

The Shifts in Lead-Lag Properties of the US Business Cycle

Joshua Brault*

Carleton University
& Ottawa-Carleton GSE

Hashmat Khan†

Carleton University
& Ottawa-Carleton GSE

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Comments Welcome

Abstract

We document novel shifts in the lead-lag properties of the US business cycle since the mid-1980s that have gone unnoticed in contemporary research. Specifically, (i) the well-known inverted-leading-indicator-property of real interest rates has completely vanished; (ii) labour productivity switched from leading positively to lagging negatively over the cycle; (iii) Labour input measures shifted from lagging labour productivity positively to leading negatively; (iv) Unemployment rate shifted from lagging productivity negatively to leading positively. Many contemporary business cycle models produce counterfactual cross-correlations. Determining the underlying sources of these shifts in the lead-lag properties is challenging yet a promising direction for future research.

Key words: Business Cycles, Cross-Correlations, Stylized Facts, Interest Rates, Productivity, Hours, Employment, Unemployment

JEL classification: E32, E47

*Department of Economics, D886 Loeb, 1125 Colonel By Drive, Carleton University, Ottawa, Canada.
E-mail: joshua.brault@carleton.ca. Tel: +1.613.520.2600 (ext 3057).

†Department of Economics, D891 Loeb, 1125 Colonel By Drive, Carleton University, Ottawa, Canada.
E-mail: hashmat.khan@carleton.ca. Tel: +1.613.520.2600 (ext 1561).

1 Introduction

We document novel *shifts* in the cross-correlations (also referred to as phase shifts) among macroeconomic variables in post-World War II US business cycles, especially after the onset of the Great Moderation period in the mid-1980s. These cross-correlations are larger in absolute magnitude than the contemporaneous correlations reflecting both the shifts in the lead-lag properties and the presence of important mechanisms not captured by comovements alone. While there is a long history at least since [Burns and Mitchell \(1946\)](#) of characterizing business cycles along this dimension, these shifts in cross-correlations have gone unnoticed in contemporary research. The main contributions of our paper are to shed light on the shifts in the lead-lag patterns of the US business cycle and to discuss important challenges for model development and evaluation for future research. To the best of our knowledge, we are the first to document the shifts in the lead-lag properties listed below.

We use the [Hodrick and Prescott \(1980, 1997\)](#) (HP) filter to obtain the cyclical component of the data. The advantage of using the HP filter is that it facilitates comparisons with the previous literature that has also used the same filter. Recently, however, [Hamilton \(2017\)](#) has proposed an alternative to the HP filter which we use to assess robustness of our empirical findings. The two time periods we consider are 1947-1984 (the pre-1985 period) and 1985-2017 (the post-1985 period). This sample split has been widely studied in the literature in the context of declining volatility and cyclicity of macroeconomic variables associated with the onset of the Great Moderation period. We summarize the four major lead-lag shifts between the pre- and post-1985 periods as follows:

1. **Real interest rates positively lag output:** Real interest rates display a *Positive Lagging Property*. They strongly lag output by three quarters with positive signs. The well known inverted-leading-indicator property of real interest rates has completely vanished.
2. **Labour productivity negatively lags output:** Labour productivity has shifted from leading the cycle with a positive sign to lagging with a negative sign. Output per hour lags by four quarters and output per person lags by five quarters.
3. **Labour inputs negatively lead labour productivity:** Total hours worked have shifted from lagging output per worker by three quarters with a positive sign to leading

by two quarters with a negative sign. Employment has shifted from lagging output per person by three quarters with a positive sign to leading by four quarters with a negative sign.

4. **Unemployment rate positively leads labour productivity:** The unemployment rate shifted from lagging both output per hour and output per person negatively by two quarters to positively leading output per worker by two quarters, and output per person by four quarters.

These surprising shifts in the lead-lag properties have gone unnoticed in contemporary business cycle research. We think that there are at least three potential reasons related to previous and current research. First, the emphasis in recent business cycle literature has been on the changes in unconditional contemporaneous correlations that indicate the pro-, counter-, or acyclical nature of a variety of macroeconomic variables, and their volatility (see, for example, [Hall \(2007\)](#), [Stiroh \(2009\)](#), [Barnichon \(2010\)](#), [Galí and van Rens \(2017\)](#), [Garin, Pries and Sims \(2018\)](#)). This literature has not examined shifts in the lead-lags properties of macroeconomic variables over the business cycle.

Second, a large body of literature, starting at least since [Backus, Kehoe and Kydland \(1992\)](#), has either motivated and/or evaluated models, based on cross-correlations and lead-lag properties.¹ The primary focus of this literature is to consider cross-correlations over the whole sample period of study.² By contrast, we focus on the shifts in cross-correlations that have occurred across the pre- and post-1985 sample split that coincides with the widely studied onset of the Great Moderation in the US economy.

Third, current research on business cycles continues to be motivated by contemporaneous

¹Contributions in the case of real interest rates-output cross-correlations, for example, are [Fiorito and Kollintzas \(1994\)](#), [Chari, Christiano and Eichenbaum \(1995\)](#), [King and Watson \(1996\)](#), [Beaudry and Guay \(1996\)](#), and [Boldrin, Christiano and Fisher \(2001\)](#). For cross-correlations related to investment see [Hornstein and Praschnik \(1997\)](#), [Gomme, Kydland and Rupert \(2001\)](#), [Fisher \(2007\)](#), [Kydland, Rupert and Sustek \(2016\)](#), [Khan and Rouillard \(2016\)](#), [Khan and Rouillard \(2017\)](#), among others). For labour productivity-output and labour productivity-hours cross-correlations see [Burnside and Eichenbaum \(1993\)](#). In the case of net exports, terms of trade, and balance of payments cross-correlations see [Backus, Kehoe and Kydland \(1992\)](#), [Backus, Kehoe and Kydland \(1994\)](#), among others. [Azariadis, Kaas and Wen \(2016\)](#) examine cross-correlations in unsecured firm credit to motivate their model of self-fulfilling credit cycles. And finally, [Beaudry, Galizia and Portier \(2017\)](#) discuss the positive correlation between their measure of capital over-accumulation prior to a recession and the subsequent severity of the recession over the period 1959-2015 in motivating their theoretical model.

²One exception is [Backus, Kehoe and Kydland \(1994\)](#) who consider pre- and post-1972 data.

correlations. We provide two examples to support our point. The first example is [Jordà, Schularick and Taylor \(2016\)](#) who emphasize the correlations between credit growth and output growth. Using their data, we calculated the cross-correlations between credit growth and output growth across the subsamples 1948-1984 and 1985-2015. As it turns out, there has been a shift in the lead-lag pattern in 14 of the 17 countries in their data.³ The second example is [Angeletos \(2017\)](#) who motivates the research agenda on frictional coordination based on comovements (contemporaneous correlations). He notes that the contemporaneous correlation between output and labour productivity is approximately zero (see Figure 2 in [Angeletos \(2017\)](#) based on the 1960-2015 period). As we show below, this assessment misses the picture revealed in the lead-lag shifts in labour productivity over the cycle since the onset of the Great Moderation period in the US economy. Specifically, while the contemporaneous correlation is close to zero even in the post-1985 period, labour productivity lags output negatively by four quarters with a cross-correlation of -0.61 . By definition, the leads and lags denote the largest absolute cross-correlations, which are often substantially larger than the contemporaneous correlations, suggesting important business cycle forces at work that are not reflected in contemporaneous correlations alone.

