“Stationary Bubbles in Nonlinear Business Cycle Models” by Robert Kollman

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Introduction

- This paper considers the occurrence of stationary bubbles in nonlinear DSGE models and suggests that such bubbles may be able to explain standard business cycle facts.
- The insights are obtained from a simple nonlinear model $E_t G(Y_{t+1}, Y_t) = 0$, where E_t denotes expectation conditional on time t information and $G : \Re^2 \rightarrow \Re$ is a function.
- Equivalently, $G(Y_{t+1}, Y_t) = \epsilon_{t+1}$, where $E_t \epsilon_{t+1} = 0$.
- To obtain the results of the paper, re-write this equation as $Y_{t+1} = \Lambda(Y_t, \epsilon_{t+1})$, where in the absence of exogenous forcing variables ϵ_{t+1} reflects unanticipated changes about Y_{t+1} that are driven by expectations about the future path $\{Y_{t+s}\}_{s>1}$.
- Here ϵ_{t+1} is referred to as the *bubble*. We note that $\{Y_{t+1}\}$ is likely to diverge for $|\Lambda_Y| > 1$ for any white noise process $\{\epsilon_{t+1}\}$.

An Example

- Consider the Cagan model where demand for real balances depend negatively on the expected inflation rate as $m_t^d - p_t = \alpha(E_t p_{t+1} - p_t)$, $\alpha < 0$. Equating demand for real balances to their supply yields

$$p_t = \lambda E_t p_{t+1} + \delta m_t, \quad \lambda = \frac{-\alpha}{1-\alpha} \text{ and } \delta = \frac{1}{\alpha}. \tag{1}$$

- We can write this as $p_t = \lambda(p_{t+1} + \epsilon_{t+1}) + \delta m_t$, where $E_t \epsilon_{t+1} = 0$ or

$$p_{t+1} = \lambda^{-1} p_t - \frac{\delta}{\lambda} m_t + \epsilon^*_{t+1}. \tag{2}$$

- Clearly, for any white noise process $\{\epsilon^*_{t+1}\}$ and any stationary $\{m_t\}$, $\{p_{t+1}\}$ is likely to diverge.

Stationary Bubbles

- The contribution of the paper is to construct $\{\epsilon_{t+1}\}$ such that the process generated by $Y_{t+1} = \Lambda(Y_t, \epsilon_{t+1})$ is stationary.
- A key requirement is that the function $\Lambda(Y_t, \epsilon_{t+1})$ is nonlinear in ϵ_{t+1} (so that $\Lambda_{\epsilon\epsilon} \neq 0$) and the distribution of ϵ_{t+1} depends on Y_t.
- Using a second-orde approximation, they show that $E_t Y_{t+1} = \Lambda(Y_t, 0) + \frac{1}{2}\Lambda_{\epsilon\epsilon}(Y_t, 0)E(\epsilon_{t+1}^2)$. If $E(\epsilon_{t+1}^2) = f(Y_t)$,

$$E_t Y_{t+1} = M(Y_t) = \Lambda(Y_t, 0) + \frac{1}{2}\Lambda_{\epsilon\epsilon}(Y_t, 0)f(Y_t),$$

which will exhibit mean-reversion if $|M'| < 1$ even if $|\Lambda_Y| > 1$.

Key Assumption

- The paper focuses only on the optimality conditions governing intertemporal choices and ignores the transversality condition (TVC).
- The rationale for doing so is because either the TVC is not relevant (OLG models) or because agents may not have the cognitive/computing power to detect deviations from it in a non-deterministic environment.
- The TVC requires that the value of the capital stock is zero at infinity and prevents the over-accumulation of capital, which is the condition for dynamic efficiency of the underlying economy.

The Long-Plosser Model

- This is the canonical RBC model with log utility, a Cobb-Douglas production function and 100% depreciation.
- The solution to the linearized version of this model yields $Z_t = K_t / Y_{t-1} = \alpha\beta$.
- The paper provides two cases of bubble equilibria in which the variable ϵ_{t+} can take on two values that depend on the value of Z_t. In particular, these values are chosen so that the support of the distribution of ϵ_{t+1} shrinks as Z_t approaches $\alpha\beta$.
- In these solutions, $Z^L_{t+1} = f(Z_t)$ is restricted as $\alpha\beta \leq f(Z_t) \leq \Lambda(Z_t, 0)$ for $Z_t \in [\alpha\beta, 1)$. Two versions of this specification are considered:
    - **Abrupt crashes**: $Z^L_{t+1} = \alpha\beta + \Delta, \Delta \in (0, 1 - \alpha\beta)$.
    - **Gradual contractions**: $Z^L_{t+1} = \alpha\beta + 0.95(Z_t - \alpha\beta)$.

