Re-use of Collateral: Leverage, Volatility, and Welfare*

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Abstract

We assess the quantitative implications of the re-use of collateral on financial market leverage, volatility, and welfare within an infinite-horizon asset-pricing model with heterogeneous agents. In our model, the ability of agents to re-use frees up collateral that can be used to back more transactions. Re-use thus contributes to the build-up of leverage and significantly increases volatility in financial markets. When introducing limits on re-use, we find that volatility is strictly decreasing as these limits become tighter, yet the impact on welfare is non-monotone. In the model, allowing for some re-use can improve welfare as it enables agents to share risk more effectively. Allowing re-use beyond intermediate levels, however, can lead to excessive leverage and lower welfare. So the analysis in this paper provides a rationale for limiting, yet not banning, re-use in financial markets.

Keywords: Heterogeneous agents, leverage, re-use of collateral, volatility, welfare.

JEL Classification Codes: D53, G01, G12, G18.

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1 Introduction

Re-use and re-hypothecation of collateral has become a major activity in financial markets. It refers to the practice of financial institutions to use collateral received in one transaction for another transaction.[1] Global collateral re-use at the onset of the financial crisis was estimated to be up to US \$6.6 trillion, and dropping to US \$3.4 trillion at the end of 2010, see Singh (2011).[2] While market participants have stressed the importance of re-use of collateral as a source of funding and market liquidity more generally, regulators and supervisors have raised various concerns about this market practice. For example, the Vice-President of the ECB, Vítor Constâncio, has stressed that "activities of re-hypothecation and re-use of securities amplified the creation of chains of inside liquidity and higher leverage".[3] Moreover, the Financial Stability Board (FSB) recently published work that analyzes the financial stability implications of collateral re-use, see FSB (2017a). Several regulatory frameworks already foresee specific rules like limits and transparency requirements for re-use of (non-cash) collateral. For example, European retail investment funds (UCITs) are banned from re-using non-cash collateral. Moreover, the EU framework for margin requirements for non-centrally cleared derivatives foresees a ban of re-use of non-cash collateral that is posted as initial margin. Given the importance of collateral re-use in financial markets and the need to inform the ongoing regulatory initiatives, it is important to develop a model framework to understand the (quantitative) implications of collateral re-use on financial market outcomes and welfare.

Building on Brumm et al. (2015), we develop an infinite-horizon asset-pricing model with heterogeneous agents that allows us to assess the quantitative implications of re-use on financial market leverage, volatility, and welfare. In our model, the ability of agents to re-use frees up collateral that can be used to back more transactions. Through this channel re-use of collateral contributes to the build-up of leverage and is found to significantly increase volatility in financial markets. When we introduce limits on the amount of collateral that agents may re-use, we find that volatility is strictly decreasing in the tightness of the re-use limit. While the effect of re-use limits on volatility is monotone, the impact on welfare is not. In the model, allowing for some re-use can improve welfare as it enables agents to share risks more effectively. Allowing re-use beyond intermediate levels, however, can lead to excessive leverage and lower welfare. Thus, the analysis in this paper provides a rationale for limiting, yet not banning, re-use in financial markets.

In the economic model, financial securities are only traded if the promised payments associated with selling these securities are backed by collateral. The amount of collateral needed to back a transaction is determined in equilibrium. Re-use of collateral is introduced by allowing agents who

[1] Institutions typically receive collateral in securities financing transactions (SFTs, e.g., reverse repo, securities lending/borrowing) or derivative transactions, and if eligible for re-use, may post it as collateral in other transactions (e.g., for repos, securities lending/borrowing, derivatives collateral) or use if for short sales. Re-hypothecation typically refers to the use of client assets to obtain funding as to finance client activities and is considered as a subset of re-use activity.

[2] In a related and more recent study, Kirk et al. (2014) document that three large US dealer banks have a strong dependency on re-use of collateral for financing their activities, estimating their total amount of collateral re-use to be approx. US \$1.3 trillion at end-2013.

[3] V. Constâncio, Beyond traditional banking: a new credit system coming out of the shadows, Speech, University of Frankfurt, 17 October 2014.

receive securities as collateral to sell these securities to other agents. As a consequence, the security can again be used to collateralize transactions allowing agents to further build up their leveraged position in the risky security. To generate collateralized borrowing in equilibrium we assume that there are two types of agents who differ in risk aversion and in their beliefs about the growth potential of the economy and the risky assets.[4] These agents have Epstein–Zin utility with identical intertemporal elasticity of substitution (IES) parameters and identical time discount factors. The agent with the low risk aversion parameter (agent 1) and optimistic beliefs is the natural buyer of risky assets and takes up leverage to finance these investments. The agent with the high risk aversion (agent 2) and pessimistic beliefs has a strong desire to insure against bad shocks and is thus willing to buy bonds thereby providing financing to the other agent. Growth rates in the economy reflect the possibility of disaster shocks as in Barro and Jin (2011). When the economy is hit by a bad shock, the leveraged agent 1 loses financial wealth. As a result, the collateral constraint forces him to reduce consumption and to sell risky assets to the risk-averse agent. These actions trigger an additional decrease in asset prices, which further reduces the wealth of agent 1—thereby reinforcing the impact of the bad shock.

Introducing re-use of collateral into the model significantly increases volatility. More collateral becomes available in financial markets, allowing agents to build up leverage beyond what is feasible in a situation where no re-use of collateral is possible. As a consequence, introducing re-use limits constrains this build-up of leverage, and as the impact of negative shocks becomes less severe, volatility drops. In our welfare analysis, we consider unanticipated changes in regulation and find that intermediate levels of re-use limits are welfare optimal: Compared to very loose or very strict regulation, the welfare of one agent is increased when transfers are chosen such that the other agent's welfare is kept constant; in some cases we even observe Pareto improvements. The relation between re-use limits and welfare is non-monotone because two counteracting forces are at play: First, the ability to re-use allows for more risk-sharing in the economy. This is generally beneficial for welfare given the agent's heterogeneity in risk aversion. Second, the heterogeneity in agents' beliefs triggers agents to build up leveraged positions beyond what is needed to optimally share risks. As the ability to re-use collateral allows agents to build up this leverage, limiting re-use has the potential to steer agents' choices towards socially optimal levels.

There is a growing policy and academic literature on the role of re-use and re-hypothecation in financial markets. Singh and Aitken (2010), Singh (2011), and Kirk et al. (2014) use publicly available data to estimate the amount of collateral re-use in financial markets. Bottazzi et al. (2012), Andolfatto et al. (2015), Maurin (2015), and Gottardi et al. (2015) present theoretical models of collateral re-use. In particular, Bottazzi et al. (2012) provides a general theory of re-use in repo markets whereas Andolfatto et al. (2015) show how re-use may benefit the provision of liquidity in financial markets. Maurin (2015) focuses on the role of re-use in completing markets, whereas Gottardi et al. (2015) discuss how re-use may impact collateral constraints and haircuts. Furthermore, Infante (2014), Infante and Vardoulakis (2018), and Eren (2014) present models that consider the

[4]We assume that agents (agree to) disagree about the probability distribution of the growth rates. The agents' beliefs deviate from the objective probabilities in opposite directions, agent 1 being optimistic and agent 2 being pessimistic.

funding role of re-use for dealer banks. Eren (2014) shows how re-use may expose a hedge fund to a dealer's default, whereas Infante (2014) and Infante and Vardoulakis (2018) consider how collateral runs may arise due to re-use. However, none of these papers provide a quantitative analysis of the implications of re-use on aggregate financial market outcomes. Furthermore, with the exception of Andolfatto et al. (2015), who discuss limits on re-use in the context of monetary policy and inflation and find that limits need to be stricter in economies with lower inflation or a lower rate of return on securities, none of these papers focusses on the implications of regulating re-use.

The remainder of this paper is organized as follows. Section 2 provides an overview of industry practices for collateral re-use. Section 3 provides a first glimpse of the effects of re-use in a simple two-period model. In Section 4 we describe our economic model and its benchmark parametrization. Section 5 presents numerical results for the impact of re-use on leverage and volatility in the benchmark economy. In Section 6 we examine the welfare implications of re-use limits. Section 7 concludes. The Appendix contains additional results.

2 An Overview of Collateral Re-Use Practices

Since the academic literature on the re-use of collateral is still very young, we begin with a brief description of re-use in industry practice. For this purpose, we first define the terms re-use and re-hypothecation. Next we provide some figures on the size of global re-use activities. Then we discuss some regulatory concerns about the effects of collateral re-use on the stability of financial markets. We complete our introduction with a look at recent initiatives at international level assessing financial stability risks and benefits associated with collateral re-use.

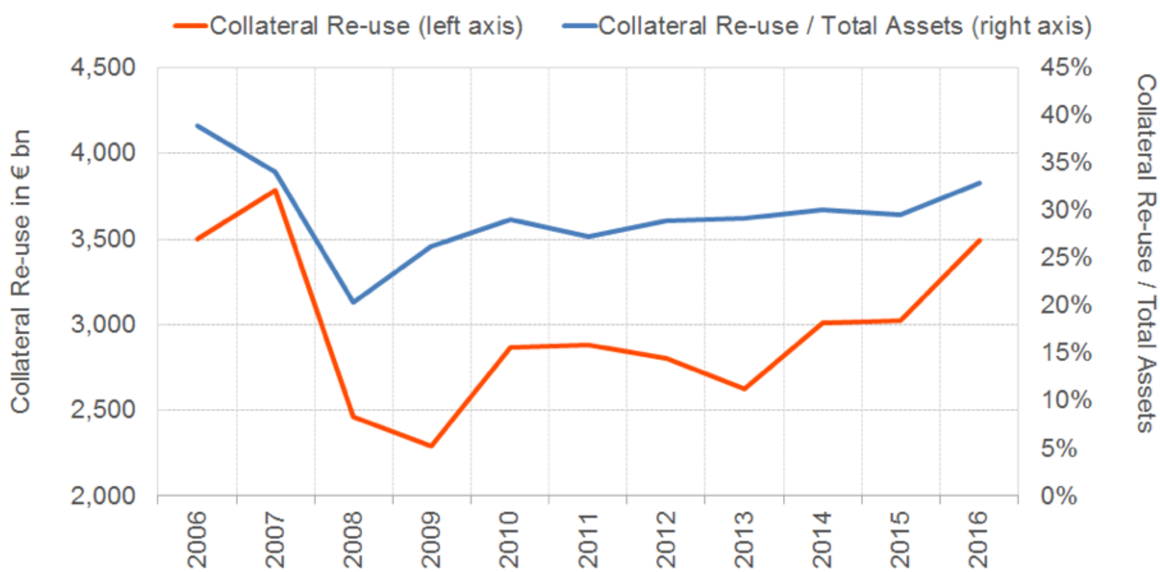
2.1 Defining Re-Use of Collateral

The Financial Stability Board (FSB) defines collateral re-use in a broad sense as "any use of assets delivered as collateral in a transaction by an intermediary or other collateral taker", see FSB (2017b). Financial institutions receive collateral in securities financing transactions (SFTs, e.g., reverse repo, securities lending/borrowing) or derivative transactions. Depending on the terms of the transaction, the collateral may be eligible for re-use by the receiving counterparty: they can use it for their own purposes (e.g., for repos, securities lending/borrowing, short sales, derivatives collateral). For example, in a repurchase transaction (repo), the counterparty providing cash (i.e. the collateral taker) can re-use the securities obtained as collateral to, among other things, pledge as collateral in a separate transaction with a third party. Often, the term "collateral re-use" is used interchangeably with the term "re-hypothecation". Again, we follow FSB (2017b) where re-hypothecation is defined narrowly as "any use by a financial intermediary of client assets". We therefore consider re-hypothecation as a subset of the broad collateral re-use concept. In a typical re-hypothecation transaction, securities that serve as collateral for a secured borrowing (e.g., a margin loan extended to a hedge fund) are further used by the intermediary to obtain funding for the initial transaction.

2.2 The Relevance of Collateral Re-Use in Financial Markets

Re-use of collateral has become a major activity in financial markets, and is a common practice across many entities in the financial system. Though incomplete data makes it difficult at this junction to pin down the exact size of global collateral re-use activity, publicly available data on collateral re-use activity of globally active banks indicates that collateral re-use plays an important role in financial markets.[5] Figure I displays the evolution of re-use activity for a set of 11 global banks. The level of collateral re-use amongst the 11 global banks amounted to around 3.8 trillion euro and approximately 30% of total assets of these financial institutions before the crisis. The time

Figure I: Collateral Re-use



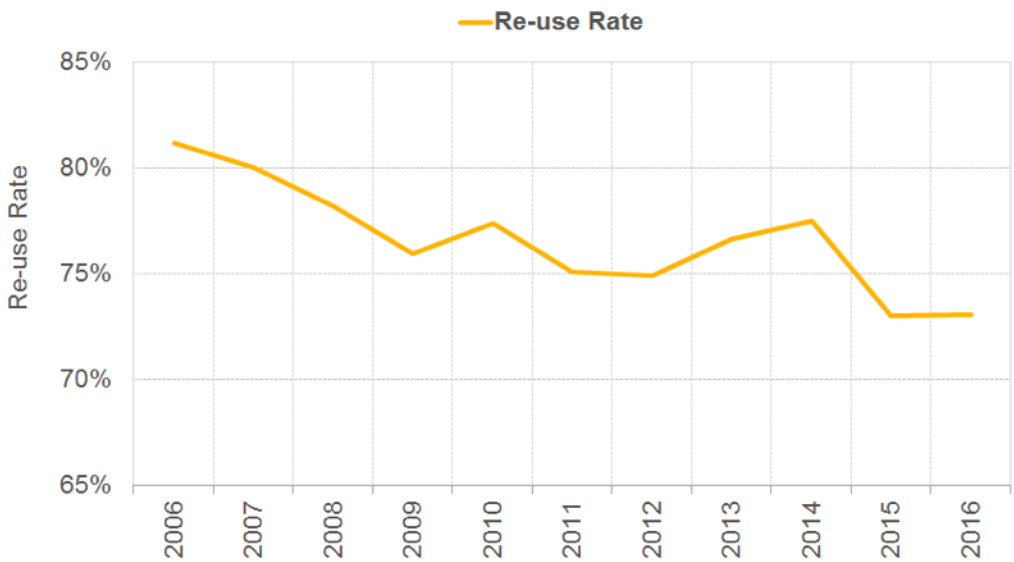
Own calculations based on 11 banks' annual reports. The institutions are Deutsche Bank, BNP Paribas, Credit Suisse, UBS, Barclays, RBS, HSBC, Nomura, Morgan Stanley, JPMorgan Chase & Co., Goldman Sachs. The figure shows collateral re-use in billion Euros and the ratio of the values of re-used collateral and total assets.

series further reveals that collateral re-use exhibits strong pro-cyclicality. The most striking fact appearing in the data is the sharp contraction in collateral re-use taking place during the global financial crisis, in 2008. Thereafter, the amount of collateral re-used by financial institutions has risen again, however, without returning to pre-crisis levels.

