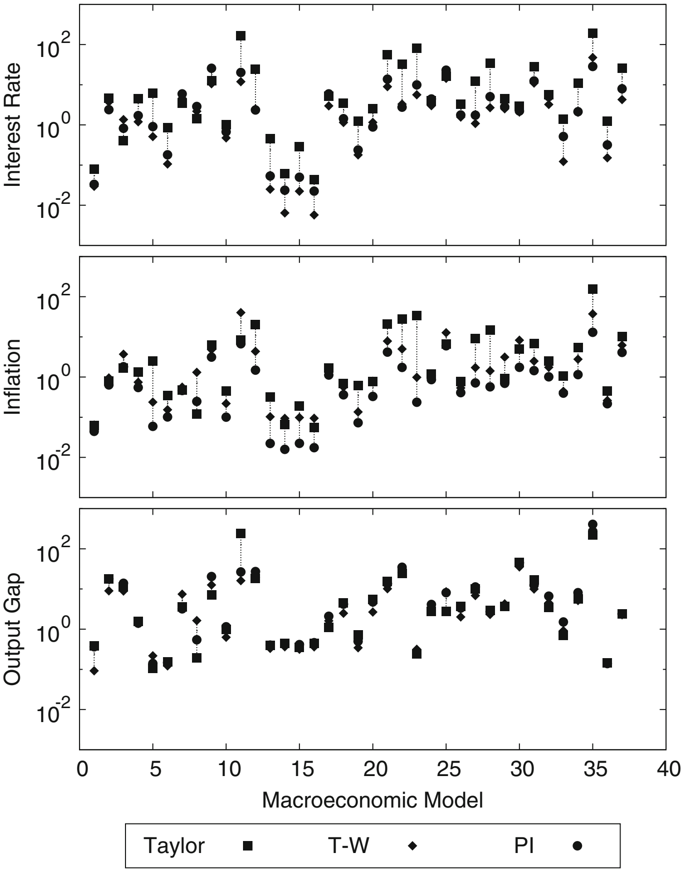

Fig. 1 The unconditional variance for interest rate, inflation and output gap from 37 macroeconomic models for three monetary policy rules: the original Taylor rule (Taylor 1993a), the 4-parameter rule of Taylor and Wieland (T–W) (Taylor and Wieland 2012), and the PI rule given by Eq. (9)

economic models[17] for small economies (models 1–8), the US (models 9–23), the euro area (models 24–30), for multiple countries (models 31–33) and other specific countries (models 34–37). In each panel we have connected the policy rule results for

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Foornote 16 continued
unity in the Taylor and Wieland (2012) rule. This feature has often been implicated as evidence of interest-rate smoothing by central banks but, as shown in our derivation, it is a simple consequence of the velocity derivation of the PI rule. Earlier research has also linked a parameter near unity for the lagged interest rate with monetary-policy robustness (Levin et al. 2003). Comparison of the other variables is complicated by the fact that Taylor and Wieland (2012) assumed $\beta_1^{(\pi)} = 0$: this was not a consequence of the tuning. The size and sign of the common parameters between these rules, however, suggests that the parameters of a PI rule tuned using macroeconomic simulation may be similar to those we obtained using historical analysis: a topic of future research.

[17] A reference for each model is provided in the Appendix below. For a discussion of the MMB software see Wieland et al. (2011).

a given macroeconomic model with a thin line to help guide the eye in comparing the relative performance of the policy rules.

Beginning with the unconditional variance of the interest rate shown in the upper panel we see that in general the Taylor–Wieland and PI policy rules outperform the original Taylor rule. The Taylor–Weiland rule outperforms overall in 29 of the 37 models examined. We also see some substantial improvement in the two rules with variances greater than 100—the Federal Reserve Board US model (Levin et al. 2003, model 11) and the model of the Brazilian economy (de Castro et al. 2011, model 35). In general, both rules improve this measure relative to the original Taylor rule; in some cases substantially.

