Macroeconomic Effects of Capital Tax Rate Changes

Saroj Bhattarai†    Jae Won Lee‡    Woong Yong Park§    Choongryul Yang¶
UT Austin        U of Virginia    Seoul National Univ.    UT Austin

Abstract

A permanent reduction in the capital tax rate from 35% to 21% generates non-trivial long-run macroeconomic effects in a standard dynamic equilibrium model. The long-run macroeconomic effects depend quantitatively on how the tax cuts are financed. Output increases by 10.8%, consumption by 6.7%, and investment by 34.7% if lump-sum transfers are cut, while they increase by only 6.1%, 2.2%, and 29% respectively, if distortionary labor tax rates are increased. Moreover, the ratio of (after-tax) capital income to labor income always increases and in fact, after-tax wages and labor income decrease in the case of labor tax rate increase. Along the transition to the new steady-state, the economy experiences a decline in consumption, output, hours, and wages. The contraction is more pronounced when labor tax rates adjust, and moreover the extent of contraction depends importantly on various nominal features of the model—the degree of nominal rigidities; the responsiveness of monetary policy; and whether the central bank allows inflation to directly facilitate government debt stabilization. We provide both analytical and numerical results with an extensive sensitivity analysis.

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Keywords: Capital tax rate; Permanent change in the tax rate; Transition dynamics; Monetary policy response; Nominal rigidities

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†Department of Economics, University of Texas at Austin, 2225 Speedway, Stop C3100, Austin, TX 78712, U.S.A. Email: saroj.bhattarai@austin.utexas.edu
‡Department of Economics, University of Virginia, PO BOX 400182, Charlottesville, VA 22904. Email: jl2rb@virginia.edu.
§Department of Economics, Seoul National University, 1 Gwanak-ro, Gwanak-gu, Seoul 08826, South Korea. Email: woongyong.park@snu.ac.kr.
¶Department of Economics, University of Texas at Austin, 2225 Speedway, Stop C3100, Austin, TX 78712, U.S.A. Email: c.yang@utexas.edu
1 Introduction

The macroeconomic effects of permanent capital tax cuts have recently become a subject of widespread discussion, spurred by the recent tax reform that reduces the corporate tax rate from 35% to 21%. Several questions have been raised. What are the long-run and the short-run effects on output, investment, and consumption? Are short-run effects different from long-run ones? Does such a tax cut lead to gains for workers in terms of wages and labor income? Will such a large tax cut be self-financing? How does the monetary policy response matter for the short-run effects of a capital tax cut? Given the nature of these questions, it is useful to pursue an analysis through the lens of a quantitative dynamic structural model. Moreover, since the tax reform is large-scale, it is imperative to consider general equilibrium effects.

This paper addresses these questions using a standard equilibrium macroeconomic model.1 We show analytically and numerically that capital tax cuts, as expected, have expansionary long-run effects on the economy. In particular, with a permanent reduction of the capital tax rate from 35% to 21%, in our baseline calibration, output in the new steady state, compared to the initial steady state, is greater by 10.8%, investment by 34.7%, consumption by 6.7%, and wages by 8.7%. The mechanism is well understood. A reduction in the capital tax rate leads to a decrease in the rental rate of capital, raising demand for capital by firms. This stimulates investment and capital accumulation. A larger amount of capital stock, in turn, makes workers more productive, raising wages and hours. Finally, given the increase in the factors of production, output increases, which also increases consumption in the long-run.2

These quantitative estimates above are obtained in a scenario in which the government has the ability to finance the capital tax cuts in a completely non-distorting way by cutting back lump-sum transfers. When the government has to rely on distortionary labor taxes, the effectiveness of capital tax cuts is smaller. We show this result both analytically and numerically. In our baseline calibration, a permanent reduction of the capital tax rate from 35% to 21% requires an increase in the labor tax rate by 6 % points, and output in the new steady state, compared to the initial steady state, is greater by 6.1%, investment by 29%, and consumption by 2.2%.3 Moreover, after-tax wages decline by 0.3% and hours also go down in the long-run under this increase in the labor tax rate.4

1The model is essentially the neoclassical growth model used in business cycle analysis, augmented with adjustment costs in investment and prices such that we can analyze realistic transition dynamics and study the role of monetary policy response.

2Clearly, it is a simplification to assume that the corporate tax rate cut recently is a one-to-one mapping with the capital tax rate cut in the standard macroeconomic model. We certainly do not include all details of the tax code in the paper, but we nevertheless find this exercise still informative for a baseline estimate, especially as an upper bound. Even in this decidedly simple analysis, we show how details of financing and short-run effects reveal interesting insights.

3We keep debt-GDP ratio the same between the initial and the new steady-state. Debt-GDP ratio, however, is allowed to deviate from the steady-state level along the transition path, when we study short-run effects.

4While these quantitative results are based on a particular parameterization, it is a very standard one. We have however done extensive sensitivity analysis and find that within a reasonable range, the only parameter that affects the quantitative results is the elasticity of substitution between capital and labor in the production function. For instance, departing from the standard Cobb-Douglas case of a unit elasticity by considering a higher elasticity leads to a bigger expansionary effect of the capital tax rate decrease.
The aforementioned positive long-run effects on macroeconomic variables are however, not a free lunch. First, as implied above already, the capital tax cuts are not self-financing, which forces the government to take a significant amount of resources away from households, either by cutting back transfers or raising distortionary labor tax rate. Keeping debt-GDP ratio at the same initial level in the new steady-state requires a permanent 60% decline in transfers from the initial level. Alternatively, if the government chooses to maintain transfers at the current level and instead adjusts labor tax rate, as we mentioned above, the latter has to increase by 6 percentage points, which leads to a permanent decrease in after-tax labor income. Second, income inequality, measured by the ratio of (after-tax) capital income to labor income, increases.\(^5\)

The third caveat to the positive long-run effects comes from analyzing transition dynamics in the model as the economy evolves from the initial steady-state to the new steady-state. Such an analysis is important as it takes the economy a long-time (around 70 quarters in our baseline parameterization) to reach the new steady-state and moreover additionally, allows us to study short-run effects of the permanent capital tax cut reduction. During the transition, the economy experiences a decline in consumption, output, hours, and wages, regardless of how the capital tax rate cuts are financed. The decline in consumption is a consequence of the need for financing of greater capital accumulation, and combined with countercyclical markups due to sticky prices, leads to a contraction in output in the short-run, even when lump-sum transfers can fully adjust as required.

The severity of the short-run contraction depends again on how the capital tax rate cuts are financed. Additionally, it depends on various “nominal” aspects of the economy – the degree of nominal rigidities, the responsiveness of monetary policy, and whether the central bank allows inflation to facilitate government debt stabilization. The contraction is more severe when prices are more rigid, monetary policy is less aggressive in stabilizing inflation, and if capital tax cuts are financed by raising labor tax rates, rather than cutting back lump-sum transfers.\(^6\) In the situation where the government only has access to distortionary labor taxes, we consider the central bank directly allowing inflation to facilitate government debt stabilization along the transition. In this interesting scenario, the rise of inflation in the short-run naturally helps reduce the extent of short-run contraction.

Our paper is related to several strands to the literature. We undertake a positive analysis, assessing the macroeconomic effects of a given reduction in the capital tax rate, but it is related to classic normative analysis of the optimal capital tax rate in Chamley (1986) and Judd (1985). Additionally, our analysis of the central bank allowing inflation to directly facilitate debt stabilization when the government has access to only distortionary labor taxes is related to the normative analysis in Sims (2001). We implement this scenario using a rules based positive description of interest rate policy, as in Leeper (1991), Sims (1994), and Woodford (1994) for instance.\(^7\)

\(^5\)The latter may increase or decrease in the long-run, depending on how capital tax cuts are financed, yet certainly decreases in the short-run with rigid prices – even when the government only adjusts transfers.

\(^6\)As one example, when labor tax rates are raised, there is a contraction in output even under fully flexible prices. This contraction is more severe with more rigid prices.

\(^7\)In this case, the central bank does not follow the Taylor principle. Bhattacharai, Lee, and Park (2016) analytically
In terms of analyzing the long-run effects of changes in the capital tax rate in a standard equilibrium macroeconomic model, our paper is closest to Trabandt and Uhlig (2011). Our contribution in terms of steady-state analysis is to show both analytically and numerically how the effects are different depending on whether non-distortionary or distortionary sources of government financing are available. Additionally, we fully study transition dynamics and show the key role played by various nominal aspects of the model, such as the extent of sticky prices and the nature and response of monetary policy.

There is by now a fairly large literature in the dynamic stochastic general equilibrium modeling tradition that assesses the effects of changes in distortionary tax rate changes and of fiscal policy generally. For instance, among others, Forni, Monteforte, and Sessa (2009) study transmission of various fiscal policy, including government spending and transfer changes in a quantitative model. Sims and Wolff (2017) additionally study state-dependent effects of tax rate changes. This work often studies effects of transitory and small changes in the tax rate while our main focus is on the long-run effects of a permanent reduction in the capital tax rate, and then on an analysis of full (nonlinear) transition dynamics following a fairly large reduction. Additionally, we provide several analytical results that help illustrate the key mechanisms on the long-run effects. Finally, our work is also motivated by the study of effects of government spending and how that depends on the monetary policy response, as highlighted recently by Christiano, Eichenbaum, and Rebelo (2011) and Woodford (2011).

Finally, while we are motivated by the particular recent episode of a permanent tax rate change, generally, our paper is influenced also by a large literature that empirically assesses the macroeconomic effects of tax policy. In particular, various identification strategies, such as narrative (Romer and Romer (2010) and statistical (Blanchard and Perotti (2002), Mountford and Uhlig (2009))) have been used to assess equilibrium effects of tax changes. Relatedly, House and Shapiro (2008) study a particular case of change in investment tax incentive. This work has focused either explicitly on temporary tax policies or does not explicitly separate out permanent changes from transitory ones.

2 Model

We now present the model, which is essentially a standard neoclassical equilibrium set-up. The model also features adjustment costs, in investment and nominal pricing, to enable a realistic study of transition dynamics. Pricing frictions also enable an analysis of the role of monetary policy for the transition dynamics.\(^8\)

\(^8\)characterize the effects of such a case in a model with sticky prices.

\(^8\)We therefore do not consider all the real and nominal frictions in Smets and Wouters (2007), that could affect transition dynamics, and instead focus on the key model features that are important to most transparently highlight the main mechanisms.
2.1 Private Sector

We start by describing the maximization problems of the private sector.

2.1.1 Household

The representative household’s problem is to

$$\max_{\{C_t, H_t, K_{t+1}, I_t, B_t\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, H_t) \right\}$$

subject to the flow budget constraint

$$(1 + \tau^C) P_tC_t + P_t I_t + B_t = (1 - \tau^H) W_t H_t + R_{t-1} B_{t-1} + (1 - \tau^K) R^K_t K_t + P_t \Phi_t + P_t S_t$$

and the capital accumulation technology

$$K_{t+1} = (1 - d) K_t + \left(1 - S \left( \frac{I_t}{I_{t-1}} \right) \right) I_t$$

where $E$ is the expectation operator, $C_t$ is consumption, $H_t$ is hours, $I_t$ is investment, $K_t$ is the stock of capital, $B_t$ is nominal risk-less one-period government bonds, and $\Phi_t$ is profits from firms. $P_t$ is the aggregate price level, $W_t$ is nominal wages, $R_t$ is the nominal interest rate, and $R^K_t$ is the rental rate of capital. $S_t$ is lump-sum transfers from the government, $\tau^C$ is the tax rate on consumption, $\tau^H$ is the tax rate on wage income, and $\tau^K$ is the tax rate on capital income. $\beta$ is the discount factor and $d$ is the rate of depreciation of the capital stock. The period utility $U(C_t, H_t)$ and investment adjustment cost $S \left( \frac{I_t}{I_{t-1}} \right)$ have standard properties, which are detailed later.

2.1.2 Firms

The model has final goods firms and intermediate goods firms.

**Final goods firms** Competitive final goods firms produce aggregate output $Y_t$ by combining a continuum of differentiated intermediate goods using a CES aggregator $Y_t = \left( \int_{0}^{1} Y(i)^{\frac{\theta - 1}{\theta}} di \right)^{\frac{1}{\theta - 1}}$, where $\theta$ is the elasticity of substitution between intermediate goods indexed by $i$. The corresponding optimal price index $P_t$ for the final good is $P_t = \left( \int_{0}^{1} P_t(i)^{1-\theta} \right)^{\frac{1}{1-\theta}}$, where $P_t(i)$ is the price of intermediate goods and the optimal demand for $Y_t(i)$ is

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\theta} Y_t.$$  \hfill (1)
The final good is used for private and government consumption as well as investment.

**Intermediate goods firms** Intermediate goods firms indexed by \( i \) produce output using a CRS production function

\[
Y_t(i) = F(K_t(i), A_t H_t(i))
\]

(2)

where \( A_t \) represents exogenous economy-wide labor augmenting technological progress. The gross growth rate of technology is given by \( a_t \equiv \frac{A_t}{A_{t-1}} = \bar{a}. \) Firms rent capital and hire labor in economy wide competitive factor markets. Intermediate good firms also face price adjustment cost

\[
\Xi \left( \frac{P_t(i)}{P_{t-1}(i)} \right) Y_t
\]

that has standard properties detailed later.

Intermediate good firms problem is to

\[
\max \left\{ P_t(i), Y_t(i), H_t(i), K_t(i) \right\} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} P_t \Phi_t(i) \right\}
\]

subject to (1) and (2), where \( \Lambda_t \) is the marginal utility of nominal income and flow profits \( \Phi_t(i) \) is given by

\[
\Phi_t(i) = \frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} H_t(i) - \frac{R^K_t K_t(i)}{P_t} - \Xi \left( \frac{P_t(i)}{P_{t-1}(i)} \right) Y_t.
\]

2.2 Government

We now describe the constraints on the government and how it determines monetary and fiscal policy.

2.2.1 Government budget constraint

The government flow budget constraint, written by expressing fiscal variables as ratio of output, is given by

\[
\frac{B_t}{P_t Y_t} + \left( \tau^C_t C_t + \tau^H_t W_t H_t + \tau^K_t R^K_t K_t \right) = R_{t-1} \frac{B_{t-1}}{P_{t-1} Y_{t-1}} \frac{1}{Y_t} Y_{t-1} + \frac{G_t}{Y_t} + \frac{S_t}{Y_t}
\]

where \( G_t \) is government spending on the final good.\(^{11}\)

2.2.2 Monetary policy

Monetary policy is given by a simple interest-rate feedback rule

\[
\frac{R_t}{R} = \left( \frac{\pi_t}{\bar{\pi}} \right) \phi
\]

\(^{10}\)Steady-state of a variable \( x \) is denoted by \( \bar{x} \) throughout. As we discuss later, we restrict preferences and technology such that the model is consistent with a balanced growth path.

\(^{11}\)We introduce government spending in the model for a realistic calibration. As we discuss later, government spending to GDP is held fixed throughout in the model.
where $\phi \geq 0$ is the feedback parameter on inflation \( (\pi_t = \frac{P_t}{P_{t-1}}) \), $\bar{R}$ is the steady-state value of $R_t$, and $\bar{\pi}$ is the steady-state value of $\pi_t$. When $\phi > 1$, the standard case, the Taylor principle is satisfied. When $\phi < 1$, which we will also consider, inflation response will play a direct role in government debt stabilization along the transition.

### 2.2.3 Fiscal policy

We consider a permanent change in the capital tax rate $\tau_t^K$ in period 0, where the economy is in the initial steady-state.\(^{12}\) Consumption tax rate and $G_t / Y_t$ do not change from their initial steady-state values in any period. Moreover, in the long-run, debt-to-GDP, $\frac{B_t}{PY_t}$, stays at the same level as in the initial steady-state. We will study both long-run effects of such permanent changes in the capital tax rate, as well as full transition dynamics as the economy evolves towards the new steady-state.

For long-run effects, we consider two different fiscal policy adjustments that ensure that debt-to-GDP stays at the same initial level in the new steady-state. First, only lump-sum transfers $\frac{S_t}{Y_t}$ adjust. Second, only labor tax rates $\tau_t^H$ adjust.

For transition dynamics, where the behavior of the monetary authority generally matters, we consider three different types of fiscal/monetary adjustment. Two of them are analogous to the above. First, again only lump-sum transfers adjust to maintain $\frac{B_t}{PY_t}$ constant at each point in time. Second, only labor tax rates $\tau_t^H$ adjust following the simple feedback rule

\[
\tau_t^H - \bar{\tau}^H_{\text{new}} = \psi \left( \frac{B_{t-1}}{P_{t-1}Y_{t-1}} - \frac{B}{PY} \right)
\]

where $\psi \geq 1 - \beta$ is the feedback parameter on outstanding debt, $\bar{\tau}^H_{\text{new}}$ is the new steady-state value of $\tau_t^H$, and $\frac{B}{PY}$ is the (initial and new) steady-state value of $\frac{B_t}{PY_t}$.\(^{13}\) In both these cases, we have the monetary policy rule satisfying the Taylor principle, $\phi > 1$, which thereby implies that inflation plays no direct role in debt stabilization.

Third, labor taxes adjust, but not sufficiently enough, as $0 < \psi < 1 - \beta$, and inflation partly plays a direct role in government debt stabilization, as $\phi < 1$. Thus, in this third case, we allow debt stabilization, along the transition, to occur partly through distortionary labor taxes and partly through inflation.\(^{14}\)

### 2.3 Equilibrium

The (monopolistically) competitive equilibrium is standard, given the maximization problems of the private sector and the monetary and fiscal policy described above. Moreover, we consider a symmetric equilibrium across firms, where all firms set the same price and produce the same amount

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\(^{12}\)In an extension, in the Appendix, we will consider a case where the change is anticipated to occur in future.

\(^{13}\)This rule implies a gradual debt-to-GDP adjustment towards the steady-state. The Appendix reports a slightly more stark case of labor taxes adjusting to ensure that debt-to-GDP is the same as the initial level every period.

\(^{14}\)While we consider these various fiscal/monetary adjustment scenarios to investigate how results depend on different policy choices, our analysis is not normative as in the Ramsey policy tradition.
of output. Goods and factor markets clear in equilibrium.\footnote{15
The aggregate market clearing condition for goods is then given by
\[ Y_t = C_t + G_t + I_t + \Xi \left( \frac{P_t}{P_{t-1}} \right) Y_t. \]
}

The economy features a balanced growth path. Thus, we normalize variables growing along
the balanced growth path by the level of technology. Fiscal variables, as mentioned above, are
normalized by output. We use the notation, for instance, \( \bar{Y}_t = \frac{Y_t}{A_t} \) and \( \bar{b}_t = \frac{B_t}{P_t Y_t} \) to denote these
stationary variables. We also use the notation \( T^C_t, T^H_t, \) and \( T^K_t \) to denote (real) consumption,
labor, and capital tax revenues. Nominal variables are denoted in real terms in small case letters,
for instance, \( w_t = \frac{W_t}{P_t} \). All the equilibrium conditions are derived and given in detail in the
Appendix.

3 Results

We now present our results. We start with the parameterization, then discuss the long-run effects,
and finally, present results on transition dynamics that allows us to assess short-run effects.