In light of the new lead-lag stylized facts listed above, an immediate question is: how do the properties of simulated data from existing Dynamic Stochastic General Equilibrium (DSGE) models compare with their empirical counterparts? We provide a few selected examples. For real interest rate dynamics we consider simulated data from [Iacoviello \(2005\)](#), [Smets and Wouters \(2007\)](#), and [Basu and Bundick \(2017\)](#), respectively. Our rationale is that these models have frictions and shocks that are embedded in a many contemporary DSGE models, and therefore, provide a useful reference point. For labour productivity and labour input dynamics, we consider simulated data from the models in [Galí and van Rens \(2017\)](#) and [Garin, Pries and Sims \(2018\)](#), respectively. Our rationale is that since these models successfully explain the decline in the procyclicality of labour productivity after the mid-1980s, they provide a natural benchmark to determine their intrinsic lead-lag properties

³These results are available upon request. If we consider HP filtered data using a smoothing parameter of 6.25 as recommended by [Ravn and Uhlig \(2002\)](#) for annual data, then 11 of the 17 countries exhibit a lead-lag switch in the [Jordà, Schularick and Taylor \(2016\)](#) data set.

relative to the stylized facts reported above. Finally, for the unemployment rate and labour productivity dynamics we consider simulated data from [Barnichon \(2010\)](#). Our rationale is that this model studies the change in contemporaneous correlation of unemployment and labour productivity since the mid-1980s, and is well-suited to examine the cross-correlations between the same two variables.

As we discuss in detail, our comparative analysis reveals that all the models we consider produce counterfactual lead-lag properties (both qualitatively and quantitatively) relative to their empirical counterparts in the post-1985 data. By extension, we hypothesize that our assessment applies to a wide class of contemporary DSGE models. This finding raises many important challenges and suggests promising areas for future business cycle research.

The rest of this paper is organized as follows. Section 2 presents the shifts in the lead-lag properties along four dimensions. Section 3 assesses robustness of the stylized facts. Section 4 provides a discussion. Section 5 concludes.

2 Shifts in Lead-Lag Properties

In this section we present each of the four properties listed above in more detail. We also provide a comparison of lead-lag properties when looking through the lens of recent DSGE models. Some of these models have focussed on changes in contemporaneous correlations since the onset of the Great Moderation period in the mid-1980s.

2.1 Data

We use quarterly data obtained from the Federal Reserve Bank of St. Louis Economic Database (FRED).⁴ We employ two different measures of output in the paper, namely, real Gross Domestic Product (GDP) and nonfarm business sector real output, to allow a better comparison with the model-based results in the literature. The empirical findings are robust to either measure. Our baseline measure of real interest rate is the 3-month treasury bill secondary market rate minus one period ahead ex-post inflation, where inflation is defined

⁴A detailed description of the series is provided in the appendix.

as the annualized log difference of the GDP deflator. We consider alternative measures of the real interest rate which are described in Table 4 in the appendix. The two measures of labour productivity correspond to nonfarm business sector real output per hour and person, respectively. Total hours and employment are the hours and employment of all persons in the nonfarm business sector, respectively. Finally, the unemployment rate is defined as the civilian unemployment rate.

We perform standard transformations of the variables prior to examining the cross-correlations. Specifically, we take the natural log of all variables (this excludes real interest rates and the unemployment rate because they are in percent units). Throughout our analysis, the HP filter smoothing parameter for quarterly data is 1600. We present the main empirical findings in the figures below. Table 1 presents all the cross-correlations that we discuss in this section.

2.2 Real Interest Rates Positively Lag Output

Figure 1 shows the cross-correlations between HP filtered output, Y_t , and, leads and lags of the HP filtered real interest rate, R_t . The results are nearly identical if we do not HP filter the real interest rate.⁵ Specifically, $\text{Corr}(Y_t, R_{t+k}) : k = \{-5, -4, \dots, 0, \dots, 4, 5\}$. The largest correlation in absolute terms determines the lead-lag property of a given series relative to another. This cross-correlation is represented by the solid black dots. The intersection with the dashed vertical line indicates the values of the contemporaneous correlations.

Panel (a) shows that in the pre-1985 period, the real interest rate was strongly negatively correlated with future output, and was countercyclical. This is the well-known Inverted Leading Indicator Property (ILP) of real interest rates documented by King and Watson (1996).⁶ In sharp contrast, the ILP has completely vanished in the post-1985 data. The real interest rate lags output by three quarters with a positive sign. The real interest rate is also strongly procyclical. We refer to this shift in the business cycle dynamics of real interest rates in the post-1985 data as the *Positive Lagging Property* (PLP).

⁵As shown in Table 4.

⁶Similar properties of interest rates dynamics are documented in Fiorito and Kollintzas (1994) (Table 3), Chari, Christiano and Eichenbaum (1995) (Figure 3), and Beaudry and Guay (1996) (Table 2).

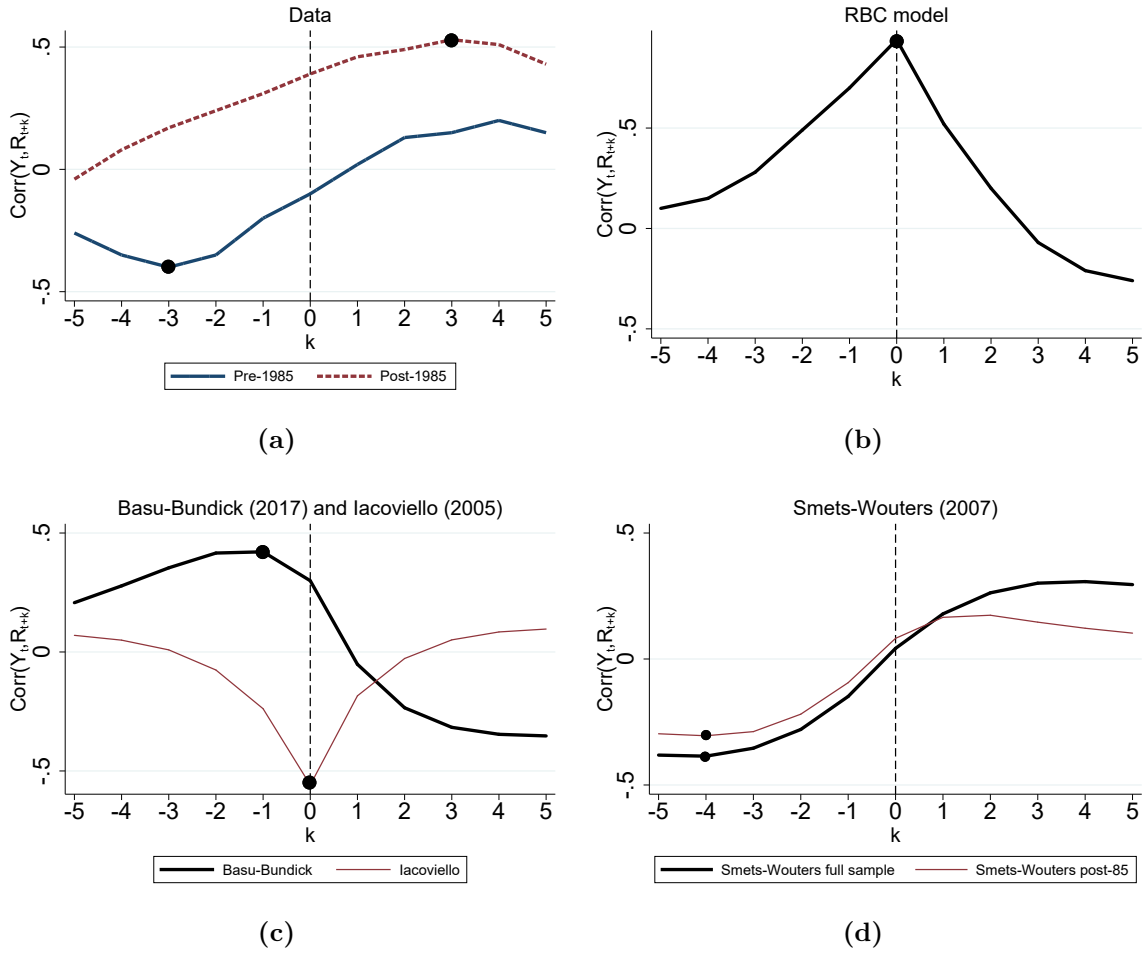


Figure 1: **Cross-correlations between output and real interest rate**

Note: k denotes the number of leads (negative values) or lags (positive values) between real interest rate and output, Y_t . The correlations at the black dashed line represent the contemporaneous correlations. Black solid circles denote the largest cross- or contemporaneous correlation that occurs at the k th lead/lag, $\max |\{\text{Corr}(Y_t, R_{t+k}), k = -5, -4, \dots, 0, \dots, 4, 5\}|$. Both actual and model-simulated output are HP filtered.