Figure II reports the evolution of the average re-use rate for the set of 11 global banks. The re-use rate is defined as re-used collateral divided by collateral received that is eligible for re-use. The re-use rate reflects banks' intensity of re-use. Even during the financial crisis 2007–09, the average re-use rate remained above 70%.

[5] See Singh (2014) for a similar exercise using a different methodology.

Figure II: Collateral Re-use Rate



Own calculations based on 11 banks' annual reports. The institutions are Deutsche Bank, BNP Paribas, Credit Suisse, UBS, Barclays, RBS, HSBC, Nomura, Morgan Stanley, JPMorgan Chase & Co., Goldman Sachs. The figure shows the proportion of re-used collateral relative to the collateral received that is eligible for re-use.

2.3 Financial Stability Risks Associated with Collateral Re-Use

While market participants have stressed the importance of re-use of collateral as a source of funding and market liquidity more generally, regulators and supervisors have raised various concerns about this market practice. For example, the same piece of collateral may be used to back a chain of transactions and may therefore contribute to system-wide leverage. This concern has, for example, been raised by Vice-President of the ECB, Vítor Constâncio, who stressed that "activities of re-hypothecation and re-use of securities amplified the creation of chains of inside liquidity and higher leverage."[6] More specifically, the use of the same security as underlying collateral for different transactions increases the sum of exposures in the financial system and, and as result, creates leverage across the intermediation chain. The evidence presented in Figures I and II suggests that re-use practices may be significant drivers of financial system leverage. The apparent cyclical behavior of aggregate re-use activities suggests that re-use may also contribute to pro-cyclicality in the financial sector. In good times, market participants tend to be more willing to allow counterparties to re-use the collateral, increasing market liquidity and lowering the cost of capital. However, in stressed market conditions, market participants become more sensitive to counterparty risk and hence, may refuse the re-use of their collateral, amplifying strains already present in markets (see also FSB (2017b)). Moreover, FSB (2017b) highlights the issue of interconnectedness arising from chains of transactions involving the re-use of collateral. Large exposures amongst financial institutions create

[6] V. Constâncio, Beyond traditional banking: a new credit system coming out of the shadows, Speech, University of Frankfurt, 17 October 2014.

a risk of contagion, though it is acknowledged that securities financing transactions usually involve small net exposures because of daily margining. Nevertheless, the unwinding of a transaction by one institution may trigger the unwinding of transactions by other institutions, and lead to the propagation of shocks through the financial system. In the context of re-hypothecation it has been argued that it creates the possibility of a run on a prime-broker if there are concerns about its credit worthiness, and therefore clients have an incentive to withdraw their assets from their prime brokers.[7] This risk appears, at least to some extent, to have materialized around the failures of Bear Stearns and Lehman Brothers, where hedge funds moved their assets from their prime brokers. These actions exacerbated the financing problems faced by the two entities which had relied on the availability of client assets for the financing of their activities.[8]

2.4 The Regulatory Framework and Ongoing Initiatives

When it comes to the regulatory framework for collateral re-use, it helps to distinguish between broader re-use of collateral and the narrower set of client asset re-hypothecation. On collateral re-use more broadly, no general restrictions exist. There are, however, a few cases where limits or bans on re-use exist for specific entities or transactions. One example relates to retail investment funds (UCITS funds) in the EU. The ESMA Guidelines on ETFs and other UCITS issues[9] foresee that UCITS do not re-use non-cash collateral they receive and set conditions for re-investment of cash collateral. A second example relates to initial margin posted in OTC derivatives. The EU regulatory technical standards (RTS) on margin requirements for non-centrally cleared derivatives foresees that re-use of non-cash collateral collected as initial margin shall not be permitted. In the international context, the FSB has recently published two papers related to collateral re-use. FSB (2017b) analyzes the financial stability implications of collateral re-use as well as the benefits of collateral re-use for financial markets. This document furthermore describes a number of post-crisis regulatory reforms related both directly and indirectly to the re-use of collateral and mitigating at least some of the risks described above. In particular, rules affecting collateral transactions like repos may indirectly limit the extent to which entities may re-use collateral. For example, FSB (2017b) highlights that the Basel III Leverage Ratio (LR) framework indirectly incentivizes banks to keep re-use activity below excessive levels. The LR framework triggers a regulatory capital for any additional funding raised by re-using collateral, and therefore it is considered as an important brake to address risk related to the contribution of re-use of collateral to the build-up of leverage in the banking sector, see FSB (2017b). While the FSB sees no need for immediate additional regulatory action, it considers that appropriately monitoring collateral re-use at the global level will be an important step towards obtaining a clearer understanding of global collateral re-use activities. Hence, to enhance monitoring of global collateral re-use activity, the FSB published a paper that sets out methodology for measuring

[7] See Duffie (2010) for an extensive discussion of this issue and Infante and Vardoulakis (2018) for a theoretical model exploring this risk channel.

[8] See Aragon and Strahan (2012) for an analysis of the impact the failure of Lehman Brothers had on its hedge fund clients.

[9] See `https://www.esma.europa.eu/sites/default/files/library/2015/11/esma-2014-0011-01-00_en_0.pdf`, last accessed May 28, 2018.

non-cash collateral re-use, and describes the related data elements that national authorities would report to the FSB (see FSB (2017a)). This FSB paper further defines various metrics that could be used to monitor financial stability risks associated with collateral re-use.

To address financial stability concerns related to re-hypothecation of client assets, the FSB issued several recommendations in 2013 (see FSB (2013)). These aim at reducing client uncertainty about the extent to which assets have been re-hypothecated and the treatment in case of bankruptcy, and at limiting re-hypothecation of client assets (without an offsetting indebtedness) to financial intermediaries subject to adequate regulation of liquidity risk. Moreover, the FSB recommended that prime brokers/banks shall use that amount of client assets only that they need for the purpose of financing client activities, representing an aggregate limit on the amount of client assets they can re-hypothecate. Without this aggregate limit, prime brokers could in theory use all client assets eligible for re-hypothecation, including for their own activities and irrespective of the aggregate level of financing their set of clients requires.

With regard to existing regulatory measures on re-hypothecation, in the EU the recently agreed Securities Financing Transaction Regulation (SFTR) imposes minimum market-wide conditions to be met on re-hypothecation of client assets such as prior consent, disclosure of the risks and consequences of re-hypothecation, and transfer of the financial instruments from the account of the client. Furthermore, in the EU no aggregate nor client-individual limit on the amount of client assets available for re-hypothecation exists and is instead agreed between the two counterparties. This is in contrast to US framework where an individual limit of 140% of the client's indebtedness applies, next to an aggregate limit in line with the aforementioned FSB recommendation. However, specifically to EU investment funds, custodians of UCITs assets are prohibited from re-using (including, but not limited to, transferring, pledging, selling, and lending) fund's assets for their own account (see FSB (2017a)).

3 A Simple Two-period Model

In this section we present a simple two-period model to illustrate some of the key qualitative effects of collateral re-use. Our objective is to develop some initial intuition for our welfare results in the calibrated infinite-horizon economy.

3.1 The Physical Economy

Consider a two-period model with two agents, $h = 1, 2$. In period 0, the economy is in state $s = 0$. There are two possible states in period 1, $s = 1, 2$. In state s, agent h receives a total endowment $\omega_s^h > 0$, $s = 0, 1, 2$. The aggregate endowment, $\omega_s = \omega_s^1 + \omega_s^2$, has the property $\omega_2 > \omega_0 > \omega_1$. So, we can call state 2 "good" and state 1 "bad". The endowments of the two agents are comprised of two parts; first, identical individual incomes, $e^h = \frac{1}{2}(1 - \delta)\omega$; and second, in state 0, identical individual portions of the dividends from a "Lucas tree", $\frac{1}{2}\delta\omega_0$, as a result of each agent initially holding half a share of the tree. So, the tree is in unit net supply and total dividends constitute the portion $\delta \in (0, 1)$ of the aggregate endowment, $d_s = \delta\omega_s$.

In period 0, the agents can trade a risk-free bond (asset 1) in zero net supply—in addition to the tree (asset 2). We denote the period-0 asset prices by q_1, q_2, and the agents' post-trading asset holdings by $\theta^h = \left(\theta_1^h, \theta_2^h\right)$. Agent h's budget constraints are

$$
\begin{aligned}
c_0^h &= \frac{1}{2}\omega_0 - q_1\theta_1^h - q_2\left(\theta_2^h - \frac{1}{2}\right), \\
c_s^h &= \frac{1}{2}(1-\delta)\omega_s + \theta_1^h + \theta_2^h\delta\omega_s, \quad s = 1, 2.
\end{aligned}
$$

Note that $\theta_2^h - \frac{1}{2}$ denotes the net trade of tree shares by agent h in period 0.

To present the main welfare result in the simplest possible framework, we assume that agent 1 is risk-neutral. His utility function is

$$U^1(c) = c_0 + \pi^1 c_1 + (1-\pi^1)c_2,$$

with $\pi^1 \in (0,1)$ denoting his subjective probability for state $s = 1$. Contrary to agent 1, agent 2 is risk-averse. She has log utility,

$$U^2(c) = \ln c_0 + \pi^2 \ln c_1 + (1-\pi^2)\ln c_2,$$

with a subjective probability $\pi^2 \in (0,1)$ for state 1.

3.2 Arrow–Debreu Equilibrium

Without any additional constraints, markets are complete and there is a unique Arrow–Debreu equilibrium. The first-order conditions of agent 1's utility maximization problem determine the three state prices (p_0, p_1, p_2) in the economy

$$p_0 = 1,\ p_1 = \pi^1,\ p_2 = 1 - \pi^1.$$

Solving agent 2's utility maximization problem at these state prices yields her consumption allocation,

$$
\begin{aligned}
c_0^2 &= \frac{1}{2}\left(\omega_0^2 + \pi^1\omega_1^2 + (1-\pi^1)\omega_2^2\right), \\
c_1^2 &= \frac{\pi^2}{2\pi^1}\left(\omega_0^2 + \pi^1\omega_1^2 + (1-\pi^1)\omega_2^2\right), \\
c_2^2 &= \frac{1-\pi^2}{2(1-\pi^1)}\left(\omega_0^2 + \pi^1\omega_1^2 + (1-\pi^1)\omega_2^2\right).
\end{aligned}
$$

Agent 1's first-order conditions also determine the two asset prices in period 0,

$$q_1 = 1,\ q_2 = \pi^1\delta\omega_1 + (1-\pi^1)\delta\omega_2.$$

We assume that both states in period 1 are equally likely. The risk-neutral agent 1 has correct beliefs, so $\pi^1 = \frac{1}{2}$. The risk-averse agent 2 is pessimistic and believes that the bad state is more likely than the good state, so $\pi^2 > \frac{1}{2}$. Observe that this assumption immediately yields $c_1^2 > c_0^2 > c_2^2$ for agent 2's consumption allocation.

To simplify the subsequent analysis of collateral re-use, we consider a specific parameterization of the two-period economy with

$$\omega_s = (1, 0.9, 1.1),\ \delta = 0.4,\ \pi^2 = \frac{21}{40}.$$

The resulting consumption allocations in equilibrium are

$$c^1 = \left(\frac{1}{2}, \frac{15}{40}, \frac{25}{40}\right) \quad \text{and} \quad c^2 = \left(\frac{1}{2}, \frac{21}{40}, \frac{19}{40}\right).$$

Equilibrium asset prices and portfolios are

$$q = \left(1, \frac{2}{5}\right) \quad \text{and} \quad \theta^1 = \left(-\frac{3}{4}, \frac{19}{8}\right), \quad \theta^2 = \left(\frac{3}{4}, -\frac{11}{8}\right),$$

respectively.

3.3 Collateral Constraints

The Arrow–Debreu equilibrium assumes that agents will honor any debt in the second period. We now remove this assumption and instead impose collateral constraints on the two agents. The agents can hold only portfolios that do not give them an incentive to default in period 1—that is, the net portfolio payoff of each agent must be positive in both states,

$$\theta_1^h + \theta_2^h \delta \omega_s \geq 0, \quad s = 1, 2.$$

If the Arrow–Debreu equilibrium portfolios satisfy these constraints, then the equilibrium prices and portfolios also constitute the equilibrium of the collateral-constrained economy.

In our parameterization, the risk-neutral agent 1 is long in the tree and so, if any, the collateral constraint in the bad state, $s = 1$, may bind. The risk-averse agent 2 is short in the tree and so the constraint in the good state, $s = 2$, is tighter than that in the bad state. For the specific values,

$$\theta_1^1 + \theta_2^1 \delta \omega_1 = \frac{21}{200} \quad \text{and} \quad \theta_1^2 + \theta_2^2 \delta \omega_2 = \frac{29}{200}.$$

Therefore, the Arrow–Debreu equilibrium is also the equilibrium of the economy with collateral constraints. Simply put, the collateral constraints are sufficiently "loose" and so do not prevent the complete-markets equilibrium.