3.1 Calibration

We use the following general functional forms for preferences and technology such that the model
is consistent with a balanced growth path\footnote{16
King, Plosser, and Rebelo (2002) describes these restrictions on preferences and technology.}

\[
U(C_t, H_t) \equiv \frac{C_t^{1-\eta} \left( 1 - \tilde{\omega}^{1-\eta} (H_t)^{1+\varphi} \right)^\eta - 1}{1 - \eta},
\]

\[
F(K_t(i), A_t H_t(i)) \equiv \left( \lambda K_t(i)^{\frac{\varepsilon - 1}{\varepsilon}} + (1 - \lambda) (A_t H_t(i))^{\frac{\varepsilon - 1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon - 1}}
\]
and standard functional forms for the investment and price adjustment costs

\[
S \left( \frac{I_t}{I_{t-1}} \right) \equiv \frac{\xi}{2} \left( \frac{I_t}{I_{t-1}} - \bar{a} \right)^2, \quad \Xi \left( \frac{P_t}{P_{t-1}} \right) \equiv \frac{\kappa}{2} \left( \frac{P_t}{P_{t-1}} - \bar{\pi} \right)^2.
\]

Table 1 contains numerical values we used for the parameters of the model. The parameteri-
ization is standard, and we provide references or justification for values we pick from the literature
in Table 1. In the baseline, we use separable preferences that imply log utility \( (\eta = 1) \) and a
Cobb-Douglas production technology \( (\varepsilon = 1) \) as this specification is the most widely used in busi-
ness cycle papers, as well as a modest, unit Frisch elasticity of labor supply \( (\frac{1}{\varphi} = 1) \). Additionally,
across various fiscal adjustment scenarios and preference and technology functions specifications,
we normalize initial hours to be 0.25 in the initial steady-state by appropriately adjusting the
scaling parameter \( \tilde{\omega} \). Finally, for parameterization of steady-state of fiscal variables, debt-to-GDP,
government spending-to-GDP, and taxes-to-GDP, we use data to compute long-run values. The Appendix describes the data in detail. We then calibrate the steady-state markup to obtain a 35% capital tax rate initially. The implied initial levels of labor tax rate and consumption tax rates are 26.7% and 1.26% respectively. In the Appendix, we also show results where we consider several variations around our baseline parameterization, especially those related to preferences and technology parameters as well as policy rules.

3.2 Long-run effects of permanent tax rate changes

We now present the long-run/steady-state effects of permanent changes in the capital tax rate. As we mentioned above in Section 2.2.3, we consider two different fiscal policy to ensure that the government debt-to-GDP ratio is at the same level in the long-run. The first is by (non-distortionary) transfer adjustment, which we take as the starting point. We then look at how a distortionary adjustment of labor tax rate alters results.

The numerical results are summarized in Figure 1. While our focus numerically is on a reduction of the capital tax rate from 35% to 21%, which are clearly shown with colored dots in that figure, we show the entire range of tax rate changes for completeness. Moreover, we present several analytical results, that clarify the mechanisms and also help get ball park estimates.

3.2.1 Lump-sum transfer adjustment

Capital tax cuts, as expected, have expansionary long-run effects on the economy. We first present some analytical results and then proceed to numerical analysis. It is useful to state as an assumption a mild restriction on government spending in steady-state as given below.\footnote{This restriction is very mild, and is just to ensure that government spending in steady-state is not very high. For instance, for a case of not an unrealistically high markup and separable preferences, this holds for any reasonable parameterization of government spending in steady-state.}

**Assumption 1.** \[ G < 1 - \frac{\theta - 1}{\theta} \left( \frac{\bar{\alpha} - (1 - d)(1 - \bar{a})}{\bar{\alpha} - (1 - d)} \right) \left( 1 - \bar{\tau}_K^n \right) = 1 - \frac{1}{X} \left( \frac{\bar{\tau}_K^n}{\bar{\tau}_K} \right) \text{ in the initial steady-state.} \]

Then, we can show that a permanent capital tax rate cut leads to an increase in output, consumption, investment, and wages, and a decline in the rental rate of capital in the model. We state this formally below in Lemma 1.

**Lemma 1.** Fix \( \bar{\tau}_H^n \) and \( \bar{\tau}_b \). With lump-sum transfer adjustment and a Cobb-Douglas production function (\( \varepsilon = 1 \)),

1. Rental rate of capital is increasing, while capital to hours ratio, wage, hours, capital, investment, and output are decreasing in \( \bar{\tau}_K^n \).
2. Under Assumption 1, consumption is also decreasing in \( \bar{\tau}_K^n \).

**Proof.** See Appendix B.2.
Intuition for this result is well-understood. A reduction in the capital tax rate leads to a decrease in the rental rate of capital, raising firms’ demand for capital.\textsuperscript{18} This stimulates investment and capital accumulation. The capital-to-labor ratio increases as a result. A relatively larger amount of capital stock, in turn, makes workers more productive, raising wages and hours. Given the increase in the factors of production, output increases, which also raises consumption unless the steady-state ratio of government spending-to-GDP is unrealistically very high, as ruled out by Assumption 1.\textsuperscript{19}

Additionally, we can also derive fully exact solution for the change in macroeconomic quantities and factor prices, as well as an approximate solution for small changes in the capital tax rates that are intuitive to understand and sign. We state this formally below in Proposition 1. Note that the results below are in terms of changes from the original steady-state.

Proposition 1. Let $\tau^K_{\text{new}} = \tau^K + \Delta (\tau^K)$. With lump-sum transfer adjustment and a Cobb-Douglas production function ($\varepsilon = 1$), relative changes of various variables from their initial steady-states are the following:

\[
\frac{\ddot{r}^K_{\text{new}}}{\ddot{r}^K} = \left( 1 - \frac{\Delta (\tau^K)}{1 - \tau^K} \right)^{-1}, \quad \frac{\ddot{w}_{\text{new}}}{\ddot{w}} = \left( 1 - \frac{\Delta (\tau^K)}{1 - \tau^K} \right)^{1 - \lambda} \frac{\ddot{w}}{\ddot{w}},
\]

\[
\frac{\ddot{K}_{\text{new}}}{\ddot{K}} = \frac{\ddot{I}_{\text{new}}}{\ddot{I}} = \left( 1 - \frac{\Delta (\tau^K)}{1 - \tau^K} \right)^{1 - \lambda} \frac{\ddot{K}_{\text{new}}}{\ddot{K}} = \left( 1 - \frac{\Delta (\tau^K)}{1 - \tau^K} \right)^{1 - \lambda} \frac{\ddot{I}_{\text{new}}}{\ddot{I}},
\]

and

\[
\frac{\ddot{C}_{\text{new}}}{\ddot{C}} = \left( 1 + \frac{\ddot{I}}{\ddot{H}} \left( \frac{\ddot{C}}{\ddot{H}} (1 - \tau^K) \right)^{-1} \right) \frac{\ddot{Y}_{\text{new}}}{\ddot{Y}}
\]

where $\Omega = \bar{\omega} \bar{H}^{1 - \phi} \frac{\lambda \eta}{1 - \lambda} \left( \frac{\bar{a} - (1 - d)}{\bar{a} - (1 - d)} \right) \frac{1 + \lambda C}{1 - \tau^K} > 0$. Moreover, for small changes in the capital tax rate

\textsuperscript{18}In steady-state, for separable preferences, the after tax rate of return on capital is directly pinned down in the model by the discount rate, rate of growth, and the rate of depreciation: $\left( 1 - \tau^K \right) \ddot{r}^K = \ddot{a} - (1 - d)$. Then clearly, with $\ddot{r}^K$ falling, $\ddot{r}^K$ will increase.

\textsuperscript{19}In such a case, government consumption or investment crowds out private consumption.
\( \Delta (\bar{\tau}^K) \), the percent changes of these variables from their initial steady-states are:

\[
\ln \left( \frac{\bar{r}_{new}}{\bar{r}^K} \right) = \Delta (\bar{\tau}^K), \\
\ln \left( \frac{\bar{w}_{new}}{\bar{w}} \right) = -\left( \frac{\lambda}{1-\lambda} \right) \Delta (\bar{\tau}^K), \\
\ln \left( \frac{\bar{K}_{new}}{\bar{K}} \right) = \ln \left( \frac{\bar{I}_{new}}{\bar{I}} \right) = -M_K \Delta (\bar{\tau}^K), \\
\ln \left( \frac{\bar{H}_{new}}{\bar{H}} \right) = -\frac{1}{1+\varphi} \Delta (\bar{\tau}^K), \\
\ln \left( \frac{\bar{Y}_{new}}{\bar{Y}} \right) = -M_Y \Delta (\bar{\tau}^K), \\
\ln \left( \frac{\bar{C}_{new}}{\bar{C}} \right) = -M_C \Delta (\bar{\tau}^K),
\]

and

\[
\ln \left( \frac{\bar{I}_{new}}{\bar{I}} \right) = -M_K \Delta (\bar{\tau}^K), \\
\ln \left( \frac{\bar{H}_{new}}{\bar{H}} \right) = -\frac{\Omega}{1+\varphi} \Delta (\bar{\tau}^K), \\
\ln \left( \frac{\bar{Y}_{new}}{\bar{Y}} \right) = -M_Y \Delta (\bar{\tau}^K),
\]

where

\[
M_K = \frac{1}{(1-\lambda)(1-\tau^K)} + \frac{\Omega}{1+\varphi} > 0, \\
M_Y = \frac{\lambda}{(1-\lambda)(1-\tau^K)} + \frac{\Omega}{1+\varphi} > 0 \\
and M_C = M_Y - \frac{\Omega}{H} (\bar{C}^0 (1-\tau^K))^{-1}.
\]

Under Assumption 1, \( M_C > 0 \).

Proof. See Appendix B.3.

Proposition 1 provides a simple representation of the model solution that helps us understand the mechanism even further. As is standard, the effects on factor prices and capital to labor ratio depend only on the production side parameters. For the level of aggregate quantities (output, consumption and investment), however, the proposition shows that the key step, in the aforementioned channel, is in fact how labor hours respond, \( \bar{H}_{new} \). This implies that preference parameters, such as the intertemporal elasticity of substitution and the Frisch elasticity of labor supply, generally matter for the effectiveness of a capital tax cut – although we show below they do not change the results significantly for the range of values considered. Moreover, given the importance of hours response, the proposition naturally leads us to a conjecture that a capital tax cut would have a smaller effect if the labor tax rate needed to adjust, which we prove formally in the next subsection. Finally, the solution also reveals that the effectiveness of a tax reform depends on the economy’s current tax rates. When the economy is initially farther away from the non-distortionary case (i.e. when \( \tau^K \), \( \tau^H \) and \( \tau^C \) are currently high), a given capital tax cut will have a stronger long-run effect.

Next we show results numerically in Figure 1 under the baseline calibration. For a reduction of the capital tax rate from 35% to 21%, output increases by 10.8% relative to the initial steady state, investment by 34.7%, consumption by 6.7%, and wages by 8.7%.

\(^{20}\)Output for example increases by the same amount (in percentage from the initial steady-state) as pre-tax labor income.

\(^{21}\)Our baseline calibration uses separable preferences that imply log utility (\( \eta = 1 \)). Thus it is actually more restrictive than the condition required for our analytical results above. But we do this as baseline first because it is often the benchmark in studies and second, our analytical results for the labor tax rate adjustment case hold only for the separable case. Moreover, the numerical results are shown for all large changes as well, for which it is difficult to get intuitive expressions in the labor tax rate adjustment scenario. Note that these are % changes from the initial steady-state. Thus the metric is the same as in Proposition 1. If we use the exact formula in Proposition 1, the solution is the same as here, while the approximate solution in Proposition 1 that is useful for ballpark estimates would give an increase in output by 11.1%, investment by 32.6%, consumption by 7.37%, and wages by 8.7%.
The positive effects however do not come without cost, as the proposed capital tax cuts are not self-financing. As shown in Figure 1, a decrease in the capital tax rate reduces total (tax) revenues-to-GDP ratio, which in turn leads to a decline in transfers-to-GDP ratio to ensure that the government debt-to-GDP ratio stays constant in the long-run.\textsuperscript{22} This result is obtained not only because output (i.e. the denominator) increases. For the range of the capital tax rate decrease considered, the total tax revenues also decline. In particular, there is a significant decrease in capital tax revenues (about 40\% decline relative to the initial steady state), which is only partially offset by an increase in consumption and labor tax revenues. The government therefore finances such a deficit by taking resources away from household: transfers decline by roughly 60\% of the initial steady-state.\textsuperscript{23}

Furthermore, income inequality, measured by the the ratio of after-tax capital income to labor income, unambiguously increases – although both types of income increase.\textsuperscript{24} However, once the loss of transfer is accounted for, the capital tax cuts only generate a minimal gain of (earned and unearned) income for those dependent on labor income and government transfers.

We conduct extensive sensitivity analysis on our baseline parameterization and report the results in the Appendix. Using general, non-separable preferences and a range of intertemporal elasticity of substitution and Frisch elasticity does not affect the main results, especially for output, investment, and wages.\textsuperscript{25} Using a general CES production technology however, does lead to quantitatively different results. For instance, for the baseline experiment of a reduction of the capital tax rate from 35\% to 21\%, for $\varepsilon = 1.2$ (0.8) output increases by 17.2\% (6.3\%), investment by 48.1\% (24.2\%), consumption by 11.8\% (3.2\%), and wages by 13.1\% (5.2\%). Intuitively, a higher elasticity of substitution between capital and labor, compared to the Cobb-Douglas benchmark, leads to a bigger response of the capital to hours ratio for the same change in relative factor prices. Here, the wage to rental rate of capital ratio goes up with a capital tax rate cut, which means that capital to hours increases by more with a higher elasticity of substitution. Then, this will boost investment, output, and consumption further in the model. In equilibrium, hours however, increase by less for

\textsuperscript{22}As shown in the Appendix B.1 analytically, $\bar{T}_K = \bar{\tau}_K (\lambda \theta - 1) \varepsilon^{-1} \left( \frac{\tilde{a}_\theta}{\bar{\theta}} - (1 - d) \right)^{1-\varepsilon} \left( \lambda \frac{\theta - 1}{\theta} \right)^{\varepsilon}$. Then for the Cobb-Douglas production function ($\varepsilon = 1$), we have $\bar{T}_K = \bar{\tau}_K (\lambda \theta - 1)$ and so capital tax revenue-to-GDP decreases with the capital tax rate. For a general CES production function, capital tax revenue-to-GDP may decrease with the capital tax rate when the elasticity of substitution between capital and labor is greater than one. In particular, for $\bar{\tau}_K \in [0, 1]$, $\frac{\partial T_K}{\partial \tau_K} < 0$ if $\bar{\tau}_K > \frac{1}{\varepsilon}$.

We omit consumption tax revenue-to-GDP from the figure as it is very small quantitatively, but that also declines. As is clear in Figure 1, labor tax revenue-to-GDP is invariant to the capital tax rate changes, which is again due to the Cobb-Douglas production technology used in our baseline simulation. We show consumption tax revenue in the Appendix Figure D.5.

\textsuperscript{23}There is a “laffer curve” for capital tax revenues, but it occurs at very high and empirically irrelevant range, such as above 70\%. We show details on the levels of fiscal variables in the Appendix in Figure D.5.

\textsuperscript{24}Labor income does decrease in the short run as shown below. Of course, our model is one of a representative household, but this analysis of inequality is still interesting.

\textsuperscript{25}As to be expected, these variations do affect consumption and hours relatively more. Qualitatively, a higher intertemporal elasticity of substitution leads to a lower response of consumption, and as a result, lower response of output and hours. A larger Frisch elasticity leads to a bigger increase in hours and thereby of output. Our analytical results on the sign of changes however, already hold for non-separable preferences and we do not find any quantitative differences when we do these comparative statics numerically.
a higher elasticity of substitution.

### 3.2.2 Labor tax rate adjustment

We next discuss the case where labor tax rate increases in the long-run to finance the permanent capital tax rate cuts. Overall, compared to the previous benchmark, the model predicts qualitatively similar long-run effects on most of the variables – except for labor hours and for after-tax wages. Quantitatively, however, the macroeconomic effects are smaller because of distortions created by the labor tax rate increase. In fact, for small changes in the capital tax rate, we have analytical results on exactly how small these effects are and what parameters determine the differences. We elaborate on these findings by first showing some analytical results and then proceeding to numerical analysis.\(^{26}\)

Once again, a mild restriction on steady-state government spending is assumed as given below.

**Assumption 2.** \(\bar{\tilde{G}} < 1 - \frac{\bar{\alpha} - (1 - d)}{\bar{\alpha}} = 1 - \frac{1}{\chi(1 - \bar{\tau}K)}(\bar{\tau}) \) in the initial steady-state.

Then, we can show that a permanent capital tax rate cut, financed by an increase in the labor tax rate, leads to an increase in the capital-to-hours ratio and in (pre-tax) wages and a decrease in the rental rate of capital, as before.\(^ {27}\) In contrast to the lump-sum transfer benchmark, however, hours now decline in the new steady-state.

**Lemma 2.** Fix \(\bar{\tilde{S}}\) and \(\bar{\tilde{b}}\). With labor tax rate adjustment and separable preferences \((\eta = 1)\) and a Cobb-Douglas production function \((\varepsilon = 1)\),

1. Rental rate of capital is increasing, while capital to hours ratio and wage are decreasing in \(\bar{\tau}K\).
2. Under Assumption 2, hours are increasing in \(\bar{\tau}K\).

**Proof.** See Appendix B.4.

We next show analytically the required adjustment in labor tax rate in the new steady-state as well as the approximate solution for small changes in the capital tax rates that are intuitive to understand and sign.\(^ {28}\) The required adjustment in the labor tax rate is approximately given by the ratio of the capital to labor input in the production function, as the government is keeping debt-to-GDP constant and hence have to compensate the loss of capital tax revenue-to-GDP with gains in labor tax revenue. One interesting result on the approximate solution is that the effects on after-tax wage rate depends on initial level of labor tax rate relative to the other tax rates. Intuitively, a further increase in labor tax rate (to finance a capital tax cut,) when it is sufficiently

\(^{26}\)Our analytical results in this subsection are slightly less comprehensive than before and are obtained under the additional assumption of separable preferences, as clearly stated in the following Lemma and Propositions.

\(^{27}\)In fact the entire response of capital-to-hours, rental rate of capital, and (pre-tax) wages are the same between transfer and labor tax rate adjustment.

