Risky Lending, Bank Leverage and Unconventional Monetary Policy *

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This version: November 2017

Abstract

I develop a New Keynesian DSGE model with a comprehensive financial system in which banks provide funds to firms and homeowners via defaultable long-term loans. Financial intermediaries are subject to an endogenous leverage constraint, implying a link between banks balance sheet and aggregate credit conditions. In this framework, I consider two set of financial shocks affecting the two endogenous components of interest rate spreads on mortgages and corporate loans in the model: i) risk shocks affecting the volatility of idiosyncratic borrowers' risk, and ii) bank collateral shocks affecting the collateral value of banks' assets. I show that these shocks can reproduce reasonably well the behavior of several macroeconomic variables during the Great Recession, when we take into account the impact of the zero-lower-bound. In addition, I use the model to quantify the effect of the Federal Reserve's purchases of mortgage-backed securities during the last recession.

Keywords: Financial Frictions, Banking, Unconventional Monetary Policy, Zero Lower Bound

JEL Classification: E32, E44, E58, G21

*The views expressed in this paper are those of the author and do not necessarily reflect the views of the Federal Reserve Board or the Federal Reserve System. I would like to thank for their guidance on this project Mark Gertler and Jaroslav Borovicka. I would like to thank Andrea Prestipino and Gaston Navarro for the very useful discussions. I would also like to thank for their comments Andrea Ferrero, Urban Jermann, Alejandro Justiniano, Luisa Lambertini, Makoto Nakajima, Matthias Paustian, John Roberts, Tao Zha and seminar participants at the Federal Reserve Board and at the New York University. Ian de Medeiros provided excellent research assistance. An old version of this paper was titled "Risky Mortgages, Bank Leverage and Credit Policy".

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1 Introduction

The 2007-2009 financial crisis, also known as the Great Recession, was characterized by a deep erosion in the equity of financial intermediaries, and by an unprecedented turmoil in the markets for mortgages and corporate loans, as it can be seen from figure 1. The top two panels of this figure present the behavior of spreads on mortgage securities and corporate lending, while the bottom one reports the XLF bank index, an equity index capturing the stock performance of U.S. banks. The BAA spread and the primary mortgage spread, two widely used measures of credit conditions, increased by about 350 basis points and 120 basis points respectively, compared to their pre-crisis level. At the same time, also the spread on AAA-rated corporate bonds and the spread on agency mortgage-backed securities (MBS) experienced a noticeable increase. Given that AAA-rated issuers are very safe companies, and that MBS are designed to diversify away the idiosyncratic borrowers' risk, we can think of this second set of spreads as having a very limited exposure to borrowers' default risk. As a result, we can interpret the difference between the solid and dashed lines in figure 1 as potentially representing a "default premium" on mortgages and corporate debt; whereas we can interpret the AAA spread and the MBS spread as capturing a residual "liquidity premium".[1]

In the last quarter of 2008, as credit spreads reached their peak with the bankruptcy of Lehman Brothers, the federal funds rate reached zero. Around this period, the Federal Reserve announced an unprecedented expansion of its balance sheet, performed by purchasing private securities and treasury bonds. A key part of this unconventional credit policy, often dubbed "quantitative easing" (QE), consisted in purchasing MBS directly from the financial sector. As shown, in figure 2, by early 2010 the Fed MBS holdings reached \$1.1 trn, corresponding to about 8% of total household credit.[2]

In this paper I develop a New Keynesian DSGE model that is able to generate endogenous dynamics for the four spreads reported in figure 1, and links them to the aggregate balance sheet conditions of financial intermediaries. I then use two sets of financial shocks, directly affecting either the default premium or the liquidity premium of spreads on mortgages or corporate debt, to match these variables at the height of the Great Recession. After taking into account the impact of the zero-lower-bound (ZLB) and of the Fed's MBS purchases reported in figure 2, such shocks can generate macroeconomic time series very similar to the ones observed during the last financial crisis. This framework is also used to quantify the impact of unconventional monetary policy, of the ZLB and of nominal rigidities.

The model introduces in a canonical DSGE, with real and nominal rigidities, a comprehensive financial system where banks channel funds from savers towards two sets of borrowers: homeowners who need to finance house purchases and entrepreneurs who need to finance capital purchases. Bank lending occurs through defaultable long term debt, implying a wedge between the loan rates, on mortgages and business loans, and banks' required rate of return. This wedge can be considered as a default premium. In addition, banks face an agency problem like in Gertler and Karadi (2011), which results in an endogenous leverage constraint and in a wedge between their required rate of return on loans and the risk free rate. I refer to this second wedge as liquidity premium.

In the model, these two types of premia (on two types of assets), evolve both in response to exogenous financial shocks and in response to endogenous changes in fundamentals.

The expected probability of default, of mortgages and corporate loans, is positively linked to the exogenous dispersion of idiosyncratic shocks to the value of housing and capital. These two types of "risk shocks", as defined by Christiano et al. (2014), are hence useful to target changes in the default premia. In addition,

[1]A similar decomposition of corporate spreads is used, for example, in Del Negro et al. (2017b).

[2]This figure corresponds to about 10% of total outstanding mortgages.

the share of defaulting homewoners or entrepreneurs will increase endogenously with borrowers' leverage and decrease with the price of houses and capital respectively.

The liquidity premia depend instead on the collateral value of mortgages and business loans for bank funding, which, as in Gertler and Karadi (2011), is inversely related to the fraction of the asset that the banker can divert for private consumption. A "collateral shock" reduces this value for either type of security, causing the bank to reduce its supply of mortgages or business loans, increasing the related lending rates. From this perspective, we can think of this shock as affecting the assets' "funding liquidity", as defined by Brunnermeier and Pedersen (2010).[3] The decline in the market value of long term debt is also going to generate an endogenous increase in liquidity premia through a standard financial accelerator linked to the deterioration in bank net worth. This mechanism, with bank equity at its core, can generate comovements among a wide set of credit spreads and asset prices, because of the interaction of the different layers of financial frictions in the model.

In the main experiment of the paper, I show how a combination of risk shocks and collateral shocks, calibrated to deliver realistic spikes in spreads in the fourth quarter of 2008, can generate declines in GDP, consumption and investment very similar to what we observed in the Great Recession. In addition, I show how the same shocks, absent the Fed's intervention in the mortgage market, would have resulted in an impact on these variables about 50% larger. This result crucially hinges on the presence of the ZLB. In fact, according to the model, if the nominal rate were not constrained to be positive, the downturn would have been much less severe and the effect of unconventional credit policies would have been quite smaller. Finally I show how these findings highlight the role of nominal rigidities in amplifying financial shocks.

1.1 Related Literature

This paper is linked to the growing literature of DSGE models introducing financial frictions in different sectors of the economy. This literature began by focusing on agency problems in the firm sector, like, for example, in the models of Bernanke, Gertler and Gilchrist (1999), Christiano, Motto and Rostagno (2014) and Jermann and Quadrini (2012). It has then been extended to consider also frictions related to the housing market affecting borrowing households, as in the models of Iacoviello (2005), Iacoviello and Neri (2010), and Forlati and Lambertini (2011). After the 2007-2009 financial crisis, there has been a large effort to introduce the role of financial intermediaries in macroeconomic models like, for example, in Gertler and Karadi (2011), Gertler, Kiyotaki and Prestipino (2016), Brunnermeier and Sannikov (2011) and He and Krishnamurty (2013).

From a modeling perspective, this paper contributes to the literature by developing a tractable framework which combines financial frictions in the three sectors of the economy, while featuring long term defaultable loans. Following a financial shock, this environment generates novel amplification channels which produce realistic comovements among a wide set of macroeconomic and financial variables.

In terms of the quantitative results, the contribution of this paper is twofold. First, it shows how financial shocks affecting the risk premium and the liquidity premium, of spreads in the mortgage market and corporate market, can generate macroeconomic dynamics very close to what we observed in the Great Recession, when we take into account the zero lower bound. Second, the paper represents the first attempt to quantify the impact of the Federal Reserve's MBS purchases in a structural macroeconomic model.[4]

[3] This collateral shock is also similar to the financial shock considered by Jermann and Quadrini (2012).

[4] There have been several papers trying to quantify the impact of unconventional monetary policy in an econometric setting, like, for example, Gagnon et al. (2011), Hancock and Passmore (2011), and Stroebel and Taylor (2012).

As regards the agency problem affecting the financial sector, this paper builds on the framework of Gertler and Karadi (2011), who first analyzed constrained banks and unconventional monetary policy in a DSGE model, and extends their work by introducing defaultable long term mortgages and corporate loans. Compared to their paper, in which banks directly purchase capital, this model presents a more realistic characterization of the balance sheet of financial intermediaries and it also allows us to study the interaction between banks' financial accelerator and the wealth of borrowing households and firms. In addition, this paper presents a more specific description of the response enacted by the Federal Reserve during the crisis, which was focused on the market for mortgage-backed securities.

Another paper pointing to the importance of financial shocks and unconventional monetary policy during the last financial crisis is Del Negro et al. (2017a). These authors include a credit friction, affecting the fraction of assets that firms can sell when they need to finance an investment opportunity, in a New Keynesian DSGE. Then they show how shocks to this resalability constraint can have large effects in presence of the ZLB, and they use the model to assess the impact of the Federal Reserve's liquidity facilities during the Great Recession. My work can be considered complementary to this paper in several ways. The liquidity premium that I model is closer to the notion of funding liquidity (how easy it is to obtain funding using an asset as collateral) rather than to that of market liquidity (how easily an asset can be traded), as defined by Brunnermeier and Pedersen (2010). In addition, the focus of my paper is to study the impact of a different type of credit policy: large scale MBS purchases, rather than liquidity facilities. The two models deliver predictions on the impact of unconventional monetary policy, and on the role of the ZLB and nominal rigidities, which are qualitatively consistent.[5]

The rest of the paper is organized as follows. Section 2 presents the model, section 3 illustrates the main quantitative exercises and section 4 concludes.

2 The Model

I introduce a comprehensive financial system in a standard New-Keynesian framework. The model is populated by four main types of agents: impatient households (or homeowners), patient households (or savers), bankers, and entrepreneurs. As explained below, bankers and entrepreneurs are members of the patient household. At the center of the financial system there are banks which raise funds from patient households and channel them to impatient households and entrepreneurs in the form of long term mortgages and long term business loans.[6] The former type of asset is used to finance house purchases, whereas the latter is used to finance capital purchases.

Financial intermediaries face an agency problem when obtaining deposits from patient households, as in Gertler and Karadi (2011), which implies an endogenous leverage constraint and a role for aggregate bank net worth in determining the supply of funds to borrowers. As a result, banks' required expected rate of return on mortgages and loans will be higher than the risk free rate, creating a liquidity premium linked to the collateral value of each security. In addition, mortgages and loans are subject to costly default implying an additional default premium between the interest rate faced by borrowers and banks' required rate of return.

The model also includes capital producers, final good retailers and labor unions to introduce standard real

[5] Another related paper is Faria-e-Castro (2017), who studies the impact of fiscal policy during the Great Recession in a model with financial intermediaries and defaultable mortgages. However this model does not have capital, and the analysis does not take into account the ZLB.

[6] In the rest of the paper I will often refer to business loans simply as "loans".

and nominal frictions. A central bank conducts both conventional monetary policy, in the form of nominal interest rate setting, and unconventional monetary policy, in the form of purchases of mortgage securities.

2.1 Patient Households

There is a continuum of patient households that save in the form of bank deposits.[7] Similarly to Gertler and Karadi (2011), I assume that each patient household has a "family" structure with a continuum of members with measure unity: whithin each family a fraction g^b of these agents are bankers, a fraction g^e are entrepreneurs and a fraction $1 - g^b - g^e$ are workers. Within an household there is perfect consumption insurance.

Each worker l supplies differentiated labor services $\hat{N}_t(l)$ to the consumption good sector and decides the amount of deposits to provide to a bank.[8] Bankers manage a financial intermediary which finances itself with deposits and retained earnings. In order to avoid that bankers save their way out the financial constraint, I assume that with probability $1-\omega^b$ they exit the financial sector, pay dividends to the household and become workers. Exiting bankers are replaced by workers endowed with some start-up funds, which I will explain in detail later. Entrepreneurs manage a non-financial firm which finances itself with retained earnings and defaultable loans from banks. Like bankers, entrepreneurs pay dividends at an exogenous rate $1 - \omega^e$. In addition, defaulting entrepreneurs are replaced by new workers with an initial endowment.

Whenever confusion is possible, I will use hatted variables to refer to patient households as opposed to impatient ones. The representative patient household gains utility from consumption $\hat{C}_t$, housing services $\hat{X}_t$, and have disutility from labor $\hat{N}_t(l)$, according to the following preference structure:

$$\sum_{t=0}^{\infty} \hat{\beta}^t \left\{ \log\left(\hat{C}_t\right) + v\left(\hat{X}_t\right) - \frac{1}{1+\varphi_n} \int_0^1 \hat{N}_t(l)^{1+\varphi_n}\, dl \right\} \tag{1}$$

In particular, housing services are provided one-to-one by the housing good $\hat{H}_t$, priced at q_t^h. I assume that the function $v(\cdot)$ implies a constant housing demand by savers, $\hat{X}_t = \hat{H}_t = \hat{H}$, so that, as I explain below, impatient agents will always be the marginal buyers of houses. The assumption that impatient households price houses is meant to capture the segmentation in the US housing market, where there is little trading of houses between rich agents (lenders) and poor ones (borrowers). A similar assumption is also used also by Justiniano et al. (2015) and by Greenwald (2016), and has the important implication of having houses being priced by leveraged borrowing agents, hence amplifying the fluctuations in the collateral value of dwellings.[9] This stylized framework produces a set of richer lending agents whose wealth is mainly composed of capital, and a set of borrowing agents whose wealth crucially depends on house prices. This result is consistent with the finding of Mian et al. (2013), who showed that poorer and more levered households had a significantly higher marginal propensity to consume out of housing wealth during the 2007-2009 recession, whereas wealthier households' consumption did not react as much or even increased in response to the drop of house prices.

[7] I will refer to patient households also as lenders, savers or depositors.

[8] We can think of workers providing funds to financial intermediaries belonging to a different patient household family.

[9] This assumption is equivalent to assuming that savers and borrowers purchase two different types of houses, traded in two separate markets.

The patient households face the following budget constraint

$$\hat{C}_t + q_t^h \left(\hat{H}_t - \hat{H}_{t-1} \right) + B_t = \int_0^1 \hat{w}_t (l) \hat{N}_t (l) \, dl + R_t B_{t-1} + \Pi_t \tag{2}$$

where B_t are bank deposits, R_t is the risk free rate, $\hat{w}_t(l)$ is the real wage for labor input l, and Π_t are profits arising from the ownership of banks and non-financial firms. The first order condition on bank deposits implies

$$1 = E_t \hat{\Lambda}_{t,t+1} R_{t+1} \tag{3}$$

where $\hat{\Lambda}_{t,t+1} = \beta \hat{C}_t / \hat{C}_{t+1}$.

As is standard in New Keynesian models, wage rigidities are introduced by assuming that monopolistically competitive labor unions set wages on a staggered basis. Workers supply the hours demanded by firms at a specific wage. Details on the wage determination can be found in the appendix.

2.2 Impatient Households

There is a continuum of impatient households with discount factor $\beta < \hat{\beta}$.[10] Within each family there are two types of agents: workers and homeowners. Workers provide differentiated labor input $N_t(l)$ and choose housing services X_t. Homeowners choose housing and mortgage stocks, and can transfer to or receive additonal funds from their household in order to finance house purchases. Within the family there is perfect insurance in consumption and housing services.[11]

The Worker's problem: The utility of the impatient household can be expressed in terms of its aggregate consumption C_t, housing services X_t and differentiated hours $N_t(l)$ as

$$\sum_{t=0}^{\infty} \beta^t \left\{ \log(C_t) + \chi \log(X_t) - \frac{1}{1+\varphi_n} \int N_t (l)^{1+\varphi_n} \, dl \right\} \tag{4}$$

I assume that every period impatient agents can trade housing services among themselves at price r_t^h, which can be thought of as rent.[12] As a result we can write the impatient worker's budget constraint as

$$C_t + X_t r_t^h = \int w_t (l) N_t (l) \, dl + D_t^h \tag{5}$$

where $w_t(l)$ is the real wage for impatient households labor inputs and the term D_t^h represents aggregate "housing dividends". As shown below, this term is linked to the net worth of non-defaulting homeowners and to the current stock of houses and mortgage debt.

The first order condition for housing services equalizes r_t^h to the marginal rate of substitution between consumption as housing services

$$\frac{\chi}{X_t} = \frac{r_t^h}{C_t} \tag{6}$$

[10] The lower discount factor guarantees that in the steady state of the model they are willing to borrow by issuing mortgages.

[11] The decomposition of the intratemporal and intertemporal decisions of the impatient household helps to obtain a cleaner characterization of the optimal policies; but at the aggregate level this is equivalent to considering the problem of a single representative impatient household, dividing equally the amount of housing and mortgages across its members.

[12] The presence of two distinct markets for housing services and houses simplifies aggregation for impatient households because the rental rate equalizes the marginal utility from housing across agents.

