DSGE models with financial frictions: does frequency matter?*

Claudia Foroni†
Deutsche Bundesbank

Paolo Gelain‡
Federal Reserve Bank of Cleveland

Massimiliano Macellino§
Bocconi University, IGIER, and C.E.P.R.

March 2018

Abstract

We use mixed-frequency data to estimate a dynamic stochastic general equilibrium model embedded with the financial accelerator mechanism a la Bernanke et al. (1999). The use of financial variables in the estimation, available at high frequency and typically very responsive to changing economic conditions, has a large impact on the estimated parameters. As a consequence the transmission of shocks and the their relevance in explaining endogenous variables variability is deeply altered. In particular we find that the financial accelerator (decelerator) mechanism is either inverted or accentuated.

Keywords: DSGE models, Financial frictions, Mixed-frequency data.

JEL codes: C52, E32, E52

*This paper should not be reported as representing the views of the Federal Reserve Bank of Cleveland, the Board of Governors of the Federal Reserve System or the Deutsche Bundesbank. The views expressed are those of the authors and do not necessarily reflect those of Federal Reserve Bank of Cleveland, the Board of Governors of the Federal Reserve System or the Deutsche Bundesbank.

†E-mail: claudia.foroni@bundesbank.de.

‡Corresponding author. E-mail: Paolo.Gelain@clev.frb.org. Address: Federal Reserve Bank of Cleveland, 1455 East 6th Street, Cleveland 44114 Telephone number: +1 216 579 2432.

§E-mail: massimiliano.marcellino@unibocconi.it.

1 Introduction

Financial frictions acquired a lot of attention, especially starting from the Great Recession. Many researchers proved that standard DSGE models failed to capture the macroeconomic dynamics during and after the crisis. On the contrary, models which incorporate financial frictions have been able to interpret the evolution of economic activity and inflation also after 2008. Negro et al. (2015) show that when incorporating financial frictions into a standard DSGE, the model successfully predicts the dynamics of real and nominal variables as we experienced in the Great Recession. In particular, the model predicts a sharp contraction in the economic activity and, at the same time, a persistent but modest decline in inflation, for the period starting in the last quarter of 2008.

Financial frictions are, in simple words, wedges between the cost of capital and the return that investors earn from the capital. They can be measured as a gap between the returns earned by savers and the cost of funding for accessing credit, which rises because financial institutions impose higher spreads to protect against the entrepreneurs' default risk and because they ration credit (see Hall (2013)). In practice, financial frictions are included in standard DSGE models, building on the work of Bernanke et al. (1999) and Christiano et al. (2014), as in Negro et al. (2015). The assumption is that entrepreneurs borrow funds from the banks and invest those to acquire physical capital. However, the entrepreneurs are subject to idiosyncratic shocks which affect their ability to manage capital, and therefore their ability to pay back the bank loans. In order to protect themselves against the entrepreneurs' default risk, banks charge a spread over the deposit rate, when lending money. Maybe expand the references to financial frictions papers.

This way of modelling financial frictions implies having information about interest rates and financial conditions, such as spreads. These variables are typically reacting fast to economic conditions, and they are available at a high frequency. As shown by Foroni and Marcellino (2014), the mismatch between the time scale of DSGE models and the data used in their estimation translates into identification problems, estimation bias, and distortions in policy analysis. The authors find that, when looking at the Smets and Wouters (2007) model, differences in the responses to structural shocks depending on whether the model is set at a quarterly frequency or at the monthly frequency with the use of mixed frequency techniques.

Given the relevance that financial frictions acquired in the literature to explain the features of the Great Recession, with our paper we want to investigate whether the responses of the economy to structural shocks and their policy implications are still the same, once we include higher frequency information into the model. The relevance of high-frequency information is even higher in the presence of financial frictions, given that financial variables are available in real-time and the adjust frequently to news in the economy.