\(^{28}\)We do have exact solutions as well, but unlike the transfer adjustment case, they are highly unwieldy and it is difficult to get any insights.
high already, lowers after-tax wage rate. Moreover, again, hours fall, which is the result we highlight
given that it is qualitatively different.\footnote{For this case, because of opposite movement of hours and capital to hours ratio, it is not possible to provide
intuitive results on the levels of variables such as output and consumption.}

**Proposition 2.** Let \( \bar{\tau}^K_{\text{new}} = \bar{\tau}^K + \Delta (\bar{\tau}^K) \). With labor tax rate adjustment and separable preferences
\((\eta = 1)\) and a Cobb-Douglas production function \((\varepsilon = 1)\),

1. New steady-state labor tax rate is given by \( \bar{\tau}^H_{\text{new}} = \bar{\tau}^H + \Delta (\bar{\tau}^H) \) where

\[
\Delta (\bar{\tau}^H) = -\frac{\lambda}{1 - \lambda} \left( 1 + \bar{\tau}^C \left( \frac{\bar{a} - (1 - d)}{\frac{\bar{a}}{\beta} - (1 - d)} \right) \right) \Delta (\bar{\tau}^K).
\]

2. For small changes in the capital tax rate \( \Delta (\bar{\tau}^K) \), relative changes of rental rate, wage, after-tax wage, capital to hours ratio and hours from their initial steady-states are the following:

\[
\ln \left( \frac{\bar{\tau}^K_{\text{new}}}{\bar{\tau}^K} \right) = \frac{\Delta (\bar{\tau}^K)}{1 - \bar{\tau}^K}, \quad \ln \left( \frac{\bar{K}_{\text{new}}}{H_{\text{new}}} \right) = -\frac{1}{1 - \lambda} \frac{\Delta (\bar{\tau}^K)}{1 - \bar{\tau}^K},
\]

\[
\ln \left( \frac{\bar{w}_{\text{new}}}{\bar{w}} \right) = -\frac{\lambda}{1 - \lambda} \frac{\Delta (\bar{\tau}^K)}{1 - \bar{\tau}^K}, \quad \ln \left( \frac{(1 - \bar{\tau}^H_{\text{new}}) \bar{w}_{\text{new}}}{(1 - \bar{\tau}^H) \bar{w}} \right) = M_W \Delta (\bar{\tau}^K),
\]

and

\[
\ln \left( \frac{\bar{H}_{\text{new}}}{H} \right) = M_H \Delta \bar{\tau}^K,
\]

where \( M_H = \frac{1}{1 + \bar{\tau}^H} \frac{\lambda}{1 - \lambda} \left( \frac{1 - \bar{\tau}^C + \frac{\bar{a} - (1 - d)}{\beta} (\bar{\tau}^C + \bar{\tau}^H + \bar{\tau}^K - \frac{\bar{a}}{\beta})}{(1 - \bar{\tau}^H) \bar{C}} \right) \) and \( M_W = \frac{\lambda}{1 - \bar{\tau}^H} \left( \frac{\bar{a} - (1 - d)}{\beta} \bar{C} \right) - \frac{1 - \bar{\tau}^H}{1 - \bar{\tau}^K} \).

Under Assumption 2, \( M_H > 0 \). Moreover, \( M_W > 0 \) if and only if \( 1 + \frac{\bar{\tau}^C (\bar{a} - (1 - d))}{\beta (1 - d)} > \frac{1 - \bar{\tau}^H}{1 - \bar{\tau}^K} \).

**Proof.** See Appendix B.5.\qed

We are also able to compare analytically the change in macroeconomic quantities as a result of the capital tax rate cut for the two fiscal adjustment cases. We do this for the small capital tax rate adjustment approximation and prove in Proposition 3 that the increase in output, capital, investment, consumption, and hours increase by more under adjustment in lump-sum transfers compared to labor tax rate adjustment.\footnote{This result does not require Assumption 2. That is, it holds regardless of whether hours increase or decrease following a capital tax rate cut. Additionally, as seen above wages and rental rates are the same across the two fiscal adjustments, as shown in Proposition 1 and 2, and so we do not present these obvious results in Proposition 3.} Moreover, the differences in these changes for output, investment, consumption, and hours are given by the same amount. This constant difference depends intuitively and precisely on the labor supply parameter for a given change in the tax rates. A higher Frisch elasticity \((\frac{1}{\phi})\) makes workers more responsive to labor tax rates, thereby generating greater distortions, which in turn magnifies the difference. The two fiscal adjustments produce the same outcomes only if labor supply is completely inelastic \((\frac{1}{\phi} = 0)\). Moreover, as is intuitive, higher is the initial level of the labor tax rate, bigger is the difference. Thus, for the same change
in the labor tax rate, if the initial labor tax rate is higher, the increase in output, investment, consumption, and hours will be relatively smaller.

**Proposition 3.** Let \( \bar{\tau}_K^{\text{new}} = \bar{\tau}_K + \Delta (\bar{\tau}_K), \bar{\tau}_H^{\text{new}} = \bar{\tau}_H + \Delta (\bar{\tau}_H) \), and consider separable preferences \((\eta = 1)\) and a Cobb-Douglas production function \((\varepsilon = 1)\). Denote \(X_T^{\text{new}}\) and \(X_L^{\text{new}}\) as the new steady-state variables in transfer adjustment case and in labor tax rate adjustment case, respectively. For small changes in the capital tax rate \(\Delta (\bar{\tau}_K)\), for \(X \in \{\bar{C}, \bar{K}, \bar{I}, \bar{Y}, \bar{H}\}\), we get

\[
\ln \left( \frac{X_T^{\text{new}}}{X_L^{\text{new}}} \right) = -\Theta \Delta (\bar{\tau}_K) = \frac{1}{1 + \varphi} \left( \frac{1}{1 - \bar{\tau}_H} \right) \Delta (\bar{\tau}_H)
\]

where \(\Theta = \frac{1}{1 + \varphi} \left( \frac{1}{1 - \bar{\tau}_H} \right) \left( 1 + \bar{\tau}_C^C (\bar{a} - (1 - d)) \right) > 0\). In other words, generally, output, capital, investment, consumption and hours increase by more in the transfer adjustment case than in the labor tax rate adjustment case when capital tax rate is cut.

**Proof.** See Appendix B.6.

Now we show numerical results that illustrate our findings and we discuss the economics behind them. Figure 1 shows that in the long-run, to finance the reduction of the capital tax rate from 35% to 21%, labor tax rates have to increase from 26.7% to 32.7%.\(^{31}\)

The same mechanism as we described for the transfer adjustment case works, and moreover the capital-to-hours ratio, (pre-tax) wages and rental rate change by the same amount as before under the baseline specification. There continues to be an expansion in output, investment, and consumption as a result of the capital tax rate cut. The increase in output, investment, and consumption is however, less under labor tax rate adjustment – as is consistent with what we proved in Proposition 3 above for small changes. In particular, for the baseline experiment of a reduction of the capital tax rate from 35% to 21%, output increases by 6.1%, investment by 29%, and consumption by 2.2%. The comparable numbers for the benchmark were 10.8%, 34.7%, and 6.7% respectively.\(^{32}\)

The reason for the smaller boost is a decline of the after-tax wages (by 0.3% in our experiment.)\(^{33}\) This in turn leads to a decrease in labor hours in the long-run, from 0.25 to 0.244.\(^{34}\) These are the

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\(^{31}\)This is exactly the change that we show in Proposition 2 and is roughly given by the ratio of the Cobb-Douglas parameters on labor and capital inputs.

\(^{32}\)Given the small change based result in Proposition 3, it would have predicted a \(\left( \frac{1}{1 + \varphi} \right) (\bar{\tau}_C^C (\bar{a} - (1 - d)))/(32.7\%) = 4.09\%

\(^{33}\)Not one subtle result regarding small and large changes for the after tax wage rate. In Proposition 2, we show a condition under which after-tax wage decreases following a capital tax rate cut. That condition is actually not satisfied in our parameterization. But here, for a larger change, we find numerically that the after tax wage declines. For marginal changes around the initial steady-state, as can be seen in the Figure, after-tax wage does increase, as is consistent with Proposition 2 when we violate the condition on initial level of labor taxes being high enough.

\(^{34}\)Unless the steady-state ratio of government spending to output is unrealistically very high, that is it violates Assumption 2, hours decrease, as given in Proposition 2 for small changes. When hours increase, it goes together with a strong reduction in consumption. Moreover, we find numerically that even in such a case, the increase in hours is less than the case where lump-sum transfers adjust, as shown in Proposition 3 above for small changes. Note that Proposition 2 also gives basically the same result, 0.2444, as the exact numerical one above of 0.2440.
first major qualitative differences from the lump-sum transfer adjustment case, as also highlighted by Proposition 2. The decrease in hours dampens the expansionary effect of capital tax cuts on output, consumption and investment.\textsuperscript{35} Furthermore, our measure of inequality increases by more, as after-tax labor income now decreases.

We conduct extensive sensitivity analysis on our baseline parameterization and report the results in the Appendix. Using general, non-separable preferences and a range of intertemporal elasticity of substitution and Frisch elasticity does not affect the main results, especially for output, investment, and wages.\textsuperscript{36} Using a general CES production technology however, does lead to quantitatively different results, as in the transfer adjustment case. Like in that case, a higher elasticity of substitution between capital and labor, compared to the Cobb-Douglas benchmark, leads to greater capital accumulation and a higher capital to hours ratio, thereby boosting output more. Moreover, in equilibrium, hours decrease by less for a higher elasticity of substitution.

### 3.3 Transition dynamics of permanent tax rate changes

We now discuss transition dynamics associated with a permanent capital tax rate cut, from 35% to 21%. Thus, we trace the evolution of the economy as it transitions from the initial steady-state to the new steady-state. Studying transition dynamics is important as we find that it typically take a quite long time, around 70 quarters, for the economy to converge to a new steady-state following a permanent reduction in the capital rate. This allows us in particular to analyze short-run effects, which are the focus here. Compared to the long-run analysis in the previous sections, we also pay a special attention to the role of the “nominal components” in the model, which can be potentially important due to imperfect price adjustments in the short-run.

#### 3.3.1 Three different fiscal adjustments

We start with the baseline parameterization of the model, and now consider three different fiscal/monetary policy adjustment, as described in Section 2.2.3. In particular, a new policy response that we consider here is one where inflation plays a partial role in debt stabilization. The results are shown in Figure 2.

**Lump-sum transfer adjustment** Once again, the starting point is the case of non-distortionary transfer adjustment. What makes the short-run distinct from the long-run is that capital tax cuts

\textsuperscript{35}In particular, an actual drop in long-run consumption is more likely to happen than before for other parameterizations.

\textsuperscript{36}For hours, the effects of Frisch elasticity can be discerned, but within a reasonable range the effects are not much bigger than for the transfer adjustment case. Note that here, the required adjustment in labor tax rate in the new steady-state is the same across different Frisch elasticities as the labor tax to output ratio is invariant to the preference parameter. In fact, numerically, there is a boom in output even with infinite Frisch elasticity (\(\phi = 0\)) and a higher labor tax in the new steady-state, in our baseline parameterization. In this extreme version, consumption does fall in the new steady-state. In the Appendix B.7, we show analytically that the change in output is positive even under infinite Frisch elasticity.
can now generate a contractionary effect during the transition periods, which in turn has nontrivial implications for income inequality, labor income, and fiscal adjustments.

The model dynamics can be best understood as depicting transition dynamics when the capital stock initially is below the new steady-state. As mentioned before, a reduction in the capital tax rate leads to a decrease in the rental rate of capital, thereby facilitating capital accumulation via more investment. In the short-run, to finance this increase of investment, consumption in fact declines for many periods. Given this postponement of consumption, combined with sticky prices, output also falls temporarily, before rising towards the high new steady-state. The temporary contraction in output is a result of sticky prices, which renders output (partially) demand-determined and markups countercyclical in the model. Moreover, the temporary fall in output (which is coupled with increased capital stock,) leads to fall in hours. Finally, inflation is determined by forward looking behavior of firms that face adjustment costs. In particular, inflation depends on current and future real marginal costs, which are a function of wages and capital rental rate. As wage dynamics matter more and wages drop in the short-run, the path of inflation roughly follows that of wages.

In contrast to the long-run, (after-tax) labor income actually decreases in the short-run because both hours (as discussed above) and wages decrease. The decrease in wages is driven by both supply and demand forces. The drop in consumption and the rise in marginal utility of consumption raise the supply of hours for a given wage rate. On the other hand, demand declines as firms produce a smaller amount of output as discussed above.\footnote{We show later that with fully flexible prices, while consumption continues to fall in the short-run, output and hours do not fall. But, even in this case, wages fall temporarily. We discuss this fall in output that happens only with sticky prices in more detail later with clear comparative statics.}

This result suggests that the long-run positive effects of capital tax cuts come at the expense of short-run decline of labor income – even under lump-sum transfer adjustments. Furthermore, the decrease in labor income requires a larger adjustment of transfers. Transfers fall sharply and in fact decrease below the new steady-state.\footnote{In some extensions, we find that transfers might need to actually go negative.} This is because labor tax revenues fall, not just the capital tax revenue, forcing the government to take more resources away from households during the transition.

\textbf{Labor tax rate adjustment} Next, we analyze the case of labor tax rate increases. Here, labor tax rate evolves according to the tax rate rule, (4), given in Section 2.2.3.\footnote{We present results where labor tax rates adjust period by period to keep debt to output ratio constant along the transition in the Appendix D. The results are very similar compared to the ones here.} Overall, model dynamics are qualitatively similar to those in the benchmark. We still see capital accumulation, achieved by increased investment and postponement of consumption, which in turn also causes output to fall with sticky prices.

Quantitatively, however, the drop in consumption and output is larger in this case compared to the benchmark. As in the lump-sum transfer adjustment case, delayed consumption decreases hours by lowering firms’ labor demand. In addition, increased labor tax rate decreases hours even
further by discouraging workers from supplying labor. Consequently, hours in equilibrium fall much more, below even the lower new steady-state. This in turn amplifies the short-run contraction in consumption and output.\footnote{We note here that with labor tax rate adjustment, there is an output contraction even under flexible prices. We discuss this in more detail later. With sticky prices, this contraction in output is stronger, for the same countercyclical markup intuition that we gave above while discussing the lump-sum transfer adjustment case.}

**Labor tax rate and inflation adjustment** Finally, we analyze the case where labor tax rates increase, but not by enough, and inflation partly plays a role in government debt stabilization, as described in Section 2.2.3.\footnote{Note in particular that in this case, the monetary policy rule (3) does not satisfy the Taylor principle, which is coupled with a low response of the tax rate in the tax rule (4).} The main difference now compared to the pure labor tax adjustment analysis is that there is a short-run burst of inflation to help stabilize debt. This increase in inflation, as the model has nominal rigidities, helps lower the short-run contractionary effects. In particular, a key driving force is that consumption drops by less. This in turn has the effect of lowering the drop in output and wages as well.\footnote{Clearly, we can analyze a similar fiscal adjustment case where inflation plays a role in debt stabilization even with lump-sum transfer adjustment. When non-distortionary sources of revenue is possible, allowing inflation to play a role in debt stabilization might not be a very insightful experiment and so we only show this in the Appendix. But it is clear there that in this case as well, the drop in consumption, output, and wages drops by less.} In addition, a smaller increase in labor tax rates also contributes to the relatively moderate contraction.

### 3.3.2 Role of nominal components of the model

For transition dynamics, various nominal aspects of the model matter. We explore in detail below how short-run effects depend on the extent of price stickiness and the response of interest rates to inflation. For concreteness, we only focus on the benchmark case where lump-sum transfers adjust following the capital tax rate cut. For the labor tax rate adjustment case, we present results in the Appendix D.

**Role of price rigidity** Figure 3 shows comparative statics with respect to the sticky price parameter ($\kappa$).\footnote{When this parameter is 0, that is the case of fully flexible prices.} When prices are more rigid, there is a bigger short-run drop in consumption, output, and wages and a smaller drop in inflation. Intuitively, more rigid prices weaken the self-correction mechanism. When consumption and output drop in a sticky-price environment, which puts a downward pressure on marginal costs below the desired (or flexible-price) level of marginal cost (that is constant,) firms also decrease prices. The decrease in prices partially countervails the drop (in the aggregate demand.) Such countervailing effects are weaker with more rigid prices, which leads to a larger drop in consumption, output and hours. That is, when the economy is experiencing an effect akin to a negative demand shock, limiting the response of prices leads to a bigger contraction in output. Another way to get intuition for the result is that sticky prices lead to a countercyclical markup in the model. Thus, the rise in markups is behind the short-term contraction.
contraction in output, and more rigid prices lead to a bigger contraction.\textsuperscript{44} This result on stronger contractionary effects when prices are more rigid also holds under labor tax rate adjustment, which we show in the Appendix in Figure D.6.\textsuperscript{45}

**Role of monetary policy response** Figure 4 shows comparative statics with respect to the monetary policy response to inflation ($\phi$). When the response is weak, there is a bigger short-run drop in consumption, output, hours and wages and a smaller drop in inflation. Intuitively, a greater $\phi$ leads to stronger stabilization of inflation and marginal cost, which brings the economy closer to the flexible-price environment, and prevents a bigger short-run drop in the aforementioned macroeconomic variables. Again, when the economy is experiencing an effect akin to a negative demand shock, limiting the response of prices through a smaller $\phi$ leads to a bigger contraction in output. Moreover, with a weaker response to inflation, there is a higher rise in markups in the model, which leads to a bigger drop in output.\textsuperscript{46}

### 3.4 Extensions

We now discuss various extensions and sensitivity analysis on the baseline model and parameterization. For concreteness and to preserve space, unless we mention otherwise, we present these results for the case where lump-sum transfers adjust. All the Figures are in the Appendix D.

#### 3.4.1 Anticipated permanent tax rate changes

We first consider a case where the permanent tax rate change in anticipated to happen in future, in particular in four quarters. Figure D.8 shows the transition dynamics for this case, where for comparison we also show the baseline case where the change happens in the current period. As is clear, other than for fiscal variables, the responses of macroeconomic quantities and prices are basically the same.\textsuperscript{47}

#### 3.4.2 General technology and preferences

We next consider a general CES production function. As we mentioned before, this does lead to changes quantitatively, compared to the Cobb-Douglas case, and is an important extension also

\textsuperscript{44}Moreover, with constant markup as with the flexible price case, output contraction cannot happen in the short-run with lump-sum transfer adjustment as a drop in consumption will go together with a rise in hours, and thereby output. It is easy to show this analytically using the labor market equilibrium condition under flexible prices. Here, we illustrate the flexible price price case numerically. This flexible price case is also useful to give intuition from an output gap perspective. The output gap, the difference between actual output under sticky prices and the the output that would prevail under flexible prices, is negative along the transition in the short-run. Moreover, more rigid are prices, more negative is the output gap.

\textsuperscript{45}Under labor tax rate adjustment, there is a contraction in output even under flexible prices. Sticky prices make this contraction stronger, for the same countercyclical markup reason. Alternatively, the output gap is negative in the short-run along the transition, and more rigid are prices, more negative is the output gap.

\textsuperscript{46}Another way to get intuition is that a larger response of $\phi$ leads to a larger drop in a nominal rate, which in this model with nominal rigidities helps reduce the contraction in economic activity. Again, similar result holds under labor tax rate adjustment, which we show in the Appendix in Figure D.7.

\textsuperscript{47}Similar numerical results hold also for the labor tax rate adjustment case.
because our analytical results are based on the Cobb-Douglas assumption as well. Figures D.9 and D.10 show the long-run effects of a permanent capital tax rate cut for the two fiscal adjustment scenarios. As discussed before, a higher elasticity amplifies the effects of capital tax cuts and thus leads to a greater increase in the steady-state level of output, consumption and investment. The same amplification mechanism also works in the short run, which generates a greater response of the economy on the transition path. Figure D.11 presents results for the transition dynamics where lump-sum transfers adjust, and the short-term drop in consumption, output, hours, and wages is more pronounced for a higher elasticity of substitution.48

We next present results for non-separable preferences, where we consider various values of the intertemporal elasticity of substitution. The steady-state results are in Figures D.12 and D.13. Other than for hours, the differences are very minor. Figure D.14 shows transition dynamics across various values of the intertemporal elasticity of substitution. The dynamic response of consumption is quantitatively a bit higher, as expected, for a higher intertemporal elasticity of substitution.