As for patient workers, wages are set by monopolistically competitive labor unions on a staggered basis.

The Homeowner's problem: Every period a homeowner chooses an amount of housing h_t and finances it with a mortgage m_t valued at the price Q_t^m and provided by a banker. Mortgages are long term securities which promise to pay every period, in absence of default, a coupon c_m and a portion λ_m of the principal, while the remaining $(1-\lambda_m)$ remains outstanding.[13] As a result, the mortgage has an expected duration of $1/\lambda_m$.

Houses are risky in the model because of an idiosyncratic shock affecting the return on housing.[14] In particular, the realized return at time t+1 on a house for a homeowner with an idiosyncratic shock ξ_{t+1}^m will be $\xi_{t+1}^m R_{t+1}^h$, where

$$R_{t+1}^h = \frac{r_{t+1}^h + q_{t+1}^h}{q_t^h} \tag{7}$$

The shock ξ_{t+1}^m is drawn from a distribution $F_t\left(\xi_{t+1}^m, \sigma_t^m\right)$, where σ_t^m is an exogenous variable affecting the variance of the distribution, but not the mean, and evolving according to

$$\log \sigma_t^m = (1-\rho_{\sigma^m}) \log \sigma^m + \rho_{\sigma^m} \log \sigma_{t-1}^m + \varepsilon_t^{\sigma^m} \tag{8}$$

I will refer to σ_t^m as a "mortgage risk shock", since it will directly affect the expected probability of mortgage defaults, as explained shortly. In addition, $E_t\left(\xi_{t+1}^m\right) = 1$ for any σ_t^m, so that variations in mortgage risk do not affect the aggregate stock of housing available.

Mortgages are defaultable debt. Every period, after observing his ξ_t^m, a homeowner chooses whether to default on his outstanding mortgage. In case of default the homeowner exits and becomes a worker, while the bank repossesses a fraction γ_m of the house.

If we define "housing net worth", $\bar{n}_t^h$, as the difference between the return on housing and the value of outstanding mortgage debt (including interest payments)

$$\bar{n}_t^h = \xi_t^m h_{t-1} R_t^h q_{t-1}^h - (c_m + \lambda_m) m_{t-1} - (1-\lambda_m) Q_t^m m_{t-1} \tag{9}$$

we can write the budget constraint of a homeowner as

$$q_t^h h_t + d_t^h = \bar{n}_t^h + Q_t^m m_t \tag{10}$$

where d_t^h represents housing dividends. If positive this quantity represents the amount of funds that the homeowner can transfer to the household for consumption purposes, once new houses have been purchased and new funds have been borrowed. If negative, the homeowner is raising extra funds from the household to finance house purchases.

Let $V_t^h\left(h_{t-1}, m_{t-1}, \xi_t^m\right)$ be the value function of a non-defaulting homeowner with initial housing stock h_{t-1}, initial mortgage stock m_{t-1} and idiosyncratic shock ξ_t^m. Then we can write

$$V_t^h\left(h_{t-1}, m_{t-1}, \xi_t^m\right) = \max_{h_t, m_t} \left\{d_t^h + E_t \Lambda_{t,t+1} \max\left\{0, V_{t+1}\left(h_t, m_t, \xi_{t+1}^m\right)\right\}\right\} \tag{11}$$

subject to (9) and (10) . The homeowner maximizes the present discounted value of housing dividends,

[13] This tractable specification of long term loans is used also, for example, in Gomes et al. (2016).

[14] A similar shock is used also by Jeske et al. (2013) in a model with endogenous mortgage defaults.

while using the stochastic discount factor of the impatient household $\Lambda_{t,t+1} = \beta C_t / C_{t+1}$. In addition, the homeowner takes into account the option to default in the future.

As shown in the appendix, the value function is linear in housing net worth, and consequently in the housing stock, according to

$$V_t^h \left(h_{t-1}, \eta_{t-1}^m, \xi_t^m\right) = \varphi^h \bar{n}_t^h = \varphi^h \left[\xi_t^m R_t^h q_{t-1}^h - (c_m + \lambda_m)\, \eta_{t-1}^m - (1-\lambda_m)\, Q_t^m \eta_{t-1}^m\right] h_{t-1} \tag{12}$$

where we can refer to the ratio $\eta_t^m = m_t / h_t$ as to homeowner's leverage.[15]

This result implies a simple characterization of the mortgage default decision. The homeowner will default whenever his housing net worth is negative, or equivalently, whenever his idiosyncratic shock is below an endogenous threshold $\bar{\xi}_t^m \left(\eta_{t-1}^m\right)$, where

$$\bar{\xi}_t^m \left(\eta_{t-1}^m\right) = \frac{(c_m + \lambda_m) + (1-\lambda_m)\, Q_t^m}{R_t^h q_{t-1}^h} \eta_{t-1}^m \tag{13}$$

The default probability will hence be negatively correlated to current house prices, and positively correlated to the current value outstanding debt, which is a function of Q_t^m and initial leverage η_{t-1}^m. As is standard in models with endogenous default, the relationship between the default threshold and the initial leverage will imply that the mortage price will be inversely related to η_t^m, as we will see in the banker's pricing equation in section 2.4. Homeowners will internalize this effect when choosing their leverage.

Given the form of the value function, the first order conditions for h_t and η_t will be

$$\left(q_t^h - Q_t^m \eta_t^m\right) = E_t \Lambda_{t,t+1} \int_{\bar{\xi}_{t+1}^m(\eta_t^m)}^{\infty} \left[\xi_{t+1}^m R_{t+1}^h q_t^h - (c_m + \lambda_m)\, \eta_t^m - (1-\lambda_m)\, Q_{t+1}^m \eta_t^m\right] dF_t\left(\xi_{t+1}^m\right) \tag{14}$$

$$Q_t^m + \frac{\partial Q_t^m}{\partial \eta_t^m} \left[\eta_t^m - (1-\lambda_m)\, \eta_{t-1}^m \frac{h_{t-1}}{h_t}\right] = E_t \Lambda_{t,t+1} \left[1 - F_t\left(\bar{\xi}_{t+1}^m\left(\eta_t^m\right)\right)\right] \left[(c_m + \lambda_m) + (1-\lambda_m)\, Q_{t+1}^m\right] \tag{15}$$

Equation (14) equalizes the cost of a house financed with a mortgage amount $Q_t^m m_t$, on the left hand side, to the expected return on the house next period, net of the debt value, in case default does not occur. This equation links housing demand to the mortgage interest rate $1/Q_t^m$, and it captures the the channel through which financial shocks affect house prices in the model. As anticipated, the left hand side of the optimality condition for leverage, equation (15), internalizes the negative impact of higher leverage on the morgage price, $\frac{\partial Q_t^m}{\partial \eta_t^m} < 0$, which is proportional to the amount of new debt issued. The right hand side of equation (15) represents the expected cost of the mortgage, given by the interest payment and the value of future outstanding debt, in case of no default.

Aggregation for the Impatient Household: The first order conditions in equation (14) and (15) imply that h_t, η_t^m and consequently $\bar{\xi}_t^m$ will be the same for all homeowners, facilitating aggregation. In fact, if we define H_t as aggregate housing held by the impatient households, M_t as the aggregate mortgage stock, and $\bar{N}_t^h$ as their aggregate housing net worth, we can write aggregate housing dividends as

$$D_t^h = \bar{N}_t^h - H_t q_t^h + Q_t^m M_t \tag{16}$$

[15] In the appendix I show that $\varphi^h = 1$.

where

$$\bar{N}_t^h = H_{t-1}\left\{R_t^h q_{t-1}^h \int_{\bar{\xi}_t^m\left(\eta_{t-1}^m\right)}^{\infty} \xi_t^m dF_{t-1}\left(\xi_t^m\right) - \left[1 - F_{t-1}\left(\bar{\xi}_t^m\left(\eta_{t-1}^m\right)\right)\right]\left[\left(c_m + \lambda_m\right) + \left(1-\lambda_m\right) Q_t^m\right]\eta_{t-1}^m\right\} \tag{17}$$

Finally, substituting (16) in the impatient household budget constraint (5) we obtain

$$C_t + X_t r_t + H_t q_t^h = \int w_t\left(l\right) N_t\left(l\right) dl + \bar{N}_t^h + Q_t^m M_t \tag{18}$$

From this equation we see how the impatient households' consumption will depend positively on their housing net worth and on the amount of money that they can raise through mortgage debt.

2.3 Entrepreneurs

As anticipated, entrepreneurs are members of the patient household. Every period an entrepreneur purchases capital k_t, with price q_t^k, by using a business loan l_t and his retained earnings $\bar{n}_t^e$.[16][17] We can define the ratio $\eta_t^l = l_t/k_t$ as entrepreneurial leverage. As in the case of impatient households, loans are in the form of defaultable long term debt, with price Q_t^l, coupon payment c_l and principal amortization rate λ_l.

Similarly to houses, capital is risky because of an idiosyncratic shock $\xi_t^l \tilde{} F_{t-1}\left(\xi_t^l, \sigma_{t-1}^l\right)$ affecting the return on capital R_t^k, where σ_{t-1}^l affects the standard deviation of the shock at time t. At time t+1 a unit of capital with a shock ξ_{t+1}^l will deliver $\xi_{t+1}^l R_{t+1}^k$, where

$$R_{t+1}^k = \frac{r_{t+1}^k + (1-\delta)\, q_{t+1}^k}{q_t^k}$$

and r_t^k is the rental rate on capital, and δ is the capital depreciation rate. Also in this case I assume that σ_t^l varies over time according to

$$\log \sigma_t^l = \left(1-\rho_{\sigma^l}\right)\log \sigma_t^l + \rho_{\sigma^l} \log \sigma_{t-1}^l + \varepsilon_t^{\sigma^l} \tag{19}$$

I will refer to σ_t^l as a "loan risk shock", since it will directly affect the expected default probability of corporate loans. This type of risk shock is the same used, for example, in Christiano et al. (2014) to model financial frictions in non-financial firms' investment decisions.

After observing their idiosyncratic shock, entrepreneurs decide whether to default or not. In case of default they exit and return to the patient household, while the bank seizes their capital subject to default costs equal to a fraction $(1-\gamma_l)$ of the capital value. Defaulting entrepreneurs are replaced by an equal mass of new entrepreneurs. If they do not default, entrepreneurs have to pay a fraction $(1-\omega^e)$ of their net worth as dividends to patient households.[18] In addition, I assume that non-defaulting entrepreneurs can insure each other with respect to the idiosyncratic shock. As explained in detail in the appendix, this assumption is needed to equalize the marginal value of net worth across non-defaulting entrepreneurs, in order to obtain aggregation.[19]

[16] For simplicity, In the rest of the text I will refer to business loans also as simpliy "loans".

[17] I assume that entrepreneurs can not issue equity and can only rely on debt for external finance, as in, for example, Bernanke et al. (1999), Christiano et al. (2014) and Gertler and Karadi (2011).

[18] The exogenous dividend payment decision is a simplification used to guarantee that the entrepreneurial net worth does not grow indefinitely, given the excess returns arising from the costly default friction. See, for example, Bernanke et al. (1999).

[19] This assumption was not needed in the case of homeowners because they can always adjust their portfolio by raising

The entrepreneur's net worth, n_t^e, will be given by the difference between the return on capital and the value of outstanding debt

$$n_t^e = \left[\xi_t^l R_t^k q_{t-1}^k k_{t-1} - (c+\lambda)\, l_{t-1} - (1-\lambda)\, Q_{t-1}^l l_{t-1}\right] \tag{20}$$

As a result, we can write the budget constraint for the entrepreneur as

$$q_t^k k_t = \bar{n}_t^e + Q_t^l l_t \tag{21}$$

where $\bar{n}_t^e$ are retained earnings, given by

$$\bar{n}_t^e = \omega^e n_t^e \tag{22}$$

Let $V_t^e\left(k_{t-1}, l_{t-1}, \xi_t^l\right)$ be the value function of a non-defaulting entrepreneur with initial capital k_{t-1} and debt l_{t-1}, and with an idiosyncratic shock ξ_t^l. Then we can write

$$V_t^e\left(k_{t-1}, l_{t-1}, \xi_t^l\right) = \max_{k_t, l_t}\left\{(1-\omega^e)\, n_t^e + E_t\hat{\Lambda}_{t,t+1}\max\left\{0, V_{t+1}^e\left(k_t, l_t, \xi_{t+1}^l\right)\right\}\right\} \tag{23}$$

subject to (20) (21) and (22). The entrepreneur maximizes the present discounted value of dividends, while using the stochastic discount factor of the patient household and taking into account the option to default in the future.

Also in this case it can be shown that the value function is linear in the entrepreneur's net worth, according to

$$V_t^e = \varphi_t^e n_t^e = (1-\omega^e + \omega^e\kappa_t^e)\, n_t^e \tag{24}$$

where κ_t^e represents the marginal value of retained earnings, which will depend only on aggregate variables.[20]

As a result entrepreneurs default when their net worth is negative, that is when their idiosyncratic shock is below a threshold given by

$$\bar{\xi}_t^l\left(\eta_{t-1}^l\right) = \frac{\left[(c+\lambda) + (1-\lambda)\, Q_t^l\right]}{R_t^k q_{t-1}^k}\eta_{t-1}^l \tag{25}$$

which will be increasing in leverage and decreasing in the realized return on their asset, as in the case of homeowners.

The first order condition for η_t^l will be given by

$$\kappa_t^e Q_t^l + \frac{\partial Q_t^l}{\partial \eta_t^l}\left[\kappa_t^e\eta_t^l - \frac{k_{t-1}}{k_t}(1-\lambda_l)\,\varphi_t^e\eta_{t-1}^l\right] = E_t\hat{\Lambda}_{t,t+1}\varphi_{t+1}^e\left[1 - F_t\left(\bar{\xi}_{t+1}^l\left(\eta_t^l\right)\right)\right]\left[(c_l+\lambda_l) + (1-\lambda_l)\, Q_{t+1}^l\right] \tag{26}$$

This equation is very similar to the first order condition for homeowners' leverage, equation (15). Like the homeowners, entrepreneurs internalize that a higher leverage will have a negative effect on the amount they can borrow, $\frac{\partial Q_t^l}{\partial \eta_t^l} < 0$, through an increase in the expected probability of default. In addition, using

additional funds from the impatient household. A similar assumption is used also, for example, in Gertler and Kiyotaki (2010), in a model with financial frictions and idiosyncratic investment opportunities.

[20]In the appendix I show that the marginal value of retained earnings is given by

$$\kappa_t^e = \left\{\frac{1}{q_t^k - Q_t^l\eta_t^l}E_t\hat{\Lambda}_{t,t+1}\varphi_{t+1}^e\int_{\bar{\xi}_{t+1}^l}^{\infty}\left[\xi_{t+1}^l R_{t+1}^k q_t^k - \left[(c_l+\lambda_l) + (1-\lambda_l)\, Q_{t+1}^l\right]\eta_t^l\right] dF_t\left(\xi_{t+1}^l, \sigma_t^l\right)\right\}$$

and depends only on aggregate variables.

equation (21), we can write capital demand as

$$k_t = \frac{\bar{n}_t^e}{\left(q_t^k - \eta_t^l Q_t^l\right)} \tag{27}$$

Equation (27) links the entrepreneurs' capital expenditures to his net worth and to the amount of funds that he is able to raise through business loans. Such relationship is similar to the one implied by the financial accelerator of Bernanke et al. (1999), where the main difference resides in the presence of long term debt in this model.

Aggregation for the entrepreneurs: As explained in the appendix, given the model assumptions, η_t^l, and consequently Q_t^l, will be the same across all entrepreneurs, and determined by equation (26). As a result we can obtain aggregate demand for capital K_t, by aggregating equation (27) as

$$K_t = \frac{\bar{N}_t^e}{\left(q_t^k - \eta_t^l Q_t^l\right)} \tag{28}$$

where $\bar{N}_t^e$ is aggregate entrepreneurial net worth. In order to compensate for the exit of defaulting entrepreneurs, I assume that an equal mass of new entrepreneurs enter every period with startup funds proportional to the value of existing capital, according to $\left(T^e q_t^k K_{t-1}\right)/F\left(\bar{\xi}_t^l\right)$.[21] As a result, we can write aggregate net worth as

$$\bar{N}_t^e = \omega^e \left\{ R_t^k q_{t-1}^k \int_{\bar{\xi}_t^l\left(\eta_{t-1}^l\right)}^{\infty} \xi_t^l dF_{t-1}\left(\xi_t^l\right) - \left[1 - F_{t-1}\left(\bar{\xi}_t^l\left(\eta_{t-1}^l\right)\right)\right]\left[(c+\lambda) + (1-\lambda) Q_t^l\right]\eta_{t-1}^l \right\} K_{t-1} + T^e q_t^k K_{t-1} \tag{29}$$

Fluctuations in the price of capital and in the proportion of defaulting entrepreneurs will affect the evolution of $\bar{N}_t^e$, and together with variations in loan interest rates $1/Q_t^l$ and leverage, will determine aggregate capital investment through (28).

