Some Simple Bitcoin Economics

Linda M. Schilling* and Harald Uhlig**

*Ecole Polytechnique CREST and **University of Chicago

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Motivation

Coexistence of fiat moneys:

Dollar	*versus*	Bitcoin (any cryptocurrency)
CB controlled supply	*versus*	uncontrolled production

Question

- How do Bitcoin prices evolve (Speculation)?
- How do Bitcoin prices affect Monetary Policy and vice versa?

Literature

Bitcoin Pricing

- Athey et al
- Garratt and Wallace (2017)
- Huberman, Leshno, Moallemi (2017)

Currency Competition

- Kareken and Wallace (1981)

(Monetary) Theory

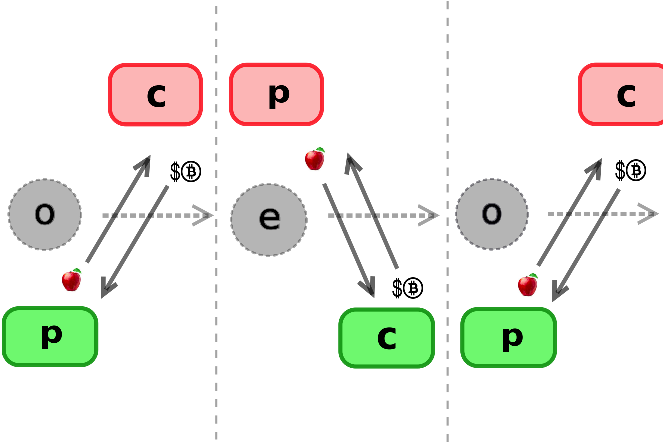
- Bewley (1977)
- Townsend (1980)
- Kyotaki and Wright (1989)
- Lagos and Wright (2005)

The Model I - Economy

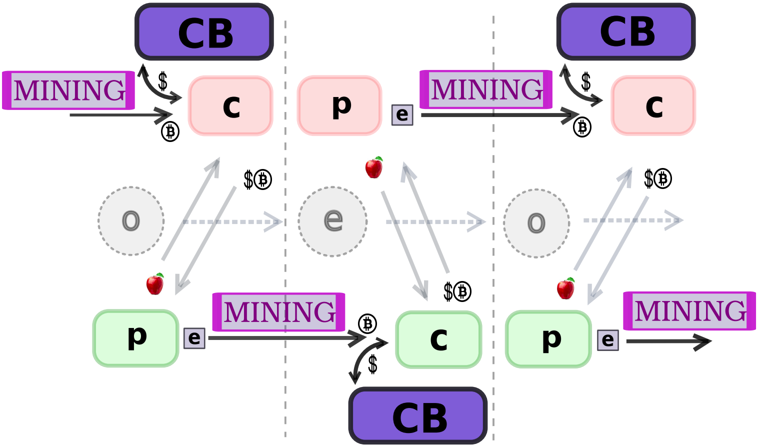
- Discrete time
- Randomness θ_t per period
- 2 types of agents:
  - red $j \in [0, 1)$, green $j \in [1, 2]$: each mass 1
  - utility from consuming: $u(\cdot)$ strictly increasing, concave
  - no money in utility function (money intrinsically worthless)
- 3 goods:
  - CONSUMPTION GOOD
    perishable/not storable
    random production $y_t \in [\underline{y}, \overline{y}]$, $\underline{y} > 0$
  - 2 FIAT MONEYS (Bitcoin, Dollar)
    storable
    equally adopted as means of payment

$$U^g = \sum_{t=0}^{\infty} \beta^t \left(\xi_t \, u(c_t) - e_t\right), \; \xi_t = \begin{cases} 0, & \text{t odd} \\ 1, & \text{t even} \end{cases}$$

Timing - Alternation



Timing - Transfers



The Model II - Moneys

- $P_t(\theta_t)$ price of consumption good in Dollar
- $Q_t(\theta_t)$ price of Bitcoin in terms of consumption good

Dollars D_t:

- CB

$$D_t = D_{t-1} + \tau_t, \quad \tau_t : \mathbf{!} P_t \equiv 1\mathbf{!}$$

Bitcoins B_t:

-

$$B_{t+1} = B_t + f(e_t), \qquad A_{t+1} = f(e_{t+1}, B_{t+1}) \geq 0$$

Equilibrium

An equilibrium is a stochastic sequence
$(A_t, [B_t, B_{t,g}, B_{t,r}], [D_t, D_{t,g}, D_{t,r}], \tau_t, (P_t, z_t, d_t), (Q_t, x_t, b_t), e_t)_{t\geq 0}$

- Utility is maximized to green and red agents.
- Prices clear market for consumption good, Dollars and Bitcoin
  - $y_t = \int_0^2 c_{t,j}\, dj$
  - $\int_0^2 z_{t,j}\, dj = \int_0^2 d_{t,j}\, dj$ $\qquad y_t = x_{t,j} + z_{t,j}$
  - $\int_0^2 x_{t,j}\, dj = \int_0^2 b_{t,j}\, dj$ $\qquad c_{t,j} = b_{t,j} + d_{t,j}$
  - $D_t = D_{t,g} + D_{t,r}$
  - $B_t = B_{t,g} + B_{t,r}$
- Central Bank control $P_t = 1$
- Budget constraints $\qquad$ Evolution money stock
  - $0 \leq b_{t,j} \leq B_{t,j} Q_t$ $\qquad B_{t+1,j} = B_{t,j} - b_{t,j}/Q_t \geq 0$
  - $0 \leq P_t d_{t,j} \leq D_{t,j}$ $\qquad D_{t+1,j} = D_{t,j} - P_t d_{t,j} \geq 0$
  - $B_{t+1,j} = B_{t,j} + x_{t,j}/Q_t + A_{t,j}(e_{t,j})$
  - $B_{t+1,j} = B_{t,j} + x_{t,j}/Q_t + A_{t,j}(e_{t,j})$

Results

Proposition (Fundamental Condition)

Assume agents use both Dollars **and** *Bitcoins to buy goods at t and $t+1$, (i.e. $x_t, x_{t+1}, z_t, z_{t+1} > 0$). Then*

$$\mathbb{E}_t[u'(c_{t+1})] \cdot Q_t = \mathbb{E}_t[u'(c_{t+1})\, Q_{t+1}]$$

If production (consumption) is constant, $Q_t = \mathbb{E}_t[Q_{t+1}]$

Proposition (Speculative Condition)

Assume that Bitcoin and Dollar prices are positive. Assume agents do not spend all Bitcoins $b_t < B_t Q_t$. Then it has to hold

$$u'(c_t) \leq \beta^2 \mathbb{E}_t \left[u'(c_{t+2}) \frac{Q_{t+2}}{Q_t} \right]$$

where this inequality holds with equality if $x_t > 0$ and $x_{t+2} > 0$.

Results

Assumption 1: For $y \in [\underline{y}, \overline{y}]$

$$u'(\overline{y}) > \beta^2 \, \mathbb{E}_t[u'(\underline{y})]$$

Proposition

Under Ass. 1, agents spend all Dollars in each period.

Assumption 2:

$$u'(\overline{y}) > \beta \, \mathbb{E}_t[u'(\underline{y})]$$

Theorem (No Bitcoin Speculation)

Given Ass 2 holds, assume Dollar and Bitcoin prices are positive. Then all Bitcoins are spent in each period.

Given Ass 2, **(Version of Kareken-Wallace)**

$$\mathbb{E}_t[u'(c_{t+1})] \cdot 1 = \mathbb{E}_t[u'(c_{t+1}) \frac{Q_{t+1}}{Q_t}], \qquad \text{for all } t$$

We know

$$cov(X, Y) = \mathbb{E}[X \cdot Y] - \mathbb{E}[X] \cdot \mathbb{E}[Y]$$

Rewrite

$$Q_t = \underbrace{\frac{\sigma_{u'(c)|t}\sigma_{Q_{t+1}|t}}{\mathbb{E}_t[u'(c_{t+1})]}}_{=\kappa_t>0} \underbrace{\frac{cov_t[u'(c_{t+1})\, Q_{t+1}]}{\sigma_{u'(c)|t}\sigma_{Q_{t+1}|t}}}_{=corr_t(u'(c_{t+1}),Q_{t+1})} + \mathbb{E}_t[Q_{t+1}]$$