Finally, we consider various values of the Frisch elasticity of labor supply. Figures D.15 and D.16, which contain steady-state results, show that the difference, as expected, is basically only seen in the behavior of hours, with a higher response for a higher Frisch elasticity. On transition dynamics, as shown in Figure D.17, a higher Frisch elasticity leads to a smaller short-term drop in consumption, output and hours when lump-sum transfers adjust. For the labor tax rate adjustment case however, it is the opposite, and we show this in Figure D.18 as it is qualitatively different. Intuitively, a decline in consumption encourages workers to supply more hours through a standard income effect. Such a shift in labor supply is bigger with a higher Frisch elasticity, which in equilibrium tends to generate a greater increase in hours, and thereby, a smaller drop in output and consumption under lump-sum transfer adjustment. When labor tax rates increase however, substitution effect also kicks in, which can produce a qualitatively different result.

3.4.3 Policy rules and parameters

We first start with comparative statics with respect to the monetary policy response to inflation parameter ($\phi$), for the case where labor tax rates adjust. Figure D.19 shows that when the response is weak, there is a bigger short-run drop in consumption, output, and wages and a smaller drop in inflation. This is the same result as for the transfer adjustment discussed above in Figure 4 and the same intuition applies.

We now consider different fiscal/labor tax rate rules. First, given the tax rate rule (4), we do comparative statics with respect to the response to debt parameter ($\psi$). As is expected, Figure D.20 shows that a larger value of this parameter leads to a bigger contraction in the short-run as labor tax rates increase more rapidly. Next, instead of using a feedback rule for labor tax rate, we let it freely adjust period-by-period, to keep debt to GDP constant throughout the transition. Figure D.21 shows that the macroeconomic implications are essentially the same as in our baseline case with a feedback rule, with some initial differences in wages due to a stronger rise in the labor

48Similar results hold for the labor tax rate adjustment.
tax rate.

Finally, we consider a case where we let inflation facilitate debt adjustment even with lump-sum transfers. In particular, note that with lump-sum transfers adjustment in Figure 2, transfers have to fall below the new steady-state in the short-run. We therefore model a situation where transfers immediately go to this new steady-state, but do not fall below that. Thus, transfers can adjust, but not by enough. Then, by using an interest rate rule that does not satisfy the Taylor Principle, we allow inflation to help with debt stabilization. This is precisely in the same spirit as the partial labor tax rate and partial inflation adjustment combination that we considered above in Section 3.3.1. Figure D.22 shows then that the results also have the same flavor as Figure 2: with the increase in inflation in this case, the drop in consumption, output, and wages is less than when only lump-sum transfers adjust.

4 Conclusion

A standard macroeconomic model predicts that a permanent reduction in the capital tax rate from 35% to 21% generates a non-trivial long-run increase in output, consumption and investment. To finance these tax cuts, the government however, needs to take a significant amount of resources away from households, either by cutting back transfers or by raising distortionary labor tax rates. In the latter case, not only are the increases in output, consumption, and investment lower, but also, after-tax wages and labor income permanently decrease while income inequality is more pronounced. We also find that other features of the economy, such as preference and technology specification/parameterization, while generally relevant in theory, do not affect the results quantitatively. The only exception is the elasticity of substitution between capital and labor in the production function: the larger is the elasticity, the greater is the long-run effect of capital tax cuts.\(^{49}\)

In contrast to the long-run analysis, a study of transition dynamics shows that in the short-run, the economy experiences a decline in consumption, output, hours, wages, and thus labor income, regardless of how the capital tax rate cuts are financed. The short-run contraction is more severe when labor tax rates (rather than transfers) adjust and when prices are more rigid. Interestingly, there is a discontinuity in the relationship between the extent of contraction and the response of monetary policy. To the extent that monetary policy satisfies the Taylor principle, a less aggressive response to inflation leads to a more severe short-run contraction. However, once monetary policy does not satisfy the Taylor principle allows inflation to play a direct role in debt stabilization along the transition, a less aggressive response to inflation helps reduce the extent of short-run contraction.

The relative simplicity of our model allows us to derive key results analytically and to illustrate various mechanisms and the role of monetary policy and rigid prices clearly. While the results are model-dependent, we regard our estimates as a useful starting point. Introducing some form of

\(^{49}\)A capital tax cut is more effective also when the consumption, labor and capital tax rates are initially large.
household heterogeneity is a potentially important extension. New positive and normative insights are likely to emerge by introducing capitalists and workers separately into the model, such that income inequality has non trivial aggregate implications. In addition, our analysis of the short-run and the long-run suggests that the proposed tax reform will have heterogeneous effects on different generations. Exploring generational heterogeneity is another interesting avenue for future research.

References


## 5 Tables and figures

### Table 1: Calibration

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<td>Steady-state growth rate</td>
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<td>Steady-state consumption tax revenue to GDP ratio</td>
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<td>Steady-state capital tax revenue to GDP ratio</td>
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<td>Labor tax rate response to debt, Labor tax rate response when inflation helps debt adjustment</td>
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**Government (Fiscal/Monetary Policy)**

<table>
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<tr>
<th>Value</th>
<th>Description</th>
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<tr>
<td>$\psi$</td>
<td>$\begin{cases} 0.05 \ 0.002 \end{cases}$</td>
<td>Labor tax rate response to debt, Labor tax rate response when inflation helps debt adjustment</td>
</tr>
</tbody>
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Figure 1: Long-run Effects of Permanent Capital Tax Rate Changes

- Consumption ($\bar{\tilde{C}}$)
- Investment ($\bar{\tilde{I}}$)
- Output ($\bar{\tilde{Y}}$)
- Hours ($\bar{\tilde{H}}$)
- Wage ($\bar{\tilde{w}}$)
- After Tax Wage ($(1 - \bar{\tau}_H)\bar{\tilde{w}}$)
- Capital Rental Rate ($\bar{\tilde{r}}_K$)
- After Tax Rental Rate ($(1 - \bar{\tau}_K)\bar{\tilde{r}}_K$)
- Capital to Hours Ratio ($\bar{\tilde{K}}/\bar{\tilde{H}}$)
- After Tax Capital to Labor Income Ratio ($(1 - \bar{\tau}_H)\bar{\tilde{K}}/(1 - \bar{\tau}_H)\bar{\tilde{w}}$)
- Labor Tax Revenue ($\bar{\tilde{T}}_H$)
- Capital Tax Revenue ($\bar{\tilde{T}}_K$)
- Transfers ($\bar{\tilde{S}}$)
- Labor Tax Rate ($\bar{\tilde{\tau}}_H$)
- Capital Tax Rate ($\bar{\tilde{\tau}}_K$)
- Debt to Output ($\bar{\tilde{b}}$)

% from initial steady-state

- Consumption: -50, -30, -10, 2.26, 6.7
- Investment: -50, -25, 0, 6.1, 10.8
- Output: 0.24, 0.244, 0.25, 0.255, 0.26, 0.27
- Hours: -60, -40, -20, 0, 8.7, 20
- Wage: -60, -40, -20, 0, 8.7, 20
- After Tax Wage: 4.2, 5.1, 6, 8, 10
- Capital Rental Rate: 2.5, 3, 3.3, 4
- After Tax Rental Rate: 3.3, 3, 2.5
- Capital to Hours Ratio: 30, 46.2, 50.3
- After Tax Capital to Labor Income Ratio: 0, 5, 10
- Labor Tax Revenue: 0, 4.3, 7.2
- Capital Tax Revenue: 10, 15
- Transfers: 0, 10, 20
- Labor Tax Rate: 1.8, 4.7, 10
- Capital Tax Rate: 36, 36.3, 37
- Debt to Output: 35, 36
Figure 2: Transition Dynamics of Permanent Capital Tax Rate Changes

- Consumption (\( \tilde{C}_t \))
- Investment (\( \tilde{I}_t \))
- Output (\( \tilde{Y}_t \))
- Hours (\( \tilde{H}_t \))
- Wage (\( \tilde{w}_t \))
- After Tax Wage (\( (1 - \tau^H)\tilde{w}_t \))
- Capital Rental Rate (\( r^K_t \))
- After Tax Rental Rate (\( (1 - \tau^K)r^K_t \))
- Inflation (\( \pi_t \))
- After Tax Capital to Labor Income Ratio (\( (1 - \tau^K)\tilde{r}_K + (1 - \tau^H)\tilde{r}_H \))
- Labor Tax Revenue (\( \tilde{T}H_t \))
- Capital Tax Revenue (\( \tilde{T}K_t \))
- Transfers (\( \tilde{S}_t \))
- Labor Tax Rate (\( \tau^H_t \))
- Capital Tax Rate (\( \tau^K_t \))
- Debt to Output (\( \tilde{b}_t \))
Figure 3: Degree of Price Rigidity: Transfers Adjustment

- Consumption $\tilde{C}_t$
- Investment $\tilde{I}_t$
- Output $\tilde{Y}_t$
- Hours $\tilde{H}_t$
- Wage $\tilde{w}_t$
- After Tax Wage $((1 - \tau)\tilde{w}_t)$
- Capital Rental Rate $\tilde{r}_K$
- After Tax Capital Rental Rate $((1 - \tau)\tilde{r}_K)$
- Inflation $\tilde{\pi}_t$
- After Tax Capital to Labor Income Ratio $((1 - \tau K)(1 - \tau H)\tilde{w}_t\tilde{H}_t)$
- Transfers $\tilde{S}_t$
- Labor Tax Rate $\tilde{\tau}_H$
- Capital Tax Rate $\tilde{\tau}_K$
- Debt to Output $\tilde{b}_t$
Figure 4: Degree of Monetary Policy Inflation Response: Transfers Adjustment

- Consumption (C_t)
- Investment (I_t)
- Output (Y_t)
- Hours (H_t)
- Wage (\tilde{w}_t)
- After Tax Wage \((1 - \tau)\tilde{w}_t\)
- Capital Rental Rate (r_K)
- After Tax Rental Rate \(((1 - \tau)\tilde{r}_K)\)
- Inflation (\pi_t)
- After Tax Capital to Labor Income Ratio \((1 - \tau)\tilde{r}_K\)
- Labor Tax Revenue (\tilde{T}_H)
- Capital Tax Revenue (\tilde{T}_K)
- Transfers (\tilde{S}_t)
- Labor Tax Rate (\tau_H)
- Capital Tax Rate (\tau_K)
- Debt to Output \((\tilde{b}_t)\)

MP Inflation Feedback(\phi = 1.2) - MP Inflation Feedback(\phi = 1.5) - MP Inflation Feedback(\phi = 1.8)
Appendix

A Model

A.1 Households

\[
\max_{\{C_t, H_t, K_{t+1}, I_t, B_t\}} \quad E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ C_t^{1-\eta} \left( 1 - \bar{\omega}^{1-\eta} (H_t)^{1+\varphi} \right)^{\eta} - 1 \right] \right\} \\
\text{s.t.} \quad (1 + \tau^C) P_t C_t + P_t I_t + B_t \\
= (1 - \tau_t^H) W_t H_t + R_{t-1} B_{t-1} + (1 - \tau_t^K) R_t^K K_t + P_t \Phi_t + P_t S_t \\
K_{t+1} = (1 - d) K_t + \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right) I_t
\]

where \( E \) is the expectation operator, \( C_t \) is consumption, \( H_t \) is hours, \( I_t \) is investment, \( K_t \) is the stock of capital, \( B_t \) is nominal risk-less one-period government bonds, and \( \Phi_t \) is profits from firms. \( P_t \) is the aggregate price level, \( W_t \) is nominal wages, \( R_t \) is the nominal one-period interest rate, and \( R_t^K \) is the rental rate of capital. \( S_t \) is lump-sum transfers from the government, \( \tau^C \) is the tax rate on consumption, \( \tau_t^H \) is the tax rate on wage income, and \( \tau_t^K \) is the tax rate on capital income. \( \beta \) is the discount factor and \( d \) is the rate of depreciation of the capital stock.

A.2 Firms

A.2.1 Final goods firms

Competitive final goods firms produce aggregate output \( Y_t \) by combining a continuum of differentiated intermediate goods using a CES production function

\[
Y_t = \left( \int_0^1 Y_t (i)^{\theta-1} di \right)^{\frac{\theta}{\theta-1}},
\]

where \( \theta \) is the elasticity of substitution between intermediate goods indexed by \( i \). The corresponding optimal price index \( P_t \) for the final good is

\[
P_t = \left( \int_0^1 P_t (i)^{1-\theta} \right)^{\frac{\theta}{1-\theta}},
\]

where \( P_t (i) \) is the price of intermediate goods and the optimal demand for \( Y_t (i) \) is

\[
Y_t (i) = \left( \frac{P_t (i)}{P_t} \right)^{-\theta} Y_t, \quad (A.1)
\]

A.2.2 Intermediate goods firms

Intermediate goods firms indexed by \( i \) produce output using a CRS production function

\[
Y_t (i) = \begin{cases} \\
K_t (i)^{\lambda} (A_t H_t (i))^{1-\lambda} \\
\left( \lambda K_t (i)^{\frac{\gamma-1}{\gamma}} + (1 - \lambda) (A_t H_t (i))^\gamma \right)^{\frac{1}{\gamma}} \\
\end{cases} \quad \text{if } \varepsilon = 1 \\
\begin{cases} \\
\left( \lambda K_t (i)^{\frac{\gamma-1}{\gamma}} + (1 - \lambda) (A_t H_t (i))^\gamma \right)^{\frac{1}{\gamma}} \\
\end{cases} \quad \text{otherwise}
\]

(A.2)
where $A_t$ represents exogenous economy-wide labor augmenting technological progress. The gross growth rate of technology is given by $a_t \equiv \frac{A_t}{A_{t-1}} = \bar{a}$. Firms rent capital and hire labor in economy wide competitive factor markets. Intermediate good firms also face price adjustment cost $\Xi \left( \frac{P_t(i)}{P_{t-1}(i)} \right) Y_t$ that has standard properties.

Firms problem is to

$$\max_{\{P_t(i), Y_t(i), H_t(i), K_t(i)\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} P_t \Phi_t(i) \right\}$$

subject to (A.1) and (A.2), where $\Lambda_t$ is the marginal utility of nominal income and flow profits $\Phi_t(i)$ is given by

$$\Phi_t(i) = \frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} H_t(i) - \frac{R_t}{P_t} K_t(i) - \Xi \left( \frac{P_t(i)}{P_{t-1}(i)} \right) Y_t.$$

### A.3 Government

#### A.3.1 Government budget constraint

The government flow budget constraint, written by expressing fiscal variables as ratio of output, is given by

$$\frac{B_t}{P_t Y_t} + \left( T_t^C + T_t^H + T_t^K \right) = R_t - B_{t-1} \frac{B_{t-1}}{P_{t-1} Y_{t-1}} \frac{1}{\pi_t} \frac{Y_{t-1}}{Y_t} + G_t + S_t$$

where

$$T_t^C = \tau^C C_t Y_t, \quad T_t^H = \tau^H W_t \frac{Y_t}{P_t Y_t} H_t, \quad T_t^K = \tau^K \frac{R_t K_t}{P_t Y_t}$$

and $G_t$ is government spending on the final good.

#### A.3.2 Monetary policy

Monetary policy is given by a simple interest-rate feedback rule

$$\frac{R_t}{\bar{R}} = \left( \frac{\pi_t}{\bar{\pi}} \right)^\phi$$

where $\phi \geq 0$ is the feedback parameter on inflation $\left( \pi_t = \frac{P_t}{P_{t-1}} \right)$, $\bar{R}$ is the steady-state value of $R_t$, and $\bar{\pi}$ is the steady-state value of $\pi_t$. When $\phi > 1$, the standard case, the Taylor principle is satisfied. When $\phi < 1$, which we will also consider, inflation response will play a direct role in government debt stabilization along the transition.

#### A.3.3 Fiscal policy

We consider a permanent change in the capital tax rate $\tau^K_t$ in period 0, where it is in the initial steady-state. Consumption tax rate and $\frac{G_t}{Y_t}$ do not change from their initial steady-state values in any period. Moreover, in the long-run, $\frac{B_t}{P_t Y_t}$ stays at the same level as in the initial steady-state.
We then consider three different types of fiscal/monetary adjustment. First, only lump-sum transfers $S_t$ adjust. Second, only labor tax rates $\tau^H_t$ adjust as given by the simple feedback rule

$$\tau^H_t - \bar{\tau}^H = \psi \left( \frac{B_{t-1}}{P_{t-1}Y_{t-1}} - \frac{B}{PY} \right)$$

where $\psi \geq 1 - \beta$ is the feedback parameter on outstanding debt, $\bar{\tau}^H$ is the (new) steady-state value of $\tau^H_t$, and $\frac{B}{PY}$ is the steady-state value of $\frac{B_t}{PY}$. In both these cases, we have the monetary policy rule satisfying the Taylor principle, $\phi > 1$. Third, labor taxes adjust, but not sufficiently enough, as $0 < \psi < 1 - \beta$, and inflation partly plays a direct role in government debt stabilization, as $\phi < 1$. Thus, in this third case, we allow debt stabilization, along the transition, to occur partly through distortionary labor taxes and partly through inflation.

### A.4 Market Clearing

$$C_t + G_t + I_t + \Xi \left( \frac{P_t}{P_{t-1}} \right) Y_t = Y_t$$

$$\int_0^1 H_t(i) \, di = H_t$$

$$\int_0^1 K_t(i) \, di = K_t$$

$$K_{t+1} = (1 - d) K_t + \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right) I_t.$$

### A.5 Nonlinear Equilibrium Conditions

In this section, we derive the equilibrium conditions that are necessary to solve the model.