2.4 Bankers

Bankers are members of patient households and the only agents able to channel funds from savers to entrepreneurs , in the form of business loans, and to impatient households, in the form of mortgages. As described above, the relationship between banks and borrowers is characterized by defaultable long term debt. Each bank j can invest in a continuum of mortgages $m_{j,t}$ and a continuum of loans $l_{j,t}$ issued by different homeowners and entrepreneurs. Potentially each of these loans could have a different leverage and consequently a different probability of default and a different price. However, as shown in the previous sections, all homeowners will choose the same leverage, and the same holds for entrepreneurs, implying that bank loans will be priced with the same Q_t^m and Q_t^l by every banker.

Absent default, the expected return on mortgages and on loans would be

$$E_t R_{t+1}^m = E_t \frac{\left(c_m + \lambda_m\right) + \left(1 - \lambda_m\right) Q_{t+1}^m}{Q_t^m} \tag{30}$$

[21]Equivalently, we can think of these startup funds as a lump sum transfer that each entrepreneur receives, as in Christiano et al. (2014).

$$E_t R^l_{t+1} = E_t \frac{(c_l + \lambda_l) + (1-\lambda_l)\, Q^l_{t+1}}{Q^l_t} \tag{31}$$

These returns also represent the one period interest rate on loans faced by borrowers in the model.

However, the presence of default implies that the banks' expected return on mortgages, $R^{m,b}_{t+1}$, and on loans, $R^{l,b}_{t+1}$, is

$$E_t R^{m,b}_{t+1} = E_t \frac{1}{Q^m_t} \left\{ [1 - F_t(\bar{\xi}^m_{t+1}(\eta^m_t))] Q^m_t R^m_{t+1} + \gamma_m \frac{q^h_t R^h_{t+1}}{\eta^h_t} \int_0^{\bar{\xi}^h_{t+1}(\eta^h_t)} \xi^m_{t+1} dF_t(\xi^m_{t+1}) \right\} = \frac{E_t \varrho^m_{t+1}(\eta^m_t)}{Q^m_t} \tag{32}$$

$$E_t R^{l,b}_{t+1} = E_t \frac{1}{Q^l_t} \left\{ [1 - F_t(\bar{\xi}^l_{t+1}(\eta^l_t))] Q^l_t R^l_{t+1} + \gamma_l \frac{q^k_t R^k_{t+1}}{\eta^l_t} \int_0^{\bar{\xi}^l_{t+1}(\eta^l_t)} \xi^l_{t+1} dF_t(\xi^l_{t+1}) \right\} = \frac{E_t \varrho^l_{t+1}(\eta^l_t)}{Q^l_t} \tag{33}$$

The terms $\varrho^i_{t+1}(\eta^i_t)$, for $i = m, l$, represent the payoff on mortages and loans with a loan-to-value ratio equal to η^m_t and η^l_t respectively. With probability $1 - F_t(\bar{\xi}^i_{t+1})$, $i = m, l$, the bank receives the interest payment and is still entitled to the future payment on the fraction of debt outstanding, as implied by the non-default returns in (30) and (31). Otherwise, when $\xi^i_{t+1} < \bar{\xi}^i_{t+1}$, for $i = m, l$, the borrower defaults and the bank can repossess the collateral, after paying mortgage default costs equal to a portion $(1 - \gamma_m)$ of the return on the house, or business loan default costs equal to a fraction $(1 - \gamma_l)$ of the return on capital. Since each bank lends to a continuum of homeowners and entrepreneurs, it can diversify away the idiosyncratic default risk, and by the law of large numbers the relized return on its mortgage holdings and loans holdings will be given by $R^{m,b}_{t+1}$ and $R^{l,b}_{t+1}$. From this perspective we can think of bank assets also as securitized assets similar, for example, to mortgage-backed securities. The returns on these securities will still vary because of aggregate risk affecting asset prices and the proportion of defaulting borrowers.

As shown below, a key feature of the model is the presence of two endogenous premia affecting the interest rates charged on mortgages and loans: a default premium will imply that $E_t R^i_{t+1} > E_t R^{i,b}_{t+1}$, for $i = m, l$; and a liquidity premium will imply that $E_t R^{i,b}_{t+1} > R_{t+1}$, for $i = m, l$.

Each bank finances its holding of mortgages and corporate loans with retained earnings $\bar{n}^b_{j,t}$, and by issuing risk-free deposits $b_{j,t}$ to patient households. As a result, we can write the budget constraint for a bank as

$$Q^l_t l_{j,t} + Q^m_t m_{j,t} = \bar{n}^b_{j,t} + b_{j,t} \tag{34}$$

We can then characterize the evolution of the net worth of an individual bank as

$$\bar{n}^b_{j,t+1} = m_{j,t} Q^m_t R^{m,b}_{t+1} + l_{j,t} Q^l_t R^{l,b}_{t+1} - b_{j,t} R_{t+1} \tag{35}$$

$$= m_{j,t} Q^m_t \left(R^{m,b}_{t+1} - R_{t+1} \right) + l_{j,t} Q^l_t \left(R^{l,b}_{t+1} - R_{t+1} \right) + \bar{n}^b_{j,t} R_{t+1} \tag{36}$$

As long as the banker makes an expected return on his assets greater than or equal to R_{t+1}, he will choose $m_{j,t}, l_{j,t}$ and $b_{j,t}$ in order to maximize the accumulated value of his net worth before it has to exit and pay dividends to the patient household, which occurs with probability $1 - \omega^b$. Hence, his value function at the end of time t, before knowing the realization of the exit random variable, is given by

$$V^b_{j,t} = E_t \sum_{i=0}^{\infty} \left(1 - \omega^b\right) \omega^b \hat{\Lambda}_{t,t+1+i} \bar{n}^b_{j,t+1+i} \tag{37}$$

As described above, banks are owned by patient households, and for this reason their stochastic discount factor enters the value function in (37). In addition, as in Gertler and Karadi (2011), I introduce an agency problem between the bank and the depositors in order to limit the amount of risky assets that the financial sector can hold, generating a wedge between the expected rate of returns on bank assets and liabilities. In particular, I assume that after raising deposits, the banker can default and divert back to his own household a fraction θ_t^l of his business loans and a fraction θ_t^m of his mortgages. If the banker does so, depositors can force him to bankruptcy and consequently to leave the banking sector forever, while recovering the remaining fractions of the assets.

As a result, the banker's problem entails the following incentive constraint, needed for patient households to provide deposits to the bank

$$V_{j,t}^b \geq \theta_t^m m_{j,t} Q_t^m + \theta_t^l l_{j,t} Q_t^l \tag{38}$$

This constraint guarantees that the value from continuing to operate the bank, the left-hand side, is larger than the value of running away with the diverted assets. In addition, I assume that θ_t^m and θ_t^l are subject to exogenous shocks according to

$$\log \theta_t^m = (1-\rho_{\theta^m}) \log \theta^m + \rho_{\theta^m} \log \theta_{t-1}^m + \varepsilon_{\theta^m,t} \tag{39}$$

$$\log \theta_t^l = (1-\rho_{\theta^l}) \log \theta^l + \rho_{\theta^l} \log \theta_{t-1}^l + \varepsilon_{\theta^l,t} \tag{40}$$

where we can think $\varepsilon_{\theta^m,t}$ and $\varepsilon_{\theta^l,t}$ as "bank collateral shocks". Such disturbances should capture changes in the collateral value of mortgages and business loans for bank funding. Hence we can think of these shocks as affecting what Brunnermeier and Pedersen (2008) define as "funding liquidity". For example, an increase in θ_t^m could be a stylized way to capture the collapse of the market for mortgage-backed securities and of securitization during the 2007-2009 financial crisis, which followed run episodes in several non-traditional banking markets.[22]

We can write the banker's value function recursively as follows

$$V_t^b\left(\bar{n}_{j,t}^b\right) = \max_{m_{j,t}, l_{j,t}, b_{j,t}} E_t \hat{\Lambda}_{t,t+1} \left\{ (1-\omega)\, \bar{n}_{j,t+1}^b + \omega V_{t+1}^b \left(\bar{n}_{j,t+1}^b\right) \right\}$$

where the maximization is subject to (38) and (35).

It can be shown that the value function for the banker is linear in net worth and can be rewritten as $V_t(\bar{n}_{j,t}^b) = \varphi_t^b \bar{n}_{j,t}^b$, where φ_t^b only depends on aggregate quantities.[23] If we define μ_t^b as the multiplier on the incentive constraint, the implied first order conditions for $m_{j,t}$ and $l_{j,t}$ are

$$E_t \hat{\Lambda}_{t,t+1} \Omega_{t+1} \left(R_{t+1}^{m,b} - R_{t+1} \right) = \mu_t^b \theta_t^m \tag{41}$$

$$E_t \hat{\Lambda}_{t,t+1} \Omega_{t+1} \left(R_{t+1}^{l,b} - R_{t+1} \right) = \mu_t^b \theta_t^l \tag{42}$$

where $\Omega_t = \left[(1-\omega^b) + \omega^b \varphi_t^b \right]$ represents the adjusted marginal value of net worth. As a result, if the constraint does not bind, $(\mu_t = 0, \Omega_t = 1)$, the expected discounted return on both bank assets should be equal to the risk-free rate. However, when the constraint binds, the bank will require a liqudity premium on loans and mortgages. Equations (41) and (42) also show that these spreads will vary both with an

[22] For models studying the effects of bank runs in a macroeconomic framework see, for example, Gertler and Kiyotaki (2015), Gertler, Kiyotaki and Prestipino (2016), and Ferrante (2017).

[23] See the appendix for a detailed solution of the problem of the financial intermediary.

endogenous tightening in the bank incentive constraint and with exogenous variations in θ_t^i for $i = m, l$. For this reason, in the quantitative experiments, I will use collateral shocks to target changes in liquidity premia during the Great Recession. In addition, the equations above imply the following no-arbitrage relationship

$$E_t \hat{\Lambda}_{t,t+1} \Omega_{t+1} \left(R_{t+1}^{m,b} - R_{t+1} \right) = \frac{\theta_t^m}{\theta_t^l} E_t \hat{\Lambda}_{t,t+1} \Omega_{t+1} \left(R_{t+1}^{l,b} - R_{t+1} \right) \tag{43}$$

Equation (43) establishes a link between the expected bank returns on loans and mortgages, which is also going to depend on the relative funding liquidity of the two securities. In particular, in steady state, if $\theta^m < \theta^l$, the excess return on mortgages will be lower than the one on loans to the productive sector.

Given the linear form of the value function, it can be shown that, when the constraint is binding, the following endogenous constraint on bank leverage will be in place

$$\left[Q_t^l l_{j,t} + \frac{\theta_t^m}{\theta_t^l} Q_t^m m_{j,t} \right] \leq \phi_t \bar{n}_{j,t}^b \tag{44}$$

where

$$\phi_t = \frac{E_t \tilde{\Lambda}_{t,t+1} R_{t+1}}{\theta_t^l - E_t \tilde{\Lambda}_{t,t+1} \left(R_{t+1}^{l,b} - R_{t+1} \right)} \tag{45}$$

and $\tilde{\Lambda}_{t,t+1} = \hat{\Lambda}_{t,t+1} \Omega_{t+1}$.

The constraint in (44) sets the value of the bank portfolio at a point such that the incentive constraint is exactly satisfied. In particular, if $\theta_t^m < \theta_t^l$, this implies a slacker limit on the bank's investment in mortgages. Also, the maximum leverage ratio will be inversely related to θ_t^l and positively related to the spread in expected returns. Equation (44) is at the heart of the standard bank financial accelerator by linking banks' asset demand to their net worth.

In addition, we can rewrite equations (41) and (42) in order to obtain the mortgage pricing equation that homeowners and entrepreneurs will internalize when choosing their optimal leverage, that is

$$1 = E_t \tilde{\Omega}_{t+1}^i R_{t+1}^{i,b} = E_t \tilde{\Omega}_{t+1}^i \frac{\varrho_{t+1}^i \left(\eta^i \right)}{Q_t^i} \quad \text{for} \quad i = m, l \tag{46}$$

where

$$E_t \tilde{\Omega}_{t+1}^i = \frac{E_t \tilde{\Lambda}_{t,t+1}}{E_t \tilde{\Lambda}_{t,t+1} R_{t+1} + \theta_t^i \mu_t} \tag{47}$$

The term $\tilde{\Omega}_{t+1}^i$, for $i = m, l$, is the adjusted stochastic discount factor that bankers use to price risky mortgages and loans. As mentioned above, a tightening of the incentive constraint and/or an increase in $\left\{ \theta_t^m, \theta_t^l \right\}$, will put downward pressure on Q_t^m and Q_t^l, increasing the interest rate charged on mortgages and business loans. Equation (46) also shows how the possibility of default introduces an additional spread between the cost of funding for banks and the one for borrowers. In fact, the form of the default thresholds in (25) and (13) implies that $\varrho_{t+1}^i \left(\eta^i \right) < Q_t^i R_{t+1}^i$, for $i = m, l$, so that

$$E_t \tilde{\Omega}_{t+1}^i R_{t+1}^i > 1 = E_t \tilde{\Omega}_{t+1}^i R_{t+1}^{i,b} \quad \text{for } i = m, l \tag{48}$$

where the right-hand side can be interpreted as the required rate of return for bankers. Therefore the price of a mortgage or of a loan will include an additional default premium that compensates financial intermediaries for the possibility of costly default.

Finally, we can use (46) to compute the derivative of the mortgage price and of the loan price with respect to leverage. In particular, we obtain

$$\frac{\partial Q_t^m}{\partial \eta_t^m} = -E_t \tilde{\Omega}_{t,t+1}^m \left\{ \frac{\partial \bar{\xi}_{t+1}^m}{\partial \eta_t^m} f_t\left(\bar{\xi}_{t+1}^m\right) \left[Q_t^m R_{t+1}^m - \gamma_m \frac{q_t^h R_{t+1}^h}{\eta_t^h} \bar{\xi}_{t+1}^m \right] + \gamma_m \frac{q_t^h R_{t+1}^h}{\left(\eta_t^m\right)^2} \int_0^{\bar{\xi}_{t+1}^m} \xi_{t+1}^m dF_t\left(\xi_{t+1}^m\right) \right\} \quad (49)$$

$$\frac{\partial Q_t^l}{\partial \eta_t^l} = -E_t \tilde{\Omega}_{t,t+1}^l \left\{ \frac{\partial \bar{\xi}_{t+1}^l}{\partial \eta_t^l} f_t\left(\bar{\xi}_{t+1}^l\right) \left[Q_t^l R_{t+1}^l - \gamma_l \frac{q_t^k R_{t+1}^k}{\eta_t^l} \bar{\xi}_{t+1}^l \right] + \gamma_l \frac{q_t^k R_{t+1}^k}{\left(\eta_t^l\right)^2} \int_0^{\bar{\xi}_{t+1}^l} \xi_{t+1}^l dF_t\left(\xi_{t+1}^l\right) \right\} \quad (50)$$

These derivatives include the impact of leverage on next period default threshold, plus the impact on the expected recovery rate in case of default. Since higher leverage implies a higher expected probability of default and a lower recovery rate, both derivatives are negative.[24]

Aggregation in the Banking Sector: Given the linearity of the incentive constraint in (44), the fact that ϕ_t only depends on aggregate quantities, and that in equilibrium all mortgages and loans will have the same leverage, we can obtain the following aggregate version of the constraint on the bank portolio

$$\left[Q_t^l L_t^b + \frac{\theta_t^m}{\theta_t^l} Q_t^m M_t^b \right] \leq \phi_t \bar{N}_t^b \quad (51)$$

where M_t^b and L_t^b represent banks' aggregate holdings of mortgages and loans, whereas $\bar{N}_t^b$ is the aggregate net worth of the financial sector. Importantly, equation (51) relates the value of assets held by intermediaries to the aggregate level of their net worth, so that any shock negatively affecting this variable will put downward pressure on Q_t^l and Q_t^m.

The evolution of aggregate net worth will be given by the wealth of the surviving bankers plus a transfer that patient households will provide to the new bankers, NW_t^e, equal to a fraction $T^b/(1-\omega^b)$ of the value of the net worth of exiting bankers

$$\bar{N}_t^b = \omega^b [R_t^m Q_{t-1}^m M_{t-1}^b + R_t^l Q_{t-1}^l L_{t-1}^b - R_t B_{t-1}] + T^b \bar{N}_{t-1}^b \quad (52)$$

From equation (52) we see how any shock affecting the realized return of the two types of loan securities will directly impact aggregate bank net worth, igniting a financial accelerator mechanism similar to the one described in Gertler and Karadi (2011). In particular, for this mechanism to operate properly, it is important that bank loans are marked to market, a feature achieved with the presence of long term debt.