In order to achieve this goal, we consider a DSGE model estimated with mixed-frequency data. Despite the literature on mixed-frequency data has grown enormously by now, there are still very few contributions in the use of mixed-frequency information in the context of structural analysis, and even fewer in the context of DSGE models. Foroni and Marcellino (2014) are the first to highlight the problems of temporal aggregation and the use of mixed frequency data in DSGE models. Second, Giannone et al. (2016) combine a DSGE framework with a nowcasting model to read timely monthly information as it becomes available. In particular, they consider a DSGE model with financial frictions as in our case, and they focus more on the forecasting properties of the model. Further, Giannone et al. (2016) keep all the parameters of the model estimated as in the quarterly frequency, and the monthly information is used only to update the estimates of the states. We are instead interested also in the estimates of the parameters per se, and we want to check whether adding more information to te data can help us in pinning down the parameters more precisely, and consequently, in having a different dynamic in the model. Another study that is combining financial market data at daily frequency along with quarterly macroeconomic data is the paper by Christensen et al. (2016), but the analysis is conducted in continuous time and with a different DSGE model.

Comparing the impulse responses obtained with and without including high-frequency information, we aim at grasping more details on the transmission channels of structural shocks in the economy, and at understanding whether the movements in the financial variables create consequences to the fundamentals of the economy or they are instead mainly noise. This can have important consequences on the policy side, giving indications to the policy makers on whether frequent movements in the financial markets need to be taken into account when taking policy decisions.

Further, on the methodological side, we improve upon Foroni and Marcellino (2014), given that we estimate our mixed-frequency model in a fully Bayesian context, consistent with what is common practice in the DSGE literature.

summary of the results

The paper is organized as follows. In Section 2 we briefly recall the basic features of the Negro et al. (2015) model. Section 3 provided the details on the estimation of the model. Section 4 presents the results we obtain and their policy implications. Section 5 concludes.

2 The model

In our analysis, we consider the model similar to the one presented in Negro et al. (2015)[1]. It is the Smets and Wouters (2007) model (SW henceforth), extended to include financial frictions as in Bernanke et al. (1999). The SW model is a medium-scale DSGE model, which includes nominal price and wage rigidities, habit formation in consumption and investment adjustment costs. Financial frictions are embedded in the SW model by Negro et al. (2015). In this set-up, the entrepreneurs need external funds, on top of their own wealth, to run their projects. However, the entrepreneurs are subject to idiosyncratic shocks to their net wealth. Banks lend the funds to the entrepreneurs, but in order to protect themselves from the shocks hitting the entrepreneurs and influencing their ability to repay, they charge a spread over the deposit rate, and this premium depends on the amount of finance required and on the borrower's net worth. A full description of the model is in Appendix A.

The model includes eight structural shocks: technology, investment-specific, risk premium, government spending, price mark-up, wage mark-up, monetary policy and risk shock.

We estimate the model for the U.S. on the same sample of Negro et al. (2015), which spans from Q1-1964 to Q3-2008. The data series are the following: real GDP growth, real consumption growth, real investment growth, real wage growth, hours worked, inflation, federal fund rate, and the spread (measured as Baa Corporate Bond Yield spread over the 10-Year Treasury Note Yield at constant maturity).

Some of this variables are available only quarterly, like GDP, investments, consumption[2] and real wages, while hours worked, inflation, federal funds rate and spread are available

[1] Differently from them, we do not consider a time-varying target inflation rate

[2] Consumption is available at monthly frequency. However, to be consistent with the other variables in the national accounts (output and investments) and use the same GDP deflator to transform them into real, we use consumption at quarterly frequency.

monthly. We will use, therefore, these series at their monthly frequency[3]. We then compare the results obtained with mixed-frequency data to those obtained from standard quarterly estimation.

3 Estimation of the mixed-frequency DSGE

The estimation of the quarterly model is conducted with Bayesian techniques. The prior specifications is taken from Negro et al. (2015). We instead focus here on the estimation of the mixed-frequency version of the DSGE model. There is a small literature pointing a the aggregation issue in the context of structural models (see Foroni and Marcellino (2014)). We briefly recall these issues in Appendix B. Here, we detail how to write our DSGE model in the state-space form. Second, we discuss our prior specification and calibrated parameters.