#### A.5.1 Firms

- Production function

$$Y_t(i) = \begin{cases} 
K_t(i) \lambda \left( A_t H_t(i) \right)^{1-\lambda} \\
\left( \lambda K_t(i) \frac{1}{e} + (1 - \lambda) \left( A_t H_t(i) \right)^\frac{1}{e} \right)^\frac{1}{e-1} 
\end{cases} \quad \text{if } \varepsilon = 1$$

$$K_t(i) \frac{1}{H_t(i)} = A_t^{1-\varepsilon} \left( \frac{W_t \lambda}{R_t^{1-\lambda}} \right)^\varepsilon$$

- Cost minimization: capital-labor ratio
• Cost minimization: nominal marginal cost

\[
MC_t = \begin{cases} 
\frac{1}{\lambda} \left( \frac{W_t}{A_t} \right)^{(1-\lambda)} R_t^K & \text{if } \varepsilon = 1 \\
\left( \lambda^\varepsilon (R_t^K)^{1-\varepsilon} + (1-\lambda)^\varepsilon \right)^{\frac{1}{1-\varepsilon}} & \text{otherwise}
\end{cases}
\]

• Profit maximization:

\[
\max_{P_t(i)} E_0 \sum_{t=0}^{\infty} \beta^t \frac{A_t}{A_0} P_t \left[ \left( \frac{P_t(i)}{P_t} - \frac{MC_t}{P_t} \right) Y_t(i) - \Xi \left( \frac{P_t(i)}{P_{t-1}(i)} \right) Y_t \right]
\]

where

\[
Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\theta} Y_t
\]

The first-order condition is:

\[
\Xi' \left( \frac{P_t(i)}{P_{t-1}(i)} \right) \frac{P_t}{P_{t-1}(i)} Y_t = (1 - \theta + \theta \frac{MC_t}{P_t(i)}) \left( \frac{P_t(i)}{P_t} \right)^{-\theta} Y_t 
\]

\[+ \beta E_t \frac{A_{t+1}}{A_t} \Xi' \left( \frac{P_{t+1}(i)}{P_t(i)} \right) \left( \frac{P_{t+1}P_{t+1}(i)}{(P_t(i))^2} \right) Y_{t+1}
\]

• Profit

\[
\Phi_t(i) = \frac{P_t(i)}{P_t} Y_t - \frac{W_t}{P_t} H_t - \frac{R_t^K}{P_t} K_t - \Xi \left( \frac{P_t(i)}{P_{t-1}(i)} \right) Y_t
\]

A.5.2 Households

• Maximization Problem:

\[
E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ C_t^{1-\eta} \left( \omega^{1-\eta} (H_t)^{1+\varphi} \right)^{\eta} - 1 \right] \right\}
\]

\[- E_0 \left\{ \sum_{t=0}^{\infty} \beta^t A_t \left\{ (1 + \tau^C) P_t C_t + P_t I_t + B_t \right\} \right\}
\]

\[+ E_0 \left\{ \sum_{t=0}^{\infty} \beta^t A_t \left\{ (1 - \tau^H) W_t H_t + R_{t-1} B_{t-1} + (1 - \tau^K) R_t K_t + P_t \int \Phi_t(i) di + P_t S_t \right\} \right\}
\]

\[+ E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \Psi_t \left\{ (1 - d) K_t + \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right) I_t - K_{t+1} \right\} \right\}
\]

32
• FOCs:

\[ C_t : \ P_t \Lambda_t (1 + \tau_C^t) = C_t^{-\eta} \left( 1 - \omega \frac{1 - \eta}{1 + \varphi} (H_t)^{1 + \varphi} \right)^{\eta} \]

\[ H_t : \ \Lambda_t (1 - \tau_H^t) W_t = \eta \omega \left( \frac{C_t}{1 - \omega \frac{1 - \eta}{1 + \varphi} (H_t)^{1 + \varphi}} \right)^{1-\eta} (H_t)\varphi \]

\[ B_t : \ \Lambda_t = \beta R_t E_t \left\{ \frac{P_t}{P_{t+1}} \right\} \]

\[ K_{t+1} : \ \Psi_t = \beta E_t \left\{ (1 - d) \Psi_{t+1} + (1 - \tau^K_{t+1}) R^K_{t+1} \Lambda_{t+1} \right\} \]

\[ I_t : \ P_t \Lambda_t = \Psi_t \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right) + \beta E_t \left\{ \Psi_{t+1} \left( \frac{I_{t+1}}{I_t} \right)^2 S' \left( \frac{I_{t+1}}{I_t} \right) \right\} \]

• Capital accumulation:

\[ K_{t+1} = (1 - d) K_t + \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right) I_t \]

A.5.3 Government and Market Clearing

• Government budget constraint

\[ \frac{B_t}{P_t Y_t} + (T^C_t + T^H_t + T^K_t) = R_{t-1} B_{t-1} \frac{1}{Y_{t-1}} Y_{t-1} + \frac{G_t}{Y_t} + \frac{S_t}{Y_t} \]

where

\[ T^C_t = \tau^C C_t Y_t, \ T^H_t = \tau^H H_t Y_t, \ T^K_t = \tau^K \frac{R^K}{P_t Y_t} K_t \]

• Resource constraint:

\[ C_t + I_t + G_t + \Xi \left( \frac{P_t}{P_{t-1}} \right) Y_t = Y_t \]

A.6 Stationary Equilibrium

We consider a symmetric equilibrium across firms, where all firms set the same price and produce the same amount of output.
A.6.1 Notations

Quantities: \( \tilde{Y}_t = \frac{Y_t}{A_t} \), \( \bar{C}_t = \frac{C_t}{A_t} \), \( \bar{I}_t = \frac{I_t}{A_t} \), \( \bar{K}_{t+1} = \frac{K_{t+1}}{A_t} \)

Prices: \( \tilde{w}_t = \frac{W_t}{P_t A_t} \), \( r^K_t = \frac{R^K_t}{P_t} \), \( \Phi_t = \frac{\Phi_t}{A_t} \), \( \pi_t = \frac{P_t}{P_{t-1}} \), \( mc_t = \frac{MC_t}{P_t} \)

Fiscal variables: \( \tilde{b}_t = \frac{B_t}{P_t Y_t} \), \( \tilde{G}_t = \frac{G_t}{Y_t} \), \( \tilde{T}_C^H = \frac{T_C^H}{Y_t} \), \( \tilde{T}_H^K = \frac{T_H^K}{Y_t} \), \( \tilde{S}_t = \frac{S_t}{Y_t} \)

Multipliers: \( \tilde{\Lambda}_t = \frac{P_t A_t^\eta \Lambda_t}{A_t} \), \( \tilde{\Psi}_t = \frac{A_t^\eta \Psi_t}{A_t} \)

A.6.2 Stationary Equilibrium Conditions

- Production function
  \[
  \bar{Y}_t = \begin{cases} 
  \bar{K}_t^{\lambda} H_t^{1-\lambda} & \text{if } \varepsilon = 1 \\
  \left( \frac{\varepsilon - 1}{\bar{K}_t^{\varepsilon - 1} + (1 - \lambda) H_t^{\varepsilon - 1}} \right)^{\frac{\varepsilon}{\varepsilon - 1}} & \text{otherwise}
  \end{cases}
  \] (A.3)

- Cost minimization: capital-labor ratio
  \[
  \frac{\bar{K}_t}{H_t} = \left( \frac{\tilde{w}_t}{r^K_t} \right)^{\frac{\lambda}{1 - \lambda}}
  \] (A.4)

- Cost minimization: nominal marginal cost
  \[
  mc_t = \begin{cases} 
  \frac{1}{\lambda^\varepsilon} \left( \frac{1}{1 - \lambda} \right)^{1-\lambda} \left( \frac{r^K_t}{\tilde{w}_t} \right)^{\lambda (1 - \lambda)} & \text{if } \varepsilon = 1 \\
  \left( \frac{\lambda^\varepsilon}{r^K_t^{1-\varepsilon}} + (1 - \lambda)^\varepsilon \right)^{\frac{1}{1-\varepsilon}} & \text{otherwise}
  \end{cases}
  \] (A.5)

- Firms’ maximization: Phillips Curve
  \[
  \Xi' (\pi_t) \pi_t = ((1 - \theta) + \theta mc_t) + \beta E_t \left\{ \frac{\bar{A}_{t+1} \bar{Y}_{t+1}}{\bar{Y}_t} (a_{t+1})^{1-\eta} \Xi' (\pi_{t+1}) \pi_{t+1} \right\}
  \] (A.6)

- Profit
  \[
  \bar{\Phi}_t = \bar{Y}_t - \hat{W}_t H_t - r^K_t \bar{K}_t - \Xi (\pi_t) \bar{Y}_t
  \] (A.7)

- Households
Marginal Utilities:

\[
\tilde{\Lambda}_t (1 + \tau^C) = \left( \frac{\tilde{C}_t}{1 - \tilde{\omega} \frac{1 - \eta}{1 + \phi} (H_t)^{1+\phi}} \right)^{-\eta} 
\]

\[
\tilde{\Lambda}_t (1 - \tau^H) \tilde{\omega} = \eta \tilde{\omega} \left( \frac{\tilde{C}_t}{1 - \tilde{\omega} \frac{1 - \eta}{1 + \phi} (H_t)^{1+\phi}} \right)^{1-\eta} (H_t)^{\phi} 
\]

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\[
\tilde{\omega}_t \frac{1 - \tau^H}{1 + \tau^C} = \eta \tilde{\omega} \frac{\tilde{C}_t (H_t)^{\phi}}{1 - \tilde{\omega} \frac{1 - \eta}{1 + \phi} (H_t)^{1+\phi}} 
\]

\[
\tilde{\Lambda}_t = \beta R_t E_t \left\{ \tilde{\Lambda}_{t+1} \left( \frac{1}{a_{t+1}} \right)^{\eta} \frac{1}{\pi_{t+1}} \right\} 
\]

\[
\tilde{\Psi}_t = \beta E_t \left\{ \left( \frac{1}{a_{t+1}} \right)^{\eta} \left( (1 - d) \tilde{\Psi}_{t+1} + (1 - \tau^K_{t+1}) \tilde{\Lambda}_{t+1} \right) \right\} 
\]

\[
\tilde{\Lambda}_t = \tilde{\Psi}_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} a_t \right] 
\]

\[
+ \beta E_t \left\{ \left( \frac{1}{a_{t+1}} \right)^{\eta} \tilde{\Psi}_{t+1} \left( \frac{I_{t+1}}{I_t} a_{t+1} \right)^2 S' \left( \frac{I_{t+1}}{I_t} a_{t+1} \right) \right\} 
\]

\[
\tilde{K}_{t+1} = (1 - d) \frac{1}{a_t} \tilde{K}_t + \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right) \tilde{I}_t 
\]

- Resource constraint

\[
\tilde{C}_t + \tilde{I}_t = \left( 1 - \Xi (\pi_t) - \tilde{G}_t \right) \tilde{Y}_t 
\]

- Government budget constraint

\[
\tilde{b}_t + \tilde{T}^C_t + \tilde{T}^H_t + \tilde{T}^K_t = R_{t-1} \tilde{b}_{t-1} \frac{1}{\pi_t a_t} \frac{\tilde{Y}_{t-1}}{Y_t} + \tilde{G}_t + \tilde{S}_t 
\]

where

\[
\tilde{T}^C_t = \tau^C \frac{\tilde{C}_t}{Y_t}, \quad \tilde{T}^H_t = \tau^H \frac{\tilde{w}_t H_t}{Y_t}, \quad \tilde{T}^K_t = \tau^K \frac{r^K \tilde{K}_t}{Y_t} 
\]
• Fiscal Policy Rules

\( \tau^H_t - \tilde{\tau}^H = \psi \left( \tilde{b}_{t-1} - \tilde{b} \right) \) \hspace{1cm} (A.16)

\( \tau^K_t = \begin{cases} \tilde{\tau}^K & \text{if } t = 0 \\ \tilde{\tau}^K_{new} & \text{if } t > 0 \end{cases} \) \hspace{1cm} (A.17)

\( \tilde{G}_t = \tilde{G} \) \hspace{1cm} (A.18)

• Monetary policy

\( \frac{R_t}{R} = \left( \frac{\pi_t}{\pi} \right)^\phi \) \hspace{1cm} (A.19)

• Government debt to GDP (or transfers):

\( \begin{cases} \tilde{b}_t = \tilde{b} & \text{if adjusting transfers} \\ \tilde{S}_t = \tilde{S} & \text{if adjusting labor tax rate} \end{cases} \) \hspace{1cm} (A.20)

• We have 18 equations from (A.3) to (A.20) and 18 variables to solve for the equilibrium:

\( \left\{ \tilde{Y}_t, \tilde{C}_t, \tilde{K}_t, \tilde{I}_t, H_t, \pi_t, mc_t, \tilde{\Phi}_t, \tilde{w}_t, r^K_t, R_t, \tilde{\Psi}_t, \tilde{b}_t, \tilde{S}_t, \tau^H_t, \tau^K_t, \tilde{G}_t \right\} \)

### A.7 Steady State

Recall that in steady-state, \( S(\bar{a}) = S'(\bar{a}) = 0 \):

From (A.6), we get

\( \bar{mc} = \frac{\theta - 1}{\theta} \).

From (A.11) and (A.12), we get

\( \bar{r}^K = \frac{\bar{a}^\eta - (1 - d)}{1 - \bar{r}^K} \). \hspace{1cm} (A.21)

Given \( \bar{r}^K \), (A.5) imply

\( \tilde{w} = \begin{cases} (1 - \lambda) \left( \bar{mc} \left( \frac{\lambda}{\bar{r}^K} \right)^\lambda \right)^{1/\lambda} & \text{if } \varepsilon = 1 \\ \left( \frac{\bar{mc}^{1-\varepsilon} - \lambda (\bar{r}^K)^{1-\varepsilon}}{(1-\lambda)^{1-\varepsilon}} \right)^{1/\varepsilon} & \text{otherwise} \end{cases} \) \hspace{1cm} (A.22)

Given \( \bar{r}^K \) and \( \tilde{w} \), we get

\( \frac{\tilde{K}}{\tilde{H}} = \left( \frac{\tilde{w}}{\bar{r}^K \frac{\lambda}{1 - \lambda}} \right)^\varepsilon \). \hspace{1cm} (A.23)
From the production function \((A.3)\), we get

\[
\frac{\ddot{Y}}{\bar{H}} = \begin{cases} 
\left(\frac{\ddot{Y}}{\bar{H}}\right)^{\lambda} & \text{if } \varepsilon = 1 \\
\lambda \left(\frac{\ddot{Y}}{\bar{H}}\right)^{\frac{\varepsilon - 1}{\varepsilon}} + (1 - \lambda) \right)^{\frac{1}{\varepsilon - 1}} & \text{otherwise}
\end{cases}
\tag{A.24}
\]

From \((A.13)\), we get

\[
\frac{\ddot{I}}{\bar{H}} = \frac{\ddot{K}}{\bar{H}} (\bar{a} - (1 - d)).
\tag{A.25}
\]

From \((A.14)\), we get

\[
\frac{\ddot{C}}{\bar{H}} = \left(1 - \ddot{G}\right) \frac{\ddot{Y}}{\bar{H}} - \frac{\ddot{I}}{\bar{H}}.
\tag{A.26}
\]

The nominal interest rate is obtained from Euler equation \((A.10)\)

\[
\ddot{R} = \frac{\ddot{a}^\eta \ddot{\pi}}{\beta}.
\]

From the government budget constraint \((A.15)\), we get

\[
\ddot{S} = \left(1 - \frac{\ddot{R}}{\bar{a}}\right) \frac{\ddot{b} - \ddot{G}}{\bar{w}} + \ddot{T}^C + \ddot{T}^H + \ddot{T}^K.
\tag{A.27}
\]

The consumption, labor income and capital income tax rates are respectively given as:

\[
\ddot{\tau}^C = \frac{\ddot{T}^C}{\ddot{Y}}, \quad \ddot{\tau}^H = \frac{1}{\bar{w}} \frac{\ddot{T}^h}{\ddot{Y}}, \quad \ddot{\tau}^K = \frac{1}{\ddot{r}^K} \frac{\ddot{T}^K}{\ddot{Y}}.
\]

We can obtain the steady-state hours \(\bar{H}\) from \((A.9)\)

\[
\bar{H} = \left(\frac{1}{\ddot{\omega} \left(\frac{1 - \eta}{1 + \varphi} + \eta \frac{1 + \ddot{\tau}^C}{1 - \ddot{r}^K} \frac{\ddot{C}}{\ddot{Y}}\right) \right)^{\frac{1}{1 + \varphi}}.
\tag{A.28}
\]

The steady-state hours depend on the preference parameter \(\ddot{\omega}\). We normalize \(\bar{H} = 0.25\) by calibrating \(\ddot{\omega}\) appropriately.

Finally, for model calibration, it is useful to express \(\ddot{r}^K\) in terms of capital tax revenue, \(\ddot{T}^K\) rather than capital tax rate \(\ddot{r}^K\). We replace \(\ddot{r}^K\) with \(\frac{1}{\ddot{r}^K} \frac{\ddot{T}^K}{\ddot{Y}}\) and then solve \(\ddot{r}^K\) as follows:

\[
\ddot{r}^K = \frac{\ddot{a}^\eta - (1 - d)}{1 - \ddot{r}^K} = \frac{\ddot{a}^\eta - (1 - d)}{1 - \frac{\ddot{T}^K}{\ddot{Y}}}
\]
which implies that \( \bar{r}^K \) is the solution of the following nonlinear equation:

\[
\bar{r}^K = \frac{\bar{a}^\eta}{\beta} - (1 - d) + \left( \frac{\bar{r}^K}{\lambda \bar{m}c} \right)^\varepsilon \bar{r}^K
\]

If \( \varepsilon = 1 \) (Cobb-Douglas), then we get

\[
\bar{r}^K = \frac{\bar{a}^\eta}{\beta} - (1 - d) \frac{1 - \bar{r}^K}{1 - \frac{\bar{r}^K}{\lambda \bar{m}c}}.
\]

**B  Proofs of Propositions**

**B.1  Steady-state ratio of capital tax revenue to output**

Notice that in steady-state, the capital tax revenue to output is

\[
\bar{T}^K = \bar{r}^K \bar{r}^K \bar{K} \bar{Y}
\]

\[
= \bar{r}^K \left( 1 - \varepsilon \left( \lambda \bar{m}c \right)^\varepsilon \right)
\]

where \( \bar{m}c = \frac{\theta^{-1}}{\theta} \). From \( \bar{r}^K = \frac{\bar{a}^\eta}{\beta} - (1 - d) \frac{1 - \bar{r}^K}{1 - \bar{r}^K} \), we get

\[
\bar{T}^K = \bar{r}^K \left( 1 - \bar{r}^K \right)^{\varepsilon - 1} \left( \frac{\bar{a}^\eta}{\beta} - (1 - d) \right)^{1 - \varepsilon} \left( \lambda \bar{m}c \right)^\varepsilon.
\]

For example, under Cobb-Douglas production function,

\[
\bar{T}^K = \bar{r}^K \lambda \bar{m}c.
\]

To show the existence of a Laffer curve, take a derivative for \( \bar{T}^K \) with respect to \( \bar{r}^K \):

\[
\frac{\partial \bar{T}^K}{\partial \bar{r}^K} = \left( \frac{\bar{a}^\eta}{\beta} - (1 - d) \right)^{1 - \varepsilon} \left( \lambda \bar{m}c \right)^\varepsilon \left[ (1 - \bar{r}^K)^{\varepsilon - 1} - (\varepsilon - 1) \bar{r}^K (1 - \bar{r}^K)^{\varepsilon - 2} \right].
\]

Notice that \( \frac{\partial \bar{T}^K}{\partial \bar{r}^K} < 0 \) if \( \bar{r}^K > \frac{1}{\varepsilon} \). Thus, for \( \bar{r}^K \in [0, 1] \), there is a Laffer curve if \( \bar{r}^K > \frac{1}{\varepsilon} \).