2.5 Intermediate Goods Producers

Intermediate goods producers are competitive and produce output to be sold to retailers at the real price P_t^m.[25] They operate a standard Cobb-Douglas technology using capital $\tilde{K}_t$, rented from the entrepreneurs,

[24] As noted by Gomes et al. (2016), these equations should include also the impact of current leverage on future leverage, $\frac{\partial \eta_{t+1}^i}{\partial \eta_t^i}$ for i=m,l, and consequently on future debt prices. Such derivative cannot be computed with standard linear approximation methods, since it requires to compute the derivative of the leverage policy functions. Gomes et al. (2016) propose an iterative algorithm to compute this term. However, given the large scale of my model, with two long term assets, and given that the model solution in the main experiment is already complicated by the presence of the zero-lower-bound, I ignore this term when solving the model. The same approach is used by Miao and Wang (2010), and can be considered as a form of approximation.

[25] As shown in the appendix, P_t^m represents also the marginal cost faced by retailers

and the combined composite labor of patient and impatient agents $\tilde{N}_t$, that is

$$Y_t = A_t \tilde{K}_t^{\alpha} \tilde{N}_t^{1-a} = A_t \tilde{K}_t^{\alpha} \left(N_t^{\mu} \hat{N}_t^{1-\mu} \right)^{1-\alpha} \tag{53}$$

where A_t represents aggregate productivity.

As in Iacoviello and Neri (2010) I assume complementarity between the labor of the two types of agents, so that the parameter μ represents the labor income share accruing to the impatient household. The first order conditions for capital and for the labor provided by patient and impatient agents are

$$r_t^k = A_t \alpha \frac{P_t^m Y_t}{\tilde{K}_t} \tag{54}$$

$$\hat{w}_t = A_t (1-\mu)(1-\alpha) \frac{P_t^m Y_t}{\hat{N}_t} \tag{55}$$

$$w_t = A_t \mu (1-\alpha) \frac{P_t^m Y_t}{N_t} \tag{56}$$

where r_t^k is the rental rate on capital and $\hat{w}_t$ and w_t are the real wages paid to the two types of agents.

2.6 Capital Producers and Nominal Rigidities

Capital producers are part of the patient household. They create new capital by using the final good as input and face convex adjustment costs in the gross rate of change in investment, $S\left(\frac{I_t}{I_{t-1}}\right) I_t$, where $S(1) = S'(1) = 0$ and $S''(1) > 0$. They sell new capital to entrepreneurs at the price q_t^k.

The model includes standard nominal frictions in the final goods market and in the labor market. Final output is a CES composite of a continuum of retail firms, with elasticity ε_p. Retail firms use intermediate goods as input, whose marginal cost is P_t^m, and are able to reset their price only with probability $1-\zeta_p$. Wage rigidities, in the market for impatient household labor and in the one for patient household labor, are introduced in a symmetric way as in Iacoviello and Neri (2010). In each of the two labor markets, a labor packer aggregates the different varieties of labor inputs into homogenous labor, by using a CES production function with elasticity ε_l. For each labor variety there is a union with monopoly power which sets wages on a staggered basis, so that a new wage is set with probability $1-\zeta_l$. A detailed description of these real and nominal frictions can be found in the appendix.

2.7 The Government

In the model the government performs traditional monetary policy and unconventional monetary policy (or credit policy). Conventional monetary policy is characterized by the following Taylor rule, subject to the zero lower bound constraint

$$i_t = \max\left\{ \left[(i_{ss}) (\pi_t)^{\kappa_\pi} \left(\frac{Y_t}{Y} \right)^{\kappa_Y} \right], 1 \right\} \tag{57}$$

where π_t represents gross inflation, and the gross nominal rate, i_t, satisfies the Fisher equation

$$i_t = R_{t+1} E_t \pi_{t+1} \tag{58}$$

The purchase of agency mortgage-backed securities was the largest unconventional monetary policy program employed by the Federal Reserve. The asset purchases were announced in November 2008 and the stock of MBS held by the Federal Reserve topped \$1.1 trn by mid-2010, representing more than 10% of outstanding mortgage securities, and approximately 8% of total household debt, as shown in figure 2. The main aim of this program was to reduce mortgage interest rates on the primary and secondary market, in order to "support housing markets and foster improved conditions in financial markets more generally."[26]

In order to capture the effects of such a policy in a simple way, I assume that the central bank is able to purchase mortgages M_t^g directly from financial intermediaries, right after they are originated, at the origination price Q_t^m.[27] Importantly, these mortgages are not subject to the agency problem between depositors and bankers. As a result, the aggregate amount of mortgages financed at time t will be given by

$$M_t = M_t^b + M_t^g \tag{59}$$

The central bank can finance this credit policy by issuing risk-free government debt to patient households, not subject to any agency problem. Given their risk-free nature, bank deposits are perfect substitutes for government debt. As a result this policy has a redistributive cost, since it transfers resources from savers towards borrowers. However, the profits arising from this activity are rebated to patient agents with lump sum transfers.

To characterize this credit policy, I consider a central bank intermediating a fraction Ψ_t^M of total assets, that is

$$M_t^g = \Psi_t^M M_t \tag{60}$$

so that the total amount of mortgages financed at time t can be also written as

$$M_t = \frac{M_t^b}{1 - \Psi_t^M} \tag{61}$$

From equation (61) we see how an increase in Ψ_t^M will imply that the constraint on the leverage of intermediaries will have a smaller impact on the aggregate amount of mortgages intermediated. For simplicity I assume that the credit policy variable Ψ_t^M is zero in steady state, and follows a simple AR process

$$\Psi_t^M = \rho_{\Psi^M} \Psi_{t-1}^M + \varepsilon_{\Psi^M,t} \tag{62}$$

where the exogenous process $\varepsilon_{\Psi^M,t}$ and the parameter ρ_{Ψ^M}, will be used to replicate an evolution of the Fed MBS portoflio similar to the one observed during the Great Recession.

2.8 Market Clearing and Aggregate Resource Constraint

The clearing of the impatient households' market for housing services and houses, implies

$$X_t = H_t = \bar{H} \tag{63}$$

Aggregate capital evolves, according to

$$K_t = (1 - \delta) K_{t-1} + I_t \tag{64}$$

[26] Federal Reserve press release from November 25, 2008.

[27] The most relevant paper modeling unconventional monetary policy in a DSGE framework is Gertler and Karadi (2011), in which, however, there is no role for a housing sector. Compared to Gertler and Karadi (2011), the two-sector framework of this model allows for a more realistic representation of the Fed's asset purchase program, which was mainly targeted at mortgage securities.

and the equilibrium in the rental market for capital requires

$$\tilde{K}_t = K_{t-1} \tag{65}$$

I define GDP, $\bar{Y}_t$, as the total output available for consumption and investment expenditures,

$$\bar{Y}_t = Y_t - (1-\gamma_m)\, q^h_{t-1} R^h_t H_{t-1} \int_0^{\bar{\xi}^m_t} \xi^m_t dF_{t-1}\left(\xi^m_t\right) - (1-\gamma_l)\, q^k_{t-1} R^k_t K_{t-1} \int_0^{\bar{\xi}^l_t} \xi^l_t dF_{t-1}\left(\xi^l_t\right) \tag{66}$$

which is given by the production of final good minus the default costs on mortgages and business loans. As a result, we can write the aggregate resource constraint as

$$\bar{Y}_t = C_t + \hat{C}_t + I_t \left[1 + S\left(\frac{I_t}{I_{t-1}}\right)\right] \tag{67}$$

3 Quantitative Results

3.1 Calibration

Table 1 summarizes the parameter values used for the numerical simulations. The model is calibrated to have a steady state in which the bank incentive constraint is always binding.[28]

Financial spreads in the model and in the data: A key goal of this paper is to create a model that can generate endogenous behaviors for the two sets of spreads reported in figure 1. Because of the frictions arising from the default option and from the banks' agency problem, the interest rates faced by homeowners and entrepreneurs, corresponding to R^m_{t+1} and R^l_{t+1} over a one period term, will incorporate two types of premia with respect to the risk free rate. In fact, up to a first order approximation the spreads on mortgages and business loans can be decomposed as

$$\underbrace{E_t\left(R^m_{t+1} - R_t\right)}_{\text{Primary Mortgage Spread}} \simeq \underbrace{E_t\left(R^m_{t+1} - R^{m,b}_{t+1}\right)}_{\text{Default Premium}} + \underbrace{E_t\left(R^{m,b}_{t+1} - R_t\right)}_{\text{Liqudity Premium (MBS Spread)}} \tag{68}$$

$$\underbrace{E_t\left(R^l_{t+1} - R_t\right)}_{\text{BAA Spread}} \simeq \underbrace{E_t\left(R^l_{t+1} - R^{l,b}_{t+1}\right)}_{\text{Default Premium}} + \underbrace{E_t\left(R^{l,b}_{t+1} - R_t\right)}_{\text{Liquidity Premium (AAA Spread)}} \tag{69}$$

I link R^m_{t+1} and R^l_{t+1} to the primary mortgage spread and to the BAA spread since these two financial series are often used as benchmark indicators of the lending rates available to home buyers and firms.[29] As shown in equations (41) and (42), the binding bank incentive constraint causes a wedge between the banker's required return on assets and the risk free rate. As mentioned above, I refer to this wedge as a funding liquidity premium, and I link this spread to the MBS spread for mortgages and to the AAA spread for corporate loans. In fact, MBS securities are obtained by pooling together thousands of different

[28] For the solution of a model with bankers a la Gertler and Karadi (2011) and an occasionally binding incentive constraint see, for example, Bocola (2015) or Gertler, Kiyotaki and Prestipino (2017).

[29] For example, the BAA spread is used in the estimation of DSGE models with frictions on the firm side by Christiano et al. (2014) and Del Negro et al. (2015).

mortgages, as done also by the bankers in the model, hence almost eliminating the idiosyncratic default risk of individual borrowers.[30] AAA bonds are issued by companies with essentially no risk of default, capturing well the residual component of the BAA spread once the default premium has been taken away.[31] In addition, as shown in equation (48), the possibility of a costly default implies an additional wedge between the expected rate on mortgages (and loans) and the banks' required rate of return on these securities. Given these decompositions, I will use model parameters governing the severity of the financial frictions in the different sectors of the economy to target specific values of these spreads.

Non-standard parameters: For impatient households, β and χ are calibrated to match the following quantities: a mortgage loan-to-value ratio, η^m, equal to 0.8, and a share of household debt to annual GDP equal to 80%.[32] Both represent a conservative estimate relative to the high household leverage experienced in the years immediately before the crisis. As regards the entrepreneur I set the transfer rate T^e in order to match a leverage $\eta^l = 0.42$, which is the same value used in Gomes et al. (2016). The dividend payout rate for both entrepreneurs and bankers is set at 5%, in line with values used in the literature.

The banker parameters θ^m,θ^l, and T^b are chosen to hit the following targets: a spread between bank return on mortgages and the risk free rate of 1%, an analogous spread on corporate loans of 1.5%, and a leverage of 8. The two spreads are in line with the precrisis values of the MBS spread and the AAA spread. The leverage target is a conservative estimate of the aggregate leverage of the financial sector, encompassing commercial banks and non-traditional bank entities, before the Great Recession.

The parameters characterizing long term mortgages and business loans are set to match the following financial values and default rates. The recovery rate parameters, γ_m and γ_l are selected to deliver a spread on mortgages of 1.5% and on corporate loans of 2.25%, and are in line with the precrisis values for the primary mortgage spread and the BAA spread. Risk shocks follow a log-normal distribution $\ln\left(\xi_t^i\right) \sim N\left(-\frac{\left(\sigma_{t-1}^i\right)^2}{2}, \left(\sigma_{t-1}^i\right)^2\right)$ for $i = m, l$, and the steady state values of σ^m and σ^l are chosen to imply an annualized default rate of 1% for mortgages, close to the average foreclosure rate reported by the Mortgage Bankers Association before the crisis; and of 1.5% on corporate loans, a number between the 1% used by Gomes et al. (2016) and the value of 2% estimated by Christiano et al. (2014). The duration of both types of loans, $1/\lambda_i$ for $i = m, l$, is set to 5 years. This same number is used by Gomes et al. (2016) for corporate debt, and is consistent with the fact that, despite having longer maturities, mortgages in the U.S. are repaid after 4 to 7 years. As in Gomes et al. (2016) I normalize the coupon payments c_m and c_l so that a default-free mortgage or corporate loan would have a price of 1 in steady state. In addition, I normalize the housing stock in order to have house prices equal to 1 in steady state.

Finally, all the financial shocks, and the credit policy rule in equation (62), are assumed to have the same persistence of 0.98. Such number is chosen to insure that in the main quantitative experiment the expected duration of the ZLB is 5 quarters. This calibration strategy is similar to the one employed by Del Negro et al. (2017a), and is supported by survey evidence from the Blue Chip Economic Indicators in 2008.[33] I also assume that the persistence parameter of the Fed MBS intervention ρ_{Ψ^M} has the same value. This

[30] The focus on the primary mortgage spread and on the MBS spread as key indicators of lending condition in the primary and secondary market is shared with Hancock and Passmore (2011).

[31] Also Del Negro et al. (2017b) use the AAA spread as a measure of liquidity in the market for corporate funding, in a DSGE model. However, in their framework this liquidity spread is modeled as an exogenous shock.

[32] The implied ratio of corporate debt to annual GDP is around 90%.

[33] In the model, obtaining a ZLB episode longer than one and a half year only with one set of shocks in one period proves to be difficult. A longer ZLB episode could be obtained with additional monetary policy shocks or with a longer sequence of financial shocks.

assumption generates a path of the MBS share intermediated by the Fed similar to the data.

Standard parameters: The discount factor of patient households is chosen to obtain a 2% real rate in steady state. The values for capital share in production and capital depreciation are standard. The parameter determining the wage income share of patient and impatient households, μ, is equal to 0.5, implying equal wage shares. The parameters governing the nominal frictions in prices and wages, and the conventional monetary policy behavior, are taken from the literature. The elasticity of capital price with respect to investment, $S^{''}$, is set to 0.75 as in Del Negro et al. (2017a), and the Frish elasticity of labor supply is set equal to one.

3.2 Model Responses to Financial Shocks

In the quantitative experiments I will focus on four financial shocks affecting different components of financial spreads on mortgages and corporate loans. The two risk shocks, σ_t^m and σ_t^l, increase the expected probability of borrowers' default by increasing the dispersion of idiosyncratic risk in houses and capital. As a result, we can think of these shocks as directly affecting the default premia in equations (68) and (69). The two collateral shocks θ_t^m and θ_t^l affect the collateral value of mortgages and business loans for bank funding, and consequently impact banks' leverage capacity. As can be seen from equations (44) and (45), a higher value of θ_t^i for $i = m, l$, implies that the bank can borrow less per a unit of its assets, hence increasing the funding liquidity premia in (68) and (69).

Figure 3 reports the model impulse responses to a risk shock and to a collateral shock in the mortgage market. Both shocks are calibrated to deliver a 50 basis points increase in the mortgage spread (first panel). To link the model-implied spread in (68) to the data in figure 1, where spreads are computed on long term securities with respect to the 10 years treasury, I report the 10 year average expected value of $E_t R_{t+1}^m - R_{t+1}$ generated by the model.[34]

A higher σ_t^m increases expected mortgage defaults and causes banks to charge a higher interest rate by decreasing Q_t^m. As a result, impatient households' borrowing through mortgages declines putting downard pressure on housing demand and house prices (not reported), and consequently eroding the net worth and consumption of these agents.[35] Through the balance sheet of financially constrained intermediaries, this shock spreads also to the corporate sector. In fact, higher mortgage defaults and lower asset prices depress bank net worth causing a reduction in business loans and an increase in corporate spreads. Capital demand drops, implying a decline in the price of capital (not reported) and consequently in entrepreneurs' net worth, igniting a self-reinforcing financial accelerator which results in lower investment. Because of lower aggregate demand and lower investment, aggregate hours and GDP decline as well. As a result, through the interaction between the financial frictions affecting homeowners, entrepreneurs and financial intermediaries, this financial shock is able to generate realistic comovements among a wide set of macroeconomic variables in the model

Through the same channels, the collateral shock on θ_t^m, has effects that are qualitatively very similar. Comparing the blue line and the red line in the top row of figure 3 we notice how, in this case, the higher mortgage spread is mainly due to an increase in the MBS spread. The lower funding liquidity of mortgages results in a drop in Q_t^m, which initiates a downturn via the interlinked declines in asset prices, bank net

[34]This approach is used also by Gertler et al. (2017) in an experiment reproducing the financial crisis. Also Del Negro et al. (2017a) mention this computation as potentially delivering a more appropriate model representation of market spreads.

[35]Aggregate consumption and housing net worth of impatient agents are labeled "C Borrowers" and "NW Borrowers" in figure 3.

worth and bank lending capacity. As shown in the top right part of figure 3, because of the relationship in equation (43), this shock generates a more direct comovement between the MBS spread and the AAA spread.

Figure 4 reports the impulse responses to the same type of shocks affecting the business loan markets, σ_t^l and θ_t^l. Similarly to the previous experiment, I target a 50 basis points increase in the BAA spread. Also in this case, the two financial shocks deliver a similar downturn, with a larger decline in investment and a more delayed decline in consumption.

