Breaking the Feedback Loop: Macroprudential Regulation of Banks' Sovereign Exposures*

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April 2018

(Preliminary)

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Abstract

This paper develops a dynamic general equilibrium model which features both endogenous bank failure risk and sovereign default risk to study the feedback loop between sovereign and banking crises. In the model, an initial shock to the banking sector contributes to an increase in public debt and sovereign risk as a result of the government bailout of failed banks. Holding high-yield, risky sovereign bonds may be attractive for surviving banks protected by limited liability, generating risk-shifting incentives which result in excessive exposure to sovereign risk. By increasing banks' failure risk and their funding costs, these exposures represent an important source of systemic spillovers, which feed back into further financial instability and depressed economic activity. The results show that the regulatory treatment of banks' sovereign exposures plays a crucial role in shaping banks' incentives to invest in sovereign debt. A counterfactual exercise is performed to assess the macroprudential implications of modifying the current regulatory framework by introducing capital requirements for banks' sovereign exposures, suggesting that they can help mitigating the negative effects of the feedback loop on financial stability and economic activity. However, they also point out to the existence of non-trivial welfare trade offs when setting the optimal requirement.

Keywords: feedback loop, financial crises, sovereign default, macroprudential policy, systemic risk, capital requirements.

JEL codes: E44, F34, G01, G21, G28

*I am indebted to Javier Suarez for his invaluable guidance and support. I also thank Mark Aguiar, Andres Almazan, Kartik Anand, Anil Ari, Huixin Bi, Guillermo Caruana, Aitor Erce, Nezih Guner, Ramon Marimon, Leonardo Martinez, David Martinez-Miera, Rafael Repullo, Juan Rubio-Ramirez, Tano Santos, Anatoli Segura, Enrique Sentana, Cesar Sosa-Padilla, Sascha Steffen, Dominik Thaler, Stijn Van Nieuwerburgh, Raf Wouters and Tanju Yorulmazer for helpful comments and suggestions, as well as participants at the Norges Bank Workshop on Nonlinear Models in Macroeconomics and Finance, Royal Economic Society (RES) Symposium of Junior Researchers at University of Sussex, ADEMU Conference on Sovereign Debt in the 21st Century at Toulouse School of Economics, CEPR Spring Symposium in Financial Economics at Imperial College Business School, and seminars at UC3M and CEMFI. Financial support from the Santander Research Chair at CEMFI is gratefully acknowledged. All existing errors are my own. Contact email: jorge.abad@cemfi.edu.es

1 Introduction

The *negative feedback loop* between banks, sovereigns and aggregate economic activity has drawn considerable attention since the onset of the European debt crisis and it has been documented in several recent papers.[1] In a nutshell, it is argued that governments' assistance to their domestic banking systems in order to avoid their collapse increased the level of sovereign debt, raising concerns regarding its sustainability. The distress in the banking system also caused a downturn in the economic activity which put even more pressure on public finances. At the same time, banks' exposure to domestic sovereign debt increased, translating the doubts about governments' solvency into further financial instability.

In this context, several voices called for changes in the regulatory treatment of banks' exposure to (domestic) sovereign debt.[2] The current regulatory framework imposes that at least a fraction of the banks' risk-weighted assets has to be financed with bank equity capital. However, as of now, it assigns zero risk weights to Euro-area sovereign debt. Furthermore, domestic government debt is exempt from existing concentration limits to single counterparties, and it is even encouraged by current liquidity regulation.[3] In a recent report on the regulatory treatment of sovereign exposures in the books of banks and insurance companies, the European Systemic Risk Board stated: *"If sovereign exposures are in fact subject to default risk, consistency with a risk-focused approach to prudential regulation and supervision requires that this default risk is taken into account"* (ESRB, 2015).

This paper develops a dynamic general equilibrium model able to address some of the elements in the discussion introduced above. The model features both endogenous bank failure risk and sovereign default risk. The interplay of these two, via the government's bailout of the banking sector and the banks' exposures to risky sovereign debt, generates a *negative feedback loop* between sovereign risk and financial instability, with important contractionary effects on economic activity. The model allows to perform counterfactual exercises regarding

[1] Lane (2012) provides a narrative of the European sovereign debt crisis. Reinhart and Rogoff (2011) document the recurrent link between sovereign and banking crises using long historical time series for a wide range of countries. Balteanu and Erce (2017) study the mechanisms through which bank and sovereign distress feed into each other using a large sample of emerging market economies over three decades,

[2] See for example Gros (2013), Weidmann (2013), Enria, Farkas and Overby (2016), and BIS (2016).

[3] Nouy (2012) provides a comprehensive review of the current regulatory treatment of sovereign exposures for banks and insurance companies.

modifications of the current regulatory treatment of banks' exposure to sovereign risk. In particular, this paper addresses the potential macroprudential implications of introducing regulatory capital requirements for banks' sovereign debt holdings.

In the model, bank failure risk stems from the exposure to both idiosyncratic and aggregate shocks, as well as from banks' holdings of sovereign debt subject to default risk. Distortions arising from banks' limited liability make investing in high-yield, risky sovereign debt attractive for banks, who enjoy high profits insofar as the government does not default and suffer losses limited to their initial equity contributions otherwise. These risk-shifting incentives result in excessive exposure to sovereign risk.

At the same time, the possibility that the government defaults not only on its outstanding stock of debt but also on its deposit insurance liabilities translates into higher bank funding costs when banks increase their exposure to the risky sovereign.[4] When depositors cannot observe the balance sheet composition of individual banks, these do not internalize the effect of their individual risk-taking choices on the funding costs of the whole banking system. This externality generates a funding cost channel which contributes to financial instability when sovereign risk increases.

Through the mechanisms described above, sovereign risk acts as an important source of systemic spillovers, by which an initial shock to a small fraction of banks can translate into system-wide instability. By disrupting banks' intermediation ability, the effects of the feedback loop have dramatic consequences for economic activity, even when the sovereign default event does not materialize ex-post. Thus, the model environment provides a rationale for macroprudential policies aimed to reduce banks' incentives to excessively expose themselves to sovereign risk.

The model is relatively parsimonious, compared to standard dynamic general equilibrium models in the literature. The reason for this is twofold. First, keeping the model simple allows to isolate the key mechanisms behind the feedback loop and to analyze possible policy changes in a tractable framework. Second, the need to use computationally intensive global solution methods restricts the size of the models that can be feasibly solved, since numerical approximation procedures in high-dimensional spaces can easily suffer from the so called curse

[4] Mäkinen, Sarno and Zinna (2018) provide evidence on the quantitative relevance of this channel.

of dimensionality.[5] In spite of this, the model is rich enough to capture and quantify many of the relevant elements analyzed in the theoretical literature about the feedback loop and documented in recent empirical work, and to provide novel insights about the implications of possible modifications of the current regulatory treatment of banks' sovereign exposures, which could potentially guide the design of macroprudential policy tools in the future.

The model is calibrated to match the magnitude and dynamics around the events of the European sovereign debt crisis. The quantitative results reveal important amplification effects resulting from the presence of the feedback loop, which could be (at least) partially mitigated by introducing positive risk weights for sovereign exposures in the calculation of the regulatory capital requirements for banks.

Under the proposed calibration of the model, the quantitative assessment of the macroprudential implications of a change in the regulatory treatment of banks' sovereign exposures evaluates the social welfare gains associated to different risk weights of sovereign debt in the calculation regulatory capital requirements, finding an interior maximum social welfare at a risk weight of 40%, for a given capital requirement of 8%. The results identify non-trivial welfare trade-offs resulting from the implementation of the proposed regulatory reform, which exhaust the potential benefits of further increasing the risk weight for sovereign exposures beyond a certain point.

The remaining of the paper is organized as follows. Section 2 discusses how the paper relates to the existing literature. Section 3 describes the model setup. Section 4 introduces the numerical solution method, describes the calibration and the main quantitative properties of the model, and provides a counterfactual exercise about the potential effects of introducing a positive risk weight for sovereign debt in the calculations of the regulatory capital requirements. Finally, Section 5 concludes. An Appendix provides data sources, the complete set of equilibrium equations, a detailed description of the numerical solution method and assess its accuracy.

[5]In order to overcome these problems, state of the art computational techniques are used. Maliar and Maliar (2014) and Fernandez-Villaverde, Rubio-Ramirez and Schorfheide (2016) provide a comprehensive survey of those techniques. Further details are provided in the Appendix.

2 Related literature

This paper connects with several strands of the literature that study the feedback effects between banks and sovereign crises, as well as with the literature on macro-financial linkages and macroprudential policies.

The existing literature identifies at least three different reasons why banks may have incentives to increase their exposure to sovereign risk during times of financial distress. First, creditor discrimination by defaulting governments may create a difference between the expected return on sovereign bonds for domestic banks and foreign investors. This difference increases during times of stress, which leads to a re-nationalization of domestic sovereign debt (see Broner, Erce, Martin and Ventura, 2014). Second, financial repression by the government in the form of moral suasion may force or incentivize banks to invest in their domestic sovereign debt. Acharya and Rajan (2013) and Chari, Dovis and Kehoe (2014) analyze this phenomenon in a theoretical framework, while Becker and Ivashina (2017) and Altavilla, Pagano and Simonelli (2017) find evidence along these lines in the context of the European sovereign debt crisis. Third, limited liability may distort banks' incentives by encouraging them to take excessive exposures to sovereign risk. Evidence of this risk shifting behavior is documented in Acharya and Steffen (2015) and Altavilla, Pagano and Simonelli (2017).[6] This paper focuses on the third of these frictions.

Previous theoretical literature analyzes the negative feedback loop between banks and sovereigns in partial equilibrium and/or static models.[7] These theoretical works shed light on the mechanisms behind the feedback loop, but cannot speak about the dynamic general equilibrium effects and therefore are not suitable for quantitative analysis. In this regard, the contribution of this paper is to embed some of the relevant elements discussed in the theoretical literature into a dynamic general equilibrium model, in order to assess the quantitative importance of the feedback loop and the macroprudential implications of modifying the current regulatory framework by introducing regulatory capital requirements for banks'

[6] Furthermore, Kirschenmann, Korte and Steffen (2017) argue that the current regulatory treatment of sovereign exposures within the European Union further encourages banks' risk shifting behavior, resulting in risk spill overs from risky periphery sovereigns to safer core countries.

[7] See for example Cooper and Nikolov (2013), Uhlig (2014), Acharya, Dreschsler and Schnabl (2014), Farhi and Tirole (forthcoming), Brunnermeier el al (2016), and Leonello (forthcoming)

sovereign exposures.

To this end, this paper builds upon the literature on macroeconomic models with financial frictions as well as the literature on sovereign default. It is most closely related to the theoretical and quantitative literature of dynamic general equilibrium models that study the effect of financial conditions on macroeconomic activity, as well as the literature on the role of macroprudential regulation on the risk-taking incentives of financial intermediaries and, in particular, the macroprudential implications of regulatory capital requirements.

