Ambiguous Business Cycles: A Quantitative Assessment

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Introduction

- What role does ambiguity aversion play in generating cyclical fluctuations and asset price movements?
- In this paper, we adopt the smooth ambiguity preferences of Klibanoff, Marinacci, and Mukerji (KMM) (2005, 2009) in a production economy framework with irreversible investment.
- Following Collard *et al.* (2016), agents are unsure about the distribution of the latent variable and cannot distinguish a process that has moderate persistence but high volatility, and one which is less volatile but highly persistent.
- Ambiguity aversion endogenously generates "doubt and pessimism." The decisions of ambiguity averse agents may be viewed in terms of the decisions of an expected utility maximizing agent with beliefs that are more uncertain and pessimistic relative to those based on inference from actual data.

Related Literature I

- Ilut and Schneider (2012) and Nimark (2014) examine business cycle phenomena in the context of the maxmin model of Gilboa and Schmeidler (1989).
- The first paper assumes a decision problem in which agents act as if they evaluate plans using a worst case probability drawn from a set of multiple prior beliefs.
- It incorporates ambiguity and changes in confidence into a standard business cycle model by assuming that "agents' set of beliefs, such as an innovation to productivity, is parametrized by an interval of means centered around zero." An increase in the width of the interval is associated with a loss of confidence, especially when the "worst case" mean becomes worse.
- One problem with this approach that ambiguity and ambiguity aversion are both modeled in terms of the size of the set of possible beliefs.

Related Literature II

- Collard *et al.* (2016) study the historical equity premium using the smooth ambiguity model in an endowment economy framework. After calibrating the ambiguity aversion parameter to match the risk-free rate, they are able to match the fist and second conditional moments of observed return dynamics.
- Jahan-Parvar and Liu (2012) and Liu and Zhang (2014) consider models with generalized recursive smooth ambiguity first proposed by Ju and Miao (2012) augmented with adjustment costs in investment and a Markov switching process for aggregate productivity growth.
- Bidder and Smith (2012) consider a version of the multiplier preferences that are robust to misspecification following Hansen and Sargent (2008) and assume that innovations to technology growth are subject to time-varying volatility.

Uncertainty

Uncertainty in this economy is assumed to driven by the stochastic behavior of productivity growth, $g_{A,t}$.
At time t, the process for the growth rate of the technology shock is given by

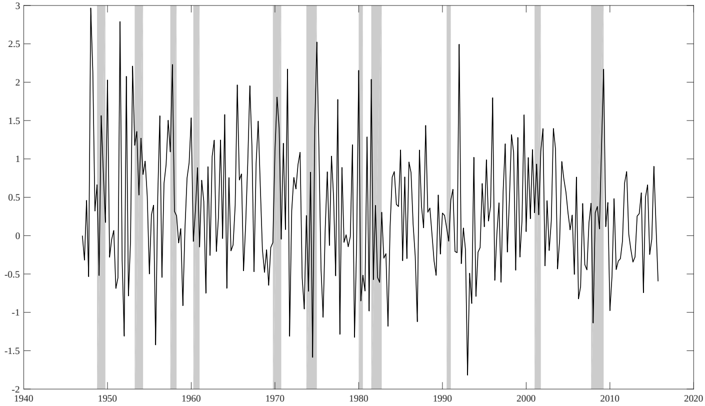
$$g_{A_k,t+1} = \bar{g} + x_{k,t+1} + \sigma_{A_k} \epsilon_{A_k,t+1}, \quad (1)$$

$$x_{k,t+1} = \rho_k x_{k,t} + \sigma_{x_k} \epsilon_{x_k,t+1}, \quad (2)$$

where $(\epsilon_{A_k,t+1}, \epsilon_{x_k,t+1})' \sim N(0, I)$ for $k = h, l$. At time t, the agent has available observations on the current and past values of the growth rate of technology, $g_{A,t}$. However, the agent does not know the process generating $x_{k,t}$ and forms beliefs about it, given prior beliefs at time 0 and the observations on $g_{A,t}, g_{A,t-1}, \ldots$.