Taken together, figure 3 and 4 show how in this model financial shocks affecting lending spreads can bring about a widespread recession with at the core a deterioration in the equity of financial intermediaries and in their leverage capacity. In order to achieve this result, the presence of long term debt is crucial. In fact, if the bank extended one period loans, the realized return on its assets would be mostly predetermined, and bank net worth would not be exposed to changes in prices of mortgages and corporate loans. The classic interaction between the leverage constraint and fire sales externalities presented in Gertler and Karadi (2011), would be almost absent, and bank net worth could potentially rise because of the higher interest rates.[36] As a result, the model would fail to generate a comovement between mortgages and corporate loans, and consequently would deliver a weaker comovement between aggregate consumption and investment.

Finally, the shocks considered in figure 3 and 4 are small enough to avoid the occurrence of a zero-lower-bound episode. In the next experiments I show how, when financial shocks interact with the ZLB they can deliver a crisis of the same magnitude of what we observed in the Great Recession.

3.3 Financial Shocks and the Great Recession

After showing that financial shocks can generate realistic comovements between financial variables and macroeconomic aggregates, I try to quantify their impact during the 2007-2009 financial crisis. In particular, I filter the combination of the four financial shocks that would generate the same peak values in spreads observed in 2008Q4, and study the path of aggregate variables implied by the model. This exercise is performed while taking into account the possibility that the nominal interest rate might hit the ZLB. To estimate the financial shocks subject to the ZLB I use a computation strategy similar to the one suggested by Anzoategui et al. (2017) and based on the piecewise linear approximation, for model with occasionally binding constraints, developed by Guerrieri and Iacoviello (2015). At the same time, I also feed into the model a shock to the rule for the Fed's MBS purchases, $\varepsilon_{\Psi^M,t}$ in order to match the approximate size of the central banks' first round of credit policy intervention in the mortgage market. I focus on the behavior of the economy from 2008Q4 until 2012Q3, right before the Federal Reserve started a second round of MBS purchases as part of "QE3".[37]

The results of this experiment are presented in figure 5 and 6. In the top row of figure 5 I report the path of financial spreads in the model and in the data, and in the bottom row I report the path of the exogenous variables implied by the initial shocks filtered in 2008Q4. The model requires a particularly large risk shock in the corporate loan market, and a significant collateral shock in the mortgage market in order to replicate the deviations of spreads from their pre-crisis values.

Figure 6 compares the behavior of macroeconomic variables in the data during the Great Recession (the black dotted line), to the one generated by the model when we use the shocks obtained in figure 5 (the blue

[36]With short term debt bank net worth would still fluctuate with the value of repossessed homes and capital, but this applies only to the small portion of defaulted loans.

[37]This is the same time period used also by Del Negro et al. (2017).

line). The paths for GDP, consumption, investment and hours represent the log-deviations of these variables from a linear trend estimated between 2000Q1 and 2012Q3, normalized to zero in 2008Q3. Bank net worth in the model is compared to the XLF index which tracks the equity values of large financial intermediaries in the US. The last panel of figure 6 compares the variable Ψ_t^M to the size of the Fed's MBS portfolio as a share of total household credit.

As we can see from the top row of figure 6, the model matches very closely the path of key macroeconomic variables. The zero-lower-bound binds for five quarters, as anticipated in the calibration section, and the model generates a drop in inflation of the same magnitude of what experienced in the financial crisis. Also the path for bank net worth is comparable with the data. The decline of this variable is larger in the model; however we have to remember that the XLF index includes mainly the shares of publicly traded banks, potentially missing the equity deterioration of shadow bank entities which was responsible for several bank run episodes in these markets.[38][39]

In figure 7, I try to decompose the effect of each of the four shocks used in the Great Recession experiment of figure 5 and 6. The nonlinearities arising from the ZLB imply that we cannot simply look at the impact of each shock innovation in isolation. Hence, for each shock I compute the impulse responses obtained by feeding all the filtered financial shocks of figure 5 apart from the one we are interested in. Then, I report in figure 7 the difference between the baseline model behavior and this impulse response as a measure of the marginal shock contribution. Because of its larger impact on bank net worth and patient household consumption, the risk shock on corporate loans seems to explain the larger share of the decline in output, consumption and investment. Christiano et al. (2014) show how this type of shock affecting non-financial firms can be a key driver of business cycles. The mortgage collateral shock is the second shock in terms of importance, a result consistent with the turmoil in the secondary markets for mortgage-backed securities which was at the center of the collapse of the shadow banking system.

3.4 Counterfactual Experiments:

Next, I try to quantify the impact of the Federal Reserve's unconventional monetary policy during the 2007-2009 financial crisis. In addition, I perform counterfactual experiments aimed at studying the role of two other key ingredients of the model: the zero-lower-bound and nominal rigidities.

3.4.1 The role of the Federal Reserve's MBS Purchases

The model presented in this paper can be used as a laboratory to quantify the effects of the Federal Reserve's intervention in the mortgage market. Figure 8 compares the impulse responses obtained in figure 5 and 6 (the blue line), with the counterfactual paths obtained when the shock to Ψ_t^M is not used to match the size of the credit intervention. The red dashed line and the black dotted line represent these alternative paths with and without the imposition of the ZLB constraint. According to the model, the drop in GDP, consumption and investment would have been about 50% larger without the MBS purchases, in presence of the ZLB. The deeper recession results from the larger increase in spreads and from the more severe deterioration of

[38] For a DSGE model with runs in the shadow banking sector see, for example, Gertler, Kiyotaki and Prestipino (2016), or Ferrante (2017).

[39] This experiment also delivers a drop in house prices of about 8% and a mortgage default rate reaching 2.5%. These number are smaller than the data (drop of about 20% and reached 4.5% respectively), possibly because the conservative household leverage used in the calibration and the data for the agency MBS spread, don't capture the market freeze in the highly leveraged subprime non-agency MBS market. In addition, corporate default rates reach about 7%, in line with the average default rate reported by Moody's. (Moody's Default and Recovery Database).

bank net worth.[40] As expected, MBS purchases are more effecting at reducing mortgage spreads, but have a strong spillover on corporate spreads as well in the model. It is also interesting to notice how part of the positive effect of quantitative easing, when the ZLB binds, comes from the positive impact on inflation, which helps to reduce the real rate and to sustain asset prices and consumption when conventional monetary policy is constrained.

In absence of the ZLB constraint, on the other hand, the recession would have been less severe than in the baseline, even without credit policy, as shown by the black dotted line. In fact, negative interest rates dictated by the Taylor rule would have implied a higher path of inflation and a substantially lower real interest rate, resulting in a smaller decline in asset prices and bank net worth, and a in lower financial spreads. Hence, even if unconventional monetary policy can potentially compensate for the lack of negative nominal rates in a ZLB regime, according to the model it would have taken a much larger intervention to completely counterbalance the effect of the ZLB. A similar result is obtained also by Del Negro et al. (2017a), who quantify the role of the Fed's liquidity facilities during the Great Recession. The next section elaborates further on this point, by analyzing more in detail the role of the ZLB and nominal frictions.

3.4.2 The role of the Zero-Lower-Bound and of Nominal Rigidities

Figure 9 compares the baseline experiment with the model responses to the same shocks (including the credit policy shock), when the ZLB is not imposed, represented by the red dashed line. The black dotted line is the same as in figure 8, and reports the model response when neither credit policy nor the the ZLB constraint are present.

As already suggested by figure 8, it appears that the ZLB plays a crucial role in amplifying the consequences of the different layers of financial frictions present in the model. Without ZLB, the decline in consumption and investment would have been more than a third smaller than what experienced in the Great Recession experiment. In addition, figure 9 implies that the positve impact of MBS purchases would have been much smaller if the Federal Funds rate could have gone negative, as shown by the similar paths of the black and red lines.

This evidence also points to the important role played by nominal rigidities in order for the model to deliver a downturn similar to the last financial crisis. Figure 10 presents the response of the model to the same shocks of the main experiment when prices and wages are perfectly flexible. In this case, the decline in investment and output is quite smaller than in the baseline, and aggregate consumption actually rises on impact. In fact, absent demand externalities, output barely moves on impact, so that aggregate consumption has to rise to compensate for the drop in investment. The initial spike in consumption is linked to a large drop in real interest rates, which prevents asset prices from declining as much as in the baseline model, causing a drop in bank net worth about 50% smaller. The implied tightening in aggregate credit conditions is much less severe in this case, as witnessed by the lower level of spreads, resulting in a higher path for investment and output. Also these results are consistent with the findings of Del Negro et al. (2017a). However, in the framework of this model, nominal frictions have an even stronger effect because of the additional interaction between asset prices, bank equity and borrowers' consumption. As shown in figure 10, the decline in the consumption of impatient households, due to lower house prices, is responsible for most of the initial decline in aggregate demand, and indirectly of output, in the baseline.

[40] The fact that bank net worth becomes negative in the first period in the absence of QE (red dashed line), can be associated with the occurrence of bank runs that are not directly modeled in this framework. For macroeconomic models with bank runs see, for example, Gertler et al. (2017) and Ferrante (2017).

To summarize, the experiments in figures 8, 9 and 10 highlight how nominal rigidities can largely amplify the effects of shocks affecting financial spreads, and how unconventional credit policy can have a substantial impact when the economy is at the zero lower bound.

4 Conclusions

In this paper, I have developed a DSGE model featuring two endogenous components of interest rate spreads on mortgages and corporate loans: a default premium, and a liquidity premium. Using this framework, I have shown that shocks affecting the two types of premia can generate a crisis of the same severity of the Great Recession, if we take into account the zero lower bound, because of the role played by banks' balance sheet in propagating and amplifying the downturn. In addition, according to the model, the Fed's intervention in the mortgage market, through purchases of mortgage backed securities, had a significant impact in preventing an even worse recession.

The results presented in this paper abstract from other amplification channels which might have operated in the financial sector. For example, a key element of the Great Recession was a run on non-traditional intermediaries, as modeled by Gertler, Kiyotaki and Prestipino (2016) or Ferrante (2016). In addition, the model does not incorporate other types of government interventions, like liquidity facilities (see Del Negro et al. (2017a)) or fiscal policy (see Faria-e-Castro (2017)).

Finally, my analysis of unconventional monetary policy is from a positive perspective, not a normative one. In fact, in the model, I only capture the redistributive cost of credit policy, transferring resources from savers to borrowers, but I do not study the possible implications for risk taking in the financial sector or in the household sector, when this intervention is anticipated. A proper quantification of these effects would be necessary to determine the optimal intervention in the mortgage market. All these are interesting topics for future research.

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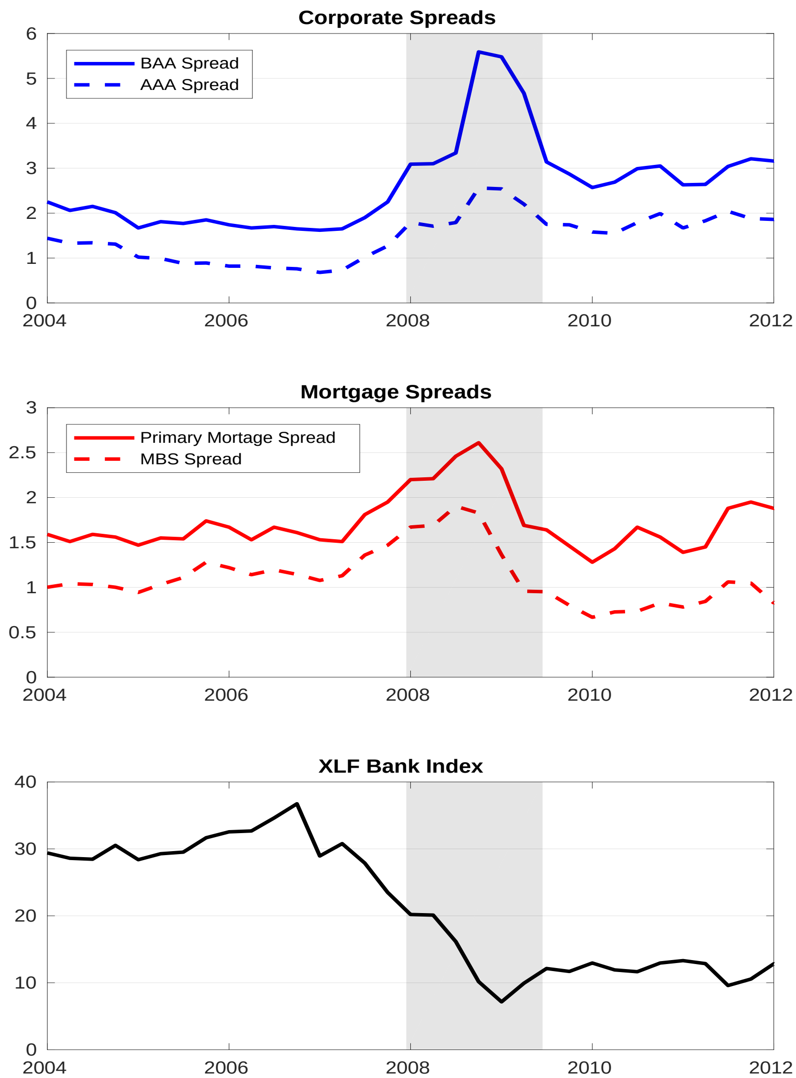
5 Tables and Figures

Table 1: Calibration

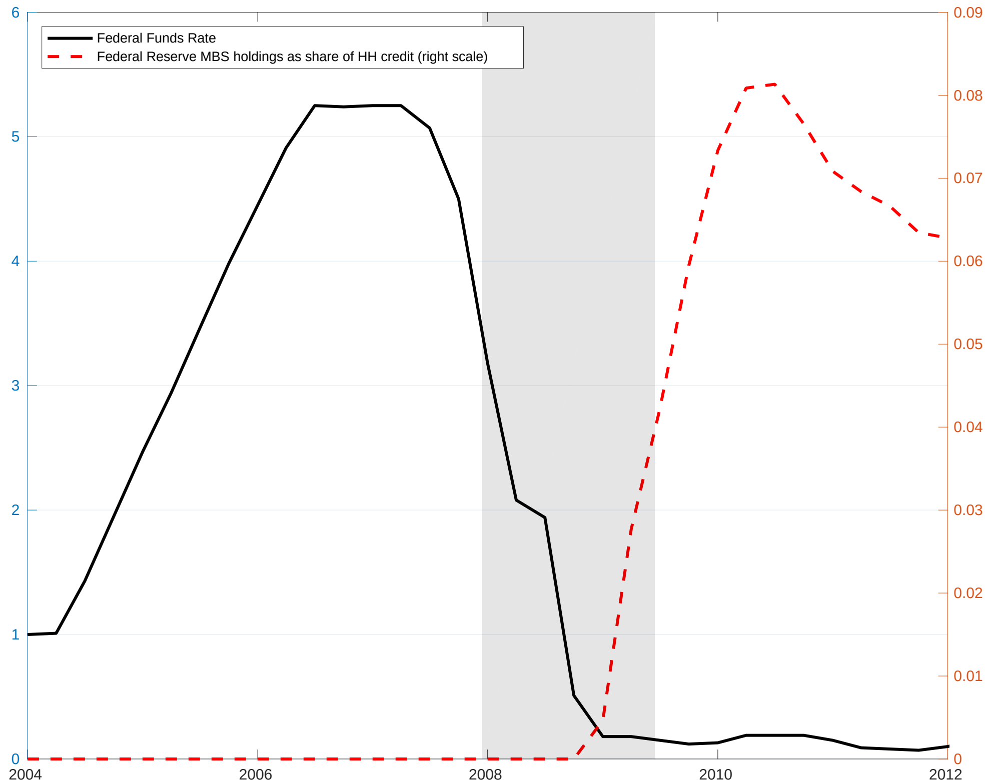
	Value	Description	Target/Source
Non-Standard Parameters			
Impatient Households			
β	.991	Discount rate impatient HH	$\eta^m = .8$
χ	.12	Housing Preference	$M/(4*Y) = 80\%$
Entrepreneurs			
T^e	.025	Transfer to new entrepreneurs	$\eta^l = 0.42$ (Gomes et al. 2016)
$1-\omega^e$	.05	Dividend payout rate	5% dividend payout
Bankers			
θ^m_{ss}	0.19	Divertable MBS	$4\left(R^{m,b} - R\right) = 1\%$
θ^l_{ss}	0.29	Divertable Capital	$4\left(R^{l,b} - R\right) = 1.5\%$
T^b	.016	Transfer to new bankers	$\phi = 8$
$1-\omega^b$	.05	Bankers exit probability	5% dividend payout
Mortgages			
$\bar{H}$	10	Housing stock	$q^h = 1$
γ_m	.52	Recovery rate	$4\left(R^m - R\right) = 1.5\%$
σ^m_{ss}	.08	Housing Risk Variance	1% foreclosure rate
c_m	.0075	Coupon rate	Normalization
λ_m	1/20	Repayment rate	5yrs Maturity
Commercial Loans			
γ_l	.55	Recovery rate	$4\left(R^l - R\right) = 2.25\%$
σ^m_{ss}	.31	Capital Risk Variance	1.5% default rate
c_l	0088	Coupon rate	Normalization
λ_l	1/20	Repayment rate	5yrs Maturity
Shock Parameters			
$\rho_{\sigma^m}, \rho_{\sigma^l}, \rho_{\theta^m}, \rho_{\theta^l}, \rho_{\Psi^M}$	0.98	Shock Persistence	ZLB for 5 quarters
Standard Parameters			
Patient Households			
$\hat{\beta}$	0.995	Discount rate patient HH	2% Real Rate
Intermediate Good Firms			
α	.33	Capital Share in Production	Standard
μ	.5	Impatient Labor Share	Standard
δ_k	.025	Capital Depreciation Rate	Standard
Capital Producing Firms			
S''	.75	Elasticity of Price to Investments	Del Negro et al. (2017)
Retail Firms			
ϵ_p	10	Elasticity of Substitution	Standard
ζ_p	.75	Probability of Fixed Price	Standard
Labor Unions			
γ_n	1	Inverse Frisch Elasticity	Standard
ϵ_w	10	Elasticity of Substitution	Standard
ζ_w	.75	Probability of Fixed Wage	Standard
Monetary Policy			
κ_π	1.5	Coeff. on Inflation	Standard
κ_y	.125	Coeff on Output	Standard