3.4 Re-Use Constraint

In a final step, we now impose an additional restriction on the agents in the economy. Short positions in the tree (asset 2) cannot be "naked"— that is, an agent must borrow the tree in order to enter a short sale. In particular, an agent can only do a short-sale of the tree, if the other agent provided him or her with tree shares as collateral. However, the agent cannot sell all of the received collateral but only a portion $\kappa \in [0, 1]$. This condition constitutes a collateral re-use constraint on the agents.

The aggregate endowments and the equilibrium consumption of the risk-averse agent 2 show exactly opposite monotonicity patterns. As a result, in equilibrium, the risk-averse agent 2 always enters into a short position in the tree (asset 2) and holds a long position in the bond (asset 1); see, for illustration, the equilibrium in the numerical example above. Therefore, it suffices to analyze a re-use constraint for agent 2 only.

When agent 1 borrows (in the bond) from agent 2, then he must pledge $\frac{-\theta_1^1}{\delta\omega_1} > 0$ shares of the tree as collateral to agent 2. And now agent 2 faces the re-use constraint

$$\theta_2^2 \geq \kappa \frac{\theta_1^1}{\delta\omega_1},$$

when she enters a short-sale in the tree. Observe that the right-hand side of the constraint is negative. It constitutes a lower bound on the possible short-sales for agent 2. If the re-use parameter is zero, $\kappa = 0$, then not only is re-use prohibited but so are short-sales of the tree.

For our illustrative parametrization, the re-use constraint simplifies to

$$-\frac{11}{8} + \kappa \frac{25}{12} \geq 0 \quad \Longleftrightarrow \quad \kappa \geq \frac{33}{50}.$$

For $\kappa \geq 0.66$, the Arrow–Debreu equilibrium also constitutes the (financial-markets) equilibrium for the economy with collateral constraints and the re-use constraint. On the contrary, for a value of the re-use parameter below 0.66, the portfolios supporting the Arrow–Debreu equilibrium violate the re-use constraint.

3.5 Constrained Equilibrium

When the risk-averse agent 2 is lending to the risk-neutral agent 1 via a bond purchase of $\theta_1^2 > 0$ units of the bond, then agent 1 must pledge as collateral

$$\frac{\theta_1^1}{\delta\omega_1} = \frac{-\theta_1^2}{\delta\omega_1}$$

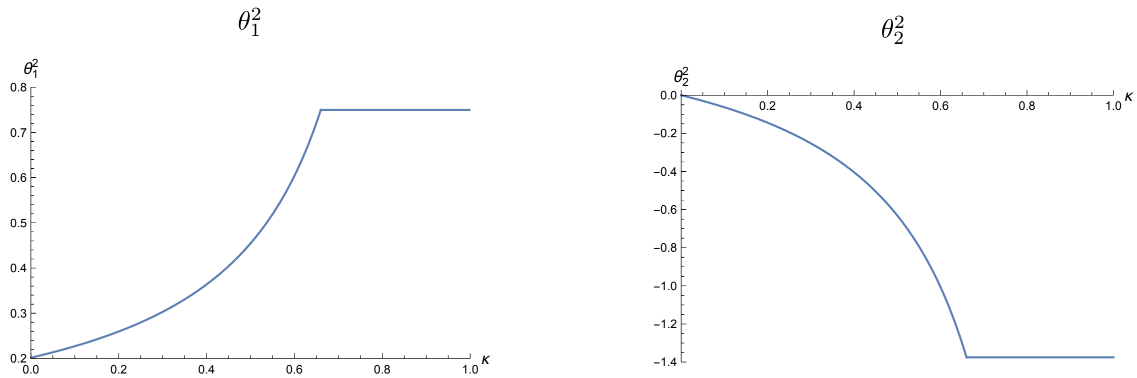
shares of the tree to agent 2. If re-use is permitted in the economy, then agent 2 can use a portion κ of this collateral for short sales. If the re-use constraint is binding, then her portfolio must satisfy the condition

$$\theta_2^2 = \kappa \frac{-\theta_1^2}{\delta\omega_1}.$$

In the economy with re-use, the risk-neutral agent 1 continues to be unconstrained for all values of the re-use parameter κ. Therefore, his first-order conditions continue to determine the asset prices and they have the same values as in the Arrow–Debreu equilibrium, $q = \left(1, \frac{2}{5}\right)$. For these prices, we can solve agent 2's utility maximization problem to determine her optimal portfolio as a function of κ. Since the asset prices do not depend on the re-use parameter, there are no price effects when κ changes its value. As a result, the feasible regions of agent 2's utility maximization problem are a weakly increasing sequence of sets in $\kappa \in [0, 1]$.

Figure III shows graphs of the risk-averse agent 2's bond and tree holdings as a function of the re-use parameter κ. (For all values of $\kappa \in [0, 1]$, the collateral constraints are non-binding.) When re-use is not permitted, $\kappa = 0$, agent 2 sells all her tree shares in period 0 and holds no shares of the tree. When κ increases, she sells as many shares of the tree short as the re-use constraint permits her to do. She then invests these funds into an increasingly larger bond position. The re-use constraint is binding for $\kappa \in [0, 0.66]$. For $\kappa > 0.66$ the re-use constraint does not bind and the holdings are identical to those of the (unconstrained) Arrow–Debreu equilibrium reported above.

Figure III: Portfolio of the risk-averse agent 2 as a function of κ



The figure shows graphs of the equilibrium holdings of the risk-averse agent 2 as a function of the re-use parameter κ. The left plot depicts her bond position, θ_1^2, and the right graph depicts her tree holding, θ_2^2. For $\kappa \leq 0.66$ the re-use constraint is binding. For $\kappa > 0.66$ the re-use constraint does not bind and the holdings are identical to those of the Arrow–Debreu equilibrium.

Figure IV(a) shows the risk-averse agent 2's consumption allocations for all three states, $s = 0, 1, 2$, as a function of κ. In autarchy, agent 2 would have to consume her endowment, $(0.5, 0.45, 0.55)$. For $\kappa = 0$, the agent can trade to a slightly different allocation, $(0.4984, 0.4716, 0.5316)$. When κ increases from zero, the agent can hold increasingly larger portfolios and thereby move her consumption allocation towards the Arrow–Debreu allocation.

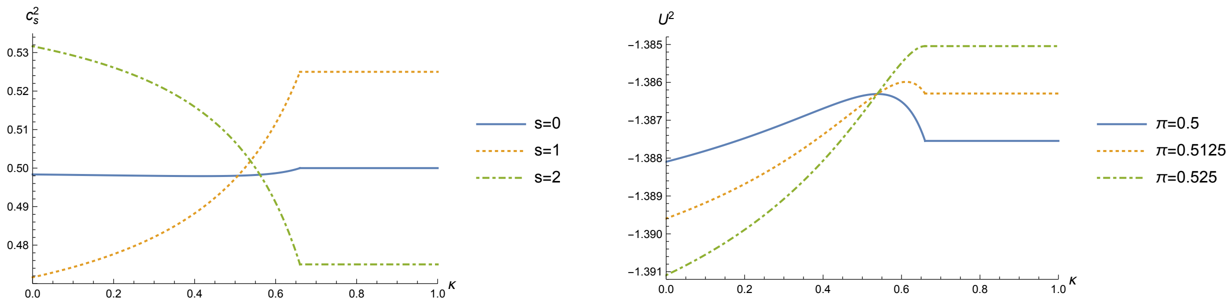
Figure IV(b) shows agent 2's utility as a function of the re-use parameter κ—evaluated for three different beliefs. The dashed–dotted line shows the agent's utility for her subjective probabitprobabilityity $\pi^2 = 0.525$ of state $s = 1$. Since the feasible regions of agent 2's utility maximization problems are an increasing sequence of sets for $\kappa \in [0, 0.66]$, her utility is increasing in κ in this region. Once the re-use constraint stops being binding, the utility is constant. The dotted line shows agent 2's utility from the consumption allocations displayed in Figure IV(a) for a smaller probability of the bad state $s = 1$, namely for $(\pi^1 + \pi^2)/2 = 0.5125$. Finally, the solid line shows her utility evaluated with the true probability $\pi^1 = 0.5$. We observe that in these two cases, her utility is maximized for an interior value of κ.

Recall that agent 2 is pessimistic and believes that the bad state 1 is more likely than the good state 2. As a result, see Figure IV(a), she trades so that she can consume more in state 1 than in state 2. But when the actual probability of state $s = 1$ is smaller than her subjective probability then she is moving too much consumption into that state. As a result, for $\kappa > 0.66$, the smaller the probability of the first state the smaller her utility.

For small values of κ near zero, we observe the opposite pattern. Here the re-use constraint is so strict that the pessimistic and risk-averse agent 2 cannot build a portfolio in order to support more

Figure IV: Consumption allocation and utility of the risk-averse agent 2 as a function of κ

(a) c_s^2 (b) U^2



The left graph in the figure depicts the risk-neutral agent 2's consumption levels for all three states, $s = 0, 1, 2$, as a function of κ. For $\kappa > 0.66$, the re-use constraint does not bind and the consumption allocation and utility level are identical to their respective Arrow–Debreu values. The right graph depicts the expected utility of agent 2 as a function of κ, evaluated for three different sets of probabilities. The dashed–dotted line shows the agent's utility for her subjective probability $\pi^2 = 0.525$ of state $s = 1$. The solid line shows her utility evaluated with the true probability $\pi^1 = 0.5$. The dotted line shows her utility evaluated with the probability $(\pi^1 + \pi^2)/2$.

consumption in the bad than in the good state. And now, when the actual probability of state $s = 1$ is smaller than her subjective probability, the agent's preference for moving consumption decreases (despite her risk aversion). And so her utility increases when the probability of $s = 1$ decreases.

In sum, when the risk-averse agent 2 is overly pessimistic—that is, her subjective probability of the bad state is larger than the true probability—then she is trying to move too much consumption into the bad state. A value of the re-use parameter prohibiting the Arrow–Debreu consumption allocation (under her subjective probabilities) now actually improves her utility (under the true probabilities). However, if κ is very small, then the re-use regulation restricts the possibility for risk-sharing between the two agents. Due to her risk aversion, agent 2 wants to smooth consumption across all three states in the economy. Too little re-use restricts risk-sharing in the economy, too much re-use results in an overly pessimistic agent hedging bad states too much. And so we observe an interior welfare maximum for agent 2 for the true probabilities for the two states $s = 1, 2$.

4 The Infinite-Horizon Model

This section introduces an infinite-horizon exchange economy with two infinitely-lived heterogeneous agents trading in a Lucas tree and a bond. Both assets can be traded on margin, in which case collateral needs to be posted. The received collateral can potentially be re-used. Both assets can be shorted. In case of the Lucas tree, naked shorting is not possible, so an agent can only short the tree if he has received it as collateral. A regulating agency limits the extend to which collateral can be re-used.

4.1 The Physical Economy

Time is indexed by $t = 0, 1, 2, \ldots$. Exogenous shocks (s_t) follow a Markov chain with support $\mathcal{S} = \{1, \ldots, S\}$ and transition matrix π. The evolution of time and shocks in the economy is represented by an infinite event tree Σ. Each node of the tree, $\sigma \in \Sigma$, describes a finite history of shocks $\sigma = s^t = (s_0, s_1, \ldots, s_t)$ and is also called a date-event. The symbols σ and s^t are used interchangeably. To indicate that $s^{t'}$ is a successor of s^t (or is s^t itself), write $s^{t'} \succeq s^t$. The expression s^{-1} refers to the initial conditions of the economy prior to $t = 0$.

At each date-event $\sigma \in \Sigma$, there is a single perishable consumption good. The economy is populated by $H = 2$ agents, $h \in \mathcal{H} = \{1, 2\}$. Agent h receives an individual endowment in the consumption good, $e^h(\sigma) > 0$, at each node. Agent h believes that the transition matrix of the Markov chain of exogenous shocks is π^h—this matrix may differ from the true transition matrix π. There is a single long-lived asset ("Lucas tree") in the economy, which we also call stock. At the beginning of period 0, each agent h owns initial holdings $\theta^h(s^{-1}) \geq 0$ of this asset. Aggregate holdings in the long-lived asset sum to one—that is, $\sum_{h \in \mathcal{H}} \theta^h(s^{-1}) = 1$. At date-event σ, agent h's (end-of-period) holding of the asset is denoted by $\theta^h(\sigma)$. The long-lived asset pays positive dividends $d(\sigma)$ in units of the consumption good at all date-events. The aggregate endowment in the economy is then

$$\bar{e}(\sigma) = d_a(\sigma) + \sum_{h \in \mathcal{H}} e^h(\sigma).$$

Agent h has preferences over consumption streams $c^h = \left(c^h(s^t)\right)_{s^t \in \Sigma}$ representable by the following recursive utility function, see Epstein and Zin (1989),

$$U^h\left(c^h, s^t\right) = \left\{ \left[c^h(s^t)\right]^{\rho^h} + \beta \left[\sum_{s_{t+1}} \pi^h(s_{t+1}|s_t) \left(U^h(c^h, s^{t+1})\right)^{\alpha^h} \right]^{\frac{\rho^h}{\alpha^h}} \right\}^{\frac{1}{\rho^h}},$$

where $\frac{1}{1-\rho^h}$ represents the inter-temporal elasticity of substitution (IES) and $1 - \alpha^h$ the relative risk aversion of the agent.