**B.2  Proof of Lemma 1**

**Proof.** From (A.21) and (A.22), we get

\[
\frac{\partial \bar{r}^K}{\partial \bar{r}^K} = \frac{\bar{r}^K}{1 - \bar{r}^K} > 0
\]

\[
\frac{\partial \bar{w}}{\partial \bar{r}^K} = -\left( \frac{\bar{w}}{\bar{r}^K} \right)^\varepsilon \left( \frac{\lambda}{1 - \lambda} \right)^\varepsilon \frac{\partial \bar{r}^K}{\partial \bar{r}^K} < 0.
\]
Let \( \tilde{k} = \frac{K}{H} \) and \( \tilde{y} = \frac{Y}{H} \). From (A.23) and (A.24), we get
\[
\frac{\partial \tilde{k}}{\partial \tilde{\tau} K} = -\tilde{\tau}^{2\epsilon - 1} \frac{1}{\epsilon - 1} \left( \frac{\tilde{r} K}{\lambda mc} \right)^{\epsilon - 1} \frac{1}{\tilde{r} K} \frac{\partial \tilde{r} K}{\partial \tilde{\tau} K} < 0
\]
\[
\frac{\partial \tilde{y}}{\partial \tilde{\tau} K} = \lambda \left( \frac{\tilde{y}}{\tilde{k}} \right)^{\frac{1}{2}} \frac{\partial \tilde{k}}{\partial \tilde{\tau} K} < 0
\]
Let \( \varepsilon = 1 \). Combining (A.22) and (A.26) with (A.28), we rewrite the steady-state hours as
\[
\tilde{H} = \left( \tilde{\omega} \left( 1 - \eta \frac{1 + \tau^C}{1 - \lambda} \left( \frac{1 - \tilde{G}}{\tilde{m}c} - \lambda \frac{\tilde{a} - (1 - d)}{\frac{\tilde{m}n}{\beta} - (1 - d)} (1 - \tilde{r} K) \right) \right) \right)^{-\frac{1}{1 + \varphi}}.
\]
Then, the partial derivative with respect to capital tax rate is
\[
\frac{\partial \tilde{H}}{\partial \tilde{\tau} K} = -\tilde{H}^{2 + \varphi} \left( \frac{\tilde{H}^{2 + \varphi}}{1 + \varphi} \right) \left( \frac{\lambda}{1 - \lambda} \frac{1 + \tau^C}{1 - \tilde{r} K} \frac{\tilde{a} - (1 - d)}{\frac{\tilde{m}n}{\beta} - (1 - d)} \right) < 0.
\]
Now, we find the partial derivatives of levels of variables. For capital, investment and output, we can easily verify that
\[
\frac{\partial \tilde{k}}{\partial \tilde{\tau} K} = \tilde{H} \frac{\partial \tilde{k}}{\partial \tilde{\tau} K} + \tilde{k} \frac{\partial \tilde{H}}{\partial \tilde{\tau} K} < 0
\]
\[
\frac{\partial \tilde{i}}{\partial \tilde{\tau} K} = \frac{\partial \tilde{k}}{\partial \tilde{\tau} K} (\tilde{a} - (1 - d)) < 0
\]
\[
\frac{\partial \tilde{Y}}{\partial \tilde{\tau} K} = \tilde{H} \frac{\partial \tilde{y}}{\partial \tilde{\tau} K} + \tilde{y} \frac{\partial \tilde{H}}{\partial \tilde{\tau} K} < 0
\]
For consumption, combining (A.24) and (A.25) with (A.26), we get
\[
\frac{\partial \tilde{C}}{\partial \tilde{\tau} K} = \left( 1 - \tilde{G} \right) \frac{\tilde{Y}}{\tilde{H}} - \tilde{\tilde{H}} \tilde{H}
\]
\[
= \left( \tilde{m}c \frac{\lambda (1 - \tau^K)}{\frac{\tilde{m}n}{\beta} - (1 - d)} \right)^{\frac{1}{1 - \lambda}} \left[ (1 - \tilde{G}) - \lambda \tilde{m}c (\tilde{a} - (1 - d)) (1 - \tilde{r} K) \right] \tilde{H}.
\]
Then, the partial derivative of consumption with respect to capital tax rate is
\[
\frac{\partial \tilde{C}}{\partial \tilde{\tau} K} = -\left( \frac{\lambda}{1 - \lambda} \frac{1}{1 - \tilde{r} K} \right) \left( \frac{\lambda \tilde{m}c (1 - \tau^K)}{\frac{\tilde{m}n}{\beta} - (1 - d)} \right)^{\frac{1}{1 - \lambda}} \left[ (1 - \tilde{G}) - \tilde{m}c (\tilde{a} - (1 - d)) (1 - \tilde{r} K) \right] \tilde{H} + \frac{\tilde{C}}{\tilde{H}} \frac{\partial \tilde{H}}{\partial \tilde{\tau} K}.
\]
Under Assumption 1, we find \( \frac{\partial \tilde{C}}{\partial \tilde{\tau} K} < 0 \).
B.3 Proof of Proposition 1

Proof. Let $\bar{\bar{\tau}}_{new} = \bar{\tau} + \Delta (\bar{\tau})$. From (A.21), we get

$$\frac{\bar{\bar{\tau}}_{new}}{\bar{\tau}} = \left( \frac{1 - \bar{\tau}}{1 - \bar{\tau}_{new}} \right) = \left( \frac{1 - \Delta (\bar{\tau})}{1 - \bar{\tau}} \right)^{-1}$$

Let $\varepsilon = 1$. From (A.22), (A.23) and (A.24), we get

$$\frac{\bar{\bar{\omega}}_{new}}{\bar{\omega}} = \left( \frac{1 - \Delta (\bar{\tau})}{1 - \bar{\tau}} \right)^{\frac{1}{1+\lambda}}$$

$$\frac{\bar{\bar{k}}_{new}}{\bar{k}} = \left( \frac{1 - \Delta (\bar{\tau})}{1 - \bar{\tau}} \right)^{\frac{1}{1-\lambda}}$$

$$\frac{\bar{\bar{y}}_{new}}{\bar{y}} = \left( \frac{1 - \Delta (\bar{\tau})}{1 - \bar{\tau}} \right)^{\frac{\lambda}{1-\lambda}}.$$

Combining (A.22) and (A.26) with (A.28), we get

$$\frac{\bar{\bar{H}}_{new}}{\bar{H}} = \left( \frac{1 - \eta}{1+\varphi} + \frac{\eta}{1-\lambda} \left( \frac{1 - \bar{\tau}C}{\bar{\tau}C - 1 - \bar{\tau}H} \right)^{\frac{1}{1-\lambda}} \right)^{-\frac{1}{1+\varphi}}$$

$$= \left( 1 + \bar{\omega} \bar{H}^{1+\varphi} \frac{\lambda \eta}{1-\lambda} \left( \frac{\bar{\tau}C}{\bar{\tau}C - 1 - \bar{\tau}H} \right)^{\frac{1}{1-\lambda}} \right) \left( \frac{1 - \Delta (\bar{\tau})}{1 - \bar{\tau}} \right)^{-\frac{1}{1+\varphi}}$$

where $\Omega = \bar{\omega} \bar{H}^{1+\varphi} \frac{\lambda \eta}{1-\lambda} \left( \frac{\bar{\tau}C}{\bar{\tau}C - 1 - \bar{\tau}H} \right)^{\frac{1}{1-\lambda}} > 0$.

Now, we find changes of levels of variables. For capital, investment and output, we can easily verify that

$$\frac{\bar{\bar{K}}_{new}}{\bar{K}} = \frac{\bar{k}_{new}}{\bar{k}} \frac{\bar{\bar{H}}_{new}}{\bar{H}} = \left( \frac{1 - \Delta (\bar{\tau})}{1 - \bar{\tau}} \right)^{\frac{1}{1+\lambda}} (1 + \Omega \Delta (\bar{\tau}))^{-\frac{1}{1+\varphi}}$$

$$\frac{\bar{\bar{I}}_{new}}{\bar{I}} = \frac{\bar{K}_{new}}{\bar{K}} = \left( \frac{1 - \Delta (\bar{\tau})}{1 - \bar{\tau}} \right)^{\frac{1}{1+\lambda}} (1 + \Omega \Delta (\bar{\tau}))^{-\frac{1}{1+\varphi}}$$
and

\[
\frac{\ddot{Y}_{\text{new}}}{\bar{Y}} = \left( \frac{\ddot{k}_{\text{new}}}{\dddot{k}} \right)^{\lambda} \frac{\dddot{H}_{\text{new}}}{H} = \left( 1 - \frac{\Delta (\ddot{\tau}^K)}{1 - \ddot{\tau}^K} \right)^{\frac{\lambda}{\lambda - 1}} (1 + \Omega \Delta (\ddot{\tau}^K))^{-\frac{1}{1 + \varphi}}.
\]

For consumption, combining (A.24) and (A.25) with (A.26), we get

\[
\frac{\dot{C}_{\text{new}}}{\dot{C}} = \left( \frac{\ddot{mc}}{\ddot{mc}_{\text{new}}} \right)^{\frac{\lambda}{\lambda - 1}} \left[ (1 - \ddot{G}) - \lambda \ddot{mc} \left( \frac{\bar{a} - (1 - d)}{\bar{a} - (1 - d)} \right) (1 - \ddot{\tau}_{\text{new}}^K) \right] \frac{\ddot{H}_{\text{new}}}{H} 
\]

\[
\frac{\dot{C}_{\text{new}}}{\dot{C}} = \left( 1 - \frac{\Delta (\ddot{\tau}^K)}{1 - \ddot{\tau}^K} \right)^{\frac{\lambda}{\lambda - 1}} \left( 1 + \frac{\lambda \ddot{mc} \left( \frac{\bar{a} - (1 - d)}{\bar{a} - (1 - d)} \right)}{(1 - \ddot{G}) - \lambda \ddot{mc} \left( \frac{\bar{a} - (1 - d)}{\bar{a} - (1 - d)} \right) (1 - \ddot{\tau}^K)} \right) \Delta \left( \ddot{\tau}^K \right) (1 + \Omega \Delta (\ddot{\tau}^K))^{-\frac{1}{1 + \varphi}}.
\]

Now for small changes in the capital tax rate \( \Delta (\ddot{\tau}^K) \), the percent changes of rental rate, wages, capital to hours ratio, output to hours ratio from their initial steady-states are:

\[
\ln \left( \frac{\ddot{\tau}^K_{\text{new}}}{\ddot{\tau}^K} \right) = - \ln \left( 1 - \frac{\Delta (\ddot{\tau}^K)}{1 - \ddot{\tau}^K} \right) \simeq \left( \frac{1}{1 - \ddot{\tau}^K} \right) \Delta (\ddot{\tau}^K).
\]

\[
\ln \left( \frac{\ddot{w}_{\text{new}}}{w} \right) = \frac{\lambda}{1 - \lambda} \ln \left( 1 - \frac{\Delta (\ddot{\tau}^K)}{1 - \ddot{\tau}^K} \right) \simeq - \left( \frac{\lambda}{1 - \lambda} \frac{1}{1 - \ddot{\tau}^K} \right) \Delta (\ddot{\tau}^K)
\]

\[
\ln \left( \frac{\ddot{k}_{\text{new}}}{\ddot{k}} \right) = \frac{1}{1 - \lambda} \ln \left( 1 - \frac{\Delta (\ddot{\tau}^K)}{1 - \ddot{\tau}^K} \right) \simeq - \left( \frac{1}{1 - \lambda} \frac{1}{1 - \ddot{\tau}^K} \right) \Delta (\ddot{\tau}^K)
\]

\[
\ln \left( \frac{\ddot{g}_{\text{new}}}{\ddot{g}} \right) = \frac{\lambda}{1 - \lambda} \ln \left( 1 - \frac{\Delta (\ddot{\tau}^K)}{1 - \ddot{\tau}^K} \right) \simeq - \left( \frac{\lambda}{1 - \lambda} \frac{1}{1 - \ddot{\tau}^K} \right) \Delta (\ddot{\tau}^K)
\]

and

\[
\ln \left( \frac{\ddot{H}_{\text{new}}}{\dddot{H}} \right) = - \frac{1}{1 + \varphi} \ln \left( 1 + \Omega \Delta (\ddot{\tau}^K) \right) \simeq - \frac{1}{1 + \varphi} (1 + \Omega) \Delta (\ddot{\tau}^K).
\]
The percent changes of levels of capital and investment from their initial steady-states are:

\[
\ln \left( \frac{\tilde{K}_{\text{new}}}{\bar{K}} \right) = \ln \left( \frac{\tilde{I}_{\text{new}}}{\bar{I}} \right) \approx - \left( \frac{1}{(1 - \lambda)(1 - \bar{\tau}K)} + \frac{\Omega}{1 + \varphi} \right) \Delta (\bar{\tau}K) \\
= -M_K \Delta (\bar{\tau}K)
\]

where \(M_K = \frac{1}{(1 - \lambda)(1 - \bar{\tau}K)} + \frac{\Omega}{1 + \varphi} > 0\). Also, the percent change of output from the initial steady-state is:

\[
\ln \left( \frac{\tilde{Y}_{\text{new}}}{\bar{Y}} \right) \approx - \left( \frac{\lambda}{(1 - \lambda)(1 - \bar{\tau}K)} + \frac{\Omega}{1 + \varphi} \right) \Delta (\bar{\tau}K) \\
= -M_Y \Delta (\bar{\tau}K)
\]

where \(M_Y = \frac{\lambda}{(1 - \lambda)(1 - \bar{\tau}K)} + \frac{\Omega}{1 + \varphi} > 0\).

Finally, the percent change of consumption from its initial steady-state is:

\[
\ln \left( \frac{\tilde{C}_{\text{new}}}{\bar{C}} \right) \approx - \left[ \frac{\lambda}{(1 - \lambda)(1 - \bar{\tau}K)} + \frac{\Omega}{1 + \varphi} - \frac{\lambda \bar{m}c \frac{(\tilde{a} - (1 - d))}{\bar{\omega} \beta - (1 - d)}}{1 - \tilde{G}} - \frac{\lambda \bar{m}c \frac{(\tilde{a} - (1 - d))}{\bar{\omega} \beta - (1 - d)} (1 - \bar{\tau}K)}{1 - \tilde{G} - \lambda \bar{m}c \frac{(\tilde{a} - (1 - d))}{\bar{\omega} \beta - (1 - d)} (1 - \bar{\tau}K)} \right] \Delta (\bar{\tau}K) \\
= -\frac{\lambda}{1 - \lambda} \left( \frac{\tilde{a} - (1 - d)}{\bar{\omega} \beta - (1 - d)} \right) \times \\
\left\{ \frac{\eta \bar{\omega} \bar{H}^{1 + \varphi} \left( 1 + \frac{\bar{c}}{\bar{H}} \right)}{1 + \varphi} + \frac{\left( 1 - \tilde{G} \right) - \bar{m}c \left( \frac{\tilde{a} - (1 - d)}{\bar{\omega} \beta - (1 - d)} \right) (1 - \bar{\tau}K)}{\left( 1 - \tilde{G} \right) - \lambda \bar{m}c \frac{(\tilde{a} - (1 - d))}{\bar{\omega} \beta - (1 - d)} (1 - \bar{\tau}K)} \right\} \Delta (\bar{\tau}K) \\
= -M_C \Delta (\bar{\tau}K).
\]

Notice that under Assumption 1, the numerator of the second term in the large bracket is greater than zero. Thus, we have \(M_C = M_Y - \frac{\tilde{C}}{\bar{C}} \left( \frac{\tilde{a} - (1 - d)}{\bar{H} \left( 1 - \bar{\tau}K \right)} \right)^{-1} > 0\).

\[\square\]

**B.4 Proof of Lemma 2**

**Proof.** Notice that rental rate of capital, wage, capital to hours ratio, and output to hours ratio are the same with the lump-sum transfers adjustment case in B.2.
Let $\varepsilon = 1$ and $\eta = 1$. To show hours are increasing in $\tau^K$, we rewrite (A.27) as the following:

$$
\ddot{\tilde{S}} = \left(1 - \frac{\ddot{R}}{\ddot{a}}\right) \ddot{b} - \ddot{G} + \ddot{\tau^C} \ddot{\tilde{C}} \frac{\ddot{C}}{\ddot{Y}} + \ddot{\tau^H} \ddot{\tilde{H}} \frac{\ddot{H}}{\ddot{Y}} + \ddot{\tau^K} \ddot{\tilde{K}} \frac{\ddot{K}}{\ddot{Y}}
$$

$$
= \left(1 - \frac{\ddot{R}}{\ddot{a}}\right) \ddot{b} - \ddot{G} + \ddot{\tau^C} \left[ \left(1 - \ddot{G}\right) - \frac{\ddot{\lambda} \ddot{m} \ddot{c} (\ddot{a} - (1 - d))}{\ddot{a} \ddot{Y} - (1 - d)} \left(1 - \ddot{\tau}^K\right) \right] + \ddot{\tau^H} \left(1 - \ddot{\lambda} \ddot{m} \ddot{c} + \ddot{\tau^K} \ddot{\lambda} \ddot{m} \ddot{c}\right)
$$

$$
1 - \ddot{\tau}^H = \frac{\left(1 - \ddot{\lambda} \ddot{m} \ddot{c} - \ddot{S} + \left(1 - \frac{\ddot{R}}{\ddot{a}}\right) \ddot{b} - \ddot{G} + \ddot{\tau^C} \left[ \left(1 - \ddot{G}\right) - \frac{\ddot{\lambda} \ddot{m} \ddot{c} (\ddot{a} - (1 - d))}{\ddot{a} \ddot{Y} - (1 - d)} \left(1 - \ddot{\tau}^K\right) \right] + \ddot{\tau^K} \ddot{\lambda} \ddot{m} \ddot{c}\right)}{(1 - \ddot{\lambda} \ddot{m} \ddot{c})}
$$

Then, from (A.28), we get

$$
\ddot{\bar{H}} = \left(\frac{\ddot{\omega} \left(1 + \ddot{\tau}^C\right) \left(1 - \ddot{G} - \frac{\ddot{\lambda} \ddot{m} \ddot{c} (\ddot{a} - (1 - d))}{\ddot{a} \ddot{Y} - (1 - d)} \left(1 - \ddot{\tau}^K\right)\right)}{(1 - \ddot{\lambda} \ddot{m} \ddot{c} - \ddot{S} + \left(1 - \frac{\ddot{R}}{\ddot{a}}\right) \ddot{b} - \ddot{G} + \ddot{\tau^C} \left[ \left(1 - \ddot{G}\right) - \frac{\ddot{\lambda} \ddot{m} \ddot{c} (\ddot{a} - (1 - d))}{\ddot{a} \ddot{Y} - (1 - d)} \left(1 - \ddot{\tau}^K\right) \right] + \ddot{\tau^K} \ddot{\lambda} \ddot{m} \ddot{c}\right)}\right)^{-\frac{1}{1+\ddot{\varphi}}}
$$