The PLP of real interest rates that we have documented also exists in the 1985I-2007IV sample, the Great Moderation period. This evidence (as shown in Table 3 in the appendix) indicates that PLP is not driven by the zero-lower-bound on the federal funds rate reached in the aftermath of the Great Recession in the US. The findings are also robust to alternative measures of the real interest rate (as shown in Table 4 in the appendix).

King and Watson (1996) presented a variety of models, including a financial frictions

model, that were all unable to account for the ILP. This has been a long-standing puzzle in the literature.

While this property was attributed to monetary shocks, [Boldrin, Christiano and Fisher \(2001\)](#) presented a technology shock driven two-sector real business cycle model with consumption habits and limited labour mobility that accounted for the ILP. Recently, [Pintus, Wen and Xing \(2017\)](#) present a model, building on [Kiyotaki and Moore \(1997\)](#), in which self-fulfilling belief shocks redistribute income away from lenders to borrowers during booms. Although their objective is to provide a theoretical rationale for the ILP, they do not provide a quantitative comparison of model-based cross-correlations with those in the data. However, the post-1985 evidence we document poses a new challenge to the research aimed at explaining ILP since it no longer exists in the data. Business cycle dynamics of real interest rates are characterized by PLP.

The right panel of [Figure 1](#) shows the cross-correlation based on the simulated data from a standard Real Business Cycle (RBC) model.⁷ Interestingly, the procyclicality of real interest rates in the post-1985 data is, at least qualitatively, consistent with that based on the simulated data from an RBC model. While it is known that the model does not produce any lead-lag pattern between the real interest rate and output, the purpose of showing it here provides a useful perspective. The same challenge that the RBC model faced in matching ILP applies to matching PLP in the post-1985 data. Moreover, the mechanisms discussed in [Boldrin, Christiano and Fisher \(2001\)](#) and [Pintus, Wen and Xing \(2017\)](#) produce ILP, and therefore, by construction, cannot explain PLP of real interest rates.

To investigate the real interest rate-output cross-correlations based on DSGE models developed more recently, we consider three models which have structural features—frictions and shocks—that are embedded in many contemporary DSGE models. These are [Iacoviello \(2005\)](#), [Smets and Wouters \(2007\)](#) and [Basu and Bundick \(2017\)](#).⁸ The bottom row in [Figure](#)

⁷We consider a frictionless version of the RBC model (See [Cooley and Hansen \(1995\)](#)).

⁸The replication code for [Iacoviello \(2005\)](#) is available at Matteo Iacoviello's website <https://www2.bc.edu/matteo-iacoviello/research.htm>. The replication code for [Smets and Wouters \(2007\)](#) is available from the *American Economic Review's* website <https://www.aeaweb.org/articles?id=10.1257/aer.97.3.586>. The replication code for [Basu and Bundick \(2017\)](#) is available at Brent Bundick's website <http://www.brentbundick.com/research.html> and Johannes

1 shows the cross-correlations based on data simulated from these models, respectively.

We first examine the cross-correlations based on the simulated data from [Iacoviello \(2005\)](#) model of housing that has two types of households—patient and impatient—and an endogenous collateral constraint faced by the impatient agent. Subsequently, this model framework has been widely used in studying the role of monetary and fiscal policies in the presence of durable (housing) goods. As shown in panel (c), the real interest rate-output cross-correlations based on the simulated data do not match either the ILP in the pre-1985 data or the PLP in the post-1985 data. The largest correlation implied by the model is contemporaneous. The model also implies that the real interest rates are highly countercyclical which is opposite to that in the post-1985 data.

Panel (d) shows the results based on the [Smets and Wouters \(2007\)](#) estimated for the full sample and over the 1984-2004 period, respectively. It is perhaps not well recognized that the [Smets and Wouters \(2007\)](#) model did in fact generate ILP. Clearly, the presence of consumption habits (as suggested in [Boldrin, Christiano and Fisher \(2001\)](#)) and other real and nominal frictions, and shocks contribute towards this property of the model. The model-based ILP is, however, counterfactual relative to the post-1985 evidence of PLP shown in panel (a).

The final example is [Basu and Bundick \(2017\)](#) who study the role of countercyclical markups, sticky prices, and monetary policy in producing contractionary comovement among macroeconomic aggregates after an increased uncertainty about the future. We generate simulated data from their model and compute the real interest rate-output cross-correlations. In the present context, although their model generates a positive contemporaneous correlation between output and the real interest rate consistent with that observed in the post-1985 data, the lead-lag pattern turns out to be counterfactual. In their model, real interest rates lead output by one quarter and with a positive sign.

Based on the comparison between the cross-correlations in the post-1985 data and the models, we conclude that a broad class of contemporary DSGE models do not match the PLP—the defining property of real interest rates over the business cycle. Identifying new

Pfeifer's github page https://github.com/JohannesPfeifer/DSGE_mod.

mechanisms to explain the positive lagging property of real interest rates is, therefore, an important research direction. In section 4 we discuss the key challenges for the DSGE models in explaining PLP over the business cycle.

2.3 Labour Productivity Negatively Lags Output

Panels (a) and (b) in Figure 2 show the cross-correlations between output and labour productivity, LP_t , where the latter variable is measured as output per hour and output per person, respectively. The cross-correlations in the figure represent $\text{Corr}(Y_t, LP_{t+k}) : k = \{-5, -4, \dots, 0, \dots, 4, 5\}$.

While the decline in the procyclicality of labour productivity has been widely discussed in the literature, the post-1985 data show a prominent inverted lagging property of labour productivity over the cycle. Labour productivity shifted from leading the cycle in the pre-1985 period with a positive sign to lagging by at least a year with a negative sign. The absolute magnitude of these cross-correlations are substantially larger than the contemporaneous correlations, indicating the presence of a strong business cycle relationship not captured by comovement alone.

A natural question is: Do models that can either qualitatively or quantitatively explain the decline in the procyclicality of labour productivity also account for the lead-lag shift in the data that we have documented in panel (a) of Figure 2? To answer this question we consider two recent contributions to the literature, Galí and van Rens (2017) and Garin, Pries and Sims (2018). Both models have successfully explained the decline in the procyclicality of labour productivity.

The main mechanism discussed in Galí and van Rens (2017) is the decline in turnover reflecting reduced hiring frictions since the mid-1980s as a force behind the vanishing procyclicality of labour productivity. Panel (c) shows the cross-correlations based on the simulated data where we set the separation rate $\delta = 0.35$ for the pre-1985 period and $\delta = 0.2$ for the post-1985 period, as in Galí and van Rens (2017).⁹

⁹The replication code for Galí and van Rens (2017) is available from Thijs van Rens's website <http://www.thijsvanrens.com/VPLP/>.