Factor utilization adjusted TFP process

Figure: The growth rate of factor utilization adjusted TFP over the period 1947-2 until 2015-4



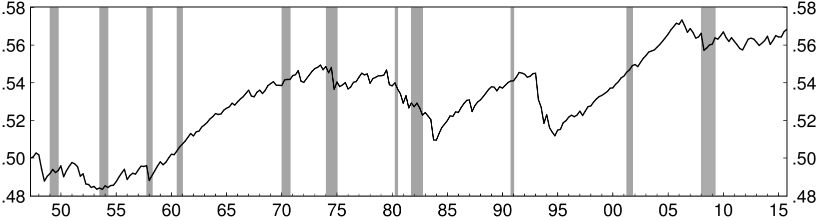
Bayesian estimation of the underlying TFP process

Table: Posterior results for the model using TFP-util for different values of ρ using the sample of 1947-2 : 1977-4

	0.25	0.30	0.65	0.70	0.85	0.90
$\bar{g}$	0.469 (0.086)	0.469 (0.086)	0.469 (0.086)	0.469 (0.086)	0.469 (0.086)	0.469 (0.086
σ_A	0.945 (0.075)	0.946 (0.071)	0.949 (0.063)	0.950 (0.063)	0.952 (0.062)	0.953 (0.062
σ_x	0.046 (0.120)	0.044 (0.110)	0.056 (0.080)	0.054 (0.073)	0.040 (0.049)	0.033 (0.040
ρ	0.250	0.300	0.650	0.700	0.850	0.900
Max.	-167.40	-167.40	-167.40	-167.40	-167.40	-167.40
Mar.	-169.70	-169.73	-170.16	170.20	-170.32	-170.38

- The inference was carried out with 60,000 draws where the first 10,000 are used as burn-in sample. We kept every $5^t h$ draw, which yields a sample of 10,000 draws from the ergodic distribution.
- The results in Table 1 confirms our findings on the (in)-ability to identify the exact value of the persistence parameter. Regardless of the values of the ρ ranging from 0.20 to 0.90, the value of maximum likelihood only changes after the second digit and the change in the marginal likelihoods are very minor.

Figure: Evolution of the probability of the model with $\rho = 0.30$ being the true DGP over the sample 1947:2:2015-4



KMM Preferences

The state at date t is denoted $s^t = (s_0, s_1, \cdots, s_t)$, where $s_t \in \Upsilon_t$. The agent is uncertain about the stochastic process governing the probabilities on the event tree. This uncertainty is indexed by the parameter $\theta \in \Theta$, which denotes the set of unobservable parameters.

$$V_{s^t}(f) = u(f(s^t)) + \beta\phi^{-1}\left[\int_{\Theta}\phi\left(\int_{\Upsilon_{t+1}} V_{(s^t,s_{t+1})}(f)d\pi_\theta(s_{t+1}|s^t)\right)d\mu(\theta|s^t)\right],$$

where $V_{s^t}(f)$ is a recursively defined direct value function, $u(\cdot)$ characterizes attitudes towards risk, β is a discount factor, $\phi(\cdot)$ is a function characterizing the agent's ambiguity attitude, and $\mu(\cdot|s^t)$ denotes the Bayesian posterior.

The Social Planner's Problem

The transformed indirect value function for the social planner's problem for the power-power specification is given by

$$\hat{J}(\hat{k}_t, \hat{\mu}_t) = \max_{\hat{c}_t, l_t, \hat{i}_t} \left\{ \frac{(\hat{c}_t^{\nu} l_t^{1-\nu})^{1-\gamma} - 1}{1-\gamma} + \beta \left[E_{\hat{\mu}_t} \left(E_{x_t}(\hat{J}(\hat{k}_{t+1}, \hat{\mu}_{t+1}) \exp(\nu(1-\gamma)) g_{A_k,t+1})) \right)^{1-\alpha} \right]^{\frac{1}{1-\alpha}} \right\}$$

subject to $\hat{c}_t + \hat{i}_t \leq \hat{k}_t^a$, $\exp(g_{A,t+1})\hat{k}_{t+1} = (1-\delta)\hat{k}_t + \hat{i}_t$, $l_t + h_t \leq 0$, $\hat{i}_t \geq 0$, and given the law of motion for beliefs to be discussed below.