Early work by Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999) underscored the amplification effects that financial frictions have in business cycle fluctuations. Since the onset of the global financial crisis, several recent papers further extended these frameworks by incorporating financial intermediaries into otherwise standard dynamic general equilibrium models. Some of these papers (for example, Gertler and Karadi, 2011; Gertler and Kiyotaki, 2011; Gertler, Kiyotaki and Queralto, 2011) emphasize the role of bankers' net worth and its dynamics as a financing constraint that financial intermediaries face, arising from the fact that bankers can divert a fraction of the funds under their management.

Other recent papers, like Martinez-Miera and Suarez (2014), Clerc (2015), and Mendicino, Nikolov, Supera and Suarez (forthcoming), which are closely related to this paper in the way banks are modeled, attribute these financing constraints to the existence of regulatory capital requirements that try to reduce the excessive risk taking caused by limited liability and deposit insurance, and explore the effects on social welfare and other macroeconomic aggregates of increasing these capital requirements. Again, these frictions make the aggregate net worth of the banking sector a relevant variable that determine the performance of the aggregate economy. Compared to the previous literature, these papers explicitly model the possibility of bank failure, which proved to be an important element of the global financial crisis. This paper adds to the existing literature on the macroprudential implications of bank capital regulation by explicitly modeling sovereign default risk and analyzing its amplification effects and its interaction with the riskiness of the banking sector.

Some recent papers study the effect of sovereign risk on the banking sector in a macroeconomic setup. Bocola (2016) analyzes the pass-through of sovereign risk to the banking sector in an environment in which sovereign risk shocks are exogenous and the banking sector is modeled as in Gertler and Karadi (2011), and thus abstracts from limited liability, the possibility of bank failure and banks' risk-shifting incentives. Ari (2017) studies banks' risk-shifting incentives in the presence of exogenous sovereign default risk, while Faia (2017) considers the effect of banks' exposure to sovereign risk on bank funding costs and, through this channel, on economic activity. These papers, by either modeling sovereign risk as completely exogenous or by abstracting from the possibility of bailouts (or both), do not capture the potential feedback effects of bank failure risk on sovereign default. This is, none of the these papers capture the side of the feedback loop by which financial instability in the banking sector translates into higher sovereign risk. Furthermore, none of them explicitly analyzes the role of bank capital requirements for sovereign exposures. The contribution of this paper is to explicitly consider the macroeconomic consequences of the two-way feedback loop between sovereign and bank risk and to analyze of the role of capital requirements for banks' sovereign exposures in mitigating these effects.

Regarding the sovereign default literature, quantitative studies following the seminal contribution of Eaton and Gersovitz (1981), such as Aguiar and Gopinath (2006), Arellano (2008) and Mendoza and Yue (2012), analyze sovereign debt dynamics and business cycle properties of emerging economies, by incorporating the possibility of a sovereign default as the outcome of the strategic behavior of a benevolent, social welfare maximizer government. Gennaioli, Martin and Rossi (2014) present a setup in which the cost of defaulting comes from the disruption of domestic banks' balance sheet when they hold sovereign debt, affecting the government default incentives.[8] As in Bi and Traum (2012), in the model presented here the government faces a stochastic limit to the debt it can issue, which causes its default if exceeded.[9]

Lastly, this paper relates to recent efforts to solve quantitative models of financial crises using global solution methods.[10] These papers highlight the importance of non-linear dy-

[8]Some recent papers have tried to embed this mechanism in a quantitative framework. These include Perez (2015), Thaler (2017) and Sosa-Padilla (2017).

[9]The same or similar approaches to modeling sovereign default risk have been used in related papers, including some of those cited above, such as Corsetti et al (2013), Bocola (2016) and Faia (2017).

[10]See for example Brunnermeier and Sannikov (2014). Martinez-Miera and Suarez (2014), Martinez-Miera and Repullo (2017), Elenev, Landvoigt and Van Nieuwerburgh (2017), Bianchi and Mendoza (2018), and He and Krishnamurthy (forthcoming).

namics and risk premia variation, which traditional local solution methods are not able to capture and need to be taken into account in quantitative policy work. These features are particularly relevant in the context of this paper, as sovereign default episodes are inherently non-linear events and default risk causes large variations in risk premia with important consequences for macroeconomic outcomes, as shown below.

3 The Model

This section presents the model economy and each of the agents that populate it. Time is discrete and runs infinitely. The domestic economy is populated by: (i) a risk-averse infinitely-lived representative household; (ii) a continuum of (potentially) short-lived bankers who are part of the representative household; (iii) a continuum of ex-ante identical banks; (iv) a representative firm; and (v) a government. There is a single non-durable consumption good, which is also used as the numeraire and can be transformed into physical capital used for production.

(i) The representative household takes consumption and savings decisions to maximize its intertemporal expected utility. It can save in the form of government-guaranteed deposits issued by the bank or by directly holding physical capital.

(ii) Bankers are a special class of agents with exclusive access to the opportunity of investing their net worth as banks' inside equity capital. They accumulate wealth until they retire, when they transfer it to the representative household and are replaced by new bankers.

(iii) Banks are perfectly competitive and subject to limited liability. They borrow from households and issue equity among bankers in order to comply with a regulatory capital requirement, which effectively constrains their intermediation ability. They invest both in physical capital and in risky sovereign debt.

(iv) A perfectly competitive firm rents physical capital and hires labor in order to produce consumption good.

(v) The government issues short-term debt to finance its deficits and the cost of the deposit insurance. It is subject to default risk which, when it materializes, imposes a write-off on the outstanding government debt and leave the government unable to honor its deposit insurance obligations. The sovereign faces a segmented credit market in that it can only borrow from its domestic banks and from a set of potential risk-averse foreign lenders with limited wealth, as in Aguiar, Chatterjee, Cole and Stangebye (2016).

The following subsections describe each of the elements of the model in detail.

3.1 Households

The representative household is infinitely lived and risk averse, and it chooses consumption and savings in order to maximize lifetime utility. It can save in the form of one-period, government-guaranteed deposits, which are remunerated with a (gross) return $\widetilde{R}^d_{t+1}$;[11] and by investing in physical capital K^h_t, which is rented to a perfectly competitive firm that combines it with labor to produce consumption good. The physical capital production technology gross return is $R^k_t = r^k_t + 1 - \delta$, which is the sum of the rental rate of capital r^k_t plus the undepreciated physical capital recovered after production takes place (with δ equal to the depreciation rate).

The representative household incurs in a management cost per unit invested in physical capital. Similarly to Gertler and Kiyotaki (2015), the management cost could reflect the comparative disadvantage of households with respect to banks in screening and monitoring investment projects. As a consequence, as in the cited paper, to the extent that the constraints on banks tighten in a recession, impairing their intermediation ability, the share of capital held by households will increase, resulting in a decrease of net output produced. The capital management cost is assumed to be increasing and convex in the total amount of capital held by the household, given by the function $h(K^h_t) = \kappa(K^h_t)^2$.

The representative household discounts future utility at a rate β and obtains utility from consumption of non-durable goods under a CRRA utility function with a risk aversion coefficient ν. It inelastically supplies one unit of labor remunerated with a wage W_t, receives

[11] The realized return on deposits $\widetilde{R}^d_t$ is equal to the promised return R^d_{t-1} minus the realized losses from bank failure Ψ_t. The convention used here is that $\widetilde{R}^d_t$ is the realized return on deposits after the realization of aggregate uncertainty in period t, while R^d_{t-1} is the promised return when the investment decisions are taken. As explained below, since deposits are insured by the government, Ψ_t will be equal to zero as long as the government does not default, and (potentially) positive otherwise.

dividend payments from bankers Π_t (which are net of the transfer of the initial endowment that is transferred to new bankers, as described below), and pays lump-sum taxes T_t.

The problem of the representative household involves choosing consumption C_t, deposit holdings D_t, and investment in physical capital K_t^h so as to maximize its expected discounted lifetime utility

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \frac{(C_{t+i})^{1-\nu}}{1-\nu}, \tag{1}$$

subject to the budget constraint:

$$C_t + D_t + K_t^h + h(K_t^h) = W_t + \widetilde{R}_t^d D_{t-1} + R_t^k K_{t-1}^h + \Pi_t - T_t. \tag{2}$$

It will be useful to define the household's net worth N_t as the relevant state variable for the household problem at the beginning of period t:

$$N_t = W_t + \widetilde{R}_t^d D_{t-1} + R_t^k K_{t-1}^h + \Pi_t - T_t. \tag{3}$$

The household's stochastic discount factor, which appears in the problem of the representative banker below, is defined as $\Lambda_{t+1} \equiv \beta \left(\frac{C_t}{C_{t+1}}\right)^{\nu}$.

3.2 Bankers

There is a continuum of measure one of bankers with accumulated net worth E_t. As in Gertler and Kiyotaki (2011), the bankers are a special class of agents that belong to the household and have exclusive access to the opportunity of investing their net worth as banks' inside equity capital. They are (potentially) short-lived, with an iid probability of retiring denoted by $1-\varphi$. When they do so, they transfer their terminal net worth to the household and are replaced by new bankers that start with an exogenous fraction ϖ of net worth of the household.

At the beginning of every period, after bankers learn whether they will continue for at least one more period, they have the possibility of transferring a fraction $1-x_t$ of their net worth to the household. Again as in Gertler and Kiyotaki (2011), the value function of the bankers is linear in the level of net worth (since, as shown below, the returns of the bank are constant returns to scale), so the marginal value of one unit of net worth can be written as:

$$v_t = 1 - x_t + x_t \mathbb{E}_t \left[\Lambda_{t+1}(1-\varphi+\varphi v_{t+1}) \widetilde{R}_{t+1}^e \right]. \tag{4}$$

The problem of the banker consists of choosing the fraction x_t of its net worth reinvested as bank equity, taking returns $\widetilde{R}^e_{t+1}$ and the stochastic discount factor of the household Λ_{t+1} as given. Note that, from the expression above, a banker will always choose to invest all of its net worth as bank equity ($x_t = 1$) as long as $v_t \geq 1$, in which case they optimally choose to postpone any dividend payments until retirement. The numerical exercise below will focus on a parameterization where $v_t \geq 1$ (and thus $x_t = 1$) holds for every period.

The dividend payments transferred to households, net of the initial endowment that households transfer to new bankers, can be described as

$$\Pi_t = (1 - x_{t-1})E_{t-1} + x_{t-1}(1 - \varphi)\widetilde{R}^e_t E_{t-1} - (1 - \varphi)\varpi N_t, , \tag{5}$$

where the first term represents the dividends paid before retirement and the second term represents the transfers of the terminal net worth of retiring bankers. The aggregate level of bankers' net worth evolves according to the following law of motion:

$$E_t = x_{t-1}\varphi\widetilde{R}^e_t E_{t-1} + (1 - \varphi)\varpi N_t, \tag{6}$$

where the first term represents the returns of the net worth of surviving bankers invested as bank equity and the second term represents the initial endowment of new bankers.

3.3 Banks

There is a continuum of measure one of perfectly competitive ex-ante identical banks. A bank lasts for one period only: it is an investment project created at t and liquidated at $t+1$. They raise deposit funds d_t from households with a promised return R^d_t, and equity capital e_t from bankers. They can invest both in physical capital k_t and in government bonds b_t.