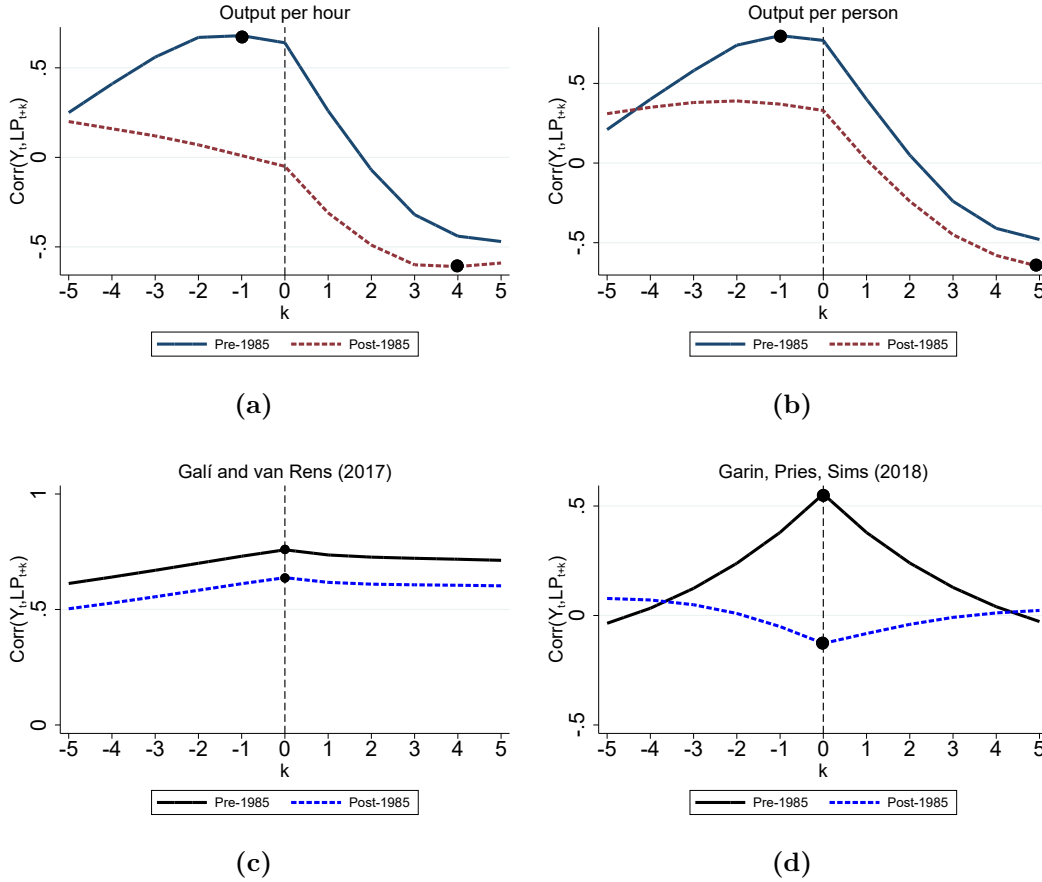


Figure 2: **Cross-correlations between output and labour productivity**

Note: k denotes the number of leads (negative values) or lags (positive values) between output and labour productivity. The correlations at the black dashed line represent the contemporaneous correlations. Black solid circles denote the largest cross- or contemporaneous correlation that occurs at the k th lead/lag, $\max\{|\text{Corr}(Y_t, LP_{t+k})|, k = -5, -4, \dots, 0, \dots, 4, 5\}$. Actual data and model data in [Garin, Pries and Sims \(2018\)](#) are HP filtered. Data from [Galí and van Rens \(2017\)](#) are unfiltered.

Consistent with the results in [Galí and van Rens \(2017\)](#), the diminished procyclicality of labour productivity is evident in panel (c). This exercise, however, also reveals that the model produces counterfactual cross-correlations for the two sub-samples. The model implies a contemporaneous correlation that is the largest in absolute value in both periods. We, therefore, conclude that the same mechanism—the decline in turnover—as developed in [Galí and van Rens \(2017\)](#), cannot account either qualitatively or quantitatively for (i) the switch from labour productivity leading output to lagging output and (ii) the switch in sign

from positive to negative as shown in panel (a) of Figure 2.

The second example is [Garin, Pries and Sims \(2018\)](#) who develop a model in which they show that the importance of sectoral shocks relative to aggregate shocks can account for the decline in the procyclicality of labour productivity in the US economy. We use the simulated data from their model to compute cross-correlations between labour productivity and output.¹⁰ Panel (d) of Figure 2 shows the cross-correlations for the pre- and post-1985 period. While the model can clearly account for the decline in procyclicality of labour productivity, it produces a counterfactual cross-correlation pattern between output and labour productivity. The cross-correlations based on the simulated data are very close to zero for the post-1985 calibration in [Garin, Pries and Sims \(2018\)](#).

There is little work in the literature that has addressed the lead-lag properties of labour productivity over the US business cycle. An early contribution by [Burnside and Eichenbaum \(1993\)](#) discussed the ability of the factor-loading model to generate the dynamic correlations between labour productivity and output.¹¹ The presence of factor hoarding behaviour, however, causes labour productivity to lead output instead of lagging, as noted in [Burnside \(1998\)](#). This means that factor-hoarding is unlikely to explain the negatively lagging labour productivity over the business cycle in the post-1985 period.

To summarize, existing models and mechanisms that have been used to explain the declining procyclicality of labour productivity do not account for its shift in the lead-lag property over the business cycle. In light of the new evidence from the post-1985 period shown in panel (a), explaining the shift in labour productivity from leading positively to lagging negatively is an important direction for future research. We discuss this point further in section 4.

2.4 Labour Inputs Negatively Lead Labour Productivity

The dynamic relationship between total hours worked and productivity features prominently in a large body of business cycle research (see, for example, [Benhabib, Rogerson and Wright](#)

¹⁰We thank Eric Sims and Julio Garin for providing us with the replication code for their paper.

¹¹See Figure 3 in [Burnside and Eichenbaum \(1993\)](#) based on US data from 1955I - 1992IV. The discussion of cross-correlations is omitted in the published version ([Burnside and Eichenbaum \(1996\)](#)).

(1991), [Christiano and Eichenbaum \(1992\)](#), [Galí \(1999\)](#) for early contributions). Panels (a) and (b) in [Figure 3](#) show two sets of cross-correlations, respectively. The first is between total hours worked, H_t , and output per hour. The second is between total employment, E_t , and output per worker. In the pre-1985 data, total hours worked lagged output per hour by three quarters and with a positive sign, $\text{Corr}(LP_t, H_{t+3}) = 0.62$. In the post-1985 data, however, this relationship has switched with total hours worked leading output per hour by two quarters. Employment also leads output per person by four quarters. Both of these cross-correlations have a negative sign, $\text{Corr}(LP_t, H_{t-2}) = -0.67$ and $\text{Corr}(LP_t, E_{t-4}) = -0.63$, respectively. The contemporaneous correlations between labour input and labour productivity measures have also switched signs from positive to negative. In particular, switching from $\text{Corr}(LP_t, H_t) = 0.21$ in the pre-1985 sample to -0.53 in the post-1985 sample for output per hour. Similarly, switching from $\text{Corr}(LP_t, E_t) = 0.24$ to -0.27 for output per person.