3.1 Mixed-frequency specification of our model

We focus on the log-linearized DSGE model, whose solution can be cast in state-space form, where the low-frequency series are then considered as high-frequency series with missing observations. In particular, the solution of the model can be written as:

$$y_t = A(\theta)s_t \quad (1)$$

$$s_t = B(\theta)s_{t-1} + C(\theta)u_t, \quad (2)$$

where y_t is a $Nx1$ vector of observables, s_t is a $kx1$ state vector, u_t is a $Nx1$ vector of shocks. All the elements depend on θ, the structural parameters of the model. Eq. (2) characterizes the DSGE model solution, and in our mixed-frequency specification t represents a time unit equal to one month. Note that he solution of the DSGE model does not change, independently of the interpretation of t. Eq. (1) maps the model variables into the observable variables. This equation is the one that needs to be adapted in the mixed-frequency set-up, given that not all the variables are observable every period.

[3] The financial market series are available at an even higher frequency. However, in order not to complicate the analysis even further, and not to introduce a noise component, we consider these series as monthly.

The monthly variables enter the measurement equations in the standard way, as:

$$\begin{aligned}
\text{Hours worked} &= \bar{l} + 100 l_t, \\
\text{Inflation} &= \pi_* + 100\pi_t, \\
\text{FFR} &= R_* + 100 R_t, \\
\text{Spread} &= SP_* + 100 E_t[\tilde{R}{}^k_{t+1} - R_t],
\end{aligned}$$

where $\bar{l}$ represents the mean of hours, π_* and R_* measure the steady-state level of inflation and fed funds rate. All variables are measured in percent.

Special attention needs to be drawn on the quarterly variables which enter the measurement equation. Here we discuss the case of flow variables, given that our quarterly series are output, investment, consumption and wages. An extensive discussion on the topic of aggregation is the measurement equations is provided by Pfeifer (2013).

We take output as an example. What we observe is the quarterly value in levels, and this can be considered as the sum of an unobserved monthly output over the three months of the quarter:

$$Y_{q,t} = Y_{m,t} + Y_{m,t-1} + Y_{m,t-2}. \tag{3}$$

However, what we are finally interested in are the variables as they enter in the measurement equations. Therefore, we need to construct the measure for the log-linearized system and consider a growth trend explicitly. In fact in the model by Negro et al. (2015) all non-stationary variables are detrended by $Z_t = e^{\gamma t + \frac{1}{1-\alpha}\tilde{z}_t}$, where γ is the steady-state growth of the economy, and $\tilde{z}_t$ is the linearly detrended log productivity process that follows an AR(1) process as law of motion. The growth rate of Z_t, in deviations from γ, is denoted by z_t.

For our purposes, then, let us define what we observed in the data, in terms of growth rate of quarterly output:

$$\Delta Y^{obs}_{q,t} = \log(Y^{obs}_{q,t}) - \log(Y^{obs}_{q,t-3}), \tag{4}$$

which is observed every third month.

What we need to do is to link the output growth to the model variables. We recall that the variables in the model need to be defined as detrended, that is we define $y_{q,t} = \frac{Y_{q,t}}{Z_t}$. With this definition in mind, we can rewrite Eq. (4) as:

$$
\begin{aligned}
\Delta Y_{q,t}^{obs} &= \log(y_{q,t} Z_t) - \log(y_{q,t-3} Z_{t-3}) \\
&= \hat{y}_{q,t} - \hat{y}_{q,t-3} + \log\left(\frac{Z_t}{Z_{t-3}}\right) \\
&= \hat{y}_{q,t} - \hat{y}_{q,t-3} + \log\left(\frac{Z_t}{Z_{t-1}} \frac{Z_{t-1}}{Z_{t-2}} \frac{Z_{t-2}}{Z_{t-3}}\right) \\
&= \hat{y}_{q,t} - \hat{y}_{q,t-3} + z_t + z_{t-1} + z_{t-2}. \qquad (5)
\end{aligned}
$$

Eq. (5) is the measurement equation we look for. However, in order to implement that, we need still one more step and define $\hat{y}_{q,t}$. In order to do that, we need to go to our definition of the quarterly variable in terms of the monthly unobserved one, as in Eq. (3), and combine this with the definition of detrended variables. We obtain that:

$$
y_{q,t} Z_t = y_{m,t} Z_t + y_{m,t-1} Z_{t-1} + y_{m,t-2} Z_{t-2},
$$

and from here

$$
\begin{aligned}
y_{q,t} &= y_{m,t} + y_{m,t-1} \frac{Z_{t-1}}{Z_t} + y_{m,t-2} \frac{Z_{t-2}}{Z_{t-1}} \frac{Z_{t-1}}{Z_t} \\
&= y_{m,t} + y_{m,t-1} \frac{1}{z_t} + y_{m,t-2} \frac{1}{z_{t-1}} \frac{1}{z_t}. \qquad (6)
\end{aligned}
$$