4.2 Financial Markets and Collateral

At each date-event, agents can engage in security trading of two assets, the long-lived stock and a one-period bond. Agent h can buy $\theta^h(\sigma)$ shares of the long-lived asset at node σ for a price $q(\sigma)$. Importantly, agents can short-sell long-lived assets. We assume that short positions in the long-lived asset cannot be "naked", meaning that agents must cover their short sales by having borrowed the long-lived asset.[10] In addition to the long-lived asset, there is a single one-period bond available for trade; this bond is in zero net supply and its face value is one unit of the consumption good in the subsequent period. Agent h's (end-of-period) holding of this bond at date-event σ is denoted

[10] Note that this assumption is in line with short selling regulations in the EU and the US (see, e.g., `https://www.esma.europa.eu/regulation/trading/short-selling`)

by $\phi^h(\sigma)$, and the price of the bond at this date-event by $p(\sigma)$. Agents can also take up debt by shorting this bond.

The agents can default on short positions in the long-lived asset or the short-lived bond at any time without any utility penalties or loss of reputation. Therefore, to enter a short position in either one of the two assets, agents must back up their promised payments by collateral. Since there are only two assets in the economy, an agent who borrows by assuming a short position in the long-lived asset must hold a long position in the bond as collateral. And vice versa, an agent who shorts the bond must hold a long position of the long-lived asset as collateral.

Specifically, if an agent borrows by short-selling the stock, $\theta^h(s^t) < 0$, then that agent is required to hold a sufficient amount of collateral in the bond. The difference between the value of the collateral holding in the bond, $p(s^t)\phi^h(s^t) > 0$, and the current value of the loan, $-q(s^t)\theta^h(s^t)$, is the amount of capital the agent puts up to obtain the loan. A margin requirement, $m_s(s^t) \in [0, 1]$, enforces a lower bound on the value of this capital relative to the value of the collateral,

$$m_s(s^t)\left(p(s^t)\phi^h(s^t)\right) \leq q(s^t)\theta^h(s^t) + p(s^t)\phi^h(s^t). \tag{1}$$

Similarly, if an agent borrows by short-selling the bond, $\phi^h(s^t) < 0$, then that agent is required to hold a sufficient amount of collateral in the long-lived asset. The difference between the value of the collateral holding in the long-lived asset, $q(s^t)\theta^h(s^t) > 0$, and the current value of the loan, $-p(s^t)\phi^h(s^t)$, is the amount of capital the agent puts up to obtain the loan. A margin requirement, $m_l(s^t) \in [0, 1]$, enforces a lower bound on the value of this capital relative to the value of the collateral,

$$m_l(s^t)\left(q(s^t)\theta^h(s^t)\right) \leq q(s^t)\theta^h(s^t) + p(s^t)\phi^h(s^t). \tag{2}$$

Since there are no penalties for default, an agent who sold the stock at date-event s^t defaults on his or her promise at a successor node s^{t+1} whenever the initial promise exceeds the current value of the collateral—that is, whenever

$$-\theta^h(s^t)\left(q(s^{t+1}) + d(s^{t+1})\right) > \phi^h(s^t).$$

Similarly, an agent who sold the bond at date-event s^t defaults on his or her promise at a successor node s^{t+1} whenever the initial promise exceeds the current value of the collateral—that is, whenever

$$-\phi^h(s^t) > \theta^h(s^t)\left(q(s^{t+1}) + d(s^{t+1})\right).$$

In this paper, margin requirements are sufficiently large so that no default occurs in equilibrium and defaultable bonds are not traded. In Brumm et al. (2015) this restriction is an equilibrium outcome: following Geanakoplos (1997) and Geanakoplos and Zame (2002), Brumm et al. (2015) assume that, in principle, bonds with any margin requirement may be traded in equilibrium, yet show that with moderate default costs only risk-free bonds are traded.

In the economy, margin requirements are determined endogenously. Financial markets determine margin requirements, $m_s(s^t)$ and $m_l(s^t)$, that are just large enough to prevent default in equilibrium. Specifically, for borrowing with the long-lived asset against the bond, market-determined margin requirements $m_s(s^t)$ are the lowest possible margins that still ensure no default in the subsequent period,

$$m_s(s^t) = 1 - \frac{q(s^t)}{\max_{s_{t+1}} \{q(s^{t+1}) + d(s^{t+1})\} \, p(s^t)}.$$

Substituting this margin requirement into inequality (1) leads to the inequality

$$-\theta^h(s^t) \max_{s_{t+1}} \left\{ q(s^{t+1}) + d(s^{t+1}) \right\} \leq \phi^h(s^t).$$

Similarly, for borrowing with the bond against the long-lived asset, market-determined margin requirements $m_l(s^t)$ are the lowest possible margins that still ensure no default in the subsequent period,

$$m_l(s^t) = 1 - \frac{p(s^t) \cdot \min_{s_{t+1}} \left\{ q(s^{t+1}) + d(s^{t+1}) \right\}}{q(s^t)}.$$

Substituting this margin requirement into inequality (2) leads to the inequality

$$-\phi^h(s^t) \leq \theta^h(s^t) \min_{s_{t+1}} \left\{ q(s^{t+1}) + d(s^{t+1}) \right\}.$$

This margin requirement makes the bond risk-free by ensuring that a short-seller will never default on his or her promise.

4.3 Re-use of Collateral

In our economy, when collateral is posted, the agent receiving it has the right to re-use the collateral for his own purposes, either using it as collateral for another transaction or (short-)selling it.[11] We denote the amount of collateral received by agent h by $\theta^h_{received}(s^t)$, and the amount of collateral re-used by $\theta^h_{reused}(s^t)$. We furthermore assume that a regulatory agency can set a limit $\kappa(s^t) \in [0, 1]$ on the fraction of the received collateral that can be re-used by agents. This leads to the following re-use constraint for agent h:

$$\theta^h_{reused}(s^t) \leq \kappa(s^t) \cdot \theta^h_{received}(s^t). \tag{3}$$

Note that, given the assumption that naked shorting is ruled out in our framework, allowing for re-use only renders possible short-sales in the long-lived asset. Setting the re-use parameter $\kappa(s^t)$ equal to zero would lead to a situation where not only re-use is ruled out but also short-selling is banned completely. Using the collateral constraint (2) and the re-use constraint (3) we can now determine the maximum short position agents can assume. First of all, we need to determine how much collateral agent h receives, $\theta^h_{received}(s^t)$. The collateral constraint (2) that the other agent, $-h$, faces for a short position in the bond dictates how much collateral agent $-h$ has to post and thus how much agent h receives:

$$\theta^{-h}(s^t) \geq \frac{p(s^t) \cdot \max\{0, -\phi^{-h}(s^t)\}}{(1 - m_l(s^t))\, q(s^t)} = \theta^h_{received}(s^t). \tag{4}$$

[11] In financial markets, a significant amount of collateral is supplied including a transfer of title (e.g., in repo transactions) that implies that for the length of the transaction the counterparty receiving the collateral becomes the owner of the collateral. It is, therefore, free to use it as collateral in a different transaction or to sell it.

The lender, agent h, can reuse a portion $\kappa(s^t)$ of this collateral for his own purposes. If he uses it for short-selling, we have $-\theta^h(s^t) = \theta^h_{reused}(s^t)$. Using (3) and (4), we obtain a lower bound on the position of agent h in the long-lived asset,

$$\theta^h(s^t) \geq -\kappa(s^t) \frac{p(s^t) \max\{0, -\phi^{-h}(s^t)\}}{(1 - m_l(s^t))\, q(s^t)}. \tag{5}$$

Observe that for $\phi^{-h}(s^t) < 0$ obviously $\phi^h(s^t) > 0$; the lower bound is negative and agent h is permitted to short the long-lived asset. Essentially he sells a portion of the collateral owned by agent $-h$ but pledged to him. Note that the re-use constraint (5) is also valid for $\phi^h(s^t) \leq 0$ and then reduces to $\theta^h(s^t) \geq 0$. If the agent does not own collateral in the short-lived bond, then he cannot short the long-lived asset.

4.4 Financial Markets Equilibrium with Collateral

We are now in the position to formally define a financial markets equilibrium. Equilibrium values of a variable x are denoted by $\bar{x}$.

Definition 1 *A financial markets equilibrium for an economy with regulated re-use limits,* $(\kappa(\sigma))_{\sigma\in\Sigma}$*, initial shock* s_0*, and initial asset holdings* $(\theta^h(s^{-1}))_{h\in\mathcal{H}}$ *is a collection of agents' portfolio holdings and consumption allocations as well as security prices and margin requirements,* $\Big(\big(\bar{\theta}^h(\sigma), \bar{\phi}^h(\sigma), \bar{c}^h(\sigma)\big)_{h\in\mathcal{H}}; \bar{q}(\sigma), \bar{p}(\sigma); \bar{m}_l(\sigma), \bar{m}_s(\sigma)\Big)_{\sigma\in\Sigma}$*, satisfying the following conditions:*

(1) Markets clear:

$$\sum_{h\in\mathcal{H}} \bar{\theta}^h(\sigma) = 1 \quad \text{and} \quad \sum_{h\in\mathcal{H}} \bar{\phi}^h(\sigma) = 0 \quad \text{for all } \sigma \in \Sigma.$$

(2) For each agent h*, the choices* $\big(\bar{\theta}^h(\sigma), \bar{\phi}^h(\sigma), \bar{c}^h(\sigma)\big)$ *solve the agent's utility maximization problem,*

$$\begin{aligned}
\max_{\theta\geq 0,\phi,c\geq 0} U_h(c) \quad & \text{s.t.} \quad \text{for all } s^t \in \Sigma \\
c(s^t) + \theta^h(s^t)\bar{q}(s^t) + \phi^h(s^t)\bar{p}(s^t) \quad &= \quad e^h(s^t) + \phi(s^{t-1}) + \theta^h(s^{t-1})\left(\bar{q}(s^t) + d(s^t)\right), \\
\bar{m}_l(s^t)\bar{q}(s^t)\theta^h(s^t) \quad &\leq \quad \bar{q}(s^t)\theta^h(s^t) + \bar{p}(s^t)\phi^h(s^t), \\
\bar{m}_s(s^t)\bar{p}(s^t)\phi^h(s^t) \quad &\leq \quad \bar{q}(s^t)\theta^h(s^t) + \bar{p}(s^t)\phi^h(s^t), \\
\theta^h(s^t)\bar{q}(s^t)\left(1 - m_l(s^t)\right) \quad &\geq \quad -\kappa(s^t)\cdot\bar{p}(s^t)\max\{0, -\bar{\phi}^{-h}(s^t)\}.
\end{aligned}$$

(3) For all s^t*, the margin requirements satisfy*

$$\begin{aligned}
\bar{m}_l(s^t) &= 1 - \frac{\bar{p}(s^t)\cdot\min_{s_{t+1}}\left\{\bar{q}(s^{t+1}) + d(s^{t+1}\right\}}{\bar{q}(s^t)}, \\
\bar{m}_s(s^t) &= 1 - \frac{q(s^t)}{\max_{s_{t+1}}\{q(s^{t+1}) + d(s^{t+1})\}\, p(s^t)}.
\end{aligned}$$

4.5 The Calibration

We determine the parameters of the model in a two-step procedure. The first set of parameters, mainly governing the exogenous endowment process, is taken from external sources, while the second set of parameters, mainly relating to preferences and beliefs, is chosen such that simulations of the model match moments of US asset pricing data.

We consider a growth economy with stochastic growth rates. The aggregate endowment at date-event s^t grows at the stochastic rate $g(s_{t+1})$ which only depends on the new shock $s_{t+1} \in \mathcal{S}$, thus

$$\frac{\bar{e}(s^{t+1})}{\bar{e}(s^t)} = g(s_{t+1})$$

for all date-events $s^t \in \Sigma$. There are four different realizations for $g(s_t)$. We declare the first to be a "disaster". We calibrate the disaster shock based on data from Barro and Ursúa (2008). A disaster is defined as a drop in aggregate consumption of more than 15 percent, which has a probability of 2.2 percent and an average size of 28 percent (see Table 10 in Barro and Ursúa (2008)). Following Barro (2009)), we choose transition probabilities such that the four exogenous shocks are i.i.d. The non-disaster shocks are then calibrated such that their average growth rate is 2 percent and their standard deviation matches the data on typical business cycle fluctuations which have a standard deviation of about 2 percent. Table I provides the resulting growth rates and their probabilities. Because of their respective size, we call these realizations as follows: disaster, recession, normal times, and boom.

Table I: Growth rates and their probabilities

	Disaster	Recession	Normal Times	Boom
growth rate g	0.72	0.96	1.02	1.08
probability $\pi(g)$	0.022	0.054	0.870	0.054

We assume that the Lucas tree pays dividends which are, for simplicity, proportional to aggregate endowments, $d(s^t) = \mathfrak{d}\bar{e}(s^t)$, $\mathfrak{d} \geq 0$. In our baseline calibration, the dividend share is 10 percent, i.e. $\mathfrak{d} = 0.10$—roughly equal to the value of dividends relative to the total income of stockholders in the US. The remaining ninety percent share of aggregate income is received by the two agents as endowments. We abstract from idiosyncratic income shocks because it is difficult to disentangle idiosyncratic and aggregate shocks in a model with only two types of agents. Agent 1's endowment is given by

$$e^1(s^t) = \tilde{e}^1\left(\bar{e}(s^t) - d(s^t)\right),$$

where $\tilde{e}^1$ parametrizes the share of non-dividend endowments that goes to agent 1. This parameter is determined in the calibration procedure summarized in Tables II and III below.

There are $H = 2$ agents in the economy. We assume that agents disagree about the likelihood of a disaster. Given the assumption that a disaster occurs about once every 50 years this seems a reasonable assumption. In order to keep the number of parameters as small as possible we assume that the agents' beliefs deviate from the objective probabilities in opposite directions, agent 1 being optimistic and agent 2 being pessimistic. Agent 1 believes that the probability of a disaster is $(1-\delta^d)$ times its objective probability, while agent 2 believes it is $(1+\delta^d)$ times its objective probability. The remaining probability weight is distributed among the other growth realizations according to their objective probability weights. We calibrate the disagreement parameter δ^d as described below.