The banks in this economy represent a consolidation of financial intermediaries and capital producing firms. Investment in physical capital uses a bank-specific production technology that transforms one unit of consumption good into ω effective units of capital, as in Bernanke, Gertler and Gilchrist (1999). Individual banks' idiosyncratic productivity ω is log-normally distributed with mean one and cross-sectional standard deviation σ, independent across time and across banks, making banks' returns heterogeneous ex-post. These idiosyncratic shocks can represent exposure to sources of risk resulting from geographic or sectoral specialization, which might, in turn, stem from specific knowledge of bankers on certain regions or sectors that are subject to heterogeneous shocks.

Banks' investment in physical capital is also subject to aggregate risk, which takes the form of a large and infrequent iid depreciation shock denoted by $\psi_t \in \{0,1\}$, whose realization is unknown when the investment decisions are taken.[12] When the shock realizes ($\psi_t = 1$), which occurs with probability π, a fraction λ of the continuum of banks sees its stock of capital fully depreciate after production takes place.[13] The expected return of capital conditional on the realization of the aggregate shock ψ_t can be written as $r_t^k + (1-\delta)(1-\lambda\psi_t)$. These shocks affect banks' returns, effectively raising bank failure when they realize, as shown below.

Banks face liquidity management costs $m(d_t, b_t)$ which are increasing in the amount of deposits issued and decreasing in the amount of the government bonds they hold.[14,15] A functional form compatible with these assumptions which will be used in the numerical exercise is $m(d_t, b_t) = \phi\left(\frac{d_t}{b_t}\right) d_t$.

Banks are subject to limited liability, which means that the equity payoffs generated by a bank at time $t+1$ are given by the positive part of the difference between the returns from its assets (net of liquidity management costs) and the repayments due to its deposits:

$$\max\left\{\widetilde{R}_{t+1}^k \omega k_t + \widetilde{R}_{t+1}^b b_t - R_t^d d_t - m(d_t, b_t), 0\right\}. \tag{7}$$

If the returns from its assets (net of liquidity management costs) are greater than the repayments due to its deposits, the difference is paid back to the bank's equity holders. Otherwise,

[12]The aggregate shock could be modeled as a persistent process at the small cost of adding an extra state variable. However, as shown in the numerical results of Section 4, even non-persistent aggregate shocks can have very persistent effects on the model economy.

[13]The nature of this shock can be interpreted to be similar to the capital quality shocks in Gertler and Karadi (2011), or the systemic shock in Martinez-Miera and Suarez (2014). Unlike in the latter paper, the exposure of banks to this shock is assumed to be exogenously given. This is done for simplicity, since the interest of this paper is on the endogenous exposure of banks to sovereign risk.

[14]The role of government bonds in reducing banks' liquidity management costs could be justified in a model in which banks receive a random stream of intra-period liquidity shocks. Having access to a liquid asset (government bonds) would allow banks to meet deposit withdrawals without having to sell other less liquid assets under fire-sale prices. The liquidity role of public debt has been analyzed in the theoretical literature, for instance in Woodford (1990) and Holmstrom and Tirole (1998).

[15]The reason for introducing liquidity management costs comes from the result derived in Repullo and Suarez (2004) which states that one-period lived perfectly competitive banks subject to limited liability that could invest in two different risky assets would optimally specialize in one of them, unless there exist intermediation costs that imply some complementarity between the two assets. In the logic of the model presented here, the complementarity comes from the different degrees of liquidity of each asset, as discussed above.

the bank equity is written down to zero and its assets are taken over by the deposit insurance scheme, which repays the principal and interests in full to the deposit holders (as long as the government does not default; otherwise, the assets of the failed banks are seized directly by the depositors).

The idiosyncratic return of the investment technology implies that the banks which draw a value of ω below a (stochastic) threshold will default every period. The threshold is given by

$$\overline{\omega}_{t+1} = \frac{R_t^d d_t + m(d_t, b_t) - \widetilde{R}_{t+1}^b b_t}{\widetilde{R}_{t+1}^k k_t}. \tag{8}$$

Taking as given the marginal value of one unit of the bankers' net worth v_t and the bankers' stochastic discount factor $\Omega_{t+1} \equiv \Lambda_{t+1}(1 - \varphi + \varphi v_{t+1})$, as well as the promised return of deposits R_t^d and the assets stochastic return, the representative bank chooses the portfolio allocation (k_t, b_t) and liability structure (d_t, e_t) that solve the following problem:

$$\underset{(k_t,b_t,d_t,e_t)}{\text{Max}} \quad \mathbb{E}_t \Omega_{t+1} \max \left\{ \widetilde{R}_{t+1}^k \omega k_t + \widetilde{R}_{t+1}^b b_t - R_t^d d_t - m(d_t, b_t), 0 \right\} - v_t e_t, \tag{9}$$

subject to the balance sheet constraint

$$k_t + b_t = d_t + e_t. \tag{10}$$

Furthermore, banks are subject to a regulatory capital requirement, which imposes that at least a fraction γ of the banks' risk-weighted assets has to be financed with bank capital. Government bond holdings are subject to a risk weight of ι, while investment in physical capital is subject to a risk weight normalized to one:

$$e_t \geq \gamma(k_t + \iota b_t). \tag{11}$$

If deposits are cheaper than equity financing, which always happens in equilibrium under parameterization presented in Section 4, the capital requirement is binding.

3.4 Firms

A standard, perfectly competitive representative firm rents physical capital K_t (remunerated at a rate r_t^k) and hires labor L_t (remunerated at a rate W_t) in order to produce consumption good under a Cobb-Douglas function where α is the elasticity of capital. Its profit-maximization problem is:

$$\underset{(K_t,L_t)}{\text{Max}} \quad K_t^{\alpha} L_t^{1-\alpha} - r_t^k K_t - W_t L_t. \tag{12}$$

3.5 Government

A government issues short-term debt to finance its deficits. There is a stochastic limit to the level of sovereign debt to which the government can commit to repay, which follows a logistic distribution, similarly to Bi and Traum (2012) and Bocola (2016). This stochastic limit depends on the level of debt outstanding (as a fraction of net output Y_t) so that, when such limit is exceeded, the government defaults.[16] The government default event at the end of period t is represented by the binary variable $s_{t+1} \in \{0, 1\}$ and the probability of the government default in each period is

$$p_t \equiv \text{Prob}(s_{t+1} = 1 | B_t/Y_t) = \frac{\exp(\eta_1 + \eta_2(B_t/Y_t))}{1 + \exp(\eta_1 + \eta_2(B_t/Y_t))}. \tag{13}$$

If the government does not default ($s_{t+1} = 0$), it pays back the promised (gross) return R_t^b per unit of debt to its creditors and the deposit insurance liabilities DI_t to the banks' depositors. If it defaults ($s_{t+1} = 1$), it writes off a fraction $\theta \in [0, 1]$ of its outstanding stock of debt and it is unable to honor its deposit insurance liabilities. The realized return of the government bonds can be expressed as

$$\widetilde{R}_{t+1}^b = (1 - \theta s_{t+1}) R_t^b. \tag{14}$$

The budget constraint of the government states that, each period, the issuance of one-period bonds B_t has to be equal to the debt service $(1 - \theta s_t) R_{t-1}^b B_{t-1}$, the cost of the deposit insurance scheme $(1 - s_t) DI_t$, and public spending G_t minus tax revenues T_t:

$$B_t = (1 - \theta s_t) R_{t-1}^b B_{t-1} + (1 - s_t) DI_t + G_t - T_t, \tag{15}$$

Tax revenues are determined according to a fiscal rule

$$T_t = \tau_y Y_t + \tau_b B_{t-1}, \tag{16}$$

where the first term can be interpreted as the automatic-stabilizer component and the second term can be interpreted as the debt-stabilizer component of tax revenues. Furthermore,

[16] As in Gertler and Kiyotaki (2015), net output is defined as output $K_t^{\alpha} L_t^{1-\alpha}$ minus the household's capital management cost $h(K_t)$.

government spending is assumed to be equal to a constant fraction g of the steady-state level of net output $\overline{Y}$ ($G_t = g\overline{Y}$).

3.5.1 Deposit insurance

When a bank fails, its equity capital is written down to zero and the deposits become a liability for the government, which has to repay principal and interests in full to the depositors. The deposit insurance scheme takes over the failed banks' assets minus resolution costs which are assumed to be a fraction μ of the assets of the bank, as in Mendicino et al. (forthcoming), resulting in a deadweight loss every time a bank defaults. If the government defaults ($s_{t+1} = 1$) it is unable to honor its deposit guarantees, and the failed banks' assets net of resolution costs are repossessed directly by the banks' debtholders, who bear the full losses.[17]

3.6 International investors

As in Aguiar et al. (2016), international financial markets are segmented, such that only a subset of foreign investors participates in the sovereign debt market. For simplicity, lenders participate in the sovereign bond market for one period and are replaced by a new set of lenders in the following period. They derive utility from final consumption C^f_{t+1} under a CRRA utility function with risk aversion parameter ν^f and solve a conventional one-period portfolio problem. They can choose between investing their exogenous endowment W^f in government bonds or in a international risk-free asset which offers them a gross return R^f (or they can also borrow at the same rate).

The problem each international investor solves is:

$$\underset{B^f_t}{\text{Max}} \quad \mathbb{E}_t \frac{(C^f_{t+1})^{1-\nu^f}}{1-\nu^f}, \tag{17}$$

subject to the budget constraint:

$$C^f_{t+1} = \widetilde{R}^b_{t+1} B^f_t + R\left(W^f - B^f_t\right). \tag{18}$$

[17]The expressions for the deposit insurance cost (B.14) and the losses for depositors (B.15) are described in the Appendix.

3.7 Market clearing

Every period, the aggregate level of bankers' net worth must equal the bank equity issued by the banks; the level of deposits supplied by the household must equal the deposits issued by the banks; the supply of government bonds must equal the bonds held by the banks and the international investors; the physical capital rented by the consumption good producing firm must equal the stock of capital held by the household and by the banks; and the firms' labor demand must equal the unit of labor inelastically supplied by the household.

3.8 Equilibrium

A competitive equilibrium is given by the policy functions for the representative household, the representative banker, the representative bank, the representative firm, and the representative international investor, such that, given prices and the realization of the shocks, the sequence of each of the agents' decisions solve their corresponding problems, the sequence of prices clears all markets, and the sequence of endogenous state variables satisfies their corresponding laws of motion. A formal definition of the competitive equilibrium is provided in the Appendix.

4 Numerical results

This section outlines the numerical solution method used, presents the baseline parameterization of the model, its main quantitative properties and provides two counterfactual exercise. The first one tries to quantify the contribution of the feedback loop by switching off the time variation of sovereign risk, assuming that the probability of default is always constant and equal to the average probability of default in the ergodic distribution of the model under the baseline parameterization. The second one analyzes the potential effects of introducing a positive risk weight for sovereign debt in the calculation of regulatory capital requirements. In particular, this section compares both the changes in the stochastic steady state of the model, the dynamic responses to a bank failure shock triggering a banking crisis, and the changes in welfare under the alternative parameterizations in each of the counterfactual exercises.