The pre-1985 evidence of total hours worked lagging output per worker is consistent with the evidence reported in [Figure 2](#) of [Burnside and Eichenbaum \(1993\)](#). They show that the factor hoarding model does a good job of matching the cross-correlations. The switch in the lead-lag property, with total hours worked leading output per worker with a negative sign in the post-1985 data, however, suggests that factor hoarding behaviour of firms cannot reconcile this evidence. We discuss this point further in [section 4](#).

We now examine the cross-correlations between labour productivity and employment through the lens of the [Galí and van Rens \(2017\)](#) and [Garin, Pries and Sims \(2018\)](#) models. Panel (c) shows the cross-correlations based on the simulated data from the [Galí and van Rens \(2017\)](#) model.¹²

The model qualitatively matches the decrease in contemporaneous correlations and the sign switch. The lead-lag pattern, however, is counterfactual. Panel (d) shows the cross-correlations based on the simulated data from [Garin, Pries and Sims \(2018\)](#). Unlike, [Galí and van Rens \(2017\)](#), their model does not capture the sign switch in the contemporaneous correlation. More importantly from the perspective of our paper, the model does not produce a

¹²The model has stationary shocks so the cross-correlations are based on unfiltered data.

lead-lag pattern between labour productivity and employment which is again counterfactual.

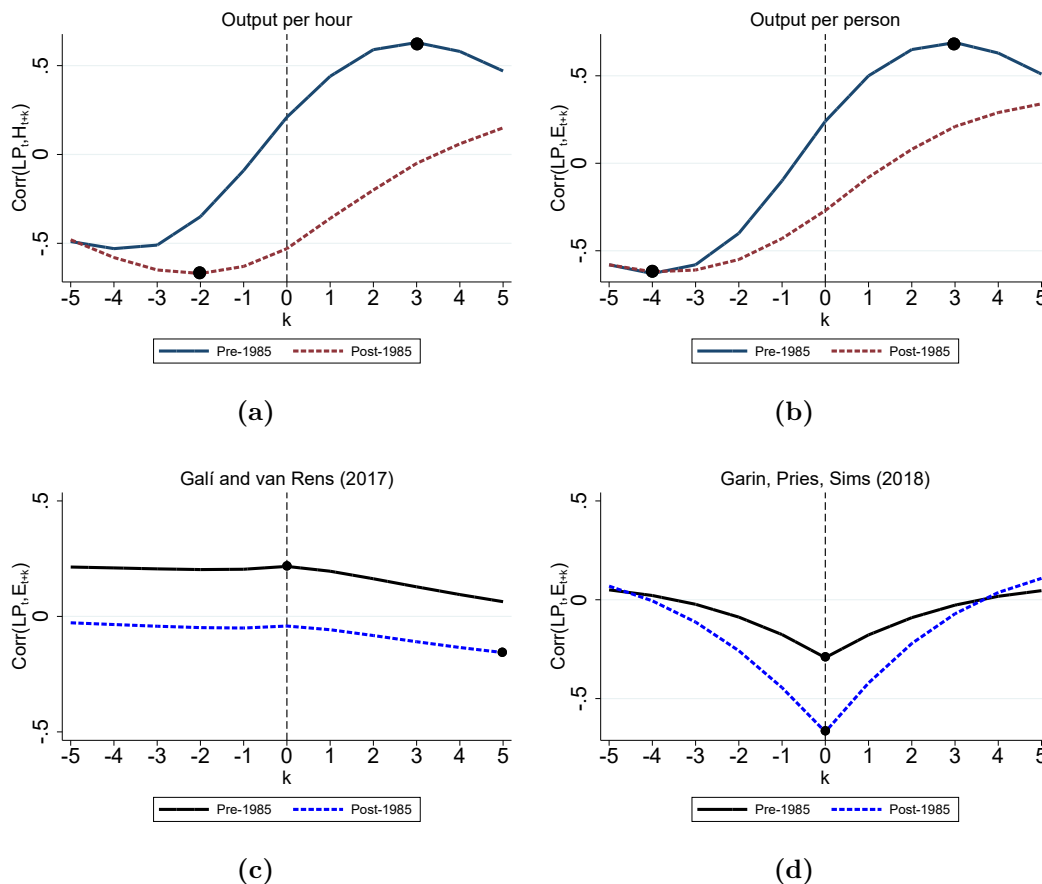


Figure 3: **Cross-correlations between labour productivity and labour inputs**

Note: k denotes the number of leads (negative values) or lags (positive values) between labour productivity and labour inputs. The correlations at the black dashed line represent the contemporaneous correlations. Black solid circles denote the largest cross- or contemporaneous correlation that occurs at the k th lead/lag, $\max\{|\text{Corr}(LP_t, \text{Labour Input}_{t+k}), k = -5, \dots, 0, \dots, 5|\}$. Actual data and simulated data from [Garin, Pries and Sims \(2018\)](#) are HP filtered. Simulated data from [Galí and van Rens \(2017\)](#) are unfiltered.

2.5 Unemployment Rate Positively Leads Labour Productivity

The relationship between labour productivity and the unemployment rate is a key component in models of search and matching (see, for example, [Mortensen and Pissarides \(1994\)](#), [Mertz \(1995\)](#), [Andolfatto \(1996\)](#), [Shimer \(2005\)](#), [Hall \(2005\)](#), among many other contributions.) Recently, [Barnichon \(2010\)](#) noted that the contemporaneous correlation between cyclical

unemployment and labour productivity over the post-WWII period switched sign in the mid-1980s: from significantly negative the correlation became significantly positive.

Panel (a) in Figure 4 shows that in the pre-1985 data the largest cross-correlation between the unemployment rate, U_t and output per worker is $\text{Corr}(U_t, LP_{t-2}) = -0.74$, indicating that unemployment lagged output per worker by two quarters with a negative sign. Increases in productivity were associated with declines in unemployment three quarters ahead. This relationship switched in the post-1985 data to $\text{Corr}(U_t, LP_{t+4}) = 0.61$. Thus, increases in the unemployment rate are associated with an increase in productivity four quarters ahead.

Barnichon (2010) also notes in passing that the cross-correlogram between unemployment and productivity look ‘dramatically different’ (p. 1015). His focus, however, is on the shift in the contemporaneous correlation between unemployment and productivity and he, therefore, does not examine if the model produces the shift in the cross-correlations which is the main focus of our paper. Interestingly, the absolute magnitude of the contemporaneous correlations in both pre-1984 and post-1984 periods that Barnichon (2010) considers are smaller than the cross-correlations. This observation reinforces our point that the focus in many recent papers has been on contemporaneous correlations, and the larger cross-correlations indicating shifts in the lead-lag patterns capturing important business cycle relationships have either remained unnoticed or have received very little attention.

We simulate data from Barnichon (2010)’s model for pre-and post-1985 periods and compute the cross-correlations between unemployment and labour productivity.¹³ Panel (c) in Figure 4 shows the model-based cross-correlations. We find that the model does not produce any lead-lag pattern for the post-1985 period. For the pre-1985 period, the model produces a lead of labour productivity over unemployment. Both of these properties are counterfactual.¹⁴

¹³We thank Regis Barnichon for providing us with the replication codes.

¹⁴The contemporaneous correlations in Figure 4 are different from those reported in Barnichon (2010). We suspect that this is because he examines the mean of a 40-quarter rolling correlation, whereas we compute the correlation coefficient for the entire sample period.