Recall that the agents have recursive utility functions (Epstein and Zin (1989)) with parameters ρ^h and α^h where $1/(1-\rho^h)$ determines the inter-temporal elasticity of substitution (IES) and $1-\alpha^h$ determines the relative risk aversion of the agent. We assume that both agents have an identical IES of 0.5. This value lies in the middle of the empirical estimates from the micro consumption literature (see, e.g. Attanasio and Weber, 1993, 1995), and is also very commonly used in the macro and public finance literature (note that it implies a coefficient of relative risk aversion of 2 with standard CRRA preferences). In finance (e.g. Barro 2009) it is often assumed that the IES is above 1. In a sensitivity analysis in the appendix we repeat the analysis assuming that agents have identical IES of 1.5.

We assume that agents have identical time discount factor β but differ in their risk aversion (in addition to the disagreement over the disaster probability). We choose the disagreement parameter, δ^d, the risk aversion parameters, α^1 and α^2, the income share of agent 1, $\tilde{e}^1$, and the discount factor of both agents, β, to match key asset pricing moments. These parameters are chosen to match the first and second moments of risk-free and risky returns as well as the price-dividend ratio from Beeler and Campbell (2012) in a calibration with unlimited re-use. There is no simple one-to-one correspondence between a parameter and a moment. For example, as we explain in detail in the next section the volatility of asset returns is jointly determined by differences in beliefs, differences in risk aversion and the income share of agent 1. We do not attempt to match the moments exactly, but rather try to find 'simple' (e.g. integer-valued) values for the parameters to match the moments approximately.

Table II: Calibration targets

target	calibrated model	data
mean equity return (in %)	5.8	5.5
mean risk-free rate (in %)	0.4	0.6
STD risky returns (in %)	18.5	20.0
STD risk-free returns (in %)	3.3	3.3
log price-dividend ratio	3.4	3.4

Table II states the moments from Beeler and Campbell (2012) that we target and the respective values generated by our calibrated model. (The abbreviation 'STD' stands for 'standard deviation'.) The base-line calibration produces realistic first and second moments of equity returns and the risk-free rate. Table III summarizes the parameter values of the baseline calibration. The values for risk aversion lie well within the range of values that are realistic and they are consistent with lab-experiments. The disagreement is rather large—however, the resulting probabilities of the disaster

Table III: Calibrated parameters

parameter	calibration
risk aversion coefficient agent 1, $1-\alpha^1$	3
risk aversion coefficient agent 2, $1-\alpha^2$	7
discount factor of both agents, β	0.94
endowment share agent 1, $\tilde{e}^1$	0.1
disagreement on disaster, δ^d	0.8

state do not seem unrealistic: one agent believes the state is very unlikely with a probability just above 0.4 percent while the other agent is overly pessimistic and believes that it has a probability of almost 4 percent. The less risk-averse agent is also the optimistic agent. This model feature is consistent with evidence from surveys that document a positive correlation between pessimism and risk-aversion (see, e.g., Dohmen et al. (2017)).

5 The Impact of Re-use on Dynamic Equilibria

In this section we demonstrate that the option to re-use collateral increases leverage in the economy and has large effects on the volatility of asset returns. Before we begin our analysis, it serves us well to recall the two-period model from Section 3.

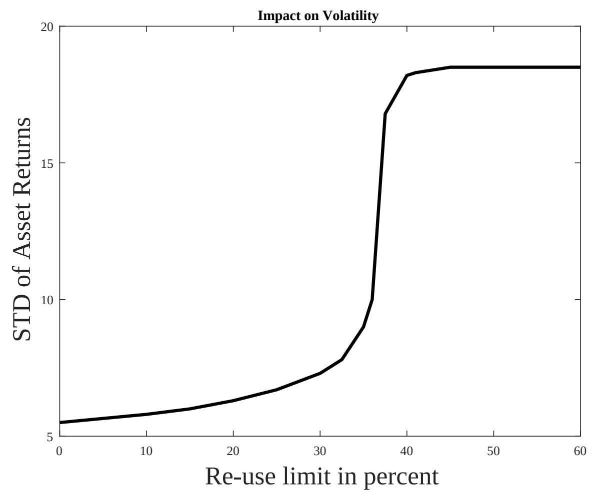
In our discussion of the two-period model, we learned that in an economy with re-use-constrained agents, an increase in κ enables the agents to hold portfolios with higher leverage. Specifically, the pessimistic and more risk averse agent 2 can assume larger short positions in the stock to finance larger investments in the safe bond. Conversely, this enables the more optimistic and less risk-averse agent 1 to buy larger stock positions, which he finances by larger debt positions in the bond. In our analysis below, we observe that increasing the re-use parameter has the same effect on equilibrium portfolios in the infinite-horizon economy. However, the more leveraged portfolios of the two agents have strong additional effects in the dynamic economy, which cannot be captured in a two-period model. The exogenous growth process including disaster and boom states leads to large changes in the wealth distribution over time, since their leveraged portfolios make both agents' financial wealth susceptible to the exogenous shocks. In a disaster state, agent 1's wealth share declines dramatically. In a boom state, particular after a long series of normal growth states, agent 2's wealth share shrinks considerably. We now document that such changes in the wealth distribution matter quantitatively and lead to large price effects.

5.1 Leverage and Volatility

We begin our analysis by examining how the equilibrium standard deviation of stock returns depends on the re-use parameter κ. Recall that for $\kappa = 0$ re-use is not allowed and for $\kappa = 1$ there are no constraints on re-use. Figure V plots the stock return volatility as a function of the re-use parameter.

The stock return volatility increases monotonically as the re-use parameter increases. Once the re-

Figure V: Stock return volatility as a function of the re-use parameter

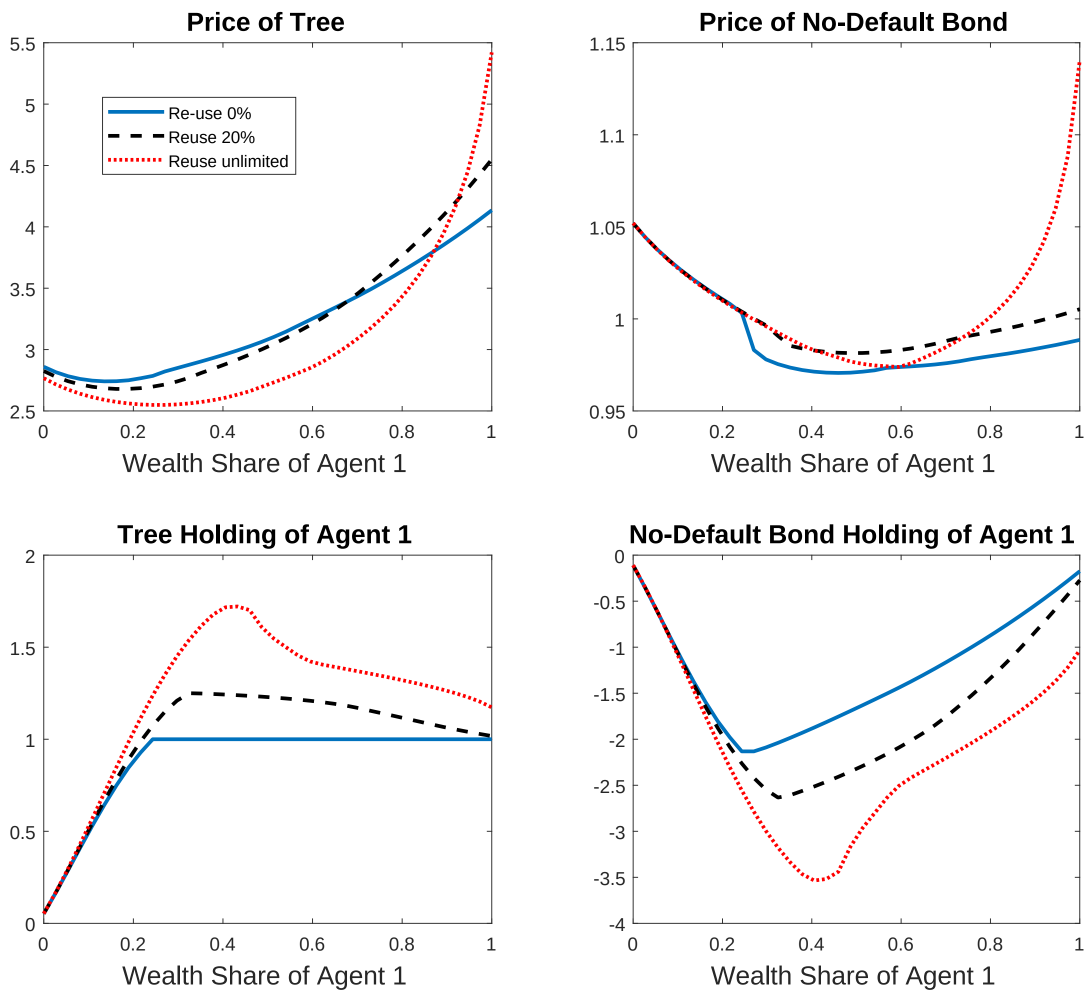


The figure plots the standard deviation (STD) of the stock return as a function of the re-use parameter κ.

use parameter reaches a value of about 45%, the re-use constraint is never binding along simulated equilibrium paths; hence, any further relaxation of the constraint does not affect the stock's return volatility. We observe not only a qualitative but also a strong quantitative effect of κ on the volatility. For $\kappa \geq 0.45$ the stock return volatility is more than three times as large as the volatility in an economy without re-use, $\kappa = 0$. A remarkably strong increase in the volatility occurs when the re-use parameter increases from 30 to 40 percent. This moderate change of κ more than doubles the volatility.

To obtain a clear understanding of the documented reaction of the volatility of returns in response to changes in the re-use parameter κ, we next work out the economic mechanism driving the wealth and asset pricing dynamics in the economy using the computed policy and pricing functions. Figure VI depicts equilibrium asset prices and asset holdings as a function of the endogenous state variable, agent 1's share of financial wealth. Just as in the two-period model, agent 1—the agent with the low risk aversion parameter and optimistic beliefs—is the natural buyer of the risky stock and borrows in the safe bond to finance his stock investments. Agent 2—the agent with the high risk aversion and pessimistic beliefs—has a strong desire to insure against bad shocks and is thus willing to provide financing to the other agent backed by collateral. These equilibrium portfolios are clearly visible in the two policy functions in the bottom row of Figure VI. For all possible values of the endogenous state variable (the wealth share of agent 1), agent 1 holds a long position in the risky stock and a short position in the risk-less bond. This portfolio structure is present for all possible values of the re-use parameter. The policy functions also clearly show the impact of the re-use parameter on the agents' portfolio positions. In the economy without re-use, $\kappa = 0$, agent 1's stock position is bounded above by the aggregate supply of one share. Once his wealth share increases

Figure VI: Price and policy functions



The figure plots the price and policy functions for three different values of the re-use parameter, $\kappa \in \{0, 0.2, 1\}$, as a function of the endogenous state variable, the wealth share of agent 1. The top row shows the stock price (left) and the bond price (right). The bottom row displays agent 1's holdings of the stock (left) and of the bond (right graph). Note that these functions are identical across all date-events since shocks in the economy are iid.

above 0.2, he holds the entire stock; his short position in the bond is then decreasing as his wealth share increases further. If re-use is permitted, then agent 1's stock position is no longer bounded above by the aggregate supply. For positive values of κ, agent 2 can now re-use the collateral (the stock), which she receives for lending to agent 1 via her bond purchases; agent 2 now sells a portion of the collateral to agent 1. As a result, agent 1's position now exceeds the aggregate supply of one share. This agent finances the additional stock purchases with further borrowing in the bond. Thus, his short position in the bond now becomes much larger than in the economy without re-use.

It is worthwhile to point out that, compared to the economy without re-use, an increase in the re-use parameter to $\kappa = 0.2$ has a comparatively modest effect on the equilibrium portfolios while moving from $\kappa = 0.2$ to $\kappa \geq 0.45$ leads to much larger effects on the portfolios. The agents' holdings become much more leveraged. This increased leverage leads to a substantial increase in the volatility of the wealth distribution, which, in turn, contributes to the drastic increase in the stock return volatility, see again Figure V above.

The top row of Figure VI shows that an increase of the re-use parameter κ affects not only the portfolio holdings but also the equilibrium asset prices. Again, we observe modest changes when we compare the economy with $\kappa = 0$ to the economy with $\kappa = 0.2$. But for $\kappa \leq 0.45$ both price functions become very steep when the optimistic agent 1 becomes richer and richer. In particular, when agent 1 is very rich, the stock prices increases drastically, while for medium and small values of the endogenous state variable the stock prices actually decreases. As a result, changes in the wealth distribution lead to much larger price fluctuations than in a tightly constrained economy. Such price effects also contribute to the behavior of the stock return volatility displayed in Figure V.