4.1 Solution method

The model is solved using global solution methods. In particular, the method used is policy function iteration (Coleman, 1990), also known as time iteration (Judd, 1998). Functions are approximated using piecewise linear interpolation, as advocated in Richter, Throckmorton and Walker (2014). A detailed description of the numerical solution method and some measures of its accuracy are provided in the Appendix.

Using global solution methods is important given the inherent non-linearities present in sovereign default models. Traditional log-linearisation methods are not able to capture the variation in risk premia (due to the certainty equivalence), which represents an important source of amplification in this model, as shown below, while higher order perturbation methods provide accurate approximations only locally, failing to capture the dynamics of models with large deviations from the steady state as the one presented here.[18] The main drawback of using global solution methods is that they are very computationally intensive, which constrains the size of the models that can be feasibly solved. This is because each additional state variable increases exponentially the size of the steady state, rendering the so called curse of dimensionality. Recent improvements in computational power and numerical solution procedures, as surveyed in Maliar and Maliar (2014) and Fernandez-Villaverde, Rubio-Ramirez and Schorfheide (2016), allow to solve increasingly complex models, but still pose a constraint that is not easily overcome.

4.2 Calibration

The calibration strategy consists of dividing the parameters into two different groups. The first group comprises parameters that are calibrated outside the model. These are typically standard parameters which are set to commonly agreed values in the business cycle literature or that are taken from related macro-banking papers. These are mostly the ones concerning the household preferences and the parameters of the aggregate production function, and some the most standard parameters in the banking side of the model. The second group comprises parameters that are specific to the model presented here, for which values are set by targeting

[18] Arouba, Fernandez-Villaverde and Rubio-Ramirez (2006), Maliar and Maliar (2014), Richter, Throckmorton and Walker (2014) or Fernandez-Villaverde, Rubio-Ramirez and Schorfheide (2016) provide a comprehensive comparison of existing solution methods for dynamic general equilibrium models.

Table 1: Baseline parameterization

Parameters calibrated outside the model		Value	Source
Subjective discount rate	β	0.99	Standard
Risk aversion	ν	2	Standard
Bankers' survival rate	φ	0.96	Bocola (2016)
Capital requirement	γ	0.08	Basel II (Clerc et al, 2015)
Risk weight of sov. bonds	ι	0.0	Basel II
Bankruptcy cost	μ	0.3	Mendicino et al (forthcoming)
Elasticity of physical capital	α	0.33	Standard
Depreciation rate of capital	δ	0.025	Standard
Write-off parameter	θ	0.55	Bocola (2016)
Intl. investors' risk aversion	ν^f	2	Same as ν (Aguiar et al, 2016)

Parameters calibrated inside the model		Value	Target
Capital mgmt. cost	κ	2.5e-4	Share of K held by households
New bankers' endowment	ϖ	0.005	Average bank return on equity
Liquidity mgmt. cost	ϕ	1e-6	Average bank exposure to sov. debt
Dispersion of iid shock	σ	0.03	Average bank failure rate
Fraction affected if ψ_t=1	λ	0.10	Fiscal cost of crises
Prob. of bank failure shock	π	0.0076	Frequency of banking crises
Govt. spending	g	0.25	Govt. final consumption expenditure
Automatic stabilizer	τ_y	0.20	Tax revenues over GDP
Debt stabilizer	τ_b	0.06	Debt over GDP
Sovereign default parameter 1	η_1	-12	Average default probability (3)
Sovereign default parameter 2	η_2	3.75	Sov. yield sensitivity to B/Y
Intl. risk-free rate	R^f	1.0088	German bond yield
Intl. investors' endowment	W^f	3	Share of debt held abroad

certain empirical moments. The parameter values and moments targeted are summarized in Table 1, while the stochastic steady state values for selected endogenous variables of the model under the baseline parameterization and their empirical counterparts are reported in Table 2.

The model is calibrated to quarterly frequency. The subjective discount rate β and the risk-aversion parameter ν of the representative household are set equal to standard values in the literature of 0.99 and 2, respectively. Similarly, the elasticity of physical capital α and its depreciation rate δ are set to 0.33 and 0.025. The capital management cost for households κ is equal to 0.00025, which implies that, in equilibrium, households directly hold around 15% of the physical capital in the economy, while the rest is held by banks.

The bankers' survival rate φ is equal to 0.96, as in Bocola (2016), and the new bankers' endowment ϖ is equal to 0.005, similar to the value in Gertler and Karadi (2011), implying an average return on equity close to 15% in annualized terms.

The capital requirement γ is set to 0.08, as in Clerc et al (2015), compatible with the full weight level of Basel I and the treatment of not rated corporate loans in Basel II and III. The risk weight of government bonds is set to zero in the baseline case, in line with the current regulatory treatment of banks' sovereign exposures. This parameter takes several different values in the counterfactual exercises performed below. The liquidity management cost is set to 1e-6, a value that guarantees an interior solution in the banks' portfolio problem and implies that sovereign bond holdings represent around 5% of banks' total assets. Again as in Clerc et al (2015) and Mendicino et al (forthcoming), the bank bankruptcy cost (the fraction of the banks' assets value that cannot recover in case of bankruptcy) is set to 0.3.

The standard deviation σ of the distribution of idiosyncratic shocks ω is equal to 0.03, which implies an average bank failure rate equal to 1%, similarly to Mendicino et al (forthcoming). The probability π that the bank failure shock realizes is equal to 0.0076, which means that, on average, it occurs once each 33 years, a frequency close to the systemic shock in Martinez-Miera and Suarez (2014). The fraction λ of banks affected when the shock realizes is set equal to 0.10.

The level of government spending as a fraction of output g is set to 0.25. The parameters governing the tax revenues τ_y and τ_b are set to 0.20 and 0.06, respectively, which imply that tax revenues equal 26% of GDP and a steady state ratio of debt-to-GDP around 30%, which matches the average in Spanish data for the period from 2000 to 2008 (excluding debt held by domestic agents other than banks).

The write-off parameter for sovereign debt θ is set to 0.55, again as in Bocola (2016), which is in line with the number Zettelmeyer, Trebesch and Gulati (2013) report for the case of the debt restructuring of Greece in 2012. The parameters of the fiscal limit distribution imply an average probability of default around 0.18%, very close to the value estimated in Bocola (2016) for the case of Italy in the context of the European sovereign debt crisis, and reproduce the sensitivity of sovereign yields to changes in the level of debt (sovereign debt spreads raise to around 500bps during the average crisis in the model, as shown below).

Table 2: Selected endogenous variables at the stochastic steady state

Variable	Model	Data
Annualized intl. risk-free rate R^f	3.5%	3.25%
Annualized return on equity R^e	14.88%	11.13%
Annualized sov. bond yield R^b	3.81%	3.38%
Annualized deposit rate R^d	3.72%	3.02%
Sovereign debt (% of GDP)	28.74%	31.51%
Share of sov. debt held abroad	61.06%	64.02%
Annualized sov. default probability	0.18%	0.19%
Share of K_t held by banks	84.7%	$\approx$ 85%
Banks' leverage (assets/equity)	13.23	13.92
Banks' sovereign exposure (% of assets)	5.49%	$\approx$ 5%

* GDP_t is defined as output Y_t minus capital management costs $h(K_t^h)$. Empircal moments correspond to Spanish data during the period from the first quarter of 2000 to the third quarter of 2008, except for the annualized sovereign default probability which corresponds to the estimate in Bocola (2016) for the case of Italy. Data sources are described in the Appendix (TBC).

Finally, the international risk-free rate R is equal to 1.008, which matches the annualized yield of one-year German bonds in the pre-crisis period. The international investors' risk-aversion parameter ν^f is set to 2, the same as for the domestic household, as in Aguiar et al (2016). The endowment W^f is set to 3, so that the share of domestic sovereign debt held abroad is around 60%, which is around the pre-crisis levels for European peripheral countries (see Merler and Pisani-Ferry, 2012).

4.3 Main results

4.3.1 Contribution of the feedback loop

In order to assess the amplification effects of the feedback loop between banks and the sovereign, this section first presents the dynamic response to a bank failure shock when sovereign default risk does not react to increases in the outstanding amount of debt and remains constant for all periods. To achieve this, the parameters governing the probability of default, η_1 and η_2, are set equal to -7.5 and 0, respectively, so that p_t becomes time invariant and equal to 0.22%, which is roughly equal to the unconditional probability of default under the baseline parameterization.