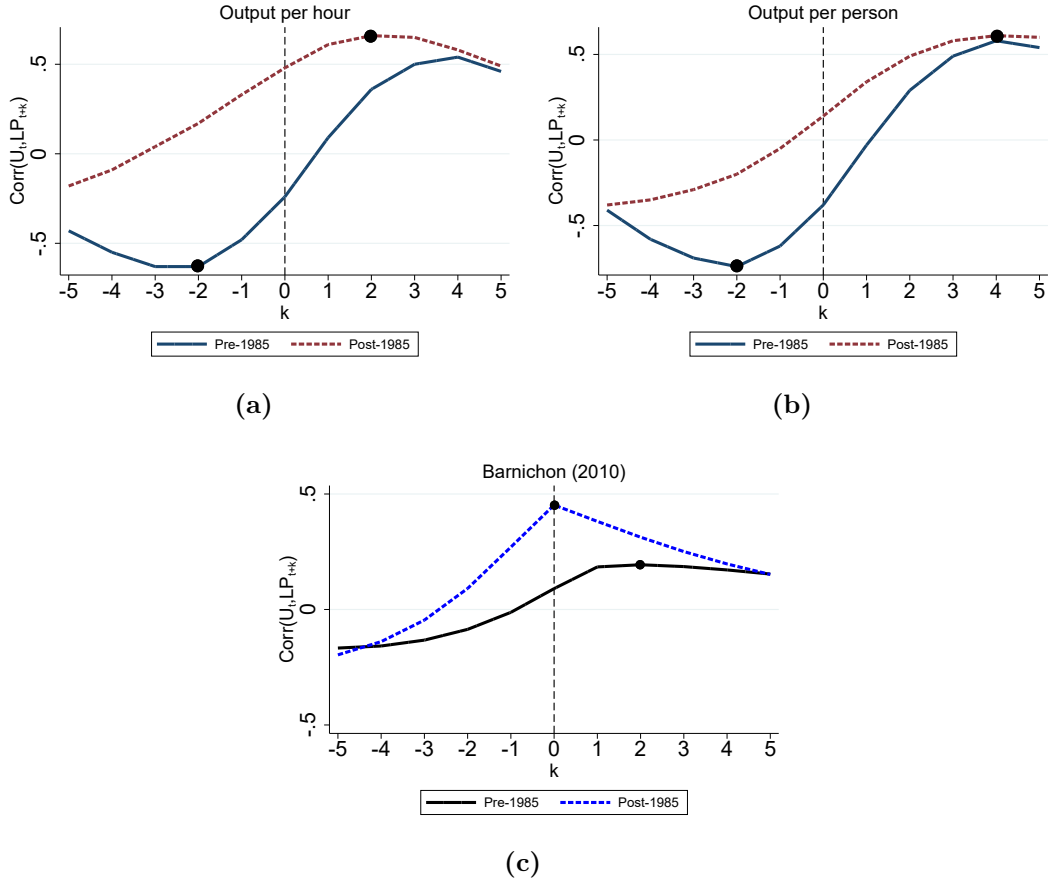


Figure 4: **Cross-correlations between the unemployment rate and labour productivity**

Note: k denotes the number of leads (negative values) or lags (positive values) between the unemployment rate and labour productivity. The correlations at the black dashed line represent the contemporaneous correlations. Black solid circles denote the largest cross- or contemporaneous correlation that occurs at the k th lead/lag, $\max\{|\text{Corr}(Y_t, LP_{t+k})|, k = -5, \dots, 0, \dots, 5\}$. Both actual and model-simulated data are HP filtered.

3 Robustness to filtering

Using the HP filtered data to present the lead-lag properties is advantageous for two reasons: first, the HP filter is arguably the most common method for obtaining the cyclical component from aggregate data, and second, it allows us to contrast the new stylized facts with those

in the previous literature.¹⁵

Recently, however, [Hamilton \(2017\)](#) has proposed an alternative to the HP filter. This new filtering method requires obtaining residuals from a regression of a variable h -periods ahead on its p most recent values as of date t .

$$y_{t+h} = \beta_0 + \sum_{j=0}^p \beta_{j+1} y_{t-j} + v_{t+h} \quad (1)$$

where the cyclical component is constructed as the residuals of (1)

$$\hat{v}_{t+h} = y_{t+h} - \left(\hat{\beta}_0 + \sum_{j=0}^p \hat{\beta}_{j+1} y_{t-j} \right) \quad (2)$$

We refer to this regression-based procedure as the hp filter, and use it to assess the robustness of the empirical findings reported in the previous section. Applying the hp filter, we use $h = 8$ and $p = 4$, which are [Hamilton \(2017\)](#)'s suggested parametric specification for detrending quarterly data. The hp results are summarized in [Table 2](#). Real interest rates exhibit ILP in the pre-1985 data, but positively lag output by three quarters in the post-1985 data. This is consistent with the results under HP filtering.

Labour productivity leads output in the pre-1985 data. However, in the post-1985 data output per hour switches to negatively lagging as in the HP filter case, while output per person continues to positively lead. Under the hp filter, total hours positively lag labour productivity in the pre-1985 sample, but negatively leads in the post-1985 data, which is again consistent with HP filtering. Total employment leads output per person in both the pre-1985 and post-1985 samples, not producing the shift in cross correlations that we observe under HP filtering. Finally, both output per person and output per hour negatively lead unemployment in the pre-1985 sample, but positively lag in the post-1985 data, consistent with HP filtering.

¹⁵Previously, [Burnside \(1998\)](#), in his comment on [Canova \(1998\)](#), has noted that that business cycle stylized facts obtained from different filters do not necessarily have to agree. He writes (on page 514): *I will argue that when the facts differ according to the filter, this simply means there are many facts to be explained.*

4 Discussion

The large absolute magnitude of the cross-correlations relative to contemporaneous correlations, their shifts, and the sign switches all point to important structural changes in business cycle mechanisms in the US economy. Three broad research questions emerge: First, how have structural changes in the US economy influenced the shifts in the lead-lag properties of macroeconomic variables studied in section 2? Second, how has the changing nature of macroeconomic shocks affected these particular shifts? Third, has the conduct of macroeconomic policies (monetary and fiscal) contributed to these shifts? We think that these are both interesting and important directions for future research.

Implications and Insights:

Here we note a few implications and insights that emerge from the findings and model comparisons in section 2.

i) Estimation of DSGE models:

Our findings have two implications for the estimation and development of DSGE models. First is that given the shifts in lead-lag properties documented in section 2, our findings caution against estimating DSGE models over the entire post WW II data. Second is that if the intrinsic (structural) properties of the DSGE model do not generate the correct cross-correlation pattern then estimation of the model parameters, and the relative importance of the driving shocks will be biased. We now discuss a few specific aspects of the findings in more detail.

ii) PLP of Real interest rates:

[Boldrin, Christiano and Fisher \(2001\)](#) provided an explanation for ILP of real interest rates. By construction, therefore, that model cannot explain the PLP in the post-1985 period. Through a set of examples, we made the point that a broad class of contemporary DSGE models featuring a variety of mechanisms also do not produce PLP. This property of real interest rates is a serious challenge to developing DSGE model-based explanations.

iii) Labour productivity, hours, and unemployment dynamics:

The business cycle dynamics in the labour market have experienced substantial shifts

since the mid-1980s. We showed that the recent models ([Galí and van Rens \(2017\)](#), [Garin, Pries and Sims \(2018\)](#)) that provide an explanation for one dimension of these dynamics—the decline in cyclical of labour productivity—produce counterfactual lead-lag cross-correlations between labour productivity and output. Similarly, a model that provides an explanation for the the sign switch in the contemporaneous unemployment-labour productivity correlation ([Barnichon \(2010\)](#)) produces no lead-lag pattern which is counterfactual. An open question is whether some modification of the basic model structure would produce the correct lead-lag pattern and sign. This seems to not be straightforward. Incorporating factor-hoarding, for example, will produce labour productivity leading labour inputs ([Burnside and Eichenbaum \(1993\)](#)) over the cycle which is counterfactual for the post-1985 period. Similarly, it appears that forces identified in [Barnichon \(2010\)](#) that account for a sharp drop in the volatility of non-technology shocks in the mid-1980s, and (ii) a decline in the response of productivity to non-technology shocks, are insufficient to explain the positive lead of unemployment over labour labour productivity.