5.2 Two Key Constraints

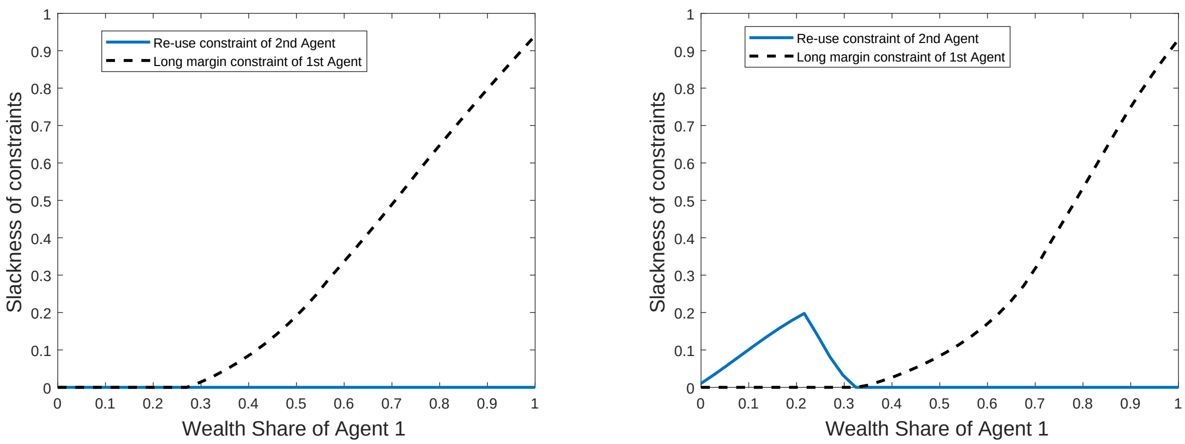
The policy functions in the bottom row of Figure VI are non-monotone with clearly visible extrema. To understand this non-monotone behavior of the policy functions, we next turn to the constraints in the agents' utility maximization problems; two constraints play important roles in equilibrium. Figure VII displays the slack in the long margin constraint of agent 1 and the slack in the re-use constraint of agent 2 for the economy without re-use (panel (a)) and for the economy with limited re-use, $\kappa = 0.2$ (panel (b)).

In the economy without reuse, $\kappa = 0$, the re-use constraint (3) is $\theta^h_{reused}(s^t) \leq 0$ and so, trivially, is always binding. This constraint implies a short-selling constraint, $\theta^2(s^t) \geq 0$, for agent 2, which is binding for agent 2, whenever agent 1 holds the entire stock. This case occurs whenever the wealth share of agent 1 exceeds a threshold of about 0.27, see also Figure VI. As agent 1's wealth share increases beyond a level of 0.27, he cannot buy more of the stock but only reduce his debt in the bond. Therefore, his long margin constraint (2) is not binding in this region of the state space. On the contrary, when agent 1 has a wealth share below the threshold of 0.27, he buys as much stock as permissible under the long margin constraint (2) and so this constraint is binding.

In the economy with limited re-use, $\kappa = 0.2$, the long margin constraint of agent 1 is binding when his wealth share is below a threshold of about 0.32. In this region of the state space, agent 1 buys as many shares of the stock as permissible under the long margin constraint (2). For wealth

Figure VII: Slack in constraints

(a) $\kappa = 0$ (b) $\kappa = 0.2$



The figure shows the slack in the re-use constraint (3) of agent 2 and the long margin constraint (2) of agent 1, respectively. For $\kappa = 0$, agent 2's re-use constraint (3) reduces to $\theta^2_{reused}(s^t) \leq 0$ and so the slack is trivially zero on the entire state space. For $\kappa = 0.2$, agent 2's re-use constraint (3) is $\theta^2_{reused}(s^t) \leq 0.2 \cdot \theta^2_{received}(s^t)$. The slack in this constraint peaks at a level of 0.2 when agent 1 holds exactly the unit net supply of the stock.

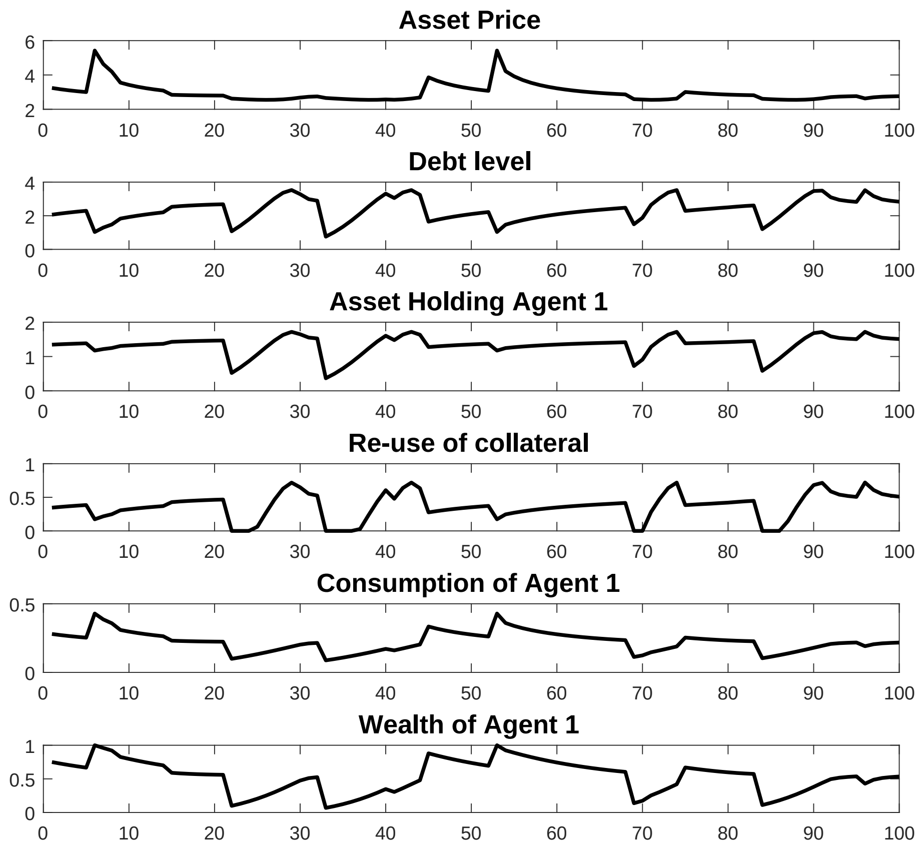
shares above this threshold, the re-use constraint of agent 2 becomes binding and then agent 1 can no longer increase his stock holding. As agent 1's wealth share increases for levels above 0.32, he can only reduce his debt in the bond, see again the bottom row of Figure VI. But then the re-use constraint (3) forces agent 2 to re-use less, which, in turn, means that agent 1 must reduce his long stock position and instead reduce his bond debt even further. Agent 1's bond holding function in Figure VI shows that it increases steeply in the region above the threshold. However, he remains leveraged on the entire domain of the endogenous state variable.

As a result of the two key constraints, leverage peaks at the wealth share where both constraints are binding. These threshold points lead to the global maxima of agent 1's stock holding function and the related global minima of his bond holding function in Figure VI. As agent 1's wealth share increases beyond these points, his long position in the stock and his short position in the bond become decreasing in the endogenous state variable.

5.3 Simulation Paths and Statistics

We continue our analysis by taking a look at equilibrium dynamics. Figure VIII shows simulated paths of key economic variables for an economy with unconstrained re-use, $\kappa = 1$. The bottom graph in the figure indicates that even over only 100 time periods the endogenous state variable, the wealth share of agent 1, may fluctuate between values close to its minimum zero and its maximum one. Put differently, the economy with unconstrained re-use, $\kappa = 1$, exhibits considerable volatility in its wealth distribution. In the vast majority of periods, agent 1's wealth share exceeds 0.2. And so, see also the graph on the bottom left of Figure VI, his stock holding exceeds one. This fact is also

Figure VIII: Simulated paths of equilibrium quantities



The figure plots simulated paths of six different equilibrium quantities over 100 periods for the economy with unconstrained re-use, $\kappa = 1$. Disaster shocks occurred in periods 22, 33, 69, and 84. The top graph entitled "Asset Price" shows a simulation path for the stock price. The graph entitled "Debt level" shows agent 1's short position in the safe bond. The graph entitled "Re-use of collateral" shows a path of agent 2's short position (in absolute terms) in the stock, which is identical to $\max\{0, \theta^1(s^t) - 1\}$.

apparent in the graph displaying agent 2's re-use of collateral. We observe that her re-use is positive for the vast majority of periods. Throughout the entire simulation, agent 1 has considerable debt (via his short position in the bond).

When the economy is hit by a disaster shock, the leveraged agent 1 loses financial wealth. In the simulation in Figure VIII, disaster shocks occur in periods 22, 33, 69, and 84. Each time agent 1's wealth share drops dramatically to values below 0.2. As a result, the long margin constraint becomes binding and forces him to reduce consumption, to reduce his stock holding, and to reduce his short position in the bond. In fact, he must reduce his stock position below one. And so, for a few time periods after each disaster, agent 2 must be long in the stock; her re-use is briefly zero.

Long periods of normal and boom shocks move the endogenous state variable in the opposite direction. Agent 1's wealth share increases and may, in fact, hit one—that is, after a series of good shocks agent 1 holds the entire wealth in the economy. The good shocks greatly increase the stock price and thereby the debt of agent 2 who is re-using the stock for short sales. As a result, her wealth share tends to zero.

Figure VIII shows that in our calibration the (realistically) high volatility of stock returns is driven to a very large extent by a high volatility in the wealth distribution. As agent 1 becomes very rich, the stock price spikes. One reason for this effect is that due to the re-use constraint the agent remains leveraged even when he is very rich. Another reason is, of course, that we assume a low elasticity of inter-temporal substitution, which leads to higher asset prices if one agent is rich.

We finish our discussion in this section with a comparison of asset-pricing moments and other key economic variables across economies with different values for the re-use parameter. Table IV reports simulation statistics for three economies; an economy in which re-use is banned entirely, $\kappa = 0$, an economy with an intermediate level of re-use, $\kappa = 0.2$, and an economy with unconstrained re-use, $\kappa = 1$.

Table IV: Simulation statistics for the model without re-use and with re-use

	no re-use ($\kappa = 0$)	some re-use ($\kappa = 0.2$)	free re-use ($\kappa = 1$)
mean wealth, agent 1 (in %)	31.4	54.1	59.9
STD wealth, agent 1 (in %)	0.9	1.8	4.8
mean re-use rate (in %)	0.0	17	32
mean bond holding, agent 1	-1.9	-2.2	-2.3
mean equity return (in %)	4.9	4.9	5.8
mean risk-free rate (in %)	1.8	1.3	0.4
STD equity returns (in %)	5.5	6.3	18.5
STD risk-free returns (in %)	1.9	1.4	3.3

The table reports simulation statistics for our economic model for three different values of the re-use parameter, $\kappa \in \{0, 0.2, 1\}$. (The abbreviation 'STD' denotes 'standard deviation'.) The re-use constraint is never binding for $\kappa \geq 0.45$. Therefore, the simulation statistics in the column for free re-use ($\kappa = 1$) are representative for all $\kappa \in [0.45, 1]$.

The results in the table reflect our description of the economic mechanisms that are present in the dynamic economy. The first row of Table IV shows that agent 1's average wealth share in the economy is increasing in the re-use parameter. At the same time, the wealth distribution is also becoming more volatile. As we discussed above, the average re-use rate, defined as re-used collateral divided by collateral received that is eligible for re-use, increases strongly with the re-use parameter κ. As a consequence, agent 1 holds, on average, a much larger short position in the bond as κ increases. The portfolios of both agents exhibit more leverage as the re-use parameter increases.

We observed in Figure V at the beginning of this section that the volatility of the stock returns is first modestly and then drastically increasing in κ. This volatility is more than three times as large in the economy with free re-use (18.5%) than in the economy without re-use (5.5%). The average equity excess return is increasing due to a decreasing risk-free rate and a modestly increasing equity return. The volatility of the risk-free rate is non-monotone, which is in line with the bond price function in the upper right graph of Figure VI.

As a final remark, we emphasize that the changes in the statistics of portfolios and prices for increasing values of κ conform with the changes in the behavior of the wealth distribution. The less risk-averse and more optimistic agent 1 holds, on average, more of the dividend-paying stock. Both his average stock holding and the average excess return are increasing in κ. Therefore, his wealth share is also increasing in κ. In addition, his wealth share also becomes more volatile due to the higher leverage in his portfolio and the higher volatility of equity returns.

6 Welfare Implications of Re-use Limits

Next we analyze the impact of re-use limits on the agents' welfare. To do so, we first have to take a stand on how to measure welfare in the presence of heterogeneous beliefs. One can, of course, take the extreme position that probabilities are part of a mathematical representation of preferences and have no meaning outside the context of this representation. If one follows this line of thought, the only reasonable way to measure agents' welfare is based on their subjective beliefs—and, measured like this, regulation of re-use makes everybody worse off.[12] Alternatively, as Gilboa et al. (2014) point out, "Savage's (1954) derivation of subjective expected utility maximization [...] is consistent with a view of probabilities and utilities as conceptually different. The view that probabilities are not empty, theoretical constructs runs throughout economics." In the recent literature, several alternative definitions of Pareto optimality under differences in beliefs have been proposed (see e.g. Brunnermeier et al. (2014), Gilboa et al. (2014) and the references therein). Following Brunnermeier et al. (2014), we say that an allocation A is (belief-neutral) Pareto-better than allocation B if under all beliefs in the convex hull of agents' subjective probabilities (all "reasonable" beliefs) the allocation makes both agents better off (perhaps after a redistribution). It turns out that under this interpretation our welfare analysis reveals a startling and robust result: the impact of tightening the re-use constraint on welfare is non-monotone and there are levels of regulation that are welfare improving both upon

[12] Although the first welfare theorem does not hold in our calibrated economy, the welfare losses from tightening binding constraints are of first order and dominate all other possible effects.

no re-use and upon unrestricted re-use.

We consider unanticipated changes of regulation that occur between two periods. Our starting point is always the setting where no re-use limits apply, $\kappa = 1$. The regulator then introduces a re-use limit with a specific $\kappa < 1$. It is important to note that the change in welfare depends both on the wealth distribution in the period before the regulatory change and on the exogenous state in the period when the regulation first applies. The wealth distribution matters because it affects both asset prices and agents' portfolio choices. As we have seen in the previous section, different levels of the re-use limit imply different price and policy functions. The exogenous state matters because it may greatly affect the wealth distribution. In light of these observations, we need to decide for which values of the endogenous and exogenous state we evaluate the change in welfare. We believe that the most reasonable point of departure are the median of the ergodic wealth distribution of the unregulated economy and the exogenous state 3, which is the mode of the distribution of shocks and represents the normal growth state. We refer to this starting point as the 'benchmark economy' in our discussion below. To demonstrate that the choice of starting point matters only quantitatively but not qualitatively, we also verify that our key results hold for the 10th and the 90th percentile of the ergodic wealth distribution.