The Optimality Conditions

Define the quantities

$$\Upsilon_t = \frac{E_{\hat{\mu}_t}\left(E_{x_t}\hat{J}(\hat{k}_{t+1},\hat{\mu}_{t+1})\exp(\nu(1-\gamma)g_{A,t+1})\right)^{-\alpha}}{\left[E_{\hat{\mu}_t}\left(E_{x_t}\hat{J}(\hat{k}_{t+1},\hat{\mu}_{t+1})\exp(\nu(1-\gamma)g_{A,t+1})\right)^{1-\alpha}\right]^{\frac{-\alpha}{1-\alpha}}} \quad (3)$$

$$\xi_t = \frac{\left(E_{x_t}\hat{J}(\hat{k}_{t+1},\hat{\mu}_{t+1})\exp(\nu(1-\gamma)g_{A,t+1}))\right)^{-\alpha}}{\left[E_{\hat{\mu}_t}\left(E_{x_t}\hat{J}(\hat{k}_{t+1},\hat{\mu}_{t+1})\exp(\nu(1-\gamma)g_{A,t+1})\right)^{-\alpha}\right]}. \quad (4)$$

Let λ_t denote the Lagrange multiplier on the aggregate resource constraint and φ_t the multiplier on the non-negativity constraint.

The Optimality Conditions (cont'd)

Intratemporal substitution condition between consumption and leisure:

$$\frac{1-\nu}{\nu}\frac{\hat{c}_t}{l_t} = (1-a)\hat{k}_t^a n_t^{-a}. \tag{5}$$

Given a solution for $\hat{k}_{t+1}$ as a function of $(\hat{k}_t, \hat{\mu}_t)$, this condition can be solved for current l_t for each $\hat{k}_t$ and beliefs $\hat{\mu}_t$.
The envelope condition:

$$\hat{J}_1(\hat{k}_{t+1}, \hat{\mu}_{t+1}) = \lambda_{t+1}\left\{a\hat{k}_{t+1}^{a-1}n_{t+1}^{1-a} + (1-\delta)\min\left(1, \Upsilon^0_{t+1}E_{\hat{\mu}_{t+1}}\left[\xi^0_{t+1}E_{x_{t+1}}\left(\frac{\hat{J}_1((1-\delta)\hat{k}_{t+1}, \hat{\mu}_{t+2})\exp((\nu(1-\gamma)-1)g_{A,t+2})}{\lambda_{t+1}}\right)\right]\right)\right\}.$$

When there is an irreversibility constraint, the marginal value of capital accounts for the fact that the irreversibility constraint may be binding next period. It is this aspect that leads to an endogenous risk premium or an option value to wait in the model with an irreversibility constraint.

The Optimality Conditions (cont'd)

Using these results, the optimal choice of investment at time t can now be written as

$$1 = \Upsilon_t^1 E_{\hat{\mu}_t} \Bigg\{ \xi_t^1 E_{x_t} \Bigg[\exp(((1-\gamma)\nu - 1) g_{A,t+1}) \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \Big(a\hat{k}_{t+1}^{a-1} n_t^{1-a} + (1-\delta)\min\Big(1, \Upsilon_{t+1}^0 E_{\hat{\mu}_{t+1}} \Big[\xi_{t+1}^0 E_{x_{t+1}} \left(\frac{\hat{J}_1((1-\delta)\hat{k}_{t+1}, \hat{\mu}_{t+2}) \exp((\nu(1-\gamma)-1) g_{A,t+2})}{\lambda_{t+1}} \right) \Big] \Big) \Big) \Bigg] \Bigg\} \quad \text{if } \hat{i}_t >$$

$$1 > \Upsilon_t^0 E_{\hat{\mu}_t} \Bigg\{ \xi_t^1 E_{x_t} \Bigg[\exp(-\gamma g_{A,t+1}) \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \Big(a\hat{k}_{t+1}^{a-1} n_{t+1}^{1-a} + (1-\delta)\min\Big(1, \Upsilon_{t+1}^0 E_{\hat{\mu}_{t+1}} \Big[\xi_{t+1}^0 E_{x_{t+1}} \left(\frac{\hat{J}_1((1-\delta)\hat{k}_{t+1}, \hat{\mu}_{t+2}) \exp(((1-\gamma)\nu - 1) g_{A,t+2})}{\lambda_{t+1}} \right) \Big] \Big) \Big) \Bigg] \Bigg\} \quad \text{if } \hat{i}_t =$$

- If the irreversibility constraint does not bind, then the marginal cost of acquiring extra capital today just equals the expected marginal value of capital tomorrow, where the expectation is taken with respect to the second order beliefs μ. However, the expectation of a future irreversibility constraint nevertheless affects the interior choice of investment today.
- If the irreversibility condition is binding today so that the choice of investment is a corner solution, then the first-order condition holds with inequality such that the marginal cost of investment today exceeds the marginal value of investment tomorrow evaluated at zero investment, $i_t = 0$.