One promising direction is to explore how structural changes in the US labour market since the 1980s have influenced the joint dynamics of labour productivity, labour inputs, and unemployment. Two examples of structural changes are the job polarization in the labour market ([Autor and Dorn \(2013\)](#)) and the displacement of labour to machines and robots ([Acemoglu and Restrepo \(2017\)](#)). In this context, the task-based view of the labour market ([Acemoglu and Autor \(2011\)](#)) can provide new insights about the aggregate labour market dynamics. [Brault and Khan \(2018\)](#) take a step in this direction.

iv) Residential investment:

Finally, we note that the well known fact that residential investment leads the cycle remains robust across pre-1985 and post-1985 data. In the post-1985 data, residential investment leads output by two quarters over the business cycle. The largest cross-correlations are $\text{Corr}(Y_t, I_{t-1}) = 0.67$ and $\text{Corr}(Y_t, I_{t-2}) = 0.69$. This finding is also robust to using the *hp* filter.

5 Conclusion

In this paper, we document surprising shifts in the lead-lag properties of the US business cycle since the onset of the Great Moderation period of the mid-1980s. We characterize four new stylized facts in terms of cross-correlations based on HP filtered cyclical data. First, real interest rates positively lag output. Second, labour productivity negatively lags output. Third, labour inputs negatively lead labour productivity, and fourth, the unemployment rate positively leads labour productivity. The large absolute magnitude of these cross-correlations relative to contemporaneous correlations suggest important business cycle forces at work that are not reflected in comovements alone. We show that a large class of contemporary DSGE models produce counterfactual lead-lag patterns and with incorrect signs. Our empirical findings raise new challenges for explaining the lead-lag shifts and suggests many promising areas for future research.

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	SD(%)	Rel Vol	ϕ_1	$x(-5)$	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$	$x(+5)$
Cross-correlations with output														
1948Q1-1984Q4														
Real interest rate	2.03	1.02	.41	-.26	-.35	-.4	-.35	-.2	-.1	.02	.13	.15	.2	.15
1985Q1-2016Q1														
Real interest rate	1.08	.99	.64	-.04	.08	.17	.24	.31	.39	.46	.49	.53	.51	.43
Cross-correlations with output														
1948Q1-1984Q4														
Output per hour	1.23	.47	.7	.25	.41	.56	.67	.68	.64	.26	-.07	-.32	-.44	-.47
Output per person	1.58	.61	.77	.21	.4	.58	.74	.8	.77	.4	.05	-.24	-.41	-.48
Total hours	2.05	.79	.88	-.36	-.21	.02	.33	.65	.89	.9	.77	.56	.31	.1
1985Q1-2016Q1														
Output per hour	.86	.58	.74	.2	.16	.12	.07	.01	-.05	-.31	-.49	-.6	-.61	-.59
Output per person	.89	.6	.74	.31	.35	.38	.39	.37	.33	.02	-.24	-.45	-.58	-.65
Total hours	1.75	1.18	.95	.06	.23	.41	.59	.75	.87	.91	.86	.75	.6	.42
Cross-correlations with output per hour														
1948Q1-1984Q4														
Total hours	2.05	1.67	.88	-.49	-.53	-.51	-.35	-.09	.21	.44	.59	.63	.58	.47
1985Q1-2016Q1														
Total hours	1.75	2.04	.95	-.48	-.58	-.65	-.67	-.63	-.53	-.36	-.2	-.05	.06	.15
Cross-correlations with output per person														
1948Q1-1984Q4														
Total employment	1.72	1.09	.89	-.58	-.63	-.58	-.4	-.1	.24	.5	.65	.69	.63	.51
1985Q1-2016Q1														
Total employment	1.46	1.65	.96	-.58	-.62	-.61	-.55	-.43	-.27	-.08	.08	.21	.29	.34
Cross-correlations with unemployment														
1948Q1-1984Q4														
Output per hour	1.23	1.35	.7	-.43	-.55	-.63	-.63	-.48	-.24	.09	.36	.5	.54	.46
Output per person	1.58	1.74	.77	-.41	-.58	-.69	-.74	-.62	-.38	-.03	.29	.49	.58	.54
1985Q1-2016Q1														
Output per hour	.86	1.24	.74	-.18	-.09	.04	.17	.33	.48	.61	.66	.65	.58	.49
Output per person	.89	1.29	.74	-.38	-.35	-.29	-.2	-.05	.14	.34	.49	.58	.61	.6

TABLE 1: Cross-correlations and volatilities using Hodrick-Prescott filter

	SD(%)	Rel Vol	ϕ_1	$x(-5)$	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$	$x(+5)$
Cross-correlations with output														
1948Q1-1984Q4														
Real interest rate	2.5	.65	.63	-.27	-.35	-.37	-.31	-.21	-.15	-.13	-.13	-.08	-.02	.02
1985Q1-2016Q1														
Real interest rate	1.83	.67	.86	.08	.17	.26	.35	.41	.5	.56	.6	.61	.6	.55
Cross-correlations with output														
1948Q1-1984Q4														
Output per hour	2.43	.5	.88	.58	.68	.74	.74	.7	.64	.42	.17	-.03	-.18	-.27
Output per person	3.24	.66	.9	.54	.67	.76	.79	.78	.74	.52	.28	.06	-.12	-.26
Total hours	3.92	.8	.85	-.16	.05	.28	.5	.74	.88	.84	.76	.66	.49	.31
1985Q1-2016Q1														
Output per hour	2.07	.57	.91	.2	.18	.14	.07	0	-.04	-.14	-.19	-.2	-.19	-.17
Output per person	1.93	.53	.89	.37	.41	.4	.35	.29	.23	.06	-.06	-.15	-.22	-.27
Total hours	3.97	1.09	.93	.39	.53	.64	.74	.83	.87	.86	.81	.72	.6	.47
Cross-correlations with output per hour														
1948Q1-1984Q4														
Total hours	3.92	1.61	.85	-.52	-.47	-.37	-.17	.07	.27	.41	.51	.57	.55	.52
1985Q1-2016Q1														
Total hours	3.97	1.92	.93	-.42	-.49	-.54	-.56	-.54	-.5	-.42	-.35	-.28	-.21	-.13
Cross-correlations with output per person														
1948Q1-1984Q4														
Total employment	3.51	1.08	.84	-.63	-.57	-.45	-.24	.02	.25	.38	.47	.53	.5	.43
1985Q1-2016Q1														
Total employment	3.33	1.72	.94	-.48	-.49	-.48	-.42	-.33	-.24	-.13	-.05	.02	.08	.11
Cross-correlations with unemployment														
1948Q1-1984Q4														
Output per hour	2.43	1.54	.88	-.66	-.71	-.73	-.69	-.59	-.45	-.22	0	.16	.27	.31
Output per person	3.24	2.06	.9	-.64	-.73	-.76	-.74	-.65	-.52	-.3	-.07	.12	.27	.35
1985Q1-2016Q1														
Output per hour	2.07	1.65	.91	-.07	-.04	.02	.08	.14	.2	.25	.26	.24	.19	.11
Output per person	1.93	1.54	.89	-.24	-.24	-.2	-.16	-.09	.01	.12	.21	.26	.29	.28

TABLE 2: Cross-correlations and volatilities using the *hp* filter

A Replication files

Replication files are available upon request.