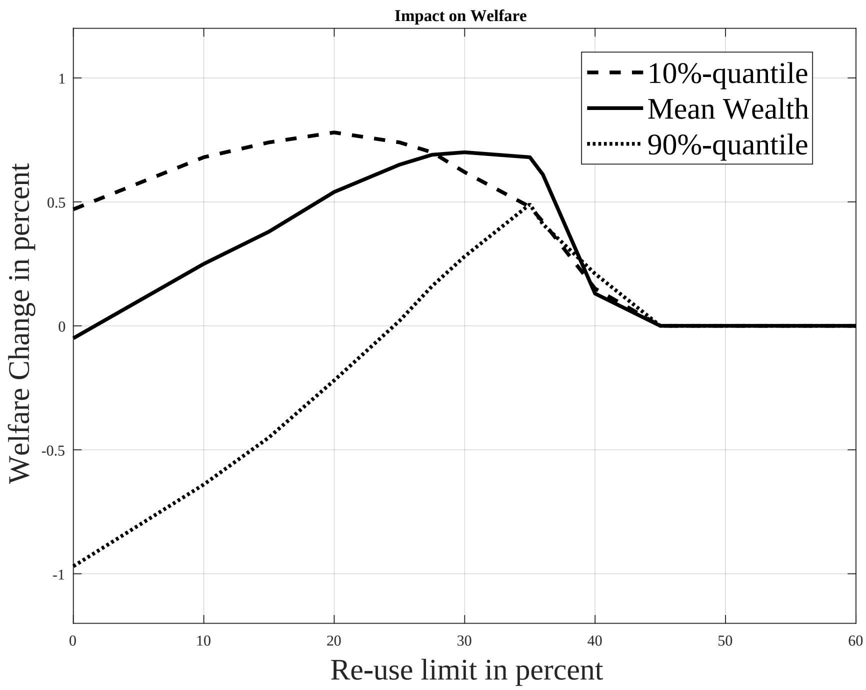
As we pointed out above, changes in the re-use limit, κ, typically have large effects on the price of the tree. Since the tree is held entirely by agent 1 this results in large welfare effects that are due to a redistribution of financial wealth. In our analysis below we report "compensated welfare gains" in the sense that we compensate for the price effect with a direct (one-time) transfer that ensures that the welfare of agent 1 is kept at the same level.

We first examine welfare effects in our baseline calibration from Section 4.5 above. In the appendix we show that the mechanism is this section is quite general and also holds for other calibrations of the model. We follow Brunnermeier et al. (2014) in that we show that regulation can lead to allocations that are Pareto-better under all reasonable beliefs. We begin our analysis with the simpler case where we only use the true beliefs to evaluate welfare. It is useful to discuss this case in detail because it makes clear how regulation can improve welfare. We then turn to the full analysis and evaluate welfare under several beliefs.

6.1 Welfare Analysis Under True Beliefs

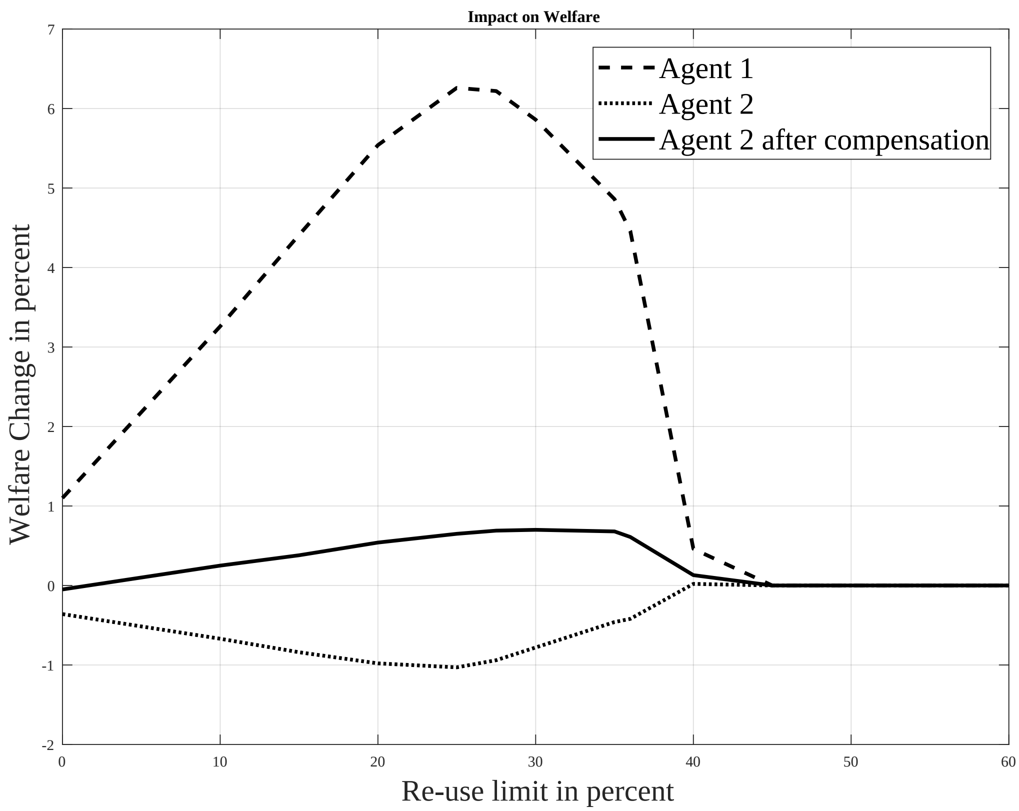
Figure IX plots the welfare effects for agent 2 of an unanticipated change in the re-use regulation when the welfare for agent 1 is kept constant by means of transfers. The interior maximum is clearly visible. Figure X plots the individual welfare changes for the two agents and also reports the post-compensation change for agent 2 for the benchmark economy. All welfare changes are exactly zero for $\kappa \in [0.45, 1]$. For such large values of the re-use parameter, the re-use constraint is never binding in long simulations. The equilibrium is identical to that for an economy with free re-use and changing κ in this region does not affect equilibrium outcomes. For values of the re-use parameter in the interval $[0.4, 0.45]$ (approximately), the regulation leads to very small positive welfare effects for both agents. The price effects are very small and both agents are slightly better off than without regulation. More interestingly, for stricter regulation with smaller values of the re-use parameter,

Figure IX: Welfare changes as a function of the re-use parameter



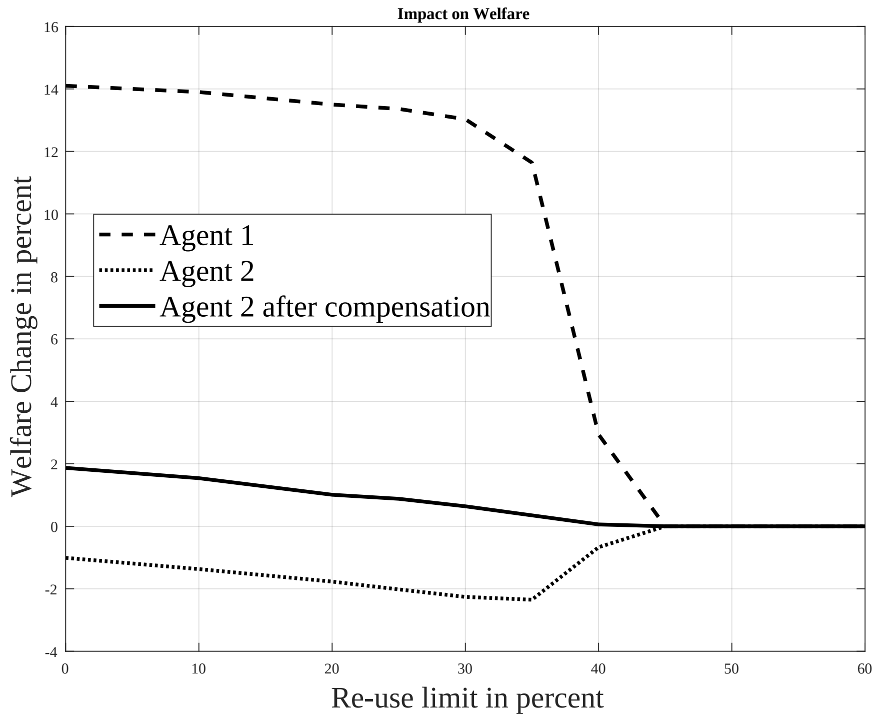
The solid line shows the welfare change for agent 2 after compensating agent 1 for the impact of the regulatory change from $\kappa = 1$ to the level of κ (in percent) on the horizontal axis. The benchmark economy for the comparison is an unregulated economy in a time period when agent 1's wealth is equal to the mean of the ergodic distribution; also, the economy enters state 3 in the subsequent period. The other two lines report the corresponding welfare changes for an unregulated economy in a time period when agent 1's wealth is equal to the 10th and 90th percentile, respectively.

Figure X: Welfare changes as a function of the re-use parameter



The figure shows the welfare changes for both agents in response to a regulatory change in the re-use parameter from $\kappa = 1$ to the level of κ on the horizontal axis. The benchmark economy for the comparison is an unregulated economy in a time period when agent 1's wealth is equal to the mean of the ergodic distribution; also, the economy enters state 3 in the subsequent period.

Figure XI: Welfare changes with identical risk-aversions



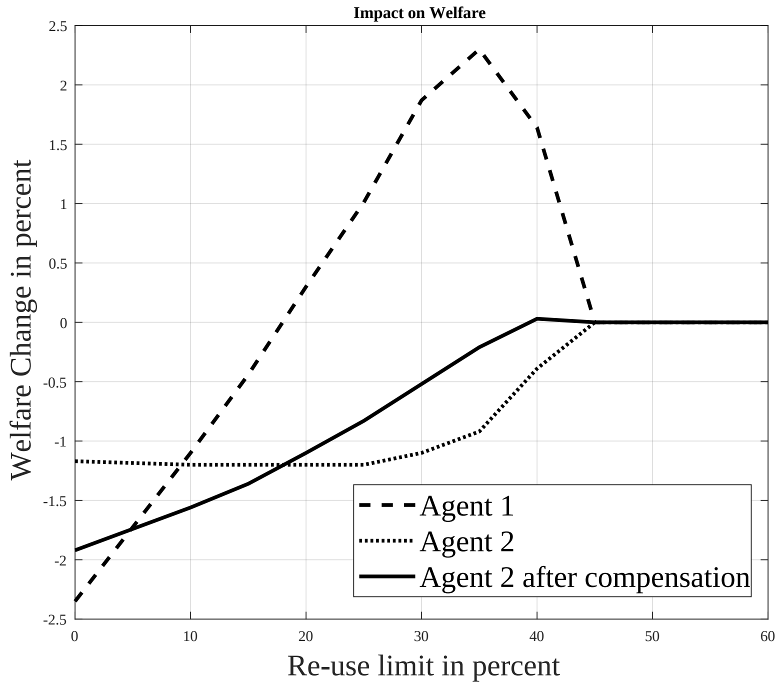
The graph shows welfare changes for the two agents in response to a regulatory change in the re-use parameter from $\kappa = 1$ to the level of κ on the horizontal axis. Both agents have identical risk aversion of 3 and the original heterogeneous beliefs.

welfare gains are substantial. Because of a large price effect (the price of the tree increases) agent 1 gains and agent 2 loses welfare. However, after compensation of agent 2, the net effect on agent 1 remains a (relatively large) positive welfare change.

Two counteracting economic forces are at play. First, the ability to re-use allows for more risk-sharing in the economy. This effect is generally beneficial for welfare given the agents' heterogeneity in risk aversion. Second, the heterogeneity in agents' beliefs triggers agents to build up leveraged positions beyond what is needed to optimally share risks (under the true beliefs). As the ability to re-use collateral allows agents to build up this leverage, limiting re-use has the potential to steer agents' choices closer to levels that are optimal under the true beliefs.

To demonstrate these two opposing economic forces, we disentangle the effects of heterogeneous risk aversion and heterogeneous beliefs in two separate experiments. In the first experiment, we set the coefficient of risk aversion of both agents to 3 and maintain their heterogeneous beliefs. Figure XI plots the individual welfare changes for the two agents in this experiment. The welfare-maximizing policy (after compensation and according to the true beliefs) is a policy prohibiting re-use. This result is not surprising. The agents have heterogeneous beliefs, which are both incorrect. Agent 1 is too optimistic and agent 2 is too pessimistic compared to the objective (and correct) beliefs.

Figure XII: Welfare changes with identical beliefs



The graph shows welfare changes for the two agents in response to a regulatory change in the re-use parameter from $\kappa = 1$ to the level of κ on the horizontal axis. Both agents have identical true beliefs and the original heterogeneous levels of risk aversion.

The availability of free re-use enables the agents to build up leveraged positions that are suboptimal (according to the true beliefs). We also note that the welfare gain of agent 2, after compensation of agent 1, is almost linear for $\kappa \in [0, 0.4]$.

In a second experiment, we set the beliefs of both agents to the true beliefs and maintain their original levels of risk aversion. Figure XII plots the individual welfare changes for the two agents in this experiment. Due to their heterogeneous levels of risk aversion but identical true beliefs, the agents trade solely for the purpose of risk-sharing. A decrease of the re-use parameter below 45% restricts the agents' ability to trade and decreases the welfare of the more risk-averse agent 2. While the less risk-averse agent 1 gains welfare for some intermediate values of κ, he also loses welfare once the re-use parameter falls below 18%. Moreover, we find that the welfare of agent 2 after compensation of agent 1 is increasing in the re-use limit for $\kappa \in [0, 0.4]$, showing a slightly convex shape for $\kappa \in [0, 0.35]$.