Figure 1: Impulse-response functions to a bank failure shock

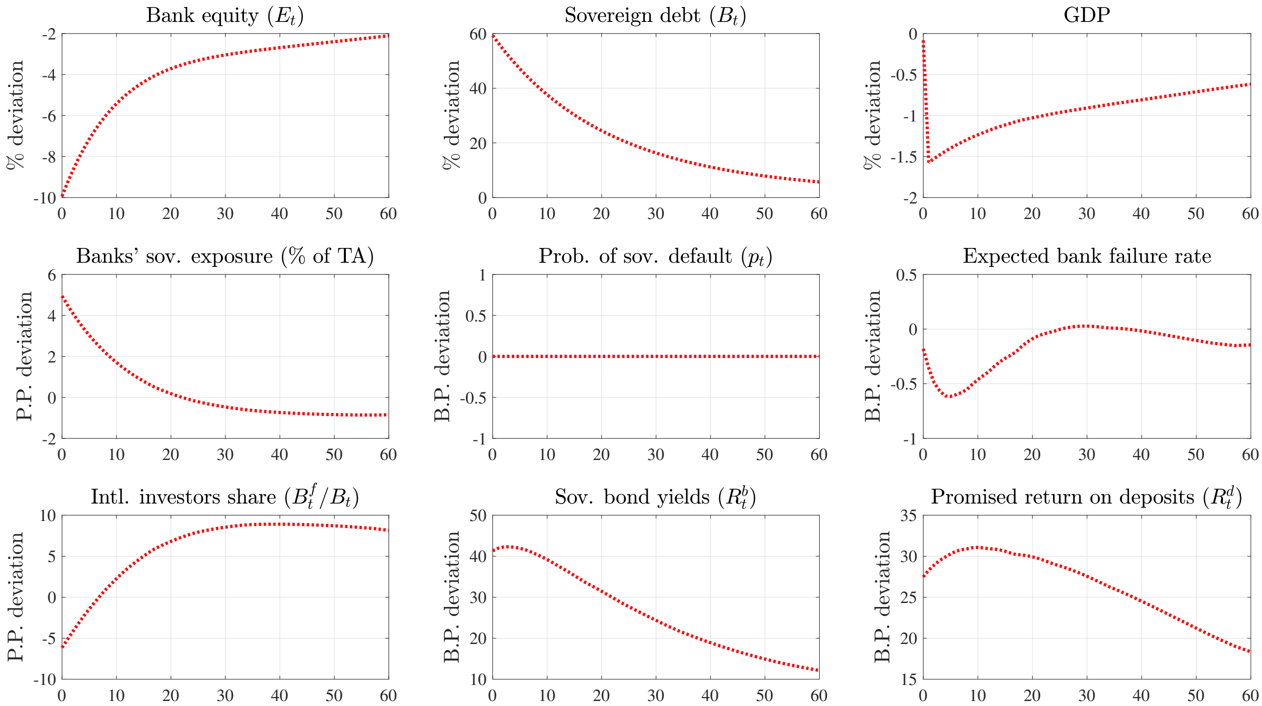
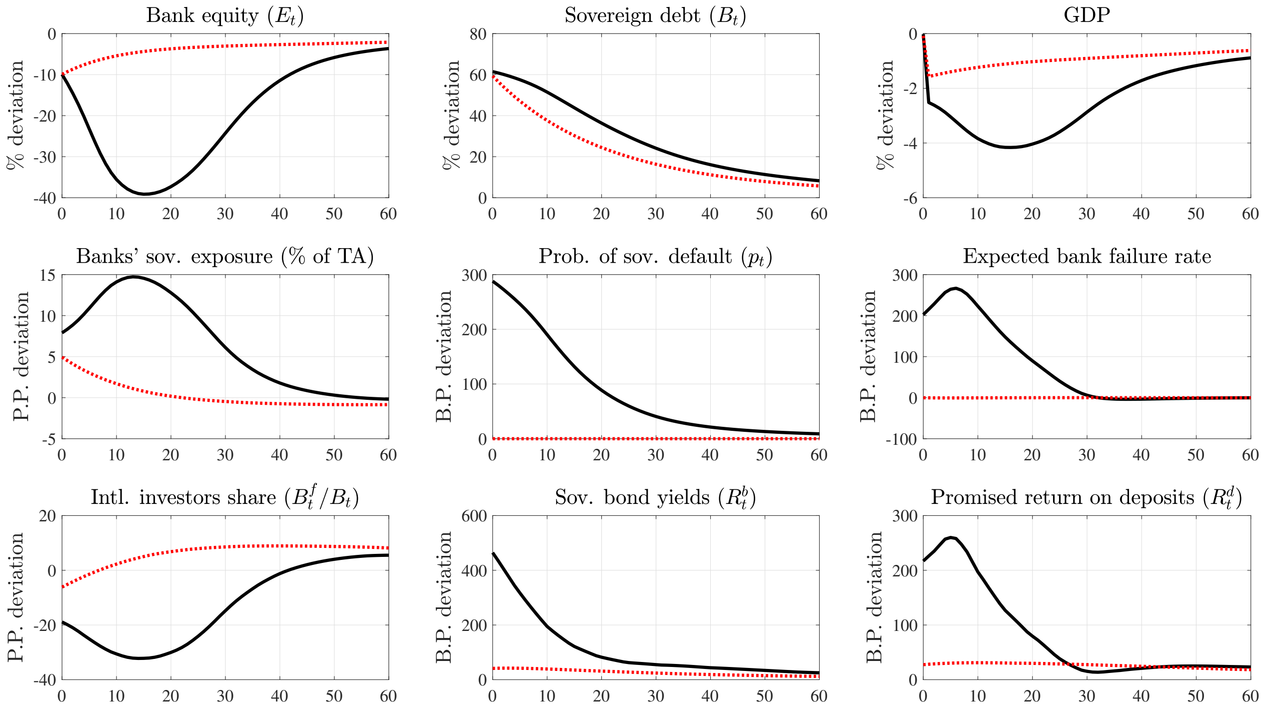


Figure 1 presents the impulse-response functions to a bank failure shock under the alternative constant-risk parameterization described above. The realization of the bank failure shock ψ_t is set equal to 1 for $t = 0$ and equal to 0 for all other t from there on. The realization of the sovereign default event s_t is equal to 0 for all t, meaning that the default event does not materialize ex-post in the simulated paths depicted.[19] Each panel represents the dynamic responses of one of the selected endogenous variables, in deviations from the stochastic steady state values in $t = -1$.

The initial shock drives up the realized bank failure rate by 10 percentage points, which translates into a 10% decrease in the level of aggregate bank equity and an increase in the outstanding sovereign debt of 60% from its initial level, due to the increase in the deposit insurance liabilities of the government. The fall in GDP (defined as total output minus the households' physical capital management cost) is caused by the shrinkage of the banks' balance sheets and the change in the composition of the owners of physical capital: since the

[19]Nevertheless, all of the agents form their expectactions taking into account the possibility that the government defaults on its obligations.

Figure 2: Impulse-response functions to a bank failure shock



decrease in aggregate bank equity constrains the ability of banks to invest in physical capital, the share of the aggregate stock that is managed by the household increases, resulting in a decrease in net output.

The increase in the stock of debt is absorbed by the banks, who increase their exposure relative to the size of their balance sheet, and by the international investors, who also increase their bond holdings in absolute terms (although the share of the total outstanding debt they hold slightly decreases). The riskiness of the sovereign bonds under this alternative parameterization, as described above, remains constant, making their promised return increase only slightly. The expected bank failure rate remains barely unchanged and so does the promised return of deposits, which increases a few basis points. The relative scarcity of bankers' net worth increases the return on equity due to the increase in the marginal product of physical capital, making the aggregare level of bank equity to quickly recover.

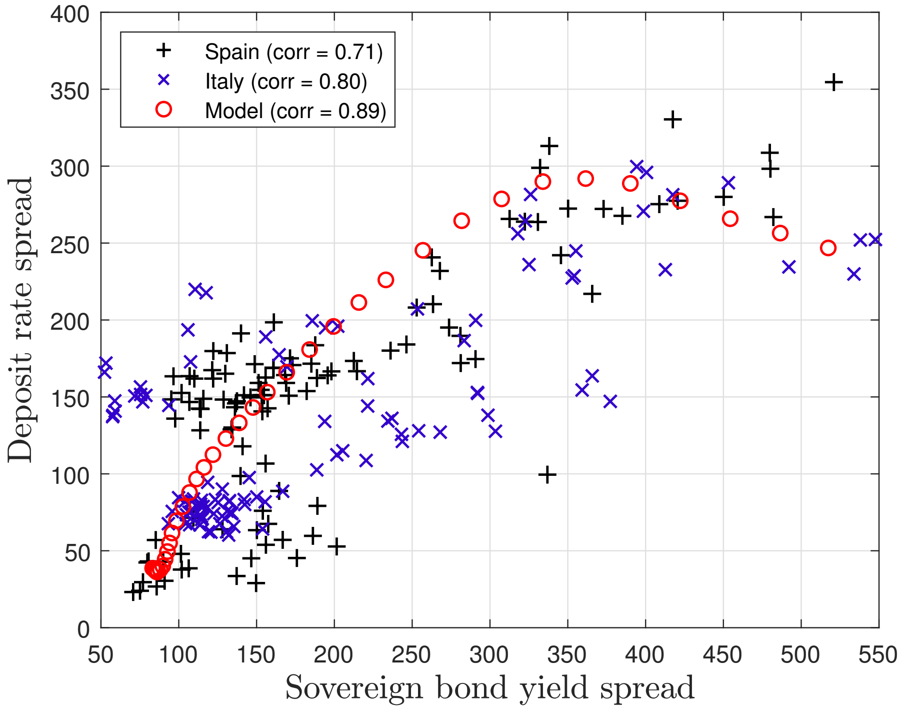
Figure 2 presents, with solid black lines, the dynamic response to the same shock under the baseline parameterization described in Table 1, where sovereign risk does react to increases in the outstanding level of debt. The dotted red lines depict the same impulse response functions as in Figure 1, when sovereign risk is time invariant and exogenously given.

Following the initial 60% increase in the level of sovereign debt, the annualized probability of default goes up by 300 bps, from an initial 0.18% (see Table 2). This sudden increase translates into a spike of the interest rate paid by the government of more than 400 bps, to which banks react by increasing their exposure by almost 10 percentage points. The increased exposure of banks to sovereign risk and their higher leverage makes the expected bank failure rate go up by more than 200 bps. As a result, the depositors, anticipating that a sovereign default, which is now much more likely, would mean the failure of the deposit insurance scheme, demand a promised return on deposits up to 200 bps higher. The increase in funding costs have a large impact on banks' profitability, making the aggregate level of bank equity go further down to a -40% of its initial level after fifteen quarters, a drop much larger than under the time-invariant sovereign risk parameterization. This has also consequences for net output since, as explained above, tighter constraints on banks' intermediation ability force the relatively inefficient households to manage an increasing share of the stock of physical capital. Furthermore, since banks' increase the deposits they borrow from households, this crowds out hoseholds' investment in physical capital, resulting in a sharper on-impact contraction of GDP than under the constant-risk scenario.

In all, these results illustrate the amplification effects that sovereign default risk has on the baking sector, representing an important source of systemic risk. As shown in Figure 2, an initial shock that affects a relatively small fraction of banks translates into system-wide instability through the endogenous contagion effect that sovereign risk has on bank failure risk, even if the default of the government does not materialize ex-post, as in the simulated trajectories depicted above.

The increase in banks' funding costs and the resulting decrease in their profitability, in addition to the high yield paid by the government bonds, encourages banks to increase their exposure to sovereign risk. Given the opacity of their balance sheets, individual banks do not internalize the effect of their increased riskiness on the funding costs of the whole banking sector. Furthermore, because of limited liability, they can enjoy the high returns from holding sovereign bonds as long as the government does not default, while suffering limited losses in case the default materializes, effectively shifting the risk to their depositors. Thus, the results

Figure 3: Sovereign yield and deposit rate spreads: model vs. data



seem to point to a potential role for macroprudential regulation in making banks internalize the effects of their sovereign exposures and in mitigating the negative effects of the feedback loop.

Figure 3 further illustrates the quantitative properties of the model in terms of its ability to fit the dynamics of the recent European sovereign debt crisis. The horizontal axis displays sovereign yield spreads in basis points, calculated as the difference between the annualized yield of 10-year Spanish and Italian government bonds and the annualized yield of 10-year German bond. The vertical axis displays deposit rate spreads, also in basis points, calculated as the difference between the annualized yield of Spanish and Italian banks' interest rate on deposits with agreed maturity of up to one year and the annualized yield of 1-year German bond.[20] The data points correspond to monthly observations for the period between 2009 and 2017. The observations from the model are obtained from the simulation of the dynamic response to a bank failure shock, as depicted in Figure 2. The simulated data points account

[20]Data sources are provided in the Appendix.

for the equivalent number of periods (36 quarters). The model does remarkably well in matching the correlation of sovereign yields and deposit rate spreads during crises, suggesting its ability to quantitatively capture the endogenous feedback effects between sovereign and bank risk.