B Data description

The data in this paper were obtained from the Federal Reserve Bank of St. Louis Economic Database (FRED) using Stata 15's import fred command (for more information, see <https://www.stata.com/new-in-stata/import-fred/>). To accommodate those not using Stata 15, the replication data are provided in an excel file. The variables used in the paper are described below (FRED codes in bold):

- Output: Real Gross Domestic Product (**GDPC1**). Billions of chained 2009 dollars, seasonally adjusted annual rate. Quarterly, 1947Q1-2017Q3.
- Output: Nonfarm Business Sector Real Output (**OUTNFB**). Index 2009=100, seasonally adjusted. Quarterly, 1947Q1-2017Q3.
- Nominal interest rate: 3-Month Treasury Bill Secondary Market Rate (**TB3MS**). Percent, not seasonally adjusted. Monthly, converted to quarterly by averaging rate per 3 months, 1947Q1-2017Q3.
- Price level: Gross Domestic Product Implicit Price Deflator (**GDPDEF**). Index 2009=100, seasonally adjusted. Quarterly, 1947Q1-2017Q3.
- Unemployment rate: Civilian Unemployment Rate (**UNRATE**). Percent, seasonally adjusted. Monthly, converted to quarterly by averaging rate per 3 months, 1948Q1-2017Q3.
- Employment: Nonfarm Business Sector Employment (**PRS85006013**). Index 2009=100, seasonally adjusted. Quarterly, 1947Q1-2017Q3.
- Total hours: Nonfarm Business Sector Hours of All Persons (**HOANBS**). Index=2009, seasonally adjusted. Quarterly, 1947Q1-2017Q3.

- Output per hour: Nonfarm Business Sector Real Output Per Hour of All Persons (**OPHNFB**). Index 2009=100, seasonally adjusted. Quarterly, 1947Q1-2017Q3.
- Output per person: Nonfarm Business Sector Real Output Per Person (**PRS85006163**). Index 2009=100, seasonally adjusted.

C Variable definitions

The following variables are defined using the initial data described above:

- Inflation = $(\ln(PRICE_t) - \ln(PRICE_{t-1})) * 400$
- Real interest rate = Nominal interest rate_t - Inflation_{t+1}

We log transform all of the variables, excluding real interest rate and unemployment rate. We use a smoothing parameter of $\lambda = 1600$ for the Hodrick-Prescott filter. Applying the Hamilton filter, we use $h = 8$ and $p = 4$, Hamilton's suggested parametric specification for detrending quarterly data.

D Pre-Great Recession phase shifts

The phase shifts presented in the main text of the paper document significant changes in lead/lag properties. One may be concerned about the extent to which these results are driven by the Great Recession. The following tables contain the dynamic correlations post-Great Moderation and pre-Great Recession,

	SD(%)	Rel Vol	ϕ_1	$x(-5)$	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$	$x(+5)$
Cross-correlations with output														
1985Q1-2007Q4														
Real interest rate	1.16	1.12	.71	-.09	0	.11	.26	.39	.47	.54	.54	.55	.52	.47
Cross-correlations with output														
1985Q1-2007Q4														
Output per hour	.82	.59	.73	.16	.07	0	-.02	-.04	-.03	-.26	-.41	-.52	-.56	-.6
Output per person	.81	.59	.73	.29	.3	.28	.3	.3	.32	.03	-.18	-.37	-.5	-.6
Total hours	1.63	1.18	.98	.2	.38	.54	.68	.78	.86	.89	.86	.79	.67	.49
Cross-correlations with output per hour														
1985Q1-2007Q4														
Total hours	1.63	1.98	.98	-.43	-.49	-.55	-.58	-.57	-.53	-.4	-.29	-.18	-.08	.02
Cross-correlations with output per person														
1985Q1-2007Q4														
Total employment	1.36	1.68	.99	-.47	-.49	-.49	-.45	-.37	-.27	-.13	0	.11	.18	.24
Cross-correlations with unemployment														
1985Q1-2007Q4														
Output per hour	.82	1.38	.73	-.16	-.04	.11	.26	.38	.47	.58	.6	.6	.59	.58
Output per person	.81	1.36	.73	-.34	-.29	-.19	-.09	.03	.16	.34	.42	.49	.55	.59

TABLE 3: Pre Great Recession cross-correlations and volatilities using Hodrick-Prescott filter

E Alternative definitions of real interest rate

In the paper, we report dynamic correlations of output and real interest rates where real interest rates are defined as the treasury bill secondary market rate minus ex post inflation at time $t + 1$ (where inflation is generated from the GDP deflator). In the following table, we use alternative definitions of real interest rates to highlight the robustness of the positive leading property.

Real interest rate (3MTB) is the reported measure in the main text of the paper, where real interest rate is the three month treasury bill secondary market rate minus inflation at $t+1$ (GDP deflator based). Real interest rate (Fed funds) is the federal funds rate minus inflation at $t+1$ (GDP deflator based). Real interest rate (CPI) is the three month treasury bill secondary market rate minus inflation at $t+1$ (CPI deflator based). Real interest rate (shadow rate) is the nominal interest rate calculated by [Wu and Xia \(2016\)](#) minus inflation at time $t+1$ (GDP deflator based). Finally, real interest rate (forecasted) is the three month treasury bill secondary market rate minus expected inflation at time t . Expected inflation data is obtained from the Survey of Professional Forecasters through the Federal Reserve Bank of Philadelphia. We do not report the forecasted dynamic correlations for the pre-1985 period due to the data only being available from 1981Q2 onwards.

	SD(%)	Rel Vol	ϕ_1	x(-5)	x(-4)	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)	x(+4)	x(+5)
Cross-correlations with output														
1948Q1-1984Q4														
Real interest rate (3MTB)	2.03	1.02	.41	-.26	-.35	-.4	-.35	-.2	-.1	.02	.13	.15	.2	.15
Real interest rate (Fed funds)	1.54	.78	.37	-.43	-.47	-.45	-.31	-.03	.15	.22	.18	.13	.1	.11
Real interest rate (CPI)	2.31	1.16	.5	-.19	-.31	-.38	-.43	-.37	-.3	-.19	-.09	-.02	.1	.09
Real interest rate (shadow rate)	1.59	.8	.43	-.49	-.5	-.45	-.34	-.07	.12	.18	.17	.13	.16	.17
Real interest rate (unfiltered)	2.84	1.43	.73	-.23	-.3	-.35	-.31	-.2	-.12	-.03	.05	.07	.1	.07
1985Q1-2016Q1														
Real interest rate (3MTB)	1.08	.99	.64	-.04	.08	.17	.24	.31	.39	.46	.49	.53	.51	.43
Real interest rate (Fed funds)	1.16	1.07	.68	-.07	.06	.17	.26	.34	.43	.51	.54	.57	.54	.46
Real interest rate (CPI)	1.9	1.75	.21	-.01	0	0	0	.06	.19	.27	.27	.27	.29	.23
Real interest rate (shadow rate)	1.21	1.11	.7	-.09	.03	.13	.22	.3	.4	.5	.55	.6	.58	.5
Real interest rate (unfiltered)	2.29	2.1	.93	.1	.17	.22	.26	.31	.35	.39	.4	.42	.4	.36
Real interest rate (forecasted)	.9	.83	.81	.11	.22	.31	.4	.45	.54	.57	.53	.5	.44	.34

TABLE 4: Cross-correlations and volatilities using alternative measures of real interest rate, HP filter