In sum, for $\kappa \in [0, 0.4]$, we observe a weakly concave decreasing welfare of agent 2 after compensation of agent 1 in the first experiment and a slightly convex increasing welfare of agent 2 after compensation of agent 1 in the second experiment. Combining these insights, it is not surprising to find the hump-shaped impact of the re-use limit on welfare in the original setting with heterogeneous beliefs and heterogeneous risk aversion.

The observed changes in the equilibrium portfolio functions for increasing values of κ have a strong impact on the wealth distribution. The less risk-averse and more optimistic agent 1 holds the dividend-paying stock (most of the time). Both his average stock holding function and the average excess return are increasing in κ. Therefore, his wealth share is also increasing in κ.

6.2 Welfare Analysis Under All Reasonable Beliefs

Until now we have measured welfare gains using the agents' value functions under true beliefs. In this section we want to argue that regulation improves welfare under all reasonable beliefs, i.e. all beliefs that are convex combinations of the two agents beliefs—the true beliefs being just one case of many. This makes the argument much more convincing, as it is quite plausible to assume that agents would agree that some belief in the convex hull of their subjective beliefs describes the actual low of motion and that agents would therefore ex ante agree to a proposed regulation that improves welfare under all such beliefs. Similarly, while it might be unrealistic to assume that the regulator's beliefs are identical to the true beliefs, it is quite plausible to assume that the regulator has beliefs that are reasonable in the above specified sense.

To present the results we first examine welfare changes under five alternative sets of beliefs. Two natural alternatives are the individual beliefs of the two agents. In addition, we consider beliefs that are the average of one of the agent's beliefs and the true beliefs. For each of these five alternatives, we compute again the peak of the welfare of agent 2 after compensation of agent 1. Table V reports the results of this first exercise. The table reports the location of the peak of the welfare function

Table V: Sensitivity to choice of the beliefs used to evaluate welfare

agent 1	average 1	benchmark	average 2	agent 2
27.5 (1.75)	27.5 (1.25)	30 (0.70)	35 (0.40)	35 (0.20)

The table reports the location of the peak of the welfare function in terms of the re-use limit κ (in percent) as well as the change in welfare (in percent) for agent 2 after compensation of agent 1 at this location (in parentheses) for our economic model for the agents' value functions under different beliefs. The entry under "benchmark" denotes the welfare change under true beliefs, the entry under "average 1" denotes the welfare change for beliefs that are the average of agent 1's and the true beliefs, and so on.

in terms of the re-use limit κ (in percent) as well as the change in welfare (in percent) for agent 2 after compensation of agent 1 at the peak (in parentheses). The entry under "benchmark" denotes the results under true beliefs, the entry under "average 1" denotes the results for beliefs that are the average of agent 1's and the true beliefs, and so on. For all five beliefs, the welfare measure has an interior maximum. The locations of the peak change only slightly in response to changes in the beliefs, while the accompanying maximal welfare gains change considerable in response to changes in the beliefs. In all of the cases, (compensated) welfare gains are positive. It turns out that they are much larger if welfare is evaluated under the beliefs of agent 1 (where they are 1.75 percent) than when they are evaluated under the beliefs of agent 2 (where they are only 0.20 percent). This is consistent with the fact that agent 2 actually re-uses the tree for short-sales. The re-use constraint is binding for her, and so, if utility is evaluated under her beliefs, the welfare loss from regulation is rather large.

We repeated the exercise for another 10 convex combination of the two agents' beliefs and observed that under any convex combination of agents' beliefs, regulating re-use delivers considerable welfare gains if we allow for compensation.

Following the concept of Pareto-efficiency in Brunnermeier et al. (2014), we can then say that a moderate constraint of re-use is Pareto-improving compared to both extremes—the situation where re-use is prohibited and the situation where re-use is not regulated.

7 Conclusion

Publicly available data on collateral re-use activity of globally active banks indicates that collateral re-use plays an important role in financial markets. While market participants have stressed the importance of re-use of collateral as a source of funding and market liquidity more generally, regulators and supervisors have raised various concerns about this market practice. The use of the same security as underlying collateral for different transactions increases the sum of exposures in the financial system and, as result, creates leverage across the intermediation chain.

Our analysis is particularly relevant in light of the growing importance of collateral in the financial system and the need to specify the rules governing the re-use of such collateral in the regulatory framework. A range of regulations already impose direct limits and restrictions on the ability of financial intermediaries to re-use collateral. Moreover, rules affecting collateral transactions like repos may also indirectly limit the extent to which entities may re-use collateral. For example, FSB (2017b) highlights that the Basel III reforms and in particular the Leverage Ratio (LR) framework are an important brake to address risk related to the contribution of re-use of collateral to the build-up of leverage in the banking sector by indirectly incentivizes banks to keep re-use activity below excessive levels. To provide information for the policy discussion on whether collateral re-use should be permitted on financial markets, and if so, to what degree, it is clearly important to develop a model framework to understand the (qualitative) implications of collateral re-use on financial market outcomes and welfare.

In this paper, we have developed a calibrated, infinite-horizon asset-pricing model with heterogeneous agents that allows us to assess the qualitative implications of re-use on financial market leverage, volatility, and welfare. In our model, the ability of agents to re-use frees up collateral that can be used to back more transactions. Through this channel, re-use of collateral contributes to the build-up of leverage in the financial system and is found to significantly increase volatility in financial markets. We have shown that the limits on the amount of collateral that agents may re-use reduces financial market volatility; in fact, the tighter the limits the lower the volatility. While the effect of re-use limits on volatility is monotone, the impact on welfare is not. In the model, allowing for some re-use can improve welfare as it enables agents to more effectively share risks. Allowing re-use beyond intermediate levels, however, can lead to excessive leverage and lower welfare. In conclusion, the analysis in this paper provides a rationale for limiting, yet not banning, re-use in financial markets. Our analysis suggests that these existing limits on collateral re-use are important to reduce financial market volatility and to contain risks related to re-use of collateral and should, therefore, remain intact.

Appendix

A Additional Proof

We show that the agents' utility maximization problem is well defined. In particular, we prove that the collateral and lower-bound constraints do not exclude reasonable portfolios.

The collateral constraint (1) only needs to hold for $\theta^h(s^t) < 0$. We now show that it is redundant for $\theta^h(s^t) \geq 0$. Note that (1) is equivalent to

$$\theta^h(s^t) \geq \frac{-(1 - m_s(s^t))p(s^t)\phi^h(s^t)}{q(s^t)}.$$

If $\theta^h(s^t) \geq 0$ and $\phi^h(s^t) \geq 0$, then the constraint holds since $m_s(s^t) \leq 1$. If $\phi^h(s^t) < 0$, the constraint cannot be binding, since in that case the collateral constraint (2) imposes the tighter constraint

$$\theta^h(s^t) \geq \frac{-p(s^t)\phi^h(s^t)}{(1 - m_l(s^t))q(s^t)}$$

for all $m_s(s^t), m_l(s^t) < 1$.

The collateral constraint (2) only needs to hold for $\phi^h(s^t) < 0$. We now show that it is redundant for $\phi^h(s^t) \geq 0$. Note that (2) is equivalent to

$$\phi^h(s^t) \geq \frac{-(1 - m_l(s^t))q(s^t)\theta^h(s^t)}{p(s^t)}.$$

If $\theta^h(s^t) \geq 0$ and $\phi^h(s^t) \geq 0$, then the constraint holds since $m_s(s^t) \leq 1$. If $\theta^h(s^t) < 0$, the constraint cannot be binding, since in that case the collateral constraint (1) imposes the tighter constraint

$$\theta^h(s^t) \geq \frac{-q(s^t)\theta^h(s^t)}{(1 - m_s(s^t))p(s^t)}$$

for all $m_s(s^t), m_l(s^t) < 1$.

B A Calibration with Large IES

While in the labor-literature most studies tend to find a IES far below 1 the macro-finance literature typically assumes and IES above 1. As a robustness check of our welfare results we therefore examine how our results depend on our choice of an IES of 0.5. For this purpose, we assume that both agents have an IES of 1.5. For this we are not able to match the second moment of equity returns observed

Table VI: Simulation statistics for the model with an IES of 1.5

	no re-use ($\kappa = 0$)	$\kappa = 0.2$	$\kappa = 0.3$	free re-use ($\kappa = 1$)
mean equity return (in %)	8.5	7.9	7.6	7.8
mean risk-free rate (in %)	5.7	3.1	1.4	0.4
STD risky returns (in %)	7.1	8.6	10.6	13.3
STD risk-free returns (in %)	1.0	2.1	1.3	2.9
mean wealth, agent 1 (in %)	41.3	58.3	71.4	78.7
STD wealth, agent 1 (in %)	0.6	1.5	2.4	3.1
mean re-use rate (in %)	0.0	18	28	33
mean bond holding, agent 1	-0.9	-1.0	-1.2	-1.2

The table reports simulation statistics for our economic model with an IES of 1.5 for four different values of the re-use parameter κ. The abbreviation 'STD' stands for 'standard deviation'.

in the data. Instead of fully re-calibrating the model we therefore only adjust the beta to 0.91 in order to approximately match the risk free rate in the data for the case of unlimited re-use.

Table VI shows first and second moments of returns for our choice of parameters. We see that the equity premium is much higher in this calibration but that the standard deviation of equity returns is significantly lower which is caused by a lower standard deviation of agent 1's financial wealth holdings. Nevertheless it seems fair to say that our parameterization produces realistic moments of asset returns. In particular it is still the case that the possibility of re-use has very large effects stock-price volatility. Our first main meme of this paper, the idea that reuse has large effects on leverage and volatility, certainly remains correct for this alternative calibration.

Figure XIII shows the analogue of VIII for this calibration with a large IES for both agents. The main advantage of this calibration is that the path for asset prices looks more "natural". In the calibration with a small IES a large part of the stock-volatility was driven by booms and spikes in asset prices. With a large IES the volatility is driven partly by booms by mainly by busts, bad shocks cause asset prices to drop substantially.

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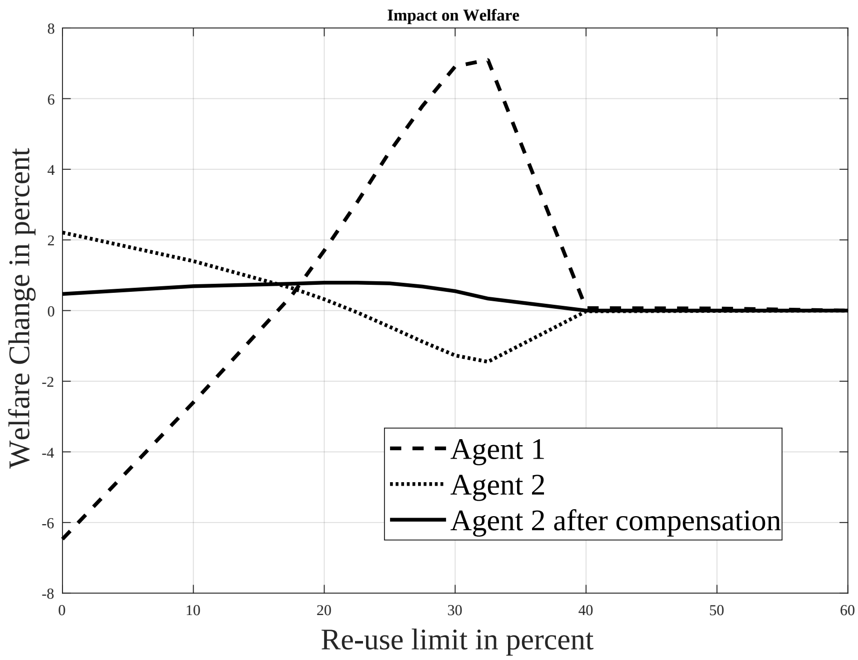
We now turn to the welfare effects of regulating re-use. As it to be expected, the effects are very similar to the ones in our benchmark calibration. In fact for this calibration there is a fairly substantial region of reuse for which a regulation is Pareto-improving (under our criterium) even without transfers.

Figure **??** is the analogue of Figure **??** above for this case of an IES greater than one. We again find an interior maximum welfare point at 22.5 percent re-use limit. The explanation for the effect

Figure XIII: Simulated paths of equilibrium quantities

The figure shows simulated paths of six different equilibrium quantities over 100 periods. Disaster shocks occurred in periods xx. The graph on re-use of collateral shows a path of agent 2's short-position in the stock, which is identical to $\max(0, \theta^1(s^t) - 1)$.

Figure XIV: Welfare changes as a function of the re-use parameter



The figure shows the welfare changes for both agents in response to a regulatory change in the re-use parameter from $\kappa = 1$ to the level of κ on the horizontal axis. The benchmark economy for the comparison is an unregulated economy in a time period when agent 1's wealth is equal to the mean of the ergodic distribution; also, the economy enters state 3 in the subsequent period.

is identical to the explanation above. Shutting down differences in risk-aversion means no reuse is always optimal while shutting down differences in beliefs means that full reuse is always optimal.

Table VII: Sensitivity to choice of the beliefs used to evaluate welfare

agent 1	average 1	benchmark	average 2	agent 2
32.5 (0.28)	27.5 (0.50)	22.5 (0.80)	12.5 (1.12)	12.5 (1.29)

The table reports the location of the peak of the welfare function in terms of the re-use limit κ (in percent) as well as the change in welfare (in percent) for agent 2 after compensation of agent 1 at this location (in parentheses) for our economic model for the agents' value functions under different beliefs. The entry under "benchmark" denotes the welfare change under true beliefs, the entry under "average 1" denotes the welfare change for beliefs that are the average of agent 1's and the true beliefs, and so on.

Finally as in Section 6.2 above we conduct the "full" welfare analysis and show that as above, following the concept of Pareto-efficiency in Brunnermeier et al. (2014), we can then say that a moderate constraint of re-use is Pareto-improving. Table VII reports the welfare effects of a regulation for 5 different values of beliefs. As above we verified the result for many other values in the convex hull.

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