Taking a partial derivative with respect to capital tax rate gives:

$$
\frac{\partial \ddot{\bar{H}}}{\partial \ddot{\tau}^K} = \frac{1 - \ddot{\tau}^H}{\ddot{\omega} (1 + \ddot{\tau}^C) \ddot{\tilde{C}} \ddot{\frac{\ddot{Y}}{\ddot{Y}}}} \left[ \ddot{\tau^C} \frac{\ddot{\lambda} \ddot{m} \ddot{c} (\ddot{a} - (1 - d))}{\ddot{a} \ddot{Y} - (1 - d)} + \ddot{\lambda} \ddot{m} \ddot{c} \left(1 - \ddot{\tau}^H\right) \right]
$$

$$
= \frac{1 - \ddot{\tau}^H}{\ddot{\omega} (1 + \ddot{\tau}^C) \ddot{\tilde{C}} \ddot{\frac{\ddot{Y}}{\ddot{Y}}}} \left[ \ddot{\tau^C} \ddot{\frac{\ddot{C}}{\ddot{Y}}} + \ddot{\frac{\ddot{\lambda} \ddot{m} \ddot{c} (\ddot{a} - (1 - d))}{\ddot{a} \ddot{Y} - (1 - d)}} \ddot{\frac{\ddot{C}}{\ddot{Y}}} - (1 - \ddot{\lambda} \ddot{m} \ddot{c} (1 - \ddot{\tau}^H)) \ddot{\frac{\ddot{C}}{\ddot{Y}}} \right]
$$

$$
= \frac{1 - \ddot{\tau}^H}{\ddot{\omega} (1 + \ddot{\tau}^C) \ddot{\tilde{C}} \ddot{\frac{\ddot{Y}}{\ddot{Y}}}} \left[ \ddot{\tau^C} \ddot{\tilde{C}} \ddot{\frac{\ddot{C}}{\ddot{Y}}} + \ddot{\tau^H} \ddot{\tilde{H}} \ddot{\frac{\ddot{H}}{\ddot{Y}}} + \ddot{\tau^K} \ddot{\tilde{K}} \ddot{\frac{\ddot{K}}{\ddot{Y}}} \ddot{\frac{\ddot{C}}{\ddot{Y}}} - (1 - \ddot{\lambda} \ddot{m} \ddot{c} (1 - \ddot{\tau}^H)) \ddot{\frac{\ddot{C}}{\ddot{Y}}} \right]
$$

Under Assumption 2, $\frac{\partial \ddot{\bar{H}}}{\partial \ddot{\tau}^K} > 0$. □
B.5 Proof of Proposition 2

Proof. Let \( \bar{\tau}_n^K = \bar{\tau}^K + \Delta (\bar{\tau}^K) \). Under the labor tax adjustment case, the steady-state labor tax rate is:

\[
\bar{\tau}^H = \frac{\bar{S} - \left[ \left( 1 - \frac{\bar{R}}{\bar{W}} \right) \bar{b} - \bar{G} + \bar{C}^C + \bar{K}^K \right]}{\bar{w}_Y^H}
\]

Then, after capital tax rate changes, the new steady-state labor tax rate is given by:

\[
\begin{align*}
\bar{\tau}_n^H &= \frac{\bar{S} - \left[ \left( 1 - \frac{\bar{R}}{\bar{W}} \right) \bar{b} - \bar{G} + \bar{C}^C \left( (1 - \bar{G}) - \frac{\lambda \bar{m} e (\bar{a} - (1 - d))}{\bar{w} - (1 - d)} \right) (1 - \bar{\tau}^K_n) \right] + \bar{\tau}^K_n \lambda \bar{m} e }{(1 - \lambda) \bar{m} e} \\
&\quad = \bar{\tau}^H - \frac{\lambda}{1 - \lambda} \left[ 1 + \bar{\tau}^C \left( (1 - \bar{G}) - \frac{\lambda \bar{m} e (\bar{a} - (1 - d))}{\bar{w} - (1 - d)} \right) \right] \Delta \left( \bar{\tau}^K_n \right) \\
&\quad = \bar{\tau}^H + \Delta \left( \bar{\tau}^H \right).
\end{align*}
\]

Let \( \varepsilon = 1 \) and \( \eta = 1 \). Notice that the relative changes of rental rate, wage and capital to hours ratio from their initial steady-states are the same with the lump-sum transfers adjustment case. For after-tax wage, from (A.22) and (B.2), we get

\[
\ln \left( \frac{1 - \bar{\tau}_n^H}{1 - \bar{\tau}^H} \frac{\bar{w}_{\text{new}}}{\bar{w}} \right) = \ln \left( \frac{1}{1 - \frac{\lambda}{1 - \bar{\tau}^H}} \left( 1 + \bar{\tau}^C \left( (1 - \bar{G}) - \frac{\lambda \bar{m} e (\bar{a} - (1 - d))}{\bar{w} - (1 - d)} \right) \right) \left( 1 - \frac{\Delta \left( \bar{\tau}^K_n \right)}{1 - \bar{\tau}^H} \right) \right).
\]

\[
\simeq \frac{\lambda}{1 - \lambda} \left( 1 - \frac{1}{1 - \bar{\tau}^H} \right) \left( 1 + \bar{\tau}^C \left( (1 - \bar{G}) - \frac{\lambda \bar{m} e (\bar{a} - (1 - d))}{\bar{w} - (1 - d)} \right) \right) - \frac{1}{1 - \bar{\tau}^H} \Delta \left( \bar{\tau}^K_n \right)
\]

where \( \mathcal{M}_W > 0 \) if \( 1 + \bar{\tau}^C \left( (1 - \bar{G}) - \frac{\lambda \bar{m} e (\bar{a} - (1 - d))}{\bar{w} - (1 - d)} \right) > \frac{1 - \frac{\bar{\tau}^H}{1 - \bar{\tau}^H}}{1 - \bar{\tau}^H} \). For hours, from (B.1), we get

\[
\begin{align*}
\frac{\bar{H}_n}{H} &= \left( (1 - \lambda) \bar{m} e - \bar{S} + \left( 1 - \frac{\bar{R}}{\bar{W}} \right) \bar{b} - \bar{C}^C \left( (1 - \bar{G}) - \frac{\lambda \bar{m} e (\bar{a} - (1 - d))}{\bar{w} - (1 - d)} \right) (1 - \bar{\tau}^K_n) \right) \frac{1}{\bar{m} e} \\
&\quad + (\bar{\tau}^K_n + \Delta \left( \bar{\tau}^K_n \right)) \lambda \bar{m} e + \frac{\bar{C}^C}{\bar{w} - (1 - d)} \left( (1 - \bar{G}) - \frac{\lambda \bar{m} e (\bar{a} - (1 - d))}{\bar{w} - (1 - d)} \right) (1 - \bar{\tau}^K_n) \lambda \bar{m} e \\
&= 1 + \left[ \frac{\lambda \bar{m} e + \frac{\bar{C}^C}{\bar{w} - (1 - d)}}{(1 - \bar{\tau}^H) (1 - \lambda) \bar{m} e} \right] \Delta \left( \bar{\tau}^K_n \right) \left( \frac{\bar{\tau}^K_n}{1 - \bar{\tau}^H} \right)
\end{align*}
\]
Then, for small changes of capital tax rate $\Delta (\bar{\tau}^K)$, we get:

$$
\ln \left( \frac{\tilde{H}_{new}}{\tilde{H}} \right) = \frac{1}{1+\varphi} \left( \frac{\lambda \bar{m}c (\bar{a} - (1-d))}{\bar{a} - (1-d)} \right) \left[ 1 - \bar{G} - \frac{\lambda \bar{m}c (\bar{a} - (1-d))(1-\bar{\tau}^K)}{\bar{a} - (1-d)} \right] \Delta (\bar{\tau}^K)
$$

$$
= \frac{1}{1+\varphi} \frac{\lambda \bar{m}c (\bar{a} - (1-d))}{\bar{a} - (1-d)} \left[ \frac{\tilde{\tau}^C \tilde{C}}{Y} + \frac{\bar{a} \eta - (1-d)}{\bar{a} - (1-d)} \left( 1 - \bar{G} \right) - \left( 1 - \bar{\tau}^K \right) \frac{\bar{k}}{Y} \bar{r}^K - (1-\lambda) \bar{m}c (1 - \bar{\tau}^H) \right] \Delta (\bar{\tau}^K)
$$

$$
= \frac{1}{1+\varphi} \frac{\lambda \bar{m}c (\bar{a} - (1-d))}{\bar{a} - (1-d)} \left[ \frac{\tilde{\tau}^C \tilde{C}}{Y} + \frac{\bar{a} \eta - (1-d)}{\bar{a} - (1-d)} \left( 1 - \bar{G} \right) - \left( 1 - \bar{\tau}^K \right) \frac{\bar{m}c (\bar{a} - (1-d))}{\bar{a} - (1-d)} \right] \Delta (\bar{\tau}^K)
$$

$$
= \frac{1}{1+\varphi} \frac{\lambda \bar{m}c (\bar{a} - (1-d))}{\bar{a} - (1-d)} \left[ \frac{\tilde{\tau}^C \tilde{C}}{Y} + \frac{\bar{a} - (1-d)}{\bar{a} - (1-d)} \left( 1 - \bar{G} \right) - \left( 1 - \bar{\tau}^K \right) \bar{c} \right] \Delta (\bar{\tau}^K)
$$

$$
M_H \Delta (\bar{\tau}^K)
$$

where $M_H > 0$ under Assumption 2.

**B.6 Proof of Proposition 3**

*Proof.* Let $\varepsilon = 1$ and $\eta = 1$. From (A.21), (A.22) and (A.23), we get

$$
\frac{\tilde{K}}{\tilde{H}} = \frac{\bar{\omega}}{\bar{r}^K} \frac{\lambda}{1 - \lambda} = \left( \frac{\lambda \bar{m}c}{\bar{a} \eta - (1-d)} \right) \left( 1 - \bar{\tau}^K \right)^{1-x}.
$$

The amount of changes in capital to hours ratio to the capital tax cut is the same in both lump-sum transfers adjustment case and labor tax rate case. In a similar way, we know that output to hours ratio, investment to hours ratio, and consumption to hours ratio change by the same amount in both cases. Thus, all the magnitudes of changes in macro quantities to capital tax cuts are determined by the hours responses. Now, we compare the changes in hours to capital tax rate changes under the transfers adjustment case with the changes under the labor tax rate adjustment case. Notice that the initial steady-states are the same in both cases. Let $\tilde{H}_{new}^T$ and $\tilde{H}_{new}^L$ denote the steady-state hours after the capital tax changes in the transfers adjustment case and in the
labor tax rate adjustment case, respectively. Then, from (A.28) and (B.2), we get

\[
\frac{\bar{H}_T}{\bar{H}^L_{new}} = \left( \frac{\frac{\lambda}{(1-\lambda)\bar{m}_c} 1+\phi C}{1+\phi \bar{H}^{new}} \left( 1 - \bar{G} - \frac{\lambda \bar{m}_c (\bar{a}-(1-d))}{\bar{\alpha}_G} \frac{1}{\bar{K}^{new}} \right) \right)^{-\frac{1}{1+\phi}}
\]

\[
= \left( 1 - \frac{\Delta (\bar{\tau}^H)}{1 - \bar{\tau}^H} \right)^{-\frac{1}{1+\phi}}
\]

\[
= \left( 1 + \frac{\lambda}{1-\lambda} \left( \frac{1}{1 - \bar{\tau}^H} \right) \left( 1 + \bar{\tau}^C (\bar{\alpha} - (1-d)) \Delta (\bar{\tau}^K) \right) \right)^{-\frac{1}{1+\phi}}
\]

For small changes in capital tax rate \( \Delta (\bar{\tau}^K) \), we get

\[
\ln \left( \frac{\bar{H}_T^{new}}{\bar{H}^L_{new}} \right) = -\frac{1}{1+\varphi} \frac{\lambda}{1-\lambda} \left( \frac{1}{1 - \bar{\tau}^H} \right) \left( 1 + \bar{\tau}^C (\bar{\alpha} - (1-d)) \right) \Delta (\bar{\tau}^K)
\]

\[
= -\Theta \Delta (\bar{\tau}^K)
\]

where \( \Theta = \frac{1}{1+\varphi} \frac{\lambda}{1-\lambda} \left( \frac{1}{1 - \bar{\tau}^H} \right) \left( 1 + \bar{\tau}^C (\bar{\alpha} - (1-d)) \right) > 0 \). Then, for the levels of output, consumption, capital and investment, the differences are the same: that is,

\[
\ln \left( \frac{\bar{Y}_T^{new}}{\bar{Y}^L_{new}} \right) = \ln \left( \frac{\bar{C}_T^{new}}{\bar{C}^L_{new}} \right) = \ln \left( \frac{\bar{K}_T^{new}}{\bar{K}^L_{new}} \right) = \ln \left( \frac{\bar{I}_T^{new}}{\bar{I}^L_{new}} \right) = \ln \left( \frac{\bar{H}_T^{new}}{\bar{H}^L_{new}} \right) = -\Theta \Delta (\bar{\tau}^K)
\]

\[
\square
\]

**B.7 Changes in Output with Infinite Frisch Elasticity**

How do the changes in output to the capital tax rate vary with the different Frisch elasticity parameters under labor tax rate adjustment case? Let \( \eta = 1 \) and \( \varepsilon = 1 \). Notice that from (B.1), we get

\[
\ln \left( \bar{Y}_T^{new} \right) = \ln \left( \bar{Y}_T^{new} \right) + \ln \left( \bar{H}_T^{new} \right)
\]

\[
= -\left( \frac{\lambda}{1-\lambda} \frac{1}{1 - \bar{\tau}^H} - \mathcal{M}_H \right) \Delta (\bar{\tau}^K)
\]
where \( \mathcal{M}_H = \frac{1}{1+\varphi} \frac{\lambda}{(1-\lambda)} \frac{(\bar{a}-(1-d))}{\frac{\theta}{\beta}-(1-d)} \left[ \bar{T} + \frac{\tilde{g}}{(\alpha)-(1-d)} \right] \left( 1 - \bar{G} - \frac{\bar{m}c(\bar{a}-(1-d))}{\frac{\theta}{\beta}-(1-d)} \right) \) and \( \bar{T} = \bar{T}^C + \bar{T}^H + \bar{T}^K \). Then, we can rewrite \( \ln \left( \frac{\bar{Y}_{\text{new}}}{Y} \right) \) as

\[
- \frac{\ln \left( \frac{\bar{Y}_{\text{new}}}{Y} \right)}{\Delta \left( \bar{T}^K \right)} = \frac{\lambda}{1-\lambda} \left[ \frac{1}{1-\bar{T}^K} - \frac{1}{1-\bar{T}^K} \right] \left[ \bar{T} C + \bar{T} H + \bar{T} K - \bar{m}c \right] + \left( 1 - \bar{G} \right) \left( 1 - \bar{G} \right) \left( 1 - \bar{T}^K \right) \left( 1 - \bar{T}^H \right) \]

Notice that under Assumption 2, the second term in the RHS is positive. Thus, \( - \frac{\ln \left( \frac{\bar{Y}_{\text{new}}}{Y} \right)}{\Delta \left( \bar{T}^K \right)} \) is increasing in \( \varphi \) and it has a lower bound at \( \frac{1}{\varphi} = \infty \). That is, for \( \Delta \left( \bar{T}^K \right) < 0 \),

\[
\frac{\lambda}{1-\lambda} \left[ \frac{1}{1-\bar{T}^K} - \frac{1}{1-\bar{T}^K} \right] \left( \bar{T} C + \bar{T} H + \bar{T} K - \bar{m}c \right) + \left( 1 - \bar{G} \right) \left( 1 - \bar{G} \right) \left( 1 - \bar{T}^K \right) \left( 1 - \bar{T}^H \right) \leq - \frac{\ln \left( \frac{\bar{Y}_{\text{new}}}{Y} \right)}{\Delta \left( \bar{T}^K \right)} \leq \frac{\lambda}{1-\lambda} \left( \frac{1}{1-\bar{T}^K} \right).
\]

The lower bound is:

\[
\frac{\lambda}{1-\lambda} \left[ \frac{1}{1-\bar{T}^K} - \frac{1}{1-\bar{T}^K} \right] \left( \bar{T} C + \bar{T} H + \bar{T} K - \bar{m}c \right) + \left( 1 - \bar{G} \right) \left( 1 - \bar{G} \right) \left( 1 - \bar{T}^K \right) \left( 1 - \bar{T}^H \right) \]

Thus, the change in output is positive even under the infinite Frisch elasticity if \( \frac{1-\bar{T}^H}{1-\bar{T}^K} > 1 + \bar{T}^C \).
C Data Appendix

We calibrate the steady-state fiscal variables using US quarterly data for the post-Volcker period from 1982:Q4 to 2008:Q2.

C.1 Debt and spending data

We use the following definitions for our debt and spending variables:

- Government debt = market value of privately held gross federal debt;
- Government expenditures = government consumption;

Note that we use a single price level, GDP deflator, for both variables.

The market value of privately held gross federal debt series was obtained from Federal Reserve Bank of Dallas and the government consumption data series was taken from National Income and Product Accounts (NIPA) tables.

C.2 Tax data

We follow a method originally based on Jones (2002). Additionally, we use the tax revenues of the federal government and local property taxes.

We use federal taxes on production and imports (lines 4 of NIPA Table 3.2) for consumption tax revenues. Let this be \( T_C \).

The average personal income tax rate is computed to get both capital tax revenues and labor tax revenues. We first compute the average personal income tax rate as

\[
\tau^P = \frac{IT}{W + PRI/2 + CI}
\]

where \( IT \) is the personal current tax revenues (line 3 of NIPA Table 3.2), \( W \) is wage and salary accruals (line 3 of NIPA Table 1.12), \( PRI \) is proprietor’s income (line 9 of NIPA Table 1.12), and \( CI \) is capital income, which is the sum of rental income (line 12 of NIPA Table 1.12), corporate profits (line 13 of NIPA Table 1.12), interest income (line 18 of NIPA Table 1.12), and \( PRI/2 \). We here regard half of proprietor’s income as wage labor income and the other half as capital income.