Moving on to the middle panel one sees the changes in the unconditional variance of the inflation rate due to changes in policy rule. As we saw in the case of interest rates above, the Taylor–Wieland and PI policy rules generally outperform the original Taylor rule, but in this panel the PI rule dominates the outperformance in 35 of the 37 macroeconomic models. We again see substantial improvement in this measure for the model of the Brazilian economy (de Castro et al. 2011, model 35) and also see large improvement for one of the US models Ireland (2011), model 23.

Turning now to the final panel we see that there is remarkably little, if any, improvement in macroeconomic model performance to be found in these new policy rules, although the Taylor–Wieland rule did outperform in 22 models. Substantial improvement is seen for one of the outlier models—the Federal Reserve Board US model (Levin et al. 2003, model 11)—but not for the model of the Brazilian economy (de Castro et al. 2011, model 35).

Comparing the policy rules across the macroeconomic models to which the Taylor–Wieland policy rule was tuned we find that for the Altig et al. (2005, models 13–16) and Smets and Wouters (2007, model 22) macroeconomic models the Taylor–Wieland rule outperforms for the interest-rate and output gap while the PI rule outperforms on inflation. Curiously, for the Taylor (1993b, model 22) macroeconomic model, the PI rule outperforms for both interest-rate and inflation while the Taylor rule outperforms for output gap.

Looking across all panels we see that in general the Taylor–Wieland and PI policy rules outperform the original Taylor rule. This outperformance is not unexpected given the extra complexity of these rules. It is interesting to note that for one macro model—the small economy model of Christoffel et al. (2009, model 8)—the original Taylor rule outperforms the others. The outperformance of the Taylor–Wieland rule in the interest-rate panel may be a result of the new model-average tuning procedure employed in its parameter estimation; a topic of current investigation. The outperformance of the PI rule in the inflation panel may reflect the greater emphasis this rule places on describing inflation and could argue against setting the lagged inflation variable to zero as seen in other rules.

4 Discussion and summary

Given that "PID control is by far the most common way of using feedback in natural and man-made systems," it is perhaps not surprising that one of the largest systems integrating the natural and the man-made—the macroeconomy—should be found to be governed by PID control.[18] While it is true that central banks have not employed control theory in the form expressed by Maxwell (1867–1868), this is because the dynamics of economies cannot yet be expressed as differential equations with analytic solutions from which controllers can be derived (Åström and Murray 2008; Bennett 1993). As demonstrated in this paper, however, central banks have employed monetary policy rules in a manner consistent both practically and theoretically with the tradition of control engineering that evolved from the work of Maxwell (1867–1868).

Most monetary policy rules have their origin in a particular macroeconomic model, however, and it is on this point that PID-based rules stand in sharp relief: their derivation can be independent of any macroeconomic model.[19] This is the primary advantage of the PID-control approach in economics: the underlying equations of motion do not need to be known to develop and deploy PID control. As with other rules the parameters of PID-based rules can be obtained either by fitting to data during periods when the central bank is thought to have followed proper monetary policy or through tuning using macroeconomic models. Our PI rule was found to generally outperform the original Taylor (1993a) rule and to perform essentially as well as a recently developed extension of the Taylor rule (Taylor and Wieland 2012) in the performance of 37 current generation macroeconomic models, and while our comparison of monetary policy rule performance employed dynamic stochastic general equilibrium (DSGE) models there is no reason in principle why one could not use models such as those discussed in Colander (2006) or built on the work of Aoki (1996, 2000) and Aoki and Yoshikawa (2007). Indeed a key feature that recommends the PID approach to policy-rule development in monetary policy is that its derivation is independent of any particular macroeconomic model and it is, thus, appropriate for use with any macroeconomic model or collection of macroeconomic models.