It is possible to compare social welfare under both scenarios, in order to quantify the loss associated to the feedback loop between sovereign and bank risk. To this end, the expected value of the household intertemporal utility is computed by averaging across a large number of simulations of the model economy. More formally, the proposed measure of welfare V_0 can be defined as

$$V_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{(C_t)^{1-\nu}}{1-\nu} \right]. \tag{19}$$

Then it is possible to represent welfare in terms of equivalent permanent consumption units by obtaining the value $\overline{C}$ that solves the equation

$$V_0 = (1-\beta) \frac{(\overline{C})^{1-\nu}}{1-\nu}. \tag{20}$$

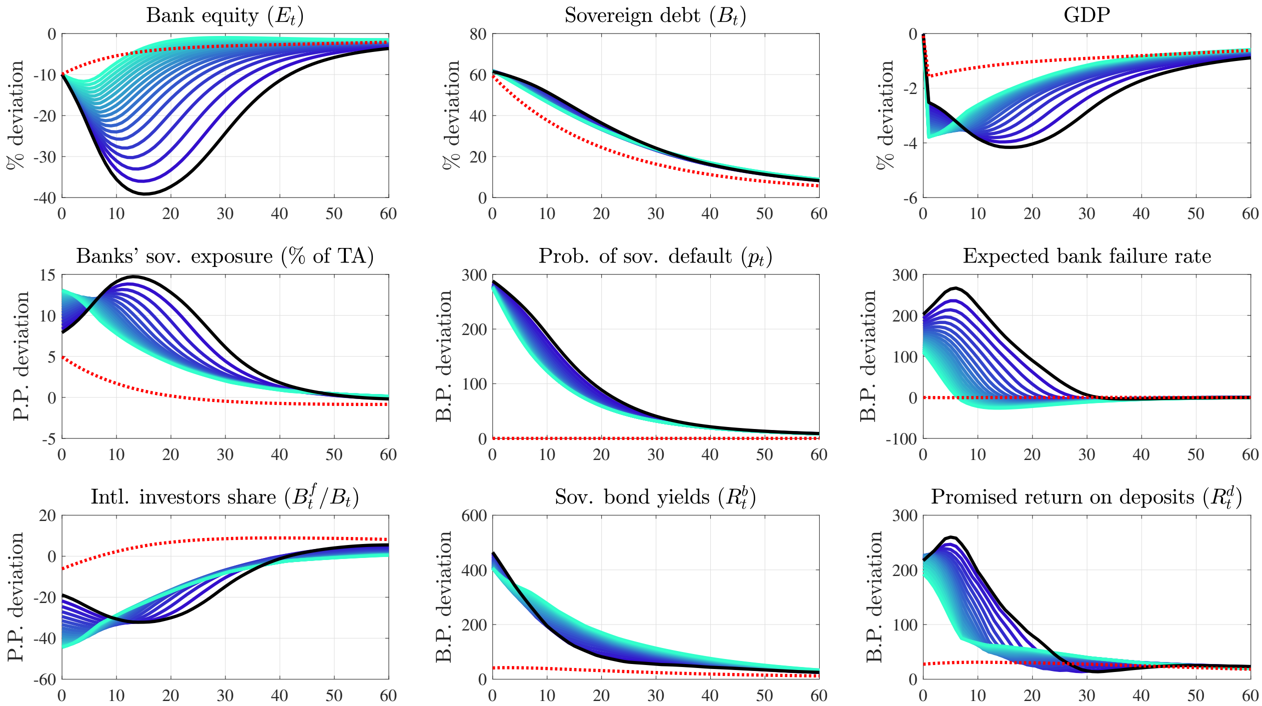
The result is that the welfare loss resulting from the feedback loop, obtained from the comparison between the model economy under the baseline calibration and the counterfactual constant-risk scenario, amounts to a decrease of 1.3% of equivalent permanent consumption units.

4.3.2 Bank capital requirements for sovereign exposures

This section analyzes the macroprudential implications of bank capital requirements for sovereign exposures. Figure 4 presents the dynamic response to the same shock under a number of parameterizations where the risk weight ι applied to banks' sovereign bond holdings in the calculation of regulatory capital requirements is increased from its initial level of zero. Each of the blue lines depict the impulse-response function under a different risk weight ι, following 5% increments, with lighter colors representing higher values, from 5% to 70%.

Increasing capital requirements for banks' sovereign exposures has two effects: first, for the same promised return, it makes investing in sovereign debt less attractive. This is because the cost of equity is higher than the cost of deposits. Furthermore, the equity losses that banks

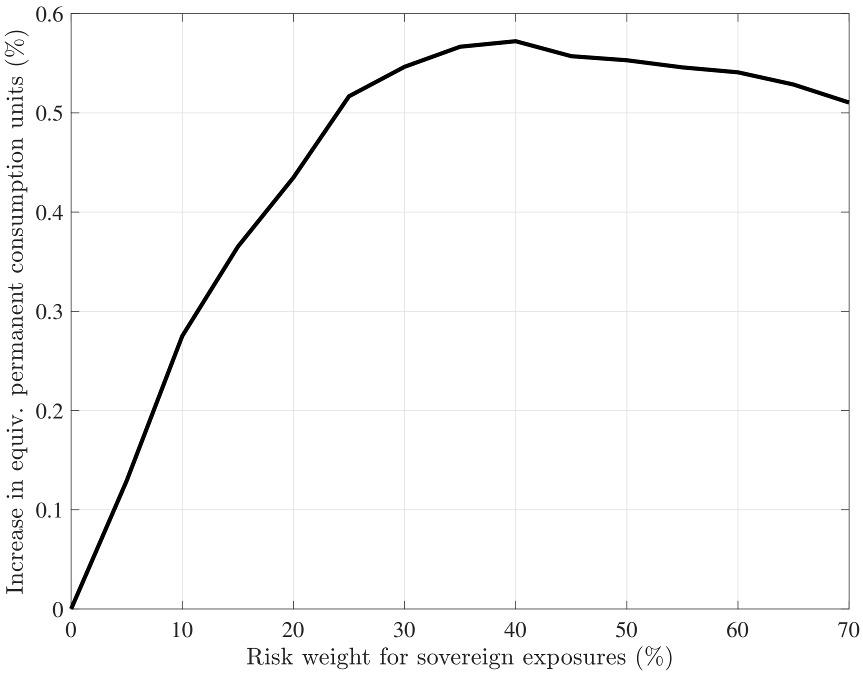
Figure 4: Impulse-response functions to a bank failure shock



would suffer in case of default are higher; this is the well-known "skin-in-the-game" effect. Second, it reduces banks' leverage, making banks effectively safer and thus decreasing the depositors losses in case of default. This translates into lower funding costs, less amplification effects and quicker recoveries from the initial shock. Each increase in the risk weight ι brings the trajectory of the response of bank equity closer to the alternative parameterization with constant sovereign risk presented in Figure 1, depicted by the red dashed lines, suggesting that capital requirements are effective in mitigating the effects of the bank-sovereign feedback loop on financial instability.

However, the benefits of increasing the risk weight for sovereign exposures do not come at no cost. First, imposing capital requirements for domestic banks' debt holdings increases the funding costs for the government. This is because domestic banks, as opposed to international investors, benefit from the liquidity services of holding sovereign bonds, and therefore demand lower returns on their bond holdings. Second, and more importantly, initial contractions in GDP become sharper at the beginning of crises. This is because banks are now required to use part of their equity to back their sovereign bond holdings, which leaves them with a lower

Figure 5: Relative welfare gains for different risk weights for sovereign exposures



amount of equity available for other purposes, effectively crowding out banks' investment in physical capital. Thus, the drop in banks' investment when equity is relatively more scarce is amplified. Nevertheless, economic activity recovers quicker than in the baseline case with zero risk weights due to the overall decrease in bank risk and the subsequent quicker recovery of aggregate bank capital.

The results above suggest the existence of non-trivial welfare trade offs resulting from increases of sovereign debt risk weights. In order to assess the socially optimal risk weight, Figure 5 presents the welfare gains in terms of equivalent permanent consumption units that are obtained for different values of ι. The results confirm that marginal departures from the zero risk weight lead to relatively large welfare gains. These gains, however, seem to exhaust when risk weights go beyond a certain point, due to the above-mentioned trade offs involved. In this numerical exercise, the point that maximizes social welfare, for a given capital requirement γ of 8%, is reached when $\iota = 40\%$, implying an increase of 0.56% equivalent permanent consumption units relative to the zero risk weight scenario.

Table 3 summarizes the stochastic steady state values for selected endogenous variables

Table 3: Selected endogenous variables at the stochastic steady state

	$\iota = 0$	$\iota = 40\%$	Diff.
Annualized return on equity R^e	14.88%	14.96%	8 bps
Annualized return of capital R^k	4.58%	4.62%	4 bps
Annualized sov. bond yield R^b	3.81%	3.91%	10 bps
Annualized deposit rate R^d	3.72%	3.75%	3 bps
Welfare (= equiv. constant consumption units)	1.449	1.458	0.57%
GDP	2.964	2.959	-0.17%
Capital-to-GDP ratio	2.28	2.27	-0.44%
Sovereign debt (% of GDP)	28.74%	28.28%	46 bps
Share of sov. debt held abroad	61.06%	77.49%	16.4 pps
Annualized sov. default probability	0.18%	0.17%	-1 bps
Share of K_t held by banks	84.7%	84.3%	-40 bps
Banks' leverage (assets/equity)	13.23	12.75	-3.63%
Banks' sovereign exposure (% of bank assets)	5.49%	3.22%	2.3 pps
Annualized default rate of banks	0.92%	0.81%	-11 bps

* GDP_t is defined as output Y_t minus capital management costs $h(K_t^h)$.

of the model under the baseline parameterization and compares them with the values for the counterfactual scenario in which the risk weight is set to the socially optimal level ($\iota = 40\%$).

5 Concluding remarks

This paper examines the *negative feedback loop* between sovereign and banking crises, and the potential effects of capital requirements for banks' sovereign exposures on mitigating it by discouraging banks' endogenous exposure to sovereign risk. To this purpose, it develops a dynamic general equilibrium model in which banks decide on their exposure to sovereign debt issued by a government subject to default risk.

One of the contributions of the model presented in this paper is that it features both endogenous bank failure risk and sovereign default risk, which have reinforcing effects on each other (what has been called the *negative feedback loop* between banks and sovereigns). The model allows to study the macroeconomic consequences of such feedback effects: the impact of an increase in bank failure on the probability of a sovereign default resulting from government guarantees, the endogenous increase in banks' exposure to sovereign risk, and the feedback effects that an increase in the sovereign default risk have on banks' solvency and their funding costs. In this sense, the possibility of a sovereign default acts as an important source of systemic risk, by which an initial shock to a small fraction of banks translates into system-wide instability.

Distortions resulting from banks' limited liability make investing in risky sovereign debt attractive for banks, who enjoy high profits insofar as the government does not default and suffer losses limited to their initial equity contributions otherwise. These risk-shifting incentives result in excessive exposure to sovereign risk. At the same time, the possibility that the government defaults not only on its outstanding stock of debt but also on its deposit insurance liabilities translates into higher funding costs for the banks when they increase their exposure to the risky sovereign. When depositors cannot observe the balance sheet composition of individual banks, these do not internalize the effect of their individual risk-taking choices on the funding costs of the whole banking system.

By disrupting banks' intermediation ability, the effects of the feedback loop have dramatic consequences for economic activity, even when the sovereign default event does not materialize ex-post. Thus, the model environment provides a rationale for macroprudential policies aimed to reduce banks' incentives to excessively expose themselves to sovereign risk.

The model is used to address some of the central issues in recent discussions about the current regulatory treatment of banks' exposure to (domestic) sovereign debt. In particular, the paper analyzes the potential macroprudential role of capital requirements for sovereign debt. The main finding is that a positive risk weight for sovereign debt in the calculation of capital requirements both reduces banks' endogenous exposure to sovereign risk and makes bank effectively safer and, consequently, helps mitigating the two-way feedback effects between banking and sovereign crises and its negative spillovers on economic activity.