Figure 1: **Bank Equity and Spreads in the Great Recession**



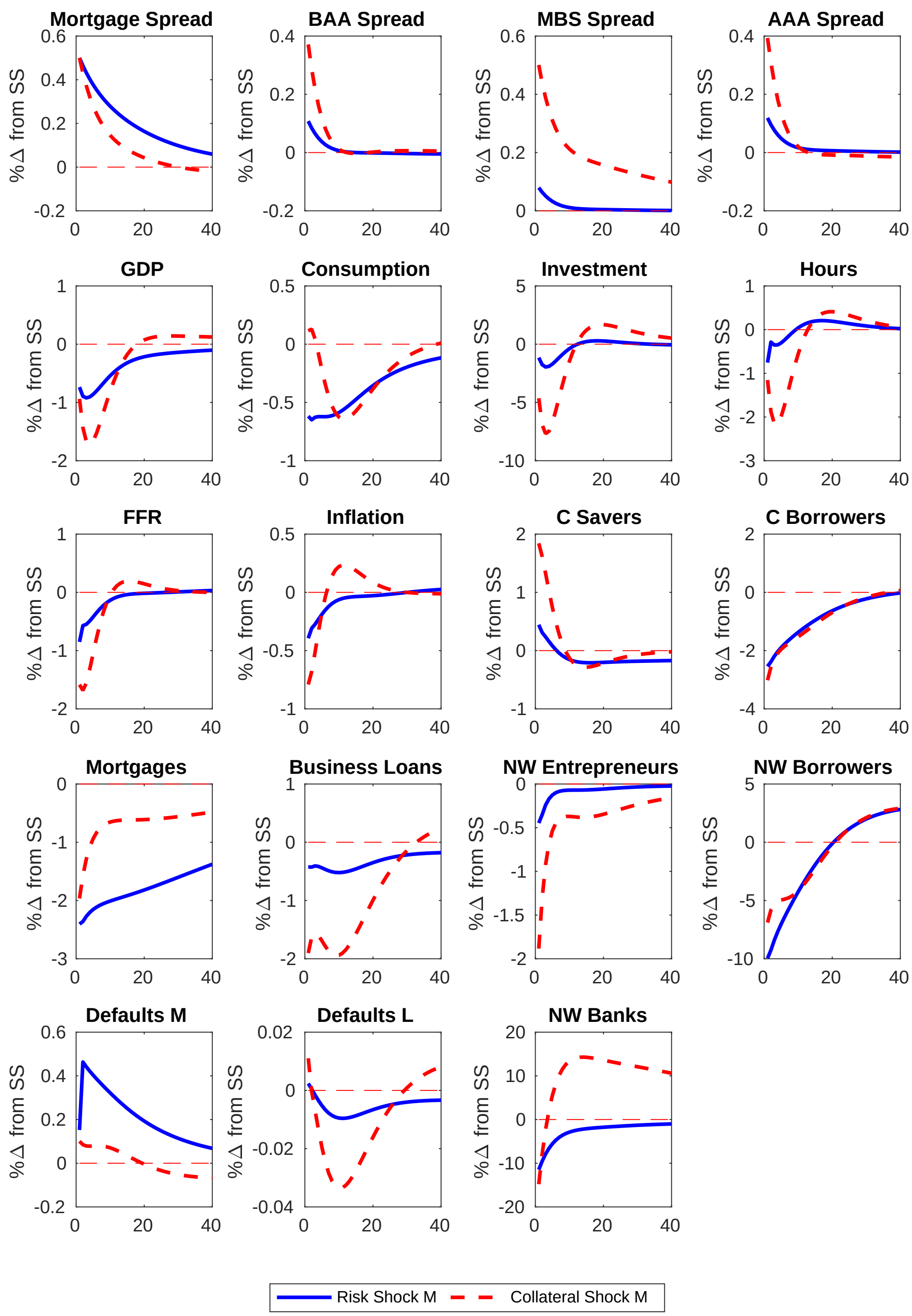
Notes: the first panel reports the Moody's Baa Corporate Bond Yield relative to the yield on the 10-Year Treasury Constant Maturity (https://fred.stlouisfed.org/series/BAA10Y) and the Moody's Aaa Corporate Bond Yield relative to the yield on the 10-Year Treasury Constant Maturity (https://fred.stlouisfed.org/series/AAA10Y). The second panel reports the difference between the primary market mortgage rate from the Freddie Mac's Primary Mortgage Market Survey and the 10-Year Treasury yield, and the difference between Fannie Mae thirty-year current-coupon MBS (Bloomberg ticker: MTGEFNCL.IND) and the 10-Year Treasury yield. The third panel reports the Financial Select Sector SPDR ETF index (XLF). All the variables are at quarterly frequency.

Figure 2: **Monetary policy and Fed's MBS purchases**



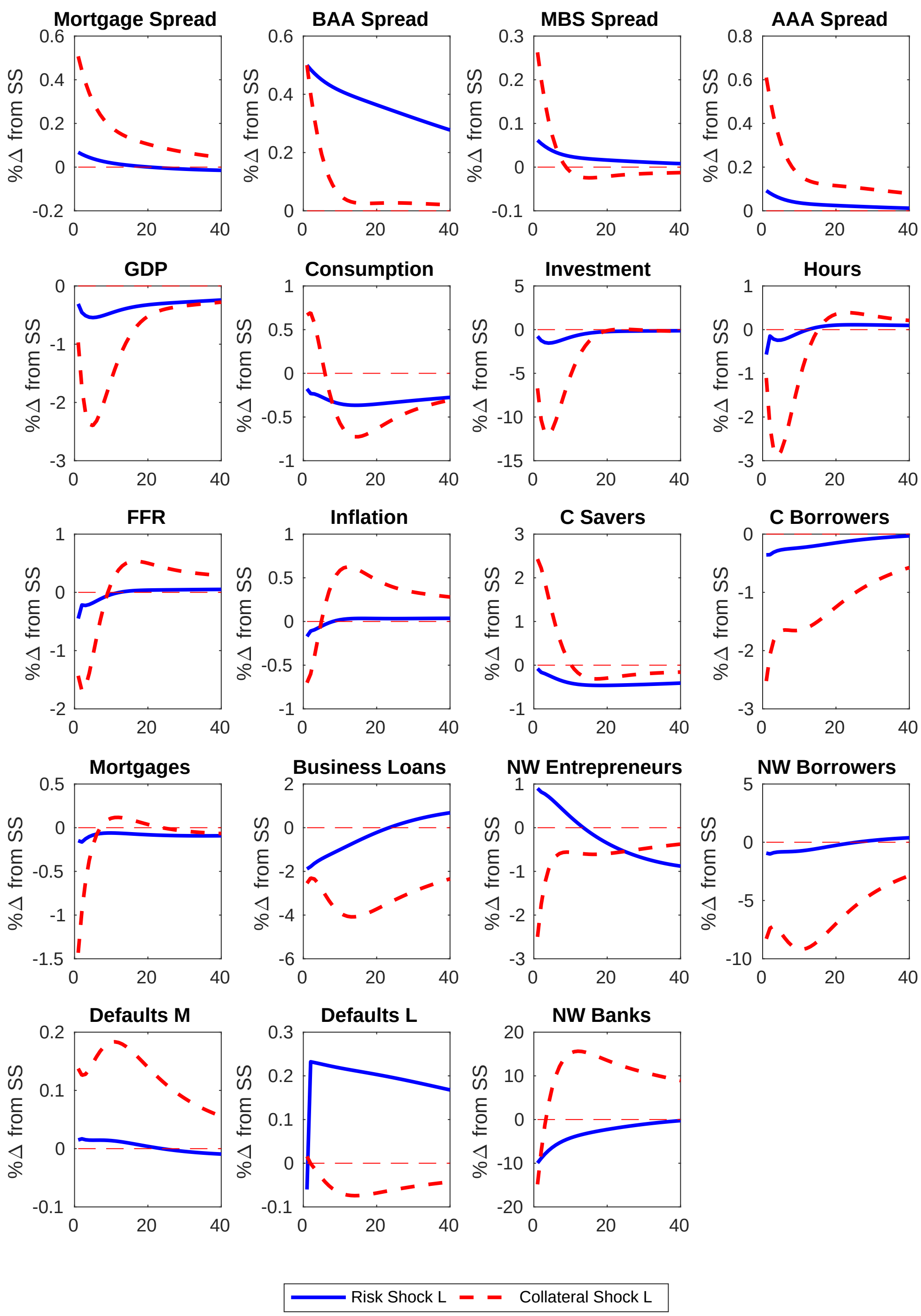
Notes: The black line represents the annualized effective Federal Funds rate. The red dashed line represents the ratio of MBS held by the Federal Reserve (https://fred.stlouisfed.org/series/MBST) and the total amount of household debt (obtained from table D.3 of the Financial Accounts of the United States).

Figure 3: **Impulse Responses to Financial Shocks in the Mortgage Market**



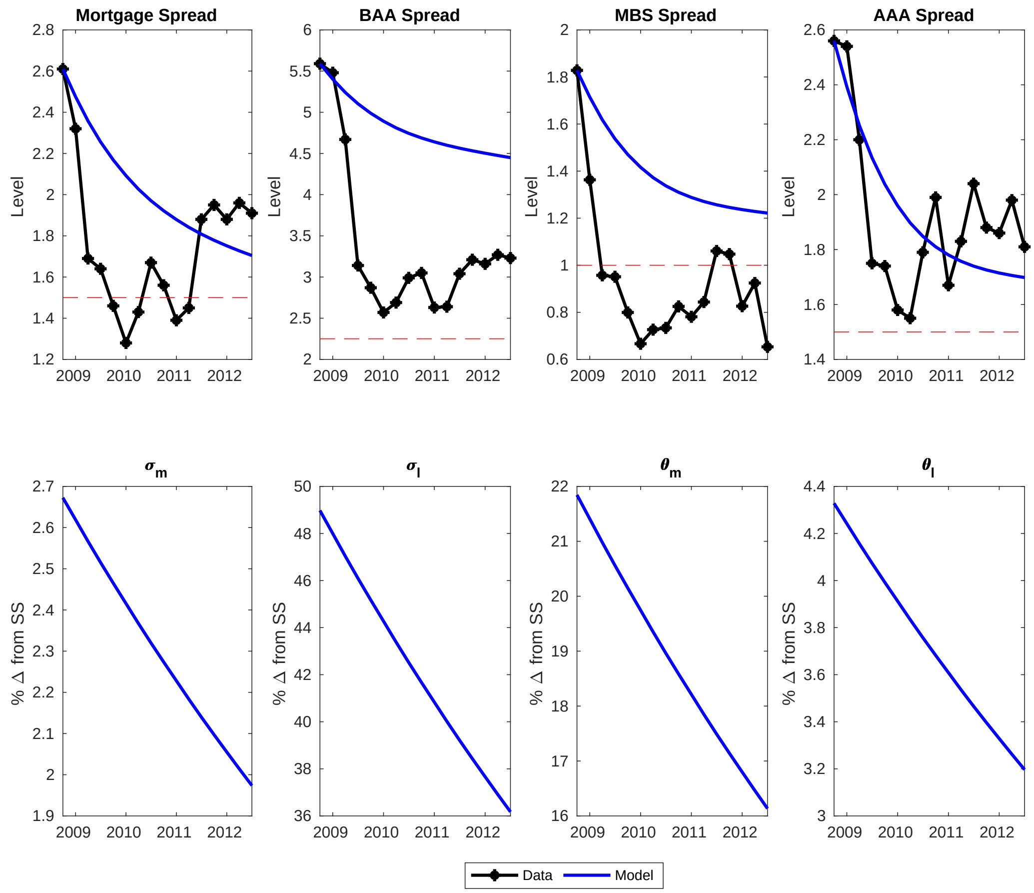
Notes: Impulse responses to a shock to σ_t^m (blue line) and to a shock to θ_t^m (red dashed line) both calibrated to deliver a 50 basis points increase in the mortgage spread.

Figure 4: **Impulse Responses to Financial Shocks in the Business Loans Market**



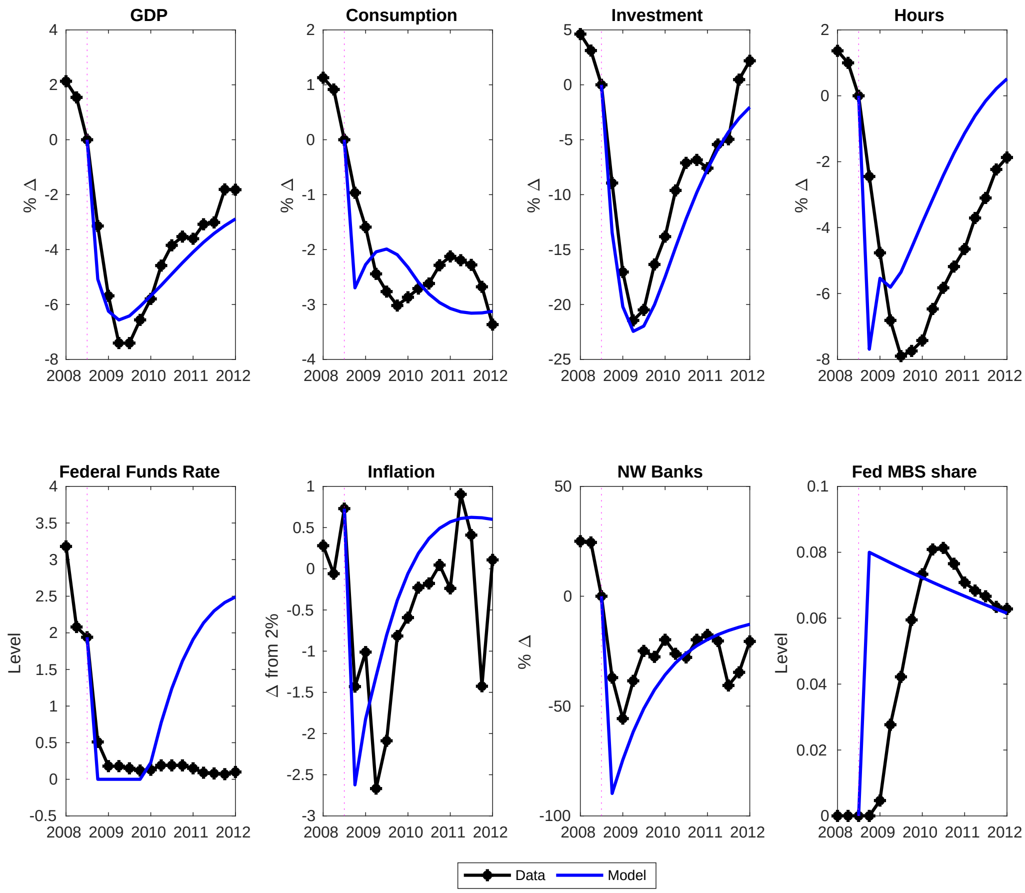
Notes: Impulse responses to a shock to σ_t^l (blue line) and to a shock to θ_t^l (red dashed line) both calibrated to deliver a 50 basis points increase in the BAA spread.