Beliefs

- *First-order uncertainty:* Given $x_{k,t}, \rho_k$ and observations on $(\hat{c}_t, \hat{i}_t, \hat{k}_t, \hat{y}_t)$, the probability distribution over $g_{A_k,t+1} \sim N(\bar{g} + \rho_k x_{k,t}, \sigma^2_{A_k} + \sigma^2_{x_k})$ denotes the typical first-order distribution $\pi_\theta(s_{t+1}|s^t)$ in the KMM formulation.
- *Second-order uncertainty:* Let $\hat{x}_{k,t} \equiv E[x_{k,t}|g_{A,1}, \ldots, g_{A,t}]$ denote the expectation of $x_{k,t}$, conditional on the history of growth rates up to t if the beliefs were updated assuming $\rho = \rho_k$ is the true data generating process. The agent's posterior beliefs are given by $\eta_t \times N(\hat{x}_{l,t}, \Omega_l)$ and $(1 - \eta_t) \times N(\hat{x}_{h,t}, \Omega_h)$, respectively, where Ω_k, $k = l, h$ denotes the steady state variance associated with the Kalman filter based on the process with $\rho = \rho_k$ and η_t shows the posterior belief on ρ_l.
- The agent's beliefs are summarized by the tuple $(\hat{x}_{l,t}, \hat{x}_{h,t}, \eta_t)$.

Endogenous Pessimism

- A distinguishing feature of the model with ambiguity aversion is that it generates endogenous uncertainty, as agents react to the exogenous sources of uncertainty in the economy.
- Recall that the optimality conditions depend on the distortion factors Υ_t and ξ_t, where these quantities arise due to the presence of ambiguity aversion.
- In this expression, ξ_t depends expectations that are taken with respect to the distribution of x_t, conditional on information of the history the technology shock $(g_{A,t}, g_{A,t_1}, \cdots)$ and, hence, is random from the view of the agent's subjective beliefs at date t.
- The function ξ_t may be viewed as the factor that create the endogenous tilting or distortion in agents' beliefs due to ambiguity aversion.

Numerical Solution Method

We use the method of value iteration with Chebyshev interpolation, which involves approximating the function $\hat{J}(\hat{k}_t, \hat{x}_{l,t}, \hat{x}_{h,t}, \eta_t)$ by a parametric function whose coefficients are determined according to a minimum residual method.

$$\hat{J}(\hat{k}_t, \hat{x}_{l,t}, \hat{x}_{h,t}, \eta_t) = \max_{\hat{c}_t, \hat{i}_t} \left\{ \frac{\hat{c}_t^{\,1-\gamma} - 1}{1-\gamma} + \right.$$

$$\beta \left[\eta_t \left(\int_{-\infty}^{\infty} \left(\int\int_{-\infty}^{\infty} \hat{J}(\hat{k}_{t+1}^{(l)}, \hat{x}_{h,t+1}^{(l)}, \hat{x}_{l,t+1}^{(l)}, \eta_{t+1}^{(l)}) \exp(g_{A_l,t+1})^{\nu(1-\gamma)} dF(\varepsilon_{l,t+1}) \right)^{1-\alpha} dF(x_{l,t}) \right) + \right.$$

$$\left. \left. (1-\eta_t) \left(\int_{-\infty}^{\infty} \left(\int\int_{-\infty}^{\infty} \hat{J}(\hat{k}_{t+1}^{(h)}, \hat{x}_{h,t+1}^{(h)}, \hat{x}_{l,t+1}^{(h)}, \eta_{t+1}^{(h)}) \exp(g_{A_h,t+1})^{\nu(1-\gamma)} dF(\varepsilon_{h,t+1}) \right)^{1-\alpha} dF(x_{h,t}) \right) \right] \right\}$$

subject to

$$\hat{c}_t + \hat{i}_t \leq \hat{k}_t^a,$$
$$\exp(g_{A,t+1})\hat{k}_{t+1} = (1-\delta)\hat{k}_t + \hat{i}_t,$$
$$\hat{i}_t \geq 0.$$

Here $\varepsilon_{k,t+1} = (\epsilon_{x_k,t+1}, \epsilon_{A_k,t+1})'$, $k = l, h$ is a 2 by 1 vector standard normal shocks and $\eta_{t+1}^{(l)}$ is the posterior probability at time $t+1$ that the model with ρ_l is the true data generating process.