In summary, the implementation of monetary policy by central banks is one of the most influential feedback mechanisms in the macroeconomy. From this perspective the use of monetary policy rules by central banks as a form of feedback control bears a striking resemblance to the most ubiquitous class of control in control engineering: proportion-integral-differential, or PID, control in general and to the more robust (to noisy data), and consequently even more ubiquitous, subset of PID control known as proportion-integral, or PI, control in particular. Motivated by the identification of the Taylor rule as a proportional controller we discovered that a number of current monetary policy rules nest structurally within PID control. With this identification we derived new PID-based monetary policy rules that have, as limiting cases, many currently popular monetary-policy rules. Unlike most policy rules that are derived from a particular model of the macroeconomy, the PID policy rule approach models in a general manner the deliberations of a central bank as they seek to realize their goals of price stability and low unemployment. PID-based rules are grounded in the engineering tradition of controlling complex systems for which the equations of motion are unknown. The PID approach to monetary policy rule development is recommended

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[18] This quote is from the 2006 edition of Åström and Murray (2008).

[19] As our derivation begins with the original Taylor rule, it is not independent of macroeconomic modelling in general. The ubiquity of the Taylor rule, however, suggests that it, like our derivation, transcends specific macroeconomic models.

by the generality of its economic derivation, its close formal relationship to existing policy rules, PI-rule performance across macroeconomic models, and its embrace of intrinsic macroeconomic-model uncertainty.

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Appendix: macroeconomic models

The macroeconomic models shown in Fig. 1 and discussed in the text above are listed below in Table 3. The model grouping labels and identifiers are those found in the Macroeconomic Model Database.

Table 3 Macroeconomic models from the macroeconomic model database (MMB) (Wieland et al. 2011) used to generate the results in Fig. 1

Macroeconomic model	Citation	MMB identifier
Small calibrated models		
1	Rotemberg and Woodford (1997)	NK_RW97
2	Levin et al. (2003)	NK_LWW03
3	Clarida et al. (1999)	NK_CGG99
4	McCallum and Nelson (1999)	NK_MCN99cr
5	Ireland (2004)	NK_IR04
6	Galí and Monacelli (2005)	NK_GM05
7	Christoffel and Kuester (2008)	NK_CK08
8	Christoffel et al. (2009)	NK_CKL09
Estimated US models		
9	Fuhrer and Moore (1995)	US_FM95
10	Orphanides and Wieland (1998)	US_OW98
11	Levin et al. (2003)	US_FRB03
12	Smets and Wouters (2007)	US_SW07
13	Altig et al. (2005)	US_ACELm
14	Altig et al. (2005)	US_ACELt
15	Altig et al. (2005)	US_ACELswm
16	Altig et al. (2005)	US_ACELswt
17	Carabenciov et al. (2008)	US_PM08fl
18	De Graeve (2008)	US_DG08
19	Christensen and Dib (2008)	US_CD08
20	Iacoviello (2005)	US_IAC05
21	Rabanal (2007)	US_RA07
22	Smets and Wouters (2007)	US_CCTW10
23	Ireland (2011)	US_IR11
Estimated euro area models		
24	Levin et al. (2003)	EA_CW05ta

Table 3 continued

Macroeconomic model	Citation	MMB identifier
25	Levin et al. (2003)	EA_CW05fm
26	Smets and Wouters (2007)	EA_SW03
27	Adolfson et al. (2007)	EA_SR07
28	Ratto et al. (2009)	EA_QUEST3
29	Christoffel et al. (2009)	EA_CKL09
30	Gelain (2010)	EA_GE10
Estimated/calibrated multi-country models		
31	Taylor (1993b)	G7_TAY93
32	Coenen and Wieland (2005)	G3_CW03
33	Laxton and Pesenti (2003)	EACZ_GEM03
Estimated models of other countries		
34	Medina and Soto (2007)	CL_MS07
35	de Castro et al. (2011)	BRA_SAMBA08
36	Lubik and Schorfheide (2007)	CA_LS07
37	Funke et al. (2011)	HK_FPP11

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