Then the capital tax revenue is

\[
T^K = \tau^P CI + CT + PT
\]

where \( CT \) is taxes on corporate income (line 7 of NIPA Table 3.2), and \( PT \) is property taxes (line 8 of NIPA Table 3.3). In NIPA, home owners are thought of as renting their houses to themselves and thus property taxes are included as taxes on rental income or capital income. The labor tax revenue is computed

\[
T^H = \tau^P (W + PRI/2) + CSI
\]
where $CSI$ is contributions for government social insurance (line 11 of NIPA Table 3.2).
D Appendix Figures

Figure D.5: Long-run Effects of Permanent Capital Tax Rate Changes on Fiscal Variables

- **Labor Tax Revenue (Level)**: \( \bar{T}_H \bar{Y} \)
- **Capital Tax Revenue (Level)**: \( \bar{T}_K \bar{Y} \)
- **Consumption Tax Revenue (Level)**: \( \bar{T}_C \bar{Y} \)
- **Total Tax Revenue (Level)**: \( (\bar{T}_H + \bar{T}_K + \bar{T}_C) \bar{Y} \)
- **Transfers (Level)**: \( \bar{S} \bar{Y} \)
- **Debt (Level)**: \( \bar{b} \bar{Y} \)
- **Government Spending (Level)**: \( \bar{G} \bar{Y} \)
- **Labor Tax Revenue to Output**: \( \bar{T}_H \bar{Y} \)
- **Capital Tax Revenue to Output**: \( \bar{T}_K \bar{Y} \)
- **Consumption Tax Revenue to Output**: \( \bar{T}_C \bar{Y} \)
- **Total Tax Revenue to Output**: \( (\bar{T}_H + \bar{T}_K + \bar{T}_C) \bar{Y} \)
- **Debt to Output**: \( \bar{b} \bar{Y} \)
- **Government Spending to Output**: \( \bar{G} \bar{Y} \)
- **Labor Tax Rate**: \( \bar{\tau}_H \)
- **Capital Tax Rate**: \( \bar{\tau}_K \)

Transfers Adjustment  
Labor Tax Rate Adjustment
Figure D.6: Degree of Price Rigidity: Labor Tax Rate Adjustment

- Consumption ($\tilde{C}_t$)
- Investment ($\tilde{I}_t$)
- Output ($\tilde{Y}_t$)
- Hours ($H_t$)
- Wage ($\tilde{w}_t$)
- After Tax Wage ($((1 - \tau^w H_t) \tilde{w}_t)$)
- Capital Rental Rate ($r^K_t$)
- After Tax Rental Rate ($((1 - \tau^K K_t) r^K_t)$)
- Inflation ($\pi_t$)
- After Tax Capital to Labor Income Ratio ($\frac{(1 - \tau^K K_t) (1 - \tau^w H_t) \tilde{w}_t H_t}{\tilde{K}_t (1 - \tau^K K_t)}$)
- Labor Tax Revenue ($\tilde{T}^H_t$)
- Capital Tax Revenue ($\tilde{T}^K_t$)
- Transfers ($\tilde{S}_t$)
- Labor Tax Rate ($\tau^H_t$)
- Capital Tax Rate ($\tau^K_t$)
- Debt to Output ($\tilde{b}_t$)

Price Adjustment Cost ($\kappa = 0$) — Price Adjustment Cost ($\kappa = 50$) — Price Adjustment Cost ($\kappa = 100$)
Figure D.7: Degree of Monetary Policy Inflation Response: Labor Tax Rate Adjustment

- Consumption ($\tilde{C}_t$)
- Investment ($\tilde{I}_t$)
- Output ($\tilde{Y}_t$)
- Hours ($\tilde{H}_t$)
- Wage ($\tilde{w}_t$)
- After Tax Wage ($((1 - \tau^H)\tilde{w}_t)$)
- Capital Rental Rate ($\tilde{r}_K^H$)
- After Tax Rental Rate ($((1 - \tau^K)\tilde{r}_K^H)$)
- Inflation ($\pi_t$)
- After Tax Capital to Labor Income Ratio ($\frac{(1 - \tau^K)\tilde{w}_t}{\tilde{Y}_t}$)
- Labor Tax Revenue ($\tilde{T}_H^H$)
- Capital Tax Revenue ($\tilde{T}_K^H$)
- Transfers ($\tilde{S}_t$)
- Labor Tax Rate ($\tau^H_t$)
- Capital Tax Rate ($\tau^K_t$)
- Debt to Output ($\tilde{b}_t$)

Legend:
- MP Inflation Feedback($\phi = 1.2$) (blue dashed line)
- MP Inflation Feedback($\phi = 1.5$) (black solid line)
- MP Inflation Feedback($\phi = 1.8$) (red dotted line)
Figure D.8: Anticipated Permanent Tax Rate Changes: Transfers Adjustment

- Consumption ($\tilde{C}_t$)
- Investment ($\tilde{I}_t$)
- Output ($\tilde{Y}_t$)
- Hours ($\tilde{H}_t$)
- Wage ($\tilde{w}_t$)
- After Tax Wage ($((1-\tau^H)\tilde{w}_t)$)
- Capital Rental Rate ($r^K_t$)
- After Tax Rental Rate ($((1-\tau^K)r^K_t)$)
- Inflation ($\pi_t$)
- After Tax Capital to Labor Income Ratio ($\left(\frac{1-\tau^K}{1-\tau^H}\right)\tilde{K}_t\tilde{w}_t\tilde{H}_t$)
- Labor Tax Revenue ($\tilde{T}_H$)
- Capital Tax Revenue ($\tilde{T}_K$)
- Transfers ($\tilde{S}_t$)
- Labor Tax Rate ($\tau^H$)
- Capital Tax Rate ($\tau^K$)
- Debt to Output ($\tilde{b}_t$)

Legend:
- Current Permanent Capital Tax Cut
- Anticipated Permanent Capital Tax Cut
Figure D.9: CES Production Function: Transfers Adjustment (Long-run)

Consumption \((\bar{C})\)

Investment \((\bar{I})\)

Output \((\bar{Y})\)

Hours \((\bar{H})\)

Wage \((\bar{w})\)

After Tax Wage \(((1 - \bar{\tau}_H)\bar{w})\)

Capital Rental Rate \((\bar{r}_K)\)

After Tax Rental Rate \(((1 - \bar{\tau}_K)\bar{r}_K)\)

Capital to Hours Ratio \((\bar{K}/\bar{H})\)

After Tax Capital to Labor Income Ratio \(((1 - \bar{\tau}_K)\bar{r}_K((1 - \bar{\tau}_H)\bar{w}))/\bar{w}\)

Labor Tax Revenue \((\bar{T}_H)\)

Capital Tax Revenue \((\bar{T}_K)\)

Transfers \((\bar{S})\)

Labor Tax Rate \((\bar{\tau}_H)\)

Capital Tax Rate \((\bar{\tau}_K)\)

Debt to Output \((\bar{b})\)

CES \((\varepsilon = 0.8)\)

CES \((\varepsilon = 1.0)\)

CES \((\varepsilon = 1.2)\)
Figure D.10: CES Production Function: Labor Tax Rate Adjustment (Long-run)

Consumption ($\bar{C}$)

Investment ($\bar{I}$)

Output ($\bar{Y}$)

Hours ($\bar{H}$)

Wage ($\bar{w}$)

After Tax Wage ($((1 - \bar{\tau}_H)\bar{w})$)

Capital Rental Rate ($\bar{r}_K$)

After Tax Rental Rate ($((1 - \bar{\tau}_K)\bar{r}_K)$)

Capital to Hours Ratio ($\bar{K}/\bar{H}$)

After Tax Capital to Labor Income Ratio ($((1 - \bar{\tau}_K)\bar{K}/(1 - \bar{\tau}_H)\bar{H})$)

Labor Tax Revenue ($\bar{T}_H$)

Capital Tax Revenue ($\bar{T}_K$)

Transfers ($\bar{S}$)

Labor Tax Rate ($\bar{\tau}_H$)

Capital Tax Rate ($\bar{\tau}_K$)

Debt to Output ($\bar{b}$)

CES ($\varepsilon = 0.8$)

CES ($\varepsilon = 1.0$)

CES ($\varepsilon = 1.2$)
Figure D.11: CES Production Function: Transfers Adjustment (Transition Dynamics)
Figure D.12: Non-separable Preference: Transfers Adjustment (Long-run)

- Consumption ($\bar{\tilde{C}}$)
- Investment ($\bar{\tilde{I}}$)
- Output ($\bar{\tilde{Y}}$)
- Hours ($\bar{\tilde{H}}$)
- Wage ($\bar{\tilde{w}}$)
- After Tax Wage ($((1 - \bar{\tau}_H)\bar{\tilde{w}})$)
- Capital Rental Rate ($\bar{\tilde{r}}_K$)
- After Tax Rental Rate ($((1 - \bar{\tau}_K)\bar{\tilde{r}}_K)$)
- Capital to Hours Ratio ($\bar{\tilde{K}}/\bar{\tilde{H}}$)
- After Tax Capital to Labor Income Ratio ($((1 - \bar{\tau}_K)\bar{\tilde{K}})/(1 - \bar{\tau}_H)\bar{\tilde{w}}\bar{\tilde{H}})$)
- Labor Tax Revenue ($\bar{\tilde{T}}_H$)
- Capital Tax Revenue ($\bar{\tilde{T}}_K$)
- Transfers ($\bar{\tilde{S}}$)
- Labor Tax Rate ($\bar{\tilde{\tau}}_H$)
- Capital Tax Rate ($\bar{\tilde{\tau}}_K$)
- Debt to Output ($\bar{\tilde{b}}$)

EIS ($1/\eta = 2.0$) EIS ($1/\eta = 1.0$) EIS ($1/\eta = 0.71$)
Figure D.13: Non-separable Preference: Labor Tax Rate Adjustment (Long-run)

- **Consumption** $(\bar{\bar{C}})$
- **Investment** $(\bar{\bar{I}})$
- **Output** $(\bar{\bar{Y}})$
- **Hours** $(\bar{\bar{H}})$

- **Wage** $(\bar{\bar{w}})$
- **After Tax Wage** $((1 - \bar{\bar{\tau}}_H)\bar{\bar{w}})$

- **Capital Rental Rate** $(\bar{\bar{r}}_K)$
- **After Tax Rental Rate** $((1 - \bar{\bar{\tau}}_K)\bar{\bar{r}}_K)$

- **Capital to Hours Ratio** $(\bar{\bar{K}}/\bar{\bar{H}})$
- **After Tax Capital to Labor Income Ratio** $((1 - \bar{\bar{\tau}}_K)\bar{\bar{r}}_K)/(1 - \bar{\bar{\tau}}_H)\bar{\bar{w}})$

- **Labor Tax Revenue** $(\bar{\bar{T}}_H)$
- **Capital Tax Revenue** $(\bar{\bar{T}}_K)$

- **Transfers** $(\bar{\bar{S}})$
- **Labor Tax Rate** $(\bar{\bar{\tau}}_H)$
- **Capital Tax Rate** $(\bar{\bar{\tau}}_K)$

- **Debt to Output** $(\bar{\bar{b}})$

EIS (1/η = 2.0) EIS (1/η = 1.0) EIS (1/η = 0.71)
Figure D.14: Non-separable Preference: Transfers Adjustment (Transition Dynamics)
Figure D.15: Frisch Elasticity: Transfers Adjustment (Long-run)

- Consumption: \( \bar{\bar{C}} \)
- Investment: \( \bar{\bar{I}} \)
- Output: \( \bar{\bar{Y}} \)
- Hours: \( \bar{\bar{H}} \)
- Wage: \( \bar{\bar{w}} \)
- After Tax Wage: \( (1 - \bar{\bar{\tau}}_H)\bar{\bar{w}} \)
- Capital Rental Rate: \( \bar{\bar{r}}_K \)
- After Tax Rental Rate: \( (1 - \bar{\bar{\tau}}_K)\bar{\bar{r}}_K \)
- Capital to Hours Ratio: \( \bar{\bar{K}}/\bar{\bar{H}} \)
- After Tax Capital to Labor Income Ratio: \( (1 - \bar{\bar{\tau}}_K)\bar{\bar{r}}_K (1 - \bar{\bar{\tau}}_H)\bar{\bar{w}} \)
- Labor Tax Revenue: \( \bar{\bar{T}}_H \)
- Capital Tax Revenue: \( \bar{\bar{T}}_K \)

Transfers: \( \bar{\bar{S}} \)
Labor Tax Rate: \( \bar{\bar{\tau}}_H \)
Capital Tax Rate: \( \bar{\bar{\tau}}_K \)
Debt to Output: \( \bar{\bar{b}} \)

Frisch Elasticity (1/\( \varphi \) = 2.0)     Frisch Elasticity (1/\( \varphi \) = 1.0)     Frisch Elasticity (1/\( \varphi \) = 0.5)
Figure D.16: Frisch Elasticity: Labor Tax Rate Adjustment (Long-run)

<table>
<thead>
<tr>
<th>Category</th>
<th>Formula</th>
<th>Initial Values</th>
<th>After Tax</th>
<th>After Tax</th>
</tr>
</thead>
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<td>-40</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Investment</td>
<td>$\bar{\tilde{I}}$</td>
<td>-60</td>
<td>-40</td>
<td>-20</td>
</tr>
<tr>
<td>Output</td>
<td>$\bar{\tilde{Y}}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Hours</td>
<td>$\bar{\tilde{H}}$</td>
<td>20</td>
<td>40</td>
<td>50</td>
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<tr>
<td>Wage</td>
<td>$\bar{\tilde{w}}$</td>
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<td>20</td>
<td>-0.3</td>
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<td>50.3</td>
<td>60</td>
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<td>Capital Rental Rate</td>
<td>$\bar{r}_K$</td>
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<td>3</td>
<td>3.3</td>
</tr>
<tr>
<td>After Tax Rental Rate</td>
<td>$(1 - \bar{r}_K)\bar{r}_K$</td>
<td>3.5</td>
<td>3.3</td>
<td>3</td>
</tr>
<tr>
<td>Capital to Hours Ratio</td>
<td>$\bar{\tilde{K}}/\bar{\tilde{H}}$</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>After Tax Capital to Labor Income Ratio</td>
<td>$(1 - \bar{\tilde{H}}\bar{\tilde{w}})/\bar{\tilde{H}}\bar{\tilde{w}}$</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Labor Tax Revenue</td>
<td>$\bar{\tilde{T}}_H$</td>
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<td>0.24</td>
<td>0.25</td>
</tr>
<tr>
<td>Capital Tax Revenue</td>
<td>$\bar{\tilde{T}}_K$</td>
<td>0.23</td>
<td>0.24</td>
<td>0.25</td>
</tr>
<tr>
<td>Transfers</td>
<td>$\bar{\tilde{S}}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Labor Tax Rate</td>
<td>$\bar{\tilde{\tau}}_H$</td>
<td>4.2</td>
<td>5.1</td>
<td>6</td>
</tr>
<tr>
<td>Capital Tax Rate</td>
<td>$\bar{\tilde{\tau}}_K$</td>
<td>4.2</td>
<td>5.1</td>
<td>6</td>
</tr>
<tr>
<td>Debt to Output</td>
<td>$\bar{\tilde{b}}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Frisch Elasticity (1/\(\phi\) = 2.0)  Frisch Elasticity (1/\(\phi\) = 1.0)  Frisch Elasticity (1/\(\phi\) = 0.5)
Figure D.17: Frisch Elasticity: Transfers Adjustment (Transition Dynamics)

- Consumption ($\tilde{C}_t$)
- Investment ($\tilde{I}_t$)
- Output ($\tilde{Y}_t$)
- Hours ($\tilde{H}_t$)
- Wage ($\tilde{w}_t$)
- After Tax Wage ($((1 - \tau)_H)\tilde{w}_t$)
- Capital Rental Rate ($\tilde{r}_K$)
- After Tax Rental Rate ($((1 - \tau)_K)\tilde{r}_K$)
- Inflation ($\pi_t$)
- After Tax Capital to Labor Income Ratio ($\frac{(1 - \tau)_K}{\tilde{r}_K}\tilde{K}_t(1 - \tau)_H\tilde{w}_t\tilde{H}_t$)
- Labor Tax Revenue ($\tilde{T}_H$)
- Capital Tax Revenue ($\tilde{T}_K$)
- Transfers ($\tilde{S}_t$)
- Labor Tax Rate ($\tau_H$)
- Capital Tax Rate ($\tau_K$)
- Debt to Output ($\tilde{b}_t$)

Symbols and equations:

- Frisch Elasticity ($1/\phi = 2.0$)
- Frisch Elasticity ($1/\phi = 1.0$)
- Frisch Elasticity ($1/\phi = 0.5$)
Figure D.18: Frisch Elasticity: Labor Tax Rate Adjustment (Transition Dynamics)

- Consumption ($\tilde{C}_t$)
- Investment ($\tilde{I}_t$)
- Output ($\tilde{Y}_t$)
- Hours ($\tilde{H}_t$)
- Wage ($\tilde{w}_t$)
- After Tax Wage ($(1 - \tau_H)\tilde{w}_t$)
- Capital Rental Rate ($r^K_t$)
- After Tax Rental Rate ($(1 - \tau^K)r^K_t$)
- Inflation ($\pi_t$)
- After Tax Capital to Labor Income Ratio ($(1 - \tau_H)\tilde{K}_t(1 - \tau_H)\tilde{w}_t$)
- Labor Tax Revenue ($\tilde{T}_H$)
- Capital Tax Revenue ($\tilde{T}_K$)
- Transfers ($\tilde{S}_t$)
- Labor Tax Rate ($\tau_H^L$)
- Capital Tax Rate ($\tau^K$)
- Debt to Output ($\tilde{b}_t$)

Frisch Elasticity ($1/\phi = 2.0$) Frisch Elasticity ($1/\phi = 1.0$) Frisch Elasticity ($1/\phi = 0.5$)
Figure D.20: Degree of Labor Tax Rate Response to Debt to Output

- Consumption ($\tilde{C}_t$)
- Investment ($\tilde{I}_t$)
- Output ($\tilde{Y}_t$)
- Hours ($\tilde{H}_t$)
- Wage ($\tilde{w}_t$)
- After Tax Wage ($(1 - \tau^H)\tilde{w}_t$)
- Capital Rental Rate ($r^K_t$)
- After Tax Rental Rate ($(1 - \tau^K)r^K_t$)
- Inflation ($\pi_t$)
- After Tax Capital to Labor Income Ratio ($(1 - \tau^K)\tilde{r}^K_t(1 - \tau^H)\tilde{w}_t\tilde{H}_t$)
- Labor Tax Revenue ($\tilde{T}^H_t$)
- Capital Tax Revenue ($\tilde{T}^K_t$)
- Transfers ($\tilde{S}_t$)
- Labor Tax Rate ($(\tau^H_t)$)
- Capital Tax Rate ($(\tau^K_t)$)
- Debt to Output ($\tilde{b}_t$)

Labor Tax Rate Rule($\psi = 0.006$), Labor Tax Rate Rule($\psi = 0.05$), Labor Tax Rate Rule($\psi = 0.5$)
Figure D.21: Labor Tax Rate Adjustment: Rule-based Adjustment v.s. Free Adjustment

- **Consumption** \((\tilde{C}_t)\)
- **Investment** \((\tilde{I}_t)\)
- **Output** \((\tilde{Y}_t)\)
- **Hours** \((\tilde{H}_t)\)
- **Wage** \((\tilde{w}_t)\)
- **After Tax Wage** \(((1 - \tau^H)\tilde{w}_t)\)
- **Capital Rental Rate** \((\tilde{r}^K)\)
- **After Tax Rental Rate** \(((1 - \tau^K)\tilde{r}^K)\)
- **Inflation** \((\pi_t)\)
- **Labor Tax Revenue** \((\tilde{T}_H)\)
- **Capital Tax Revenue** \((\tilde{T}_K)\)
- **Transfers** \((\tilde{S}_t)\)
- **Labor Tax Rate** \((\tau^H_t)\)
- **Capital Tax Rate** \((\tau^K_t)\)
- **Debt to Output** \((\tilde{b}_t)\)
Figure D.22: Transfers and Inflation Adjustment

- Consumption ($\tilde{C}_t$)
- Investment ($\tilde{I}_t$)
- Output ($\tilde{Y}_t$)
- Hours ($H_t$)
- Wage ($\tilde{w}_t$)
- After Tax Wage ($((1 - \tau_H)\tilde{w}_t)$)
- Capital Rental Rate ($r^K_t$)
- After Tax Rental Rate ($((1 - \tau_K)r^K_t)$)
- Inflation ($\pi_t$)
- After Tax Capital to Labor Income Ratio ($\frac{(1 - \tau_K)r^K_t}{(1 - \tau_H)\tilde{w}_t}$)
- Transfers ($\tilde{S}_t$)
- Labor Tax Rate ($\tau_H$)
- Capital Tax Rate ($\tau^K$)
- Debt to Output ($\tilde{b}_t$)

% from initial steady-state

Transfers Adjustment
Transfers and Inflation Adjustment