Linearizing around the state state $y_q = y_m \left(1 + \frac{1}{z} + \frac{1}{z^2}\right)$, we obtain:

$$
y_q \hat{y}_{q,t} = y_m \hat{y}_{m,t} + \frac{y_m}{z} \left(\hat{y}_{m,t-1} - z_{t-1}\right) + \frac{y_m}{z^2} \left(\hat{y}_{m,t-2} - z_{t-1} - z_{t-2}\right), \qquad (7)
$$

we can be rewritten as:

$$
\begin{aligned}
\hat{y}_{q,t} = &\frac{y_m}{y_q} \hat{y}_{m,t} + \frac{y_m}{y_q z} \left(\hat{y}_{m,t-1} - z_{t-1}\right) + \frac{y_m}{y_q z^2} \left(\hat{y}_{m,t-2} - z_{t-1} - z_{t-2}\right), \\
&= \frac{1}{\left(1 + \frac{1}{z} + \frac{1}{z^2}\right)} \hat{y}_{m,t} + \frac{1}{z} \frac{1}{\left(1 + \frac{1}{z} + \frac{1}{z^2}\right)} \left(\hat{y}_{m,t-1} - z_{t-1}\right) + \\
&\frac{1}{z^2} \frac{1}{\left(1 + \frac{1}{z} + \frac{1}{z^2}\right)} \left(\hat{y}_{m,t-1} - z_{t-1}\right) \left(\hat{y}_{m,t-2} - z_{t-1} - z_{t-2}\right). \qquad (8)
\end{aligned}
$$

Substituting Eq. (8) into Eq. (5), we get the final form of the measurement equations for quarterly variables.

More in detail, we can write the measurement equations as:

$$
\begin{aligned}
\text{Output growth} &= \gamma + 100\left(\hat{y}_{q,t} - \hat{y}_{q,t-3} + z_t + z_{t-1} + z_{t-2}\right), \\
\text{Consumption growth} &= \gamma + 100\left(\hat{c}_{q,t} - \hat{c}_{q,t-3} + z_t + z_{t-1} + z_{t-2}\right), \\
\text{Investment growth} &= \gamma + 100\left(\hat{i}_{q,t} - \hat{i}_{q,t-3} + z_t + z_{t-1} + z_{t-2}\right), \\
\text{Hours worked} &= \gamma + 100\left(\hat{w}_{q,t} - \hat{w}_{q,t-3} + z_t + z_{t-1} + z_{t-2}\right),
\end{aligned}
$$

where $\hat{y}_{q,t}$, $\hat{c}_{q,t}$, $\hat{i}_{q,t}$ and $\hat{w}_{q,t}$ are defined as in Eq. (8).

check both derivation and consistent notation (hat or not)

3.2 Prior specification

We use the same prior specification as in Negro et al. (2015). However, we need to be careful in transforming some of the prior means, dividing their value by 3, because in the mixed-frequency case we are using monthly values.

More in detail, in Table 1 we list the priors for both the quarterly and the mixed-frequency models.

add some comments on which values are transformed

4 Results

4.1 Results at quarterly frequency

First we estimate the model at quarterly frequency, in two set-ups: with and without financial frictions. In Table 2 we report the estimated parameters, and in Figures **??** to **??** we report the impulse responses. As it is easy to see, we easily replicate the standard findings of the literature: in particular we see the "financial accelerator" effect: in simple words, given that banks require entrepreneurs to pay a premium to access credit proportional to their net worth, a fall in asset prices deteriorates the entrepreneurs' balance sheets, their ability to borrow and, consequently, their investment. An adverse shock to economic activity cuts the asset prices even further, establishing a vicious circle of falling asset prices, deteriorating balance sheets, tightening financing conditions and declining investment and output.

4.2 Results with mixed-frequency data

add results

4.3 Economic implications

add results on financial accelerator

4.4 Forecasting the Great Recession

add results

5 Conclusions

References

Anderson, B. D., Deistler, M., Felsenstein, E., Funovits, B., Koelbl, L., and Zamani, M. (2016). Multivariate ar systems and mixed frequency data: g-identifiability and estimation. *Econometric Theory*, 32(4):793âĂŞ826.

Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In Taylor, J. B. and Woodford, M., editors, *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics*, chapter 21, pages 1341–1393. Elsevier.

Christensen, B. J., Posch, O., and van der Wel, M. (2016). Estimating dynamic equilibrium models using mixed frequency macro and financial data. *Journal of Econometrics*, 194(1):116 – 137.

Christiano, L. J., Motto, R., and Rostagno, M. (2014). Risk Shocks. *American Economic Review*, 104(1):27–65.

Foroni, C. and Marcellino, M. (2014). MixedâĂŘFrequency Structural Models: Identification, Estimation, And Policy Analysis. *Journal of Applied Econometrics*, 29(7):1118–1144.

Giannone, D., Monti, F., and Reichlin, L. (2016). Exploiting the monthly data flow in structural forecasting. *Journal of Monetary Economics*, 84(C):201–215.

Hall, R. E. (2013). Financial Frictions. *International Journal of Central Banking*, 9(2):155–163.

Negro, M. D., Giannoni, M. P., and Schorfheide, F. (2015). Inflation in the Great Recession and New Keynesian Models. *American Economic Journal: Macroeconomics*, 7(1):168–196.

Pfeifer, J. (2013). A Guide to Specifying Observation Equations for the Estimation of DSGE Models. *mimeo*.

Smets, F. and Wouters, R. (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, 97(3):586–606.

Table 1: Posterior mode for DSGE parameters

			Quarterly model		Mixed frequency model	
Parameter		Density	Mean	St. Dev	Mean	St. Dev
Std technology	σ_z					
Std risk premium	σ_b					
Std price mark-up	σ_{λ_f}					
Std wage mark-up	σ_{λ_w}					
Std investment specific	σ_μ					
Std spending	σ_g					
Std monetary policy	σ_{r^m}					
Std risk	σ_{σ_ω}					
Auto. technology	ρ_z					
Auto. risk premium	ρ_b					
Auto. price mark-up	ρ_λ_f					
Auto. wage mark-up	ρ_{λ_w}					
Auto. investment specific	ρ_{mu}					
Auto. government spending	ρ_g					
Auto. monetary policy	ρ_{rm}					
Auto. risk	ρ_{σ_ω}					
Arma price mark-up	η_{λ_f}					
Arma wage mark-up	η_{λ_w}					
Tech. in gov. spending	η_{gz}					
Reaction inflation	ψ_1					
Reaction output gap	ψ_2					
Reaction output gap growth	ψ_3					
Interest rate smoothing	ρ_R					
Price stickiness	ζ_p					
Wage stickiness	ζ_w					
Capital share	α					
Production fixed cost	Φ_p					
Habit formation	h					
Labour disutility	ν_l					
Price indexation	ι_p					
Discount factor	r^*					
SS inflation	pi^*					
SS tech. Growth	γ					
Invest. cdj. costs	S''					
Intertemporal elasticity	σ_c					
Wage indexation	ι_w					
Utilization costs	ψ					
SS spread	SP_*					
Elasticity of EFP w.r.t. leverage	$\zeta_{sp,b}$					
Mean hours worked	$Lmean$					