Stylized Facts: Full Sample

Table: Unconditional business cycle moments

Full Sample - 1947:1-2015:4					
Standard deviations					
	y	c	i	h	p
	1.6244	1.2621	7.4416	1.9249	0.9341
Correlations					
	y	c	i	h	p
y	1.0000	0.7742	0.8406	0.8863	-0.0640
c		1.0000	0.6943	0.6581	-0.0677
i			1.0000	0.7615	-0.0999
h				1.0000	-0.5393
p					1.0000

Stylized Facts: Restricted Sample

Table: Unconditional business cycle moments

Restricted Sample - 1980:1-2015:4					
Standard deviations					
	y	c	i	h	p
	1.2938	1.0755	6.2839	1.7830	0.9296
Correlations					
	y	c	i	h	p
y	1.0000	0.8662	0.9127	0.8645	-0.2664
c		1.0000	0.7299	0.7678	-0.2670
i			1.0000	0.8167	-0.2962
h				1.0000	-0.7147
p					1.0000

Simulations: Parameter values

Table: Parameter Values

β	Subjective discount factor	0.988
γ	Coefficient of risk aversion	0.5, 2
α	Coefficient of ambiguity aversion	0.8, 2, 5
a	Capital share	0.3
δ	Depreciation rate	0.025

Simulation Results: Ambiguity about the Persistence/Variability of the Unobserved Component of TFP Growth

	$\gamma = 0.5, \alpha = 0.8$					$\gamma = 0.5, \alpha = 5$				
Simulations conditional on $\rho_k = 0.85$										
Standard deviations										
	y	c	i	h	p	y	c	i	h	p
	1.363	0.603	6.403	0.715	0.777	1.354	0.598	6.370	0.711	0.772
Correlations										
	y	c	i	h	p	y	c	i	h	p
y	1.000	0.662	0.939	0.905	0.921	1.000	0.662	0.939	0.906	0.921
c		1.000	0.368	0.282	0.902		1.000	0.368	0.282	0.902
i			1.000	0.994	0.732			1.000	0.994	0.732
h				1.000	0.668				1.000	0.669
p					1.000					1.000
i/y					0.172					0.173
Simulations conditional on $\rho_k = 0.30$										
Standard deviations										
	y	c	i	h	p	y	c	i	h	p
	1.341	0.588	6.318	0.706	0.761	1.343	0.590	6.327	0.706	0.763
Correlations										
	y	c	i	h	p	y	c	i	h	p
y	1.000	0.660	0.940	0.907	0.921	1.000	0.661	0.940	0.907	0.921
c		1.000	0.368	0.284	0.900		1.000	0.369	0.284	0.901
i			1.000	0.994	0.735			1.000	0.994	0.734
h				1.000	0.672				1.000	0.672
p					1.000					1.000
i/y					0.172					0.174

Simulation Results: Ambiguity about the Persistence/Variability of the Unobserved Component of TFP Growth - cont'd

$\gamma = 0.8, \alpha = 5$					
Simulations conditional on $\rho_k = 0.85$					
Standard deviations					
	y	c	i	h	p
	1.242	0.602	5.127	0.532	0.781
Correlations					
	y	c	i	h	p
y	1.000	0.858	0.952	0.920	0.964
c		1.000	0.661	0.587	0.964
i			1.000	0.994	0.837
h				1.000	0.781
p					1.000
i/y	0.166				
Simulations conditional on $\rho_k = 0.30$					
Standard deviations					
	y	c	i	h	p
	1.221	0.589	5.057	0.524	0.766
Correlations					
	y	c	i	h	p
y	1.000	0.858	0.953	0.921	0.964
c		1.000	0.664	0.590	0.964
i			1.000	0.994	0.839
h				1.000	0.785
p					1.000
i/y	0.166				

The Role of EIS versus Ambiguity Aversion

How do we interpret the results in Table 4?