Under the proposed calibration of the model parameters, the quantitative results indicate that the feedback loop generates substantial amplification effects during financial crises, contributing to substantial welfare losses. The assessment of the macroprudential implications of a change in the regulatory treatment of banks' sovereign exposures evaluates the social welfare gains associated to different risk weights of sovereign debt in the regulatory capital requirements, finding an interior maximum social welfare at a risk weight of 40%, for a given capital requirement of 8%. The results identify non-trivial welfare trade-offs resulting from the implementation of the proposed regulatory reform, which exhaust the potential benefits of further increasing the risk weight for sovereign exposures beyond a certain point.

Other sets of macroprudential policies could also be analyzed in the context of the model, such as time-varying capital requirements, concentration limits to the exposure of banks to sovereign debt, or different combinations of the general regulatory capital requirement and the risk weights for sovereign debt exposures, among others.

The model could also be used to analyze the international dimension of the feedback loop. This would be particularly interesting in the context of a monetary union and could shed light on issues such as common deposit insurance mechanisms and common resolution regimes, and their effect on international risk spillovers. Conceptually, this would only require embedding the model in a multi-country setup. The main difficulty, however, would come from the computationally intensive solution methods that would be needed to solve it. Notwithstanding this, these appear to be interesting topics for a future research agenda.

References

[1] Acharya, V., I. Drechsler, and P. Schnabl (2014): "A Pyrrhic Victory? - Bank Bailouts and Sovereign Credit Risk," *Journal of Finance*, 69 (6): 2689-2739.

[2] Acharya, V. and R. Rajan (2013): "Sovereign Debt, Government Myopia, and the Financial Sector," *Review of Financial Studies*, 26 (6): 1526-1560.

[3] Acharya, V. and S. Steffen (2015): "The "Greatest" Carry Trade Ever? Understanding Eurozone Bank Risks," *Journal of Financial Economics*, 115 (2): 215-236.

[4] Aguiar, M., S. Chatterjee, H. Cole, and Z. Stangebye (2016): "Quantitative Models of Sovereign Debt Crises," in: Taylor, J. and H. Uhlig (Eds.), *Handbook of Macroeconomics*, Vol. 2, Ch. 21, 1697-1755.

[5] Aguiar, M., and G. Gopinath (2006): "Defaultable debt, interest rates and the current account," *Journal of International Economics*, 69 (1): 64-83.

[6] Altavilla, C., M. Pagano and S. Simonelli (2017): "Bank exposures and sovereign stress transmission," *Review of Finance*, 21 (6): 2103-2139.

[7] Arellano, C. (2008): "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review*, 98: 690-712.

[8] Ari, A. (2017): "Sovereign Risk and Bank Risk-Taking," manuscript, University of Cambridge.

[9] Arouba, B., J. Fernandez-Villaverde, and J. Rubio-Ramirez (2006): "Comparing solution methods for dynamic equilibrium economies," *Journal of Economic Dynamics and Control*, 30 (12): 2477-2508.

[10] Balteanu, I. and A. Erce (2017): "Linking Bank Crises and Sovereign Defaults: Evidence from Emerging Markets," European Stability Mechanism Working Papers, 22.

[11] Bank of International Settlements (2016): *86th Annual Report, 2015/16.*

[12] Becker, B. and V. Ivashina (2017): "Financial Repression in the European Sovereign Debt Crisis," *Review of Finance*, 22 (1): 83-115

[13] Bernanke, B. and M. Gertler (1989): "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, 79 (1): 14-31.

[14] Bernanke, B., M. Gertler, and S. Gilchrist (1999): "The financial accelerator in a quantitative business cycle framework," in: Taylor, J. and M. Woodford (Eds.), *Handbook of Macroeconomics*, Vol. 1 (C), Ch. 21, 1341-1393.

[15] Bi, H. (2012): "Sovereign default risk premia, fiscal limits, and fiscal policy," *European Economic Review*, 56 (3): 389-410.

[16] Bi, H. and N. Traum (2012): "Estimating Sovereign Default Risk," *American Economic Review*, 102 (3): 161-66.

[17] Bianchi, J. and E. Mendoza (2018): "Optimal Time-Consistent Macroprudential Policy," *Journal of Political Economy*, 126 (2): 588-634.

[18] Broner, F., A. Erce, A. Martin and J. Ventura (2014) : "Sovereign debt markets in turbulent times: Creditor discrimination and crowding-out effects," *Journal of Monetary Economics*, 61: 114-142.

[19] Brunnermeier, M., L. Garicano, P. Lane, M. Pagano, R. Reis, T. Santos, D. Thesmar, S. Van Nieuwerburgh, and D. Vayanos (2016): "The Sovereign-Bank Diabolic Loop and ESBies," *American Economic Review*, 106 (5): 508-12.

[20] Brunnermeier, M. and Y. Sannikov (2014): "A Macroeconomic Model with a Financial Sector," *American Economic Review*, 104 (2): 379-421.

[21] Carlstrom, C. and T. Fuerst (1997): "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis," *American Economic Review*, 87 (5): 893-910.

[22] Chari, V.V., A. Dovis and P. Kehoe (2016): "On the Optimality of Financial Repression," Federal Reserve Bank of Minneapolis, Research Department Staff Report 1000.

[23] Clerc, L., A. Derviz, C. Mendicino, S. Moyen, K. Nikolov, L. Stracca, J. Suarez, and A. Vardoulakis (2015): "Capital regulation in a macroeconomic model with three layers of default," *International Journal of Central Banking*, 11: 9-64.

[24] Coleman, W.J. (1990): "Solving the Stochastic Growth Model by Policy-Function Iteration," *Journal of Business & Economic Statistics*, 8 (1): 53-73.

[25] Cooper, R. and K. Nikolov (2013): "Government debt and banking fragility: The spreading of strategic uncertainty," NBER Working Paper Series 19278.

[26] Corsetti, G., K. Kuester, A. Meier and G. Muller (2014): "Sovereign Risk, Fiscal Policy, and Macroeconomic Stability," *The Economic Journal*, 123(566): 99-132.

[27] Eaton, J. and M. Gersovitz (1981): "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies*, 48: 289-309.

[28] Elenev, V., T. Landvoigt, and S. Van Nieuwerburgh (2017): "A Macroeconomic Model with Financially Constrained Producers and Intermediaries," , NYU Stern.

[29] Enria, A., A. Farkas and L.J. Overby (2016): "Sovereign Risk: black swans and white elephants," *European Economy*, 2016(1): 51-71.

[30] European Systemic Risk Board (2015): *ESRB report on the regulatory treatment of sovereign exposures.*

[31] Faia, E. (2017): "Sovereign Risk, Bank Funding and Investors' Pessimism," *Journal of Economic Dynamics and Control*, 79: 79-96.

[32] Farhi, E. and J. Tirole (forthcoming): "Deadly Embrace: Sovereign and Financial Balance Sheets Doom Loops," *Review of Economic Studies.*

[33] Fernandez-Villaverde, J., J. Rubio-Ramirez, and F. Schorfheide (2016): "Solution and Estimation Methods for DSGE Models," in: Taylor, J. and H. Uhlig (Eds.), *Handbook of Macroeconomics*, Vol. 2, Ch. 9, 527-724.

[34] Gertler, M., and P. Karadi (2011): "A Model of Unconventional Monetary Policy," *Journal of Monetary Economics*, 58: 17-34.

[35] Gertler, M., and N. Kiyotaki (2011): "Financial Intermediation and Credit Policy in Business Cycle Analysis," in: Friedman, B. and M. Woodford (Eds.), *Handbook of Monetary Economics*, Vol. 3, Ch. 11, 547-599.

[36] Gertler, M., N. Kiyotaki, and A. Queralto (2012): "Financial Crises, Bank Risk Exposure and Government Financial Policy," *Journal of Monetary Economics*, 59: 517-534.

[37] Gros, D. (2013): "Banking Union with a Sovereign Virus: The self-serving regulatory treatment of sovereign debt in the euro area," CEPS Papers, 7904.

[38] Gennaioli, N., A. Martin and S. Rossi (2014): "Sovereign Default, Domestic Banks, and Financial Institutions," *Journal of Finance.*

[39] He, Z. and A. Krishnamurthy (forthcoming): "A Model of Capital and Crises," *Review of Economic Studies.*

[40] Holstrom, B. and J. Tirole (1998): "Private and Public Supply of Liquidity," *Journal of Political Economy*, 106 (1): 1-40.

[41] Judd, K. (1992): "Projection methods for solving aggregate growth models", *Journal of Economic Theory*, 58 (2): 410-452.

[42] Judd, K. (1998): *Numerical Methods in Economics*, MIT Press.

[43] Kirschenmann, K., J. Korte, and S. Steffen (2017): "The Zero Risk Fallacy - Banks' Sovereign Exposure and Sovereign Risk Spillovers," manuscript.

[44] Kiyotaki, N. and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 26 (3): 49-67.

[45] Lane, P. (2012): "The European Sovereign Debt Crisis," *Journal of Economic Perspectives*, 105 (2): 211-248.

[46] Leonello, A. (forthcoming): "Government Guarantees and the Two-Way Feedback between Banking and Sovereign Debt Crises," *Journal of Financial Economics.*

[47] Mäkinen, T., L. Sarno and G. Zinna (2018): "Risky Bank Guarantees", manuscript.

[48] Maliar, L. and S. Maliar (2014): "Numerical Methods for Large Scale Dynamic Economic Models", in: Schmedders, K. and K. Judd (Eds.), *Handbook of Computational Economics*, Vol. 3, Ch. 7, 325-477, Elsevier Science.

[49] Martinez-Miera, D. and R. Repullo (2017): "Search for Yield," *Econometrica*, 85 (2): 351-378.

[50] Martinez-Miera, D. and J. Suarez (2014): "Banks' endogenous systemic risk taking," manuscript, CEMFI.

[51] Mendicino, C., K. Nikolov, D. Supera and J. Suarez (forthcoming): "Optimal dynamic capital requirements," *Journal of Money, Credit, and Banking.*

[52] Mendoza, E. and Z. Yue (2012): "A General Equilibrium Model of Sovereign Default and Business Cycles," *Quarterly Journal of Economics*, 127 (2): 889-946.

[53] Merler, S. and J. Pisani-Ferry (2012): "Who's afraid of sovereign bonds," Bruegel Policy Contribution 2012:02.

[54] Nouy, D. (2012): "Is sovereign risk properly addressed by financia regulation?," *Financial Stability Review*, Banque de France, 16: 95-106.

[55] Perez, D. (2014): "Sovereign Debt, Domestic Banks and the Provision of Public Liquidity," Job Market Paper, Stanford University.

[56] Reinhart, C. and K. Rogoff (2011): "From Financial Crash to Debt Crisis," *American Economic Review*, 101 (5): 1676-1706.

[57] Repullo, R. and J. Suarez (2004): "Loan pricing under Basel capital requirements," *Journal of Financial Intermediation*, 13 (4): 496-521.