Table 2: Posterior mode for DSGE parameters

Parameter		No financial frictions		Financial frictions	
		Quarterly	Mixed frequency	Quarterly	Mixed frequency
Std technology	σ_z	0.4621	0.1906	0.4827	0.2072
Std risk premium	σ_b	0.2163	0.0361	0.0300	0.0241
Std price mark-up	σ_{λ_f}	0.1502	0.1272	0.1846	0.1673
Std wage mark-up	σ_{λ_w}	0.2722	0.2544	0.2817	1.0828
Std investment specific	σ_μ	0.4094	0.2755	0.5652	0.1636
Std spending	σ_g	2.9521	1.7644	2.9381	1.8650
Std monetary policy	σ_{r^m}	0.2269	0.0577	0.2497	0.1492
Std risk	σ_{σ_ω}	–	–	0.0575	0.0144
Auto. technology	ρ_z	0.9604	0.9994	0.9650	0.9964
Auto. risk premium	ρ_b	0.3314	0.9372	0.9839	0.9923
Auto. price mark-up	ρ_λ_f	0.9280	0.9757	0.6870	0.9909
Auto. wage mark-up	ρ_{λ_w}	0.9710	0.9883	0.9625	0.9959
Auto. investment specific	ρ_{mu}	0.7497	0.9771	0.7358	0.9774
Auto. government spending	ρ_g	0.9801	0.9994	0.9668	0.9897
Auto. monetary policy	ρ_{rm}	0.1147	0.1371	0.0533	0.1250
Auto. risk	ρ_{σ_ω}	–	–	0.9953	0.9977
Arma price mark-up	η_{λ_f}	0.7559	0.9797	0.5961	0.0832
Arma wage mark-up	η_{λ_w}	0.8982	0.9957	0.9501	0.1337
Tech. in gov. spending	η_{gz}	0.7830	0.9184	0.8020	0.8941
Reaction inflation	ψ_1	1.9996	1.7279	1.1433	1.3563
Reaction output gap	ψ_2	0.0887	0.2648	0.0954	-0.0270
Reaction output gap growth	ψ_3	0.2306	0.4339	0.2350	0.2952
Interest rate smoothing	ρ_R	0.8459	0.9555	0.7765	0.4964
Price stickiness	ζ_p	0.6364	0.9489	0.7443	0.1984
Wage stickiness	ζ_w	0.7517	0.8605	0.8893	0.0497
Capital share	α	0.1731	0.0803	0.2083	0.1393
Production fixed cost	Φ_p	1.7085	2.0077	1.5764	2.0056
Habit formation	h	0.7104	0.1043	0.2785	0.1069
Labour disutility	ν_l	2.4892	6.7092	2.5602	6.7424
Price indexation	ι_p	0.2616	0.2766	0.4226	0.0613
Discount factor	r^*	0.1663	0.0086	0.1691	0.1400
SS inflation	pi^*	0.9262	0.2961	0.8505	0.2861
SS tech. Growth	γ	0.4071	0.1439	0.3471	0.1372
Invest. cdj. costs	S''	6.0473	0.6023	2.6698	3.3044
Intertemporal elasticity	σ_c	1.3436	1.3794	1.4143	0.3941
Wage indexation	ι_w	0.4335	0.3480	0.3061	0.3735
Utilization costs	ψ	0.6934	0.6179	0.6176	0.4608
SS spread	SP_*	–	–	1.8001	0.0822
Elasticity of EFP w.r.t. leverage	$\zeta_{sp,b}$	–	–	0.0530	0.0457
Mean hours worked	$Lmean$	-44.3288	-12.8144	-44.6400	-13.2320