- First, an increase in ambiguity aversion described by increases in α tends to have a minor impact on the behavior of quantities because agents can optimally respond to such uncertainty through their choice of investment and hours. See the earlier work of Tallarini (2000) and Backus *et al* (2014).
- This reflects the full insurance/complete markets assumption that underlies the social planner's problem used to generate the business cycle observations.
- A log-linear approximation to the model under the assumption of known persistence ($\eta_t = 0$) for the TFP growth process shows that the dynamics of the capital stock is unaffected by the ambiguity aversion parameter, α, as are the coefficients on the filtered mean, $\hat{x}_t$.
- Second, while increasing the value of γ leads to an increase in risk aversion, it also corresponds to a decline in the EIS. Thus, consumers become *less* willing to substitute consumption across periods, which decline in the volatility of the endogenous series as well as an increase in their co-movement.

Informative Priors

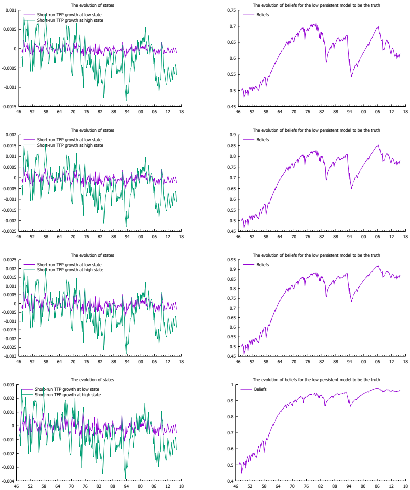
Figure: Evolution of the agent's beliefs for the low persistence model to be the true DGP over the sample 1978-1:2015-4.

PANEL A

PANEL B

PANEL C

PANEL D



Information and Learning

	Panel B					Panel E				
Simulations conditional on $\rho_k = 0.85$										
Standard deviations										
	y	c	i	h	p	y	c	i	h	p
	1.388	0.653	6.057	0.661	0.84	1.58	0.819	5.944	.613	1.042
Correlations										
	y	c	i	h	p	y	c	i	h	p
y	1.000	0.764	0.94	0.904	0.942	1.000	0.903	0.953	0.922	0.974
c		1.000	0.503	0.416	0.936		1.000	0.737	0.666	0.977
i			1.000	0.993	0.773			1.000	0.991	0.863
h				1.000	0.708				1.000	0.81
p					1.000					1.000
i/y					0.170					0.169
Simulations conditional on $\rho_k = 0.30$										
Standard deviations										
	y	c	i	h	p	y	c	i	h	p
	1.327	0.589	6.014	0.668	0.77	1.378	0.627	6.031	0.659	0.821
Correlations										
	y	c	i	h	p	y	c	i	h	p
y	1.000	0.714	0.943	0.91	0.933	1.000	0.768	0.947	0.914	0.945
c		1.000	0.445	0.361	0.918		1.000	0.526	0.444	0.936
i			1.000	0.994	0.762			1.000	0.994	0.791
h				1.000	0.701				1.000	0.73
p					1.000					1.00
i/y					0.172					0.170

Information and Learning: Quantitative Effects

- Bidder and Smith (2012) find that the model incorporating only shocks to time-varying volatility which is the source of mis-specification that leads to endogenous pessimism is able to explain 1%-16% of the variability of such series as output, consumption, investment and hours compared to the full stochastic model that contains both volatility and technology shocks.
- In our case, the volatility of investment decreases by around 5.4% (7.2%) when agents have more information about the nature of the latent process driving productivity as described by Panels B (E) of Table 5 compared to the case with uninformative priors in Table 4 when the high persistence process is the correct model of TFP growth and by 4.8% (4.5%) if the low persistence process is the correct model.
- The effect on output and consumption is to increase the difference in the consumption and output responses across the high and low persistence processes. We also find that the co-movement of all series increase when agents have more informative or less ambiguous signals.

The Risk-free rate

The risk-free rate is given by

$$1 = \beta R^f \Upsilon_t E_{\hat{\mu}_t} \left\{ \xi_t E_{x_t} \left[\exp(((1-\gamma)\nu - 1) g_{A,t+1}) \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \right] \right\}, \quad (6)$$

where $\lambda_t = \nu(\hat{c}_t^{\nu} l_t^{1-\nu})^{-\gamma} (\hat{c}_t / l_t)^{\nu-1}$ and $\hat{\mu}_t$ accounts for the second-order beliefs of agents.