[58] Richter, A., N. Throckmorton and T. Walker (2014): "Accuracy, Speed and Robustness of Policy Function Iteration," *Computational Economics*, 44 (4): 445-476.

[59] Sosa-Padilla, C. (2017): "Sovereign Defaults and Banking Crises," manuscript.

[60] Thaler, D. (2017): "Austerity to save the banks? A quantitative Model of Sovereign Default with Endogenous Default Costs and a Financial Sector," manuscript.

[61] Uhlig, H. (2014): "Sovereign Default Risk and Banks in a Monetary Union," *German Economic Review*, 15(1): 23-41.

[62] Weidmann, J. (2013): "Stop encouraging banks to buy government debt," *Financial Times*, September 30.

[63] Woodford, M. (1990): "Public Debt as Private Liquidity," *American Economic Review*, 80 (2): 382-388.

Appendix

A Data sources

TBC.

B Equilibrium equations

This Appendix presents the complete set of equilibrium equations and provides the formal definition of a competitive equilibrium.

B.1 Households

The problem of the representative household (1) results in the following optimality conditions:

$$\mathbb{E}_t[\Lambda_{t+1}\widetilde{R}^d_{t+1}] = 1, \tag{B.1}$$

$$\mathbb{E}_t\left[\Lambda_{t+1}R^k_{t+1}\right] = 1 + h'(K^h_t). \tag{B.2}$$

The household's budget constraint is given by

$$C_t + D_t + K^h_t + h(K^h_t) = W_t + \widetilde{R}^d_t D_{t-1} + R^k_t K^h_{t-1} + \Pi_t - T_t, \tag{B.3}$$

and level of the household's net worth N_t evolves according to the following law of motion:

$$N_t = W_t + \widetilde{R}^d_t D_{t-1} + R^k_t K^h_{t-1} + \Pi_t - T_t. \tag{B.4}$$

B.2 Bankers

The level of bankers' net worth E_t evolves according to the following law of motion:

$$E_t = \varphi\widetilde{R}^e_t E_{t-1} + (1-\varphi)\varpi N_t. \tag{B.5}$$

The marginal value of one unit of net worth for the bankers is

$$v_t = \mathbb{E}_t\left[\Lambda_{t+1}(1-\varphi+\varphi v_{t+1})\widetilde{R}^e_{t+1}\right]. \tag{B.6}$$

B.3 Banks

The problem of the representative bank (9) results in the following optimality conditions:

$$\mathbb{E}_t\Omega_{t+1}\left\{\widetilde{R}^k_{t+1}\left(1-\Gamma(\overline{\omega}_{t+1})\right) - \left[m^k_t + R^d_t(1-\gamma)\right]\left(1-F(\overline{\omega}_{t+1})\right)\right\} = v_t\gamma, \tag{B.7}$$

$$\mathbb{E}_t\Omega_{t+1}[\widetilde{R}^b_{t+1} - m^b_t - R^d_t(1-\gamma\iota)]\left(1-F(\overline{\omega}_{t+1})\right) = v_t\gamma\iota, \tag{B.8}$$

where

$$m^k_t \equiv \frac{\partial m(d_t,b_t)}{\partial k_t} = \phi\left[2(1-\gamma)^2(k_t/b_t) + 2(1-\gamma)(1-\gamma\iota)\right],$$

$$m^b_t \equiv \frac{\partial m(d_t,b_t)}{\partial b_t} = \phi\left[(1-\gamma\iota)^2 - (1-\gamma)^2(k_t/b_t)^2\right],$$

are the derivatives of the liquidity management cost with respect to the investment in physical capital and in sovereign bonds, respectively, and

$$\Gamma(x) = \int_0^x \omega f(\omega)\mathrm{d}\omega = \Phi\left(\frac{\log(x)-\sigma^2/2}{\sigma}\right),$$

$$F(x) = \int_0^x f(\omega)\mathrm{d}\omega = \Phi\left(\frac{\log(x) + \sigma^2/2}{\sigma}\right),$$

where $f(\omega)$ is the probability density function of the idiosyncratic shock ω and $\Phi(\cdot)$ is the cumulative distribution function of the standard normal.

The balance sheet constraint is given by

$$k_t + b_t = d_t + e_t, \tag{B.9}$$

and the regulatory capital requirement imposes that

$$e_t = \gamma(k_t + \iota b_t). \tag{B.10}$$

B.4 Firms

The problem of the representative firm (12) results in the following optimality conditions:

$$r_t^k = \frac{\alpha Y_t}{K_t}, \tag{B.11}$$

$$W_t = \frac{(1-\alpha)Y_t}{L_t}. \tag{B.12}$$

B.5 Government

The level of government debt outstanding B_t evolves according to the following law of motion:

$$B_t = (1 - \theta s_t)R_{t-1}^b B_{t-1} + (1 - s_t)DI_t + G_t - T_t. \tag{B.13}$$

The deposit insurance liabilities can be expressed as

$$DI_t = [R_{t-1}^d d_{t-1} + m(d_{t-1}, b_{t-1}) - \widetilde{R}_t^b b_{t-1}]F(\overline{\omega}_t) - \mu \widetilde{R}_t^k k_{t-1}\Gamma(\overline{\omega}_t). \tag{B.14}$$

From this expression, the loss for depositors due to banks' failure is

$$\Psi_t D_{t-1} = s_t \left\{[R_{t-1}^d d_{t-1} + m(d_{t-1}, b_{t-1}) - \widetilde{R}_t^b b_{t-1}]F(\overline{\omega}_t) - \mu \widetilde{R}_t^k k_{t-1}\Gamma(\overline{\omega}_t)\right\}. \tag{B.15}$$

B.6 International investors

The problem of the representative international investor (17) results in the following optimality condition:

$$\mathbb{E}_t\left[(\widetilde{R}_{t+1}^b - R^f)\left[\widetilde{R}_{t+1}^b B_t^f + R^f\left(W^f - B_t^f\right)\right]^{-\nu^f}\right] = 0. \tag{B.16}$$

B.7 Market clearing

Every period, the aggregate level of bankers' net worth must equal the bank equity issued by the banks:

$$E_t = e_t, \tag{B.17}$$

the level of deposits supplied by the household must equal the deposits issued by the banks:

$$D_t = d_t, \tag{B.18}$$

the supply of government bonds must equal the bonds held by the banks and the international investors:

$$B_t = b_t + B_t^f, \tag{B.19}$$

the physical capital rented by the consumption good producing firm must equal the stock of capital held by the household and by the banks:

$$K_t = K_{t-1}^h + k_{t-1}, \tag{B.20}$$

and the labor hired by the firm must equal the unit of labor inelastically supplied by the household:

$$L_t = 1. \tag{B.21}$$

B.8 Equilibrium

In equilibrium, the state of the economy at any date t can be summarized by three state variables collected in the vector $\mathbf{S} = \{N, E, B\}$: the aggregate net worth of the representative household N_t, the aggregate net worth available to the active bankers E_t, and the level of sovereign debt outstanding B_t. Formally:

Definition 1. *A competitive equilibrium is given by the policy functions for the representative bank ($k(\mathbf{S}), b(\mathbf{S}), d(\mathbf{S}), e(\mathbf{S})$), the representative household ($C(\mathbf{S}), D(\mathbf{S}), K^h(\mathbf{S})$), the representative firm ($K(\mathbf{S}), L(\mathbf{S})$) and the representative international investor ($B^f(\mathbf{S})$), which determine the actions of each of the agents for each triple $\mathbf{S} = \{N, E, B\}$, such that, given prices ($v(\mathbf{S}), R^d(\mathbf{S}), R^b(\mathbf{S}), r^k(\mathbf{S}), w(\mathbf{S})$) and the realization of the shocks:*

1. *The sequence of consumption and saving decisions $\{C_t, D_t, K_t^h\}_{t=0,1,...}$ solves the problem of the representative household, ie eq. (B.1)-(B.3).*

2. *The sequence of portfolio choices $\{k_t, b_t\}_{t=0,1,...}$ and liability structure $\{d_t, e_t\}_{t=0,1,...}$ solves the problem of the representative bank, ie eq. (B.7)–(B.10).*

3. *The sequence of input choices $\{K_t, L_t\}_{t=0,1,...}$ solves the problem of the representative firm, ie eq. (B.11)–(B.12).*

4. *The sequence of portfolio choices $\{B_t^f\}_{t=0,1,...}$ solves the problem of the representative international investor, ie eq. (B.16).*

5. *The sequence of prices $\{v_t, R_t^d, R_t^b, r_t^k, W_t\}_{t=0,1,...}$ clears the equity market, the deposits market, the physical capital market and the labor market, ie eq. (B.17)–(B.21).*

6. *The sequence of endogenous state variables $\{N_{t+1}, E_{t+1}, B_{t+1}\}_{t=0,1,...}$ satisfies the respective laws of motion, ie eq. (B.4), (B.5) and (B.13).*

C Solution method

The model is solved using global solution methods. In particular, the method used is policy function iteration (Coleman, 1990), also known as time iteration (Judd, 1998). Functions are approximated using piecewise linear interpolation, as advocated in Richter, Throckmorton and Walker (2014). A sketch of the numerical solution procedure is as follows:

1. Discretize the state variables by creating an evenly space grid, covering the relevant range of values each of them can take.

2. Select the set of policy functions. In this case, the variables chosen are $C(\mathbf{S})$, $b(\mathbf{S})$, $v(\mathbf{S})$, $R^d(\mathbf{S})$, $R^b(\mathbf{S})$.

3. Specify an initial guess for the policy functions at each point i of the state space (note that the size of the state space equals the product of all the state variable grids' sizes) and use them as candidate policy functions.

4. For each point i of the state space, plug the candidate policy functions into the equilibrium equations and calculate the value of the endogenous state variables at $t + 1$.

5. Using the value of the endogenous state variables at $t + 1$, use linear interpolation to obtain the value of the policy variables at $t + 1$ for each possible realization of the exogenous state variables.

6. Using the value of the endogenous state variables and the policy variables at $t+1$, obtain the value at $t + 1$ of the remaining variables necessary to calculate time t expectations, for each possible realization of the aggregate shocks.

7. Use a numerical root-finder to solve for the zeros of the residual equations, subject to each of the remaining equilibrium conditions. Numerical integration is needed at this step to compute expectations in the equilibrium equations. The result is a set of policy values in each point i of the state space that satisfies the equilibrium system of equations up to a specified tolerance level, which characterizes the updated policy function for the next step.

8. If the distance between the candidate policy function and the updated policy values obtained in the previous step is less than the convergence criterion for all i, then the policies have converged to their equilibrium values. Otherwise, use the updated policy functions as the new candidate and go back to step 5.

D Accuracy of the numerical solution

It is possible to assess the accuracy of the numerical solution by computing the residual errors of the equilibrium equations after simulating the model for a given sequence of the aggregate shocks using the approximated policy functions obtained by the numerical procedure described above, as proposed by Judd (1992). To this end, the model is simulated for 200,000 periods. Following standard practice, the decimal log of the absolute value of these residual errors is reported here. Figure B.1 reports the density (histogram) of these errors.

Figure D.1: Equilibrium equations' residual errors