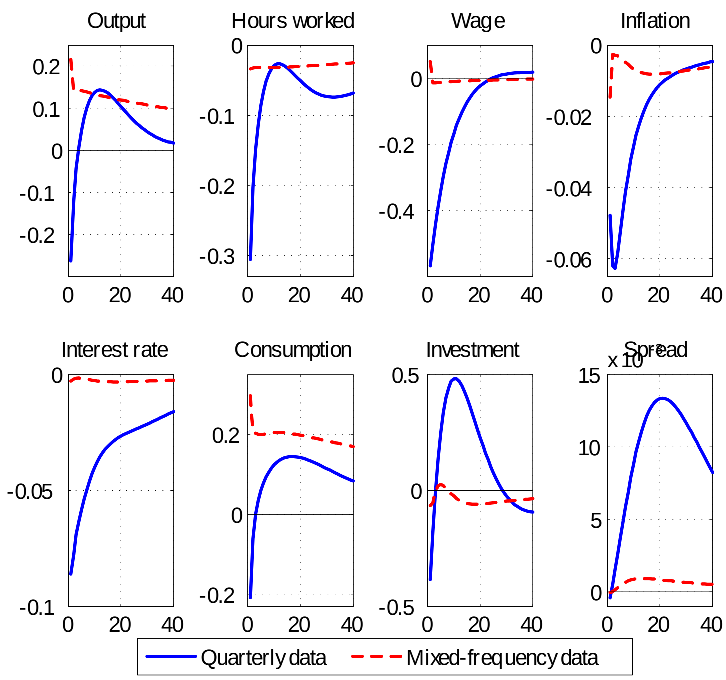

Figure 1: Technology shock

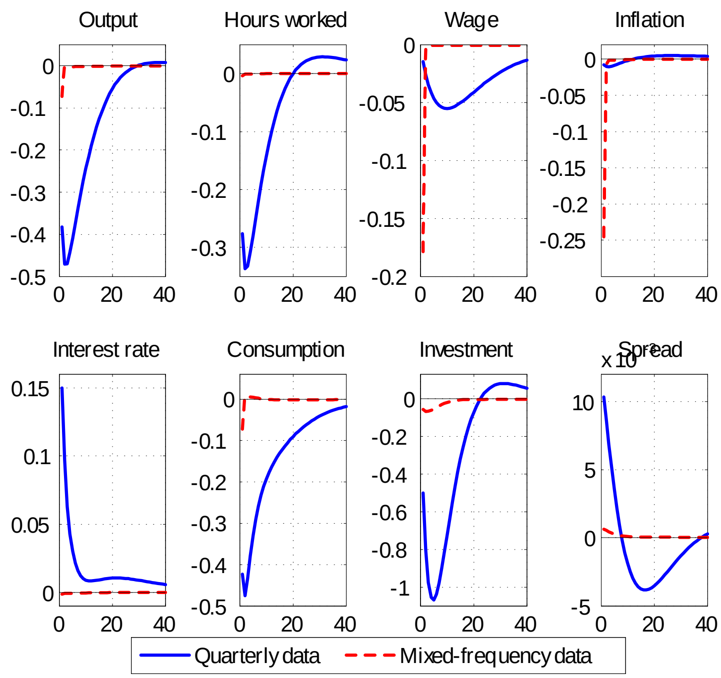

Figure 2: Monetary policy shock

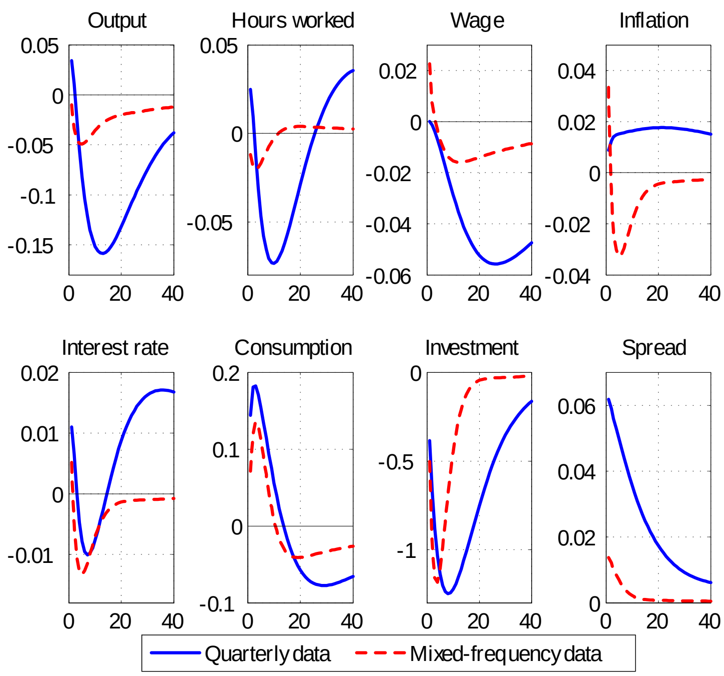

Figure 3: Risk shock

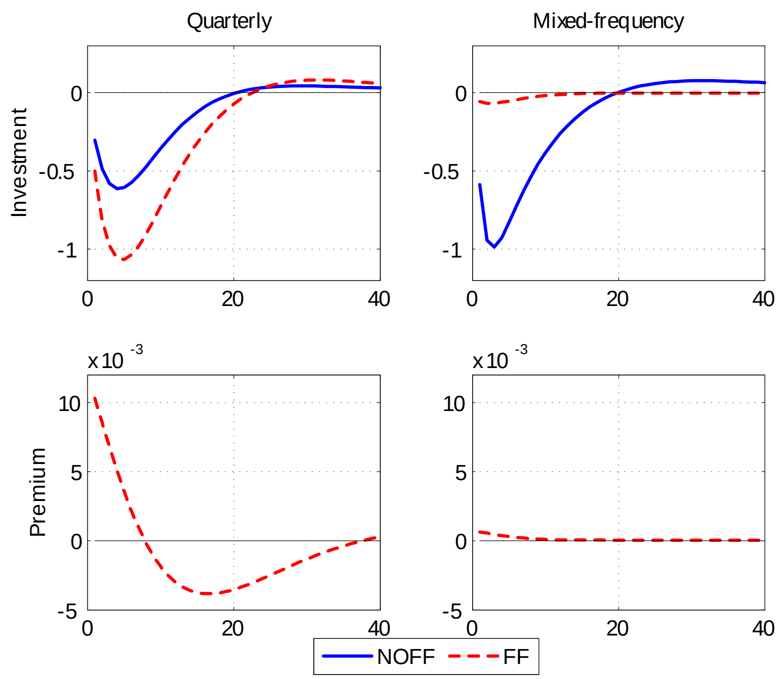

Figure 4: Accelerator mechanism. Monetary policy shock.

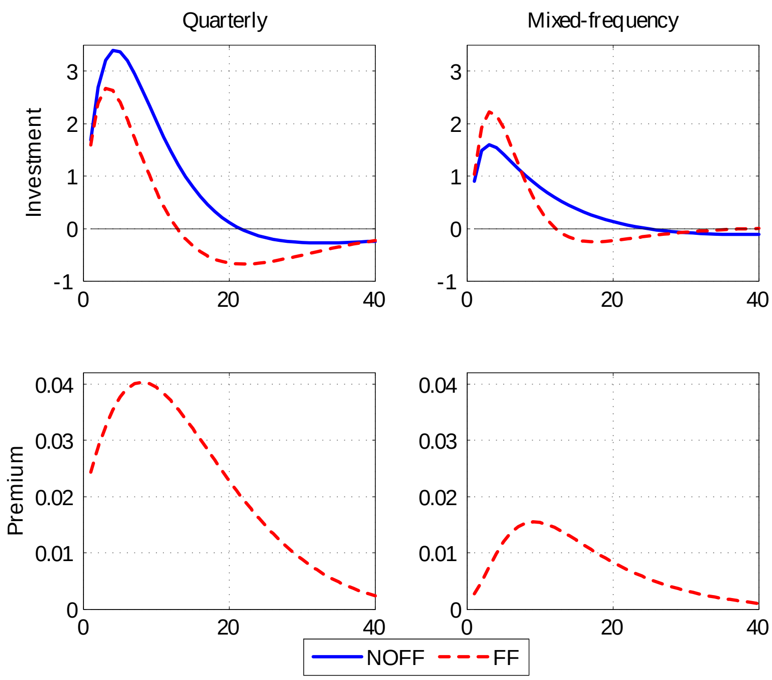

Figure 5: Accelerator mechanism. Investment specific shock.

A Model equations

add equations describing the model

B Aggregation and identification issues

As pointed out by Foroni and Marcellino (2014), temporal aggregation generates two different problems. First, since it confounds parameters across equations, it is not always possible to identify the parameters of the high-frequency model, once it has been aggregated at a lower frequency. Second, even when identification is not an issue and each parameter can be uniquely identified from a quarterly model, the common approach of considering the same structural model at a different frequency leads to different interpretations of the parameters values.

In particular, time aggregation creates non-linear combinations of the parameters which describe the monthly process, and these non-linear combinations make recovering the original parameters impossible.

To understand this issue, we assume that the the solution of the DSGE model at the monthly frequency can be written as:

$$s_t = As_{t-1} + Be_t, \tag{9}$$

where e_t is a vector of orthonormal shocks.

The equivalent quarterly aggregated process is:

$$s_t = A^3 s_{t-1} + Be_t + ABe_{t-1} + A^2 Be_{t-2}. \tag{10}$$

What the econometrician estimates is therefore a quarterly AR(1) process:

$$s_t = Cs_{t-1} + Du_t, \tag{11}$$

where u_t is also a vector of orthonormal shocks.

The identification of the monthly parameters boils down to the question whether it is possible to recover matrices A and B from the estimates of C and D. Foroni and Marcellino (2014) and Anderson et al. (2016) show that the identification is not always possible with quarterly data, while using mixed-frequency data typically implies identifiability.