Figure 5: **Financial Shocks and the Great Recession: Spreads**



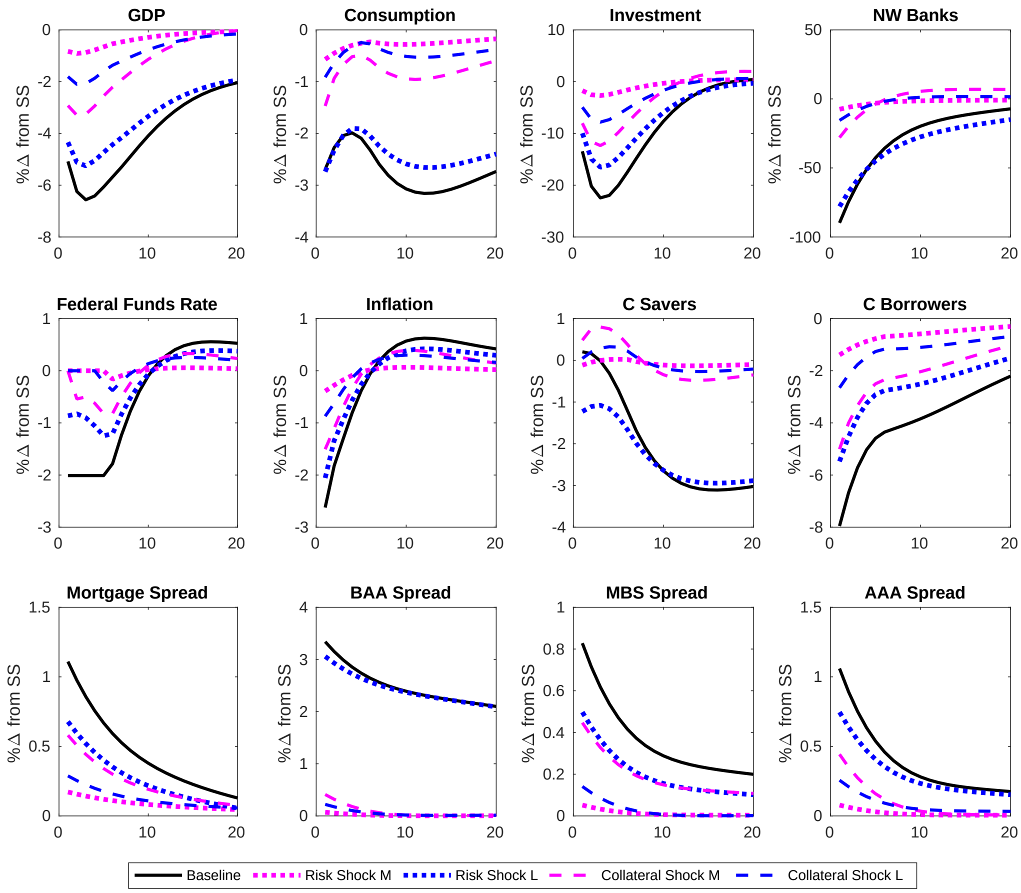
Notes: the top row of the figure reports the behavior of financial spreads from 2008Q4 until 2012Q3. The black dotted line represents the same data reported in figure 1, whereas the blue line represents the behavior of the four spreads in the model when the financial shocks in the second row are used to match the data in 2008Q4. Model spreads are computed as the 10 year average expected value of one period spreads. The red dashed line represents the steady state value of financial spreads in the model.

Figure 6: **Financial Shocks and the Great Recession: Macro Variables**



Notes: The blue line represents the model behavior when financial shocks are used to match the spreads in 2008Q4, as reported in the top part of figure 5. The dotted black line represents the following data, reported from 2008Q3 until 2012Q3. GDP is the sum of aggregate consumption and investment. Real consumption is total consumption minus durable consumption, (fred.stlouisfed.org/series/PCEC96 and PCDGCC96), expressed in 2009 chained dollars. Investment is real gross private investment (fred.stlouisfed.org/series/GPDIC1) plus durable consumption. The fourth panel reports hours from the nonfarm business sector (https://fred.stlouisfed.org/series/HOANBS). These four variables are divided by working age population and expressed in percentage log deviation from a linear trend estimated from 2000Q1 to 2012Q3, and normalized to zero in 2008Q3. The Federal Funds Rate is the annualized effective Fed Funds rate. Inflation is the annualized quarterly inflation rate obtained from the GDP deflator. The data for bank net worth is the XLF index used in figure 1, normalized to zero in 2008Q3. The Fed MBS share is obtained as the ratio of the Federal Reserve MBS holdings (https://fred.stlouisfed.org/series/MBST) and the total amount of household debt (obtained from table D.3 of the Financial Accounts of the United States).

Figure 7: **Marginal Shock Contributions**



Notes: the black line reports the same model-generated paths obtained when feeding the financial shocks used in figure 5 and 6. The other lines try to capture the marginal contribution of each shock in the following way: I compute the impulse responses of the model when using all the shocks apart from the one we are interested in; then I report the difference between the baseline and this impulse response.

Figure 8: **The Role of the Federal Reserve's MBS Purchases**

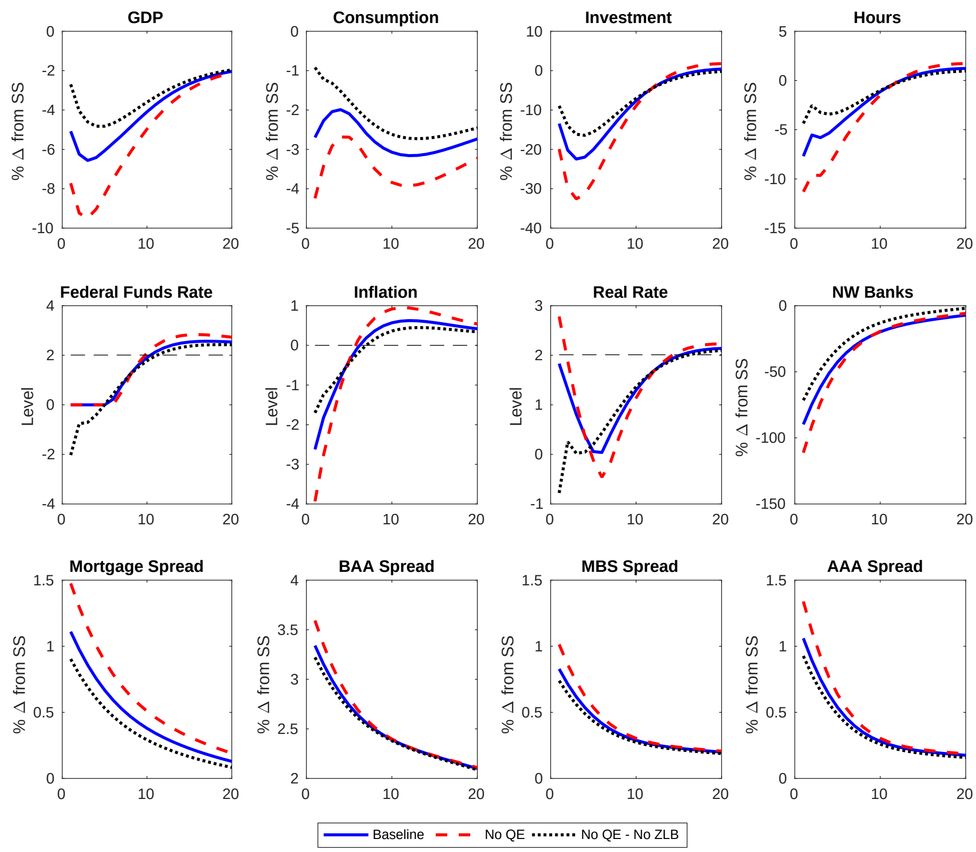


Figure 9: **The Role of the Zero Lower Bound**

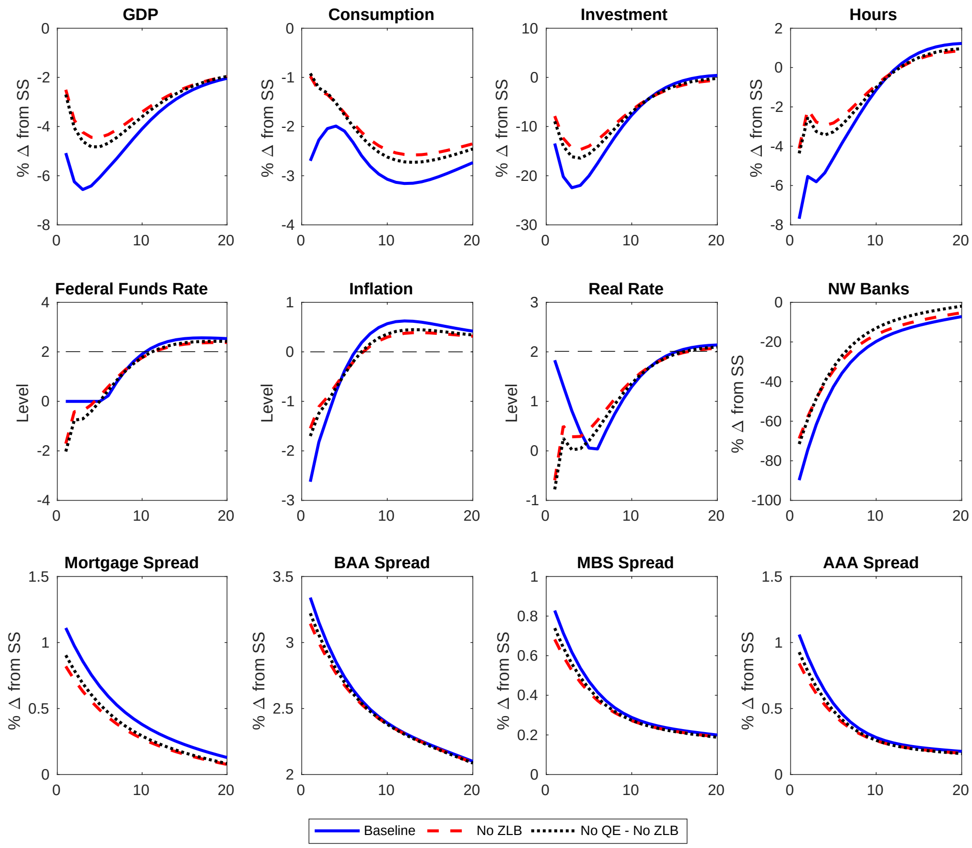
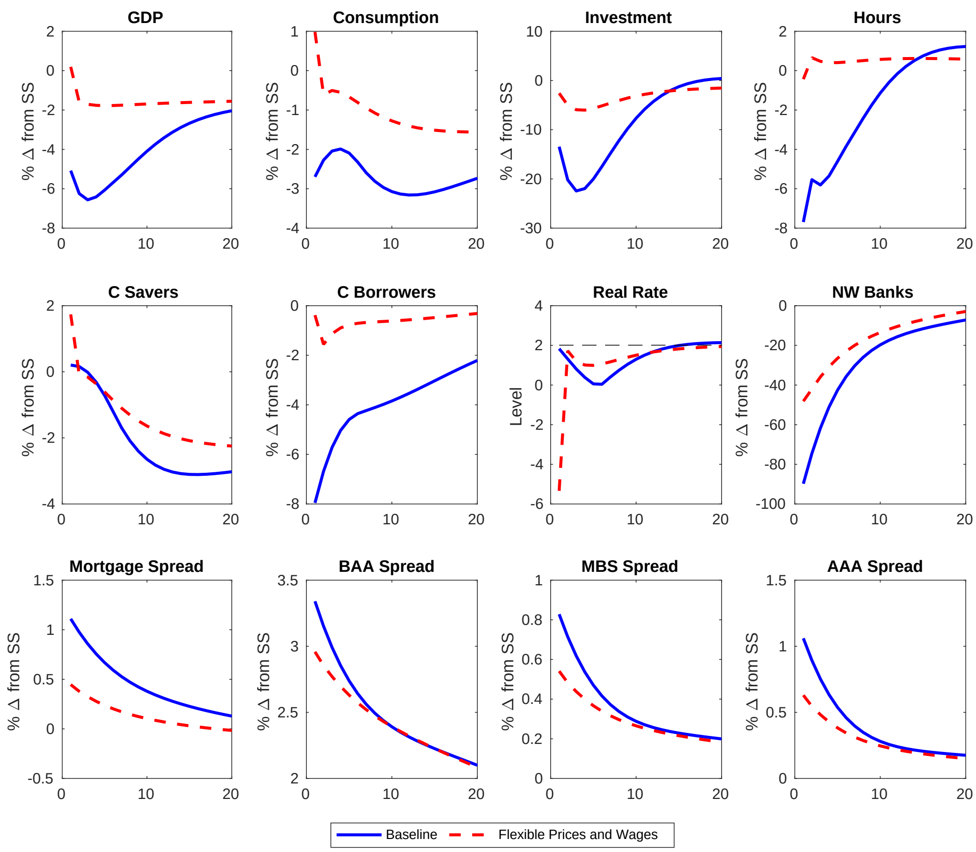


Figure 10: **The Role of Nominal Rigidities**



A Additional Derivations

In this appendix I provide additional derivations for the model.

A.1 Homeowner's Problem

The homeowner problem can be written as

$$V_t^[H]\left(h_{t-1}, m_{t-1}, \xi_t^m\right) = \max_{h_t, m_t} \left\{ d_t^[H] + E_t \Lambda_{t,t+1} \max\left\{0, V_{t+1}\left(h_t, m_t, \xi_{t+1}^m\right)\right\}\right\} \tag{A.1}$$

s.t.

$$\bar{n}_t^h = \xi_t^m h_{t-1} R_t^h q_{t-1}^h - (c_m + \lambda_m) m_{t-1} - (1-\lambda_m) Q_t^m m_{t-1} \tag{A.2}$$

$$q_t^h h_t + d_t^h = \bar{n}_t^h + Q_t^m m_t \tag{A.3}$$

I guess that the value function is linear in housing according to the following formula

$$V_t = \varphi_t^h \bar{n}_t^h = \varphi_t^h \left[\xi_t^m R_t^h q_{t-1}^h - (c_m + \lambda_m) \eta_{t-1}^m - (1-\lambda_m) Q_t^m \eta_{t-1}^m\right] h_{t-1} \tag{A.4}$$

This implies a simple default treshold

$$\bar{\xi}_t^m \left(\eta_{t-1}^m\right) = \frac{(c_m + \lambda_m) + (1-\lambda_m) Q_t^m}{R_t^h q_{t-1}^h} \eta_{t-1}^m \tag{A.5}$$

If we substitute the guess and the constraints into the problem we obtain

$$V_t^h\left(h_{t-1}, \eta_{t-1}, \xi_t^m\right) = \max_{h_t, \eta_{t-1}^h} \left\{ \begin{array}{c} \bar{n}_t^h - h_t\left(q_t^h - Q_t^m \eta_t^m\right) \\ +h_t E_t \Lambda_{t,t+1} \varphi_{t+1}^h \int_{\bar{\xi}_{t+1}^l(\eta_t^m)}^{\infty} \left[\xi_{t+1}^m R_{t+1}^h q_t^h - (c_m + \lambda_m) \eta_t^m - (1-\lambda_m) Q_{t+1}^m \eta_t^m\right] dF_t\left(\xi_{t+1}^m\right) \end{array} \right\}$$

The FOC for h_t implies

$$\left(q_t^h - Q_t^m \eta_t^m\right) = E_t \Lambda_{t,t+1} \varphi_{t+1}^h \int_{\bar{\xi}_{t+1}^l(\eta_t^m)}^{\infty} \left[\xi_{t+1}^m R_{t+1}^h q_t^h - (c_m + \lambda_m) \eta_t^m - (1-\lambda_m) Q_{t+1}^m \eta_t^m\right] dF_t\left(\xi_{t+1}^m\right) \tag{A.6}$$

and if we substitute this equality into the value function we obtain $\varphi_t^h = 1$, that is

$$V_t = \bar{n}_t^h = \left[\xi_t^m R_t^h q_{t-1}^h - (c_m + \lambda_m) \eta_{t-1}^m - (1-\lambda_m) Q_t^m \eta_{t-1}^m\right] h_{t-1} \tag{A.7}$$

As a result, equation (A.6) implies that η_t^m will be the same for all homeowners.

Next, with some algebra and by using the definition of the default threshold, we can derive the FOC for η_t^m reported in the text, that is

$$Q_t^m + \frac{\partial Q_t^m}{\partial \eta_t^m} \left[\eta_t^m - (1-\lambda_m) \eta_{t-1}^m \frac{h_{t-1}}{h_t}\right] = E_t \Lambda_{t,t+1} \left[1 - F_t\left(\bar{\xi}_{t+1}^m\left(\eta_t^m\right)\right)\right] \left[(c_m + \lambda_m) - (1-\lambda_m) Q_{t+1}^m\right] \tag{A.8}$$

This equation implies that $\frac{h_{t-1}}{h_t}$ will be the same for all non-defaulting agents, allowing for aggregation.

A.2 Entrepreneurs' Problem

As mentioned in the main text, in order to obtain simple aggregation across entrepreneurs, without having to keep track of their wealth distribution, it is necessary to add a technical assumption that allows these agents, when they do not default, to trade among each others to insure against the idiosyncratic risk. This assumption allows to equalize the marginal value of net worth across non defaulting entrepreneurs and facilitates aggregation, as shown below. A similar assumption is used also, for example, in Gertler and Kiyotaki (2010), in a model with idiosyncratic investment opportunities.

As shown in the main text, we can write the value function of a non defaulting entrepreneur as

$$V_t^e\left(k_{t-1}, l_{t-1}, \xi_t^l\right) = \max_{k_t, l_t} \left\{(1-\omega^e)\left(n_t^e\right) + E_t\hat{\Lambda}_{t,t+1} \max\left\{0, V_{t+1}^e\left(k_t, l_t, \xi_{t+1}^l\right)\right\}\right\} \tag{A.9}$$

s.t.

$$n_t^e = \left[\xi_t^l R_t^k q_{t-1}^k k_{t-1} - (c_l+\lambda_l)\, l_{t-1} - (1-\lambda_l)\, Q_{t-1}^l l_{t-1}\right] \tag{A.10}$$

$$q_t^k k_t = \bar{n}_t^e + Q_t^l l_t \tag{A.11}$$

$$\bar{n}_t^e = \omega^e\left(n_t^e\right) \tag{A.12}$$

I guess that the value function is linear in net worth, n_t^e, according to $V_t = \varphi_t^e n_t^e$, where φ_t^e depends on aggregate quantities. This implies that the default threshold will be the value $\bar{\xi}_t^l$ such that

$$\varphi_t^e\left[\bar{\xi}_t^l R_t^k q_{t-1}^k k_{t-1} - (c_l+\lambda_l)\, l_{t-1} - (1-\lambda_l)\, Q_{t-1}^l l_{t-1}\right] = 0 \tag{A.13}$$

As a result, we obtain the default threshold reported in the main text

$$\bar{\xi}_t^l\left(\eta_{t-1}^l\right) = \frac{(c_l+\lambda_l) - (1-\lambda_l)\, Q_{t-1}^l}{R_t^k q_{t-1}^k}\eta_{t-1}^l \tag{A.14}$$

If we now substitute the guess into the objective we can rewrite the problem as

$$V_t^e\left(k_{t-1}, \eta_{t-1}^l, \xi_t^l\right) = \max_{k_t, \eta_t^l}\left\{(1-\omega^e)\left(n_t^e\right) + E_t\hat{\Lambda}_{t,t+1}\varphi_{t+1}^e \int_{\bar{\xi}_t^l}^{\infty} n_{t+1}^e dF_t\left(\xi_{t+1}^l\right)\right\} \tag{A.15}$$

$$k_t\left(q_t^k - Q_t^l\eta_t^l\right) = \omega^e n_t^e \tag{A.16}$$

$$n_t^e = \left[\xi_t^l R_t^k q_{t-1}^k - (c_l+\lambda_l)\,\eta_{t-1}^l - (1-\lambda_l)\, Q_{t-1}^l\eta_{t-1}^l\right] k_{t-1} \tag{A.17}$$

If we define κ_t^e as the multiplier on the budget constraint in (A.16), we obtain the following first order conditions for k_t and η_t^l

$$\left(q_t^k - Q_t^l\eta_t^l\right)\kappa_t^e = E_t\hat{\Lambda}_{t,t+1}\varphi_{t+1}^e \int_{\bar{\xi}_{t+1}^l(\eta_t^l)}^{\infty}\left[\xi_{t+1}^l R_{t+1}^k q_t^k - (c_l+\lambda_l)\,\eta_t^l - (1-\lambda_l)\, Q_{t+1}^l\eta_t^l\right] dF_t\left(\xi_{t+1}^l\right) \tag{A.18}$$

$$(1-\lambda_l)\frac{\partial Q_t^l}{\partial\eta_t^l}\left[(1-\omega^e) + \omega^e\kappa_t^e\right]\eta_{t-1}^l = \frac{k_t}{k_{t-1}}\left\{\kappa_t^e\left[Q_t^l + \eta_t^l\frac{\partial Q_t^l}{\partial\eta_t^l}\right] - E_t\hat{\Lambda}_{t,t+1}\varphi_{t+1}^e\left(1 - F_t\left(\bar{\xi}_{t+1}^l\right)\right)\left[(c_l+\lambda_l) + (1-\lambda_l)\, Q_{t+1}^l\right]\right\} \tag{A.19}$$

From the first equation we see that if the marginal value of net worth, κ_t^e, is the same for all non-defaulting entrepreneurs, then equation (A.18) would imply that η_t^l is the same across agents. As mentioned above, one way to obtain the same κ_t^e across entrepreneurs, is to assume that they have access to some market allowing them to insure each other against the idiosyncratic shock, in case they do not default. This result can be achieved, for example, by assuming that in period t entrepreneurs can trade among each other a continuum of Arrow-Debreu securities, paying one unit of consumption good in period t+1 in case a specific realization of the idiosyncratic shock occurs and only if the entrepreneur does not default. If κ_t^e and η_t^l depend only on aggregate quantities, then equation (A.19) implies that also $\frac{k_t}{k_{t-1}}$ is going to depend only on aggregate variables.

Next, we can verify our guess by substituting the budget constraint into the value function to obtain

$$\begin{aligned} V_t^e &= \left\{1-\omega^e+\frac{\omega^e}{\left(q_t^k-Q_t^l\eta_t^l\right)}E_t\hat{\Lambda}_{t,t+1}\varphi_{t+1}^e\int_{\bar{\xi}_{t+1}^l\left(\eta_t^l\right)}^{\infty}\left[\xi_{t+1}^l R_{t+1}^k q_t^k-\left(c_l+\lambda_l\right)\eta_t^l-\left(1-\lambda_l\right)Q_{t+1}^l\eta_t^l\right]dF_t\left(\xi_{t+1}^l\right)\right\}n_t^e \quad \text{(A.20)} \\ &= \left\{1-\omega^e+\omega^e\kappa_t^e\right\}n_t^e \\ &= \varphi_t^e n_t^e \end{aligned} \tag{A.21}$$

where the second equality follows from equation (A.18). This formula corresponds to equation (24) in the main text.

A.3 Banker's Problem

The optimization of banker j can be written as

$$V_t^b\left(\bar{n}_{j,t}^b\right)=\max_{m_{j,t},l_{j,t},b_{j,t}} E_t\hat{\Lambda}_{t,t+1}\left\{\left(1-\omega^b\right)\bar{n}_{j,t+1}^b+\omega^b V_{t+1}^b\left(\bar{n}_{j,t+1}^b\right)\right\} \tag{A.22}$$

$$\text{s.t.}\quad Q_t^l l_{j,t}+Q_t^m m_{j,t}=\bar{n}_{j,t}^b+b_{j,t} \tag{A.23}$$

$$\bar{n}_{j,t+1}^b=m_{j,t}Q_t^m R_{t+1}^{m,b}+l_{j,t}Q_t^l R_{t+1}^{l,b}-b_{j,t}R_{t+1} \tag{A.24}$$

$$V_t^b\geq\theta_t^m m_{j,t}Q_t^m+\theta_t^l l_{j,t}Q_t^l \tag{A.25}$$

We start by guessing a value function of the form $V_t^b=\varphi_t^b\bar{n}_{j,t}^b$, where φ_t^b depends only on aggregate quantities. Then, if we define μ_t^b as the multiplier on the incentive constraint, the FOCs for l_t, m_t are

$$E_t\hat{\Lambda}_{t,t+1}\left[1-\omega^b+\omega^b\varphi_{t+1}^b\right]\left(R_{t+1}^{m,b}-R_{t+1}\right)=\mu_t^b\theta_t^m \tag{A.26}$$

$$E_t\hat{\Lambda}_{t,t+1}\left[1-\omega^b+\omega^b\varphi_{t+1}^b\right]\left(R_{t+1}^{l,b}-R_{t+1}\right)=\mu_t^b\theta_t^l \tag{A.27}$$

which imply the no arbitrage relationship

$$\frac{E_t\hat{\Lambda}_{t,t+1}\left[1-\omega^b+\omega^b\varphi_{t+1}^b\right]\left(R_{t+1}^{m,b}-R_{t+1}\right)}{\theta_t^m}=\frac{E_t\hat{\Lambda}_{t,t+1}\left[1-\omega^b+\omega^b\varphi_{t+1}^b\right]\left(R_{t+1}^{l,b}-R_{t+1}\right)}{\theta_t^l} \tag{A.28}$$

Plugging the guess into the value function we obtain

$$V_t^b=E_t\hat{\Lambda}_{t,t+1}\left\{\left[1-\omega^b+\omega^b\varphi_{t+1}^b\right]\left[m_{j,t}Q_t^m\left(R_{t+1}^{m,b}-R_{t+1}\right)+l_{j,t}Q_t^l\left(R_{t+1}^{l,b}-R_{t+1}\right)d\eta_t+R_{t+1}\bar{n}_{j,t}^b\right]\right\}$$

and using the relationship between the spreads, this becomes

$$V_t^b = E_t\hat{\Lambda}_{t,t+1}\left\{\left[1-\omega^b+\omega^b\varphi_{t+1}^b\right]\left[\left(R_{t+1}^{l,b}-R_{t+1}\right)\left(Q_t^l l_{j,t}+\frac{\theta_t^m}{\theta_t^l}Q_t^m m_{j,t}\right)+R_{t+1}\right]\right\}\bar{n}_{j,t}^b \tag{A.29}$$

As a result, the marginal value of net-worth will have to satisfy

$$\varphi_t^b = E_t\hat{\Lambda}_{t,t+1}\left\{\left[1-\omega^b+\omega^b\varphi_{t+1}^b\right]\left[\left(R_{t+1}^{l,b}-R_{t+1}\right)\phi_t+R_{t+1}\right]\right\} \tag{A.30}$$

where

$$\phi_t = \left[Q_t^l l_{j,t}+\frac{\theta_t^m}{\theta_t^l}Q_t^m m_{j,t}\right]/\bar{n}_{j,t}^b \tag{A.31}$$

In addition, if the constraint binds

$$\varphi_t^b\bar{n}_{j,t}^b = \left\{\theta_t^m m_{j,t}Q_t^m+\theta_t^l l_{j,t}Q_t^l\right\} \tag{A.32}$$

$$\Longrightarrow \varphi_t^b = \phi_t\theta_t^l \tag{A.33}$$

We can rewrite the last equation as

$$E_t\hat{\Lambda}_{t,t+1}\left\{\left[1-\omega^b+\omega^b\varphi_{t+1}^b\right]\left[\left(R_{t+1}^{l,b}-R_{t+1}\right)\phi_t+R_{t+1}\right]\right\} = \phi_t\theta_t^l \tag{A.34}$$

which implies the leverage quation reported in the text

$$\phi_t = \frac{E_t\tilde{\Lambda}_{t,t+1}R_{t+1}}{\theta_t^l - E_t\tilde{\Lambda}_{t,t+1}\left(R_{t+1}^{l,b}-R_{t+1}\right)} \tag{A.35}$$

where $\tilde{\Lambda}_{t,t+1} = \left[1-\omega^b+\omega^b\varphi_{t+1}^b\right]\hat{\Lambda}_{t,t+1}$.

A.4 Capital Producers

Capital producers create new capital by using the final good as input and face convex adjustment costs in the gross rate of change in investment, $S\left(\frac{I_t}{I_{t-1}}\right)I_t$, where $S(1)=S'(1)=0$ and $S''(1)>0$. They sell new capital to entrepreneurs at the price q_t^k.

Given that patient households own capital producers, they choosei investment, I_t to maximize the following

$$\max_{I_\tau}\sum_{\tau=t}^{\infty}\hat{\Lambda}_{t,\tau+1}\left\{q_t^k I_\tau - I_\tau - S(\frac{I_\tau}{I_{\tau-1}})I_\tau\right\} \tag{A.36}$$

so that the price of capital will be determined by

$$q_t^k = 1+S\left(\frac{I_t}{I_{t-1}}\right)+\frac{I_t}{I_{t-1}}f'\left(\frac{I_t}{I_{t-1}}\right)-E_t\hat{\Lambda}_{t,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right) \tag{A.37}$$

Profits, arising out of the steady state, are redistributed lump sum to patient households.

A.5 Final Good Producers and Retailers

The final output Y_t is a CES composite of a continuum of varieties produced by retail firms, owned by patient households, that employ intermediate output as input. The final good composite is

$$Y_t = \left[\int_0^1 Y_t\left(z\right)^{(\varepsilon_p-1)/\varepsilon_p}\, dz\right]^{\frac{\varepsilon_p}{\varepsilon_p-1}} \tag{A.38}$$

where $Y_t\left(z\right)$ is the output produced by firm z. Each retailer faces the demand function

$$Y_t(z) = \left(\frac{P_t\left(z\right)}{P_t}\right)^{-\varepsilon_p} Y_t \tag{A.39}$$

where the aggregate price level P_t is given by

$$P_t = \left[\int \left(P_t\left(z\right)\right)^{1-\varepsilon_p}\, dz\right]^{\frac{1}{1-\varepsilon_p}} \tag{A.40}$$

In addition, I introduce nominal rigidities by assuming that each period a firm is able to adjust its prices only with probability $(1-\zeta_p)$. As a result, the problem for the firm-setting firm is to select P_t^* to maximixe

$$E_t \sum_{i=0}^{\infty} \zeta^i \hat{\Lambda}_{t,t+1}\left[\frac{P_t^*}{P_{t+i}} - P_{t+i}^m\right] Y_{t+i}^*(z) \tag{A.41}$$

so that the first order condition will be given by

$$E_t \sum_{i=0}^{\infty} \zeta_p^i \hat{\Lambda}_{t,t+1}\left[\frac{P_t^*}{P_{t+i}} - \frac{\varepsilon_p}{\varepsilon_p-1} P_{t+i}^m\right] Y_{t+i}^*(z) \tag{A.42}$$

Finally, aggregating over (A.40) we obtain the following evolution for P_t

$$P_t = \left[(1-\zeta_p)\left(P_t^*\right)^{(1-\varepsilon_p)} + \zeta_p\left(P_{t-1}\right)^{(1-\varepsilon_p)}\right]^{\frac{1}{1-\varepsilon_p}}$$

A.6 Labor Market

I introduce nominal frictions in the labor market following Erceg,Henderson and Levine (2000). As mentioned in the main text, workers of both types of households, patient and impatient, supply differentiated labor input, $\hat{N}_t\left(l\right)$ and $N_t\left(l\right)$. Since the labor decisions are symmetrical in the labor market for hours provided by patient and impatient agents, I will solve below only the case for impatient agents.

Labor Packers: A labor packer combines the different varieties in a final labor input, which can be rented at wage W_t, according to

$$N_t = \left[\int_0^1 N_t\left(l\right)^{\frac{(\varepsilon_w-1)}{\varepsilon_w}}\, dl\right]^{\frac{\varepsilon_w}{\varepsilon_w-1}} \tag{A.43}$$

The problem of the labor packer is

$$\max_{N_t(l)} W_t N_t - \int_0^1 W_t(l) N_t(l)\, dl \tag{A.44}$$

This implies a demand function for each individual variety given by

$$N_t(l) = \left(\frac{W_t(l)}{W_t}\right)^{-\varepsilon_w} N_t \tag{A.45}$$

As a consequence we can derive the aggregate wage index as

$$W_t = \left(\int_0^1 W_t(l)^{1-\varepsilon_w}\, dl\right)^{\frac{1}{1-\varepsilon_w}} \tag{A.46}$$

Households labor decision: In both types of househol household there is perfect consumption insurance. Each worker supplies differentiated labor of type l to a union. Every period labor unions set nominal wage $W_t(l)$, taking as given the demand for their labor input. With a probability $1-\zeta_w$ the union can reset its wage, otherwise the wage stays fixed. Workers of type l are committed to supply whatever labor is demanded at that wage

Ignoring the additive preference for housing services, we can write the problem of a labor union resetting wage at time t, as that of choosing $W_t^*(l)$, to solve

$$\max_{W_t(l)} \sum_{s=0}^{\infty} (\beta\zeta_w)^s \left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} - \varphi \frac{\int N_{t+s}(l)^{1+\gamma_n}\, dl}{1+\gamma_n}\right]$$

s.t.

$$N_{t+s}(l) = \left(\frac{W_t^*(l)}{W_{t+s}}\right)^{-\varepsilon_w} N_{t+s} \tag{A.47}$$

$$C_{t+s} = \frac{W_t^*(l)}{P_{t+s}} N_{t+s}(l) + \Sigma_t \tag{A.48}$$

where Σ_t represents additional income and assets in the household budget constraint, but not affecting directly the wage decision.

The FOC for this problem is that is

$$\sum_{s=0}^{\infty} (\beta\zeta_w)^s \left\{\frac{1}{C_{t+s}} \frac{W_t^*(l)}{P_{t+s}} - \varphi \frac{\varepsilon_w}{\varepsilon_w - 1} N_{t+s}(l)^{\gamma_n}\right\} N_{t+s}(l) = 0 \tag{A.49}$$

If we define real wage as $w_t(l) = W_t(l)/P_t$, this equation can be rewritten as

$$\sum_{s=0}^{\infty} (\beta\zeta_w)^s \left\{\frac{1}{C_{t+s}} \frac{w_t^*(l)}{\pi_{t+s}} - \varphi \frac{\varepsilon_w}{\varepsilon_w - 1} \left[\left(\frac{w_t^*(l)}{\pi_{t+s} w_{t+s}}\right)^{-\varepsilon_w} N_{t+s}\right]^{\gamma_n}\right\} \left[\left(\frac{w_t^*(l)}{\pi_{t+s} w_{t+s}}\right)^{-\varepsilon_w} N_{t+s}\right] = 0 \tag{A.50}$$

where π_t is price inflation. Finally, we can write the aggregate wage indes as

$$w_t^{1-\varepsilon_w} = (1-\gamma_w)\, w_t^{*(1-\varepsilon_w)} + \gamma_w \pi_t^{\varepsilon_w - 1} w_{t-1}^{1-\varepsilon_w} \tag{A.51}$$