Assessing the Fit of the Model

- The paper provides an "empirical test" of the stationary bubble model by examining BC facts derived under te alternative assumptions.
- The version of the model with gradual contractions provides a better "fit" with more realistic BC facts compared to the version with abrupt crashes.
- Bubble phenomena arise in terms of phases when investment and output rise while consumption falls, only to fall back abruptly or gradually depending on the model specification.
- But how would we "match" the bubble or the implied bubble phenomenon to the data? How would we distinguish the bubble from other sources of shocks that have been considered in the business cycle literature?
- As an example, are there restrictions on the time paths of the endogenous series that would allow us to detect such bubble-like phenomena?

An Early Test

- In early tests of bubble phenomena such as Flood and Garber (1980) examined the solution for the price level in a Cagan-type model that depends on a fundamental component driven by the money and a bubble component that increases geometrically as a function of time.
- They work with a version of the Cagan model under perfect foresight that specifies

$$m_t - p_t = \alpha(p^e_{t+1} - p_t) + A_0. \tag{3}$$

- One solution to this difference equation is the sequence whose elements are defined by

$$p_t = \frac{1}{1-\alpha}\sum_{i=0}^{\infty}\left(\frac{-\alpha}{1-\alpha}\right)^i m_{t+i} + c\left(\frac{\alpha-1}{\alpha}\right)^t, \tag{4}$$

where $(\alpha - 1)/\alpha > 1$ since $\alpha < 0$.

Testing for Bubbles

- To test for the existence of bubbles, they multiply the term

$$c\left(\frac{\alpha-1}{\alpha}\right)^t$$

with a dummy variable which equals 1 for the period of a possible bubble, and 0 otherwise.

- They argue that the null hypothesis that there were no bubbles in these periods cannot be rejected at statistically significant levels.
- In more recent work, Philips and Yu (2011) (*Quantitative Economics*) and Philips, Shi and Yu (2015) (*IER*) provide methods for detecting and dating mildly explosive bubbles using what they term as a forward recursive regression approach.

Detecting Exploding and Collapsing Bubbles

- The interest in the Philips *et al* work is that it seeks to identify episodes when different series transit from behavior characterized by unit roots to one with mildly explosive data, which then collapse back to unit root behavior.
- Philips *et al* argue that episodes with mildly exploding data can help to identify bubble-type phenomena because the trajectories implied by unit root and mildly explosive processes differ in important ways.
- Quoting from Philips and Yu (2011): "Although a unit root process can generate successive upward movements, these movements still have a random wandering quality unlike stochastically explosive process ... ''
- Furthermore, the collapse of such bubbles also give the appearance of a form of mean reversion, suggesting that these types of processess may be useful for modeling the bubble phenomena considered in the paper.

Detecting Exploding and Collapsing Bubbles II

- The tests in questions are recursive right-sided unit tests (as opposed to the standard left-sided unit root tests against stationary alternatives) that seem effective in the detection of mildly explosive behavior and market exuberance. There are versions of the tests that allow for collapsing bubbles as well.
- Consider the empirical model

$$\Delta y_{t-1} = \alpha_{r_0,r_2} + \beta_{r_0,r_2} y_{t-1} + \sum_{i=1}^{k} \psi_{r_0,r_2} \Delta y_{t-i} + \epsilon_t .$$

- The so-called SADF test is based on the repeated estimation of the ADF model on a forward expanding sample sequence, and the test statistic is obtained as the supremum of the corresponding ADF statistic sequence as $SADF(r_0) = \sup_{r_2 \in [r_0,1]} ADF_0^{r_2}$.
- Could such tests be used to detect mildly explosive behavior that then reverts back to yield a stationary sequence, as the bubble solution does in the paper under discussion?