Application: Bitcoin price evolution II

$$Q_t = \mathbb{E}_t[Q_{t+1}] \; + \; \kappa_t \; \cdot \; corr_t(u'(c_{t+1}), Q_{t+1}),$$

Corollary

Under assumption (2), the Bitcoin price process is a

(i) **martingale** *($Q_t = \mathbb{E}_t[Q_{t+1}],$ for all t)*

$$\Leftrightarrow corr_t(u'(c_{t+1}), Q_{t+1}) = 0,$$

(ii) **supermartingale** *($Q_t \geq \mathbb{E}_t[Q_{t+1}],$ for all t)*

$$\Leftrightarrow corr_t(u'(c_{t+1}), Q_{t+1}) > 0$$

(iii) **submartingale** *($Q_t \geq \mathbb{E}_t[Q_{t+1}],$ for all t)*

$$\Leftrightarrow corr_t(u'(c_{t+1}), Q_{t+1}) < 0$$

Application: Price Convergence

Corollary (Bitcoin Price Bound)

Under Ass 2, there exists an upper bound for the Bitcoin price.

$$Q_t = \frac{b_t}{B_t} \leq \frac{b_t + d_t}{B_t} = \frac{y_t}{B_t} \leq \frac{\overline{y}}{B_0}$$

Theorem (Bitcoin Price Convergence)

Under assumption (2), assume the Bitcoin price is a sub- or a super martingale (correlation between the price and marginal utility does not switch). Then the Bitcoin price converges almost surely point wise and in L^1.

$$Q_t \to Q_\infty \text{ a.s.} \qquad \text{and } \mathbb{E}[|Q_t - Q_\infty|] \to 0$$

Monetary Policy

Market clearing: $D_t = y_t - Q_t B_t, \qquad$ for all t

Conventional: Bitcoin prices independent of central bank policies

$$D_t = D_t(Q_t)$$

Unconventional: Consider an equilibrium:

- CB maintains $P_t = 1$ independently of D_t and
- D_t set independently of production

$\Rightarrow$ CB impacts Bitcoin price

$$Q_t = \frac{y_t - D_t}{B_t}$$

- Implication: If D_t independent of production: $\mathbb{E}[Q_{t+1}] \geq Q_t$
- y_t iid: $\mathbb{P}(Q_{t+1} < s) = \mathbb{P}(y_t < B_t s + D_t)$

Conclusion

We analyze a model of currency competition in which we derive sufficient conditions such that in equilibrium

- there is no Bitcoin speculation
- evolution of Bitcoin price process is determined by its correlation with marginal utility
- we can characterize central bank policy as function of Bitcoin price evolution or vice versa

Back Up - Monetary Policy - Unconventional

Proposition (Dollar Stock evolution)

If the Dollar quantity is set independently of production, the Bitcoin price process is a submartingale, $\mathbb{E}_t[Q_{t+1}] \geq Q_t$.

Proposition (Bitcoin Price Distribution)

As the Bitcoin or Dollar quantity rises, high Bitcoin price realizations become less likely (FOSD).

Proposition

As productivity increases (in terms of FOSD), the Bitcoin price is higher in expectation.

Back-Up - Inflation

Given Ass 2, **(Version of Kareken-Wallace)**

$$\mathbb{E}_t\left[u'(c_{t+1}) \cdot \underbrace{\frac{P_t}{P_{t+1}}}_{\frac{1}{\pi_{t+1}}}\right] = \mathbb{E}_t\left[u'(c_{t+1})\,\frac{Q_{t+1}}{Q_t}\right], \qquad \text{for all } t$$

With inflation $\pi_{t+1} > 1$

Back-Up - Inflation

$\pi_{t+1} > 1$ deterministic

$$Q_t = \pi_{t+1}\,\mathbb{E}_t[Q_{t+1}] \;+\; \pi_{t+1}\,\kappa_t \cdot corr_t(u'(c_{t+1}), Q_{t+1}), \qquad (1)$$

If inflation high, Q_t can be supermartingale despite negative correlation between marginal consumption and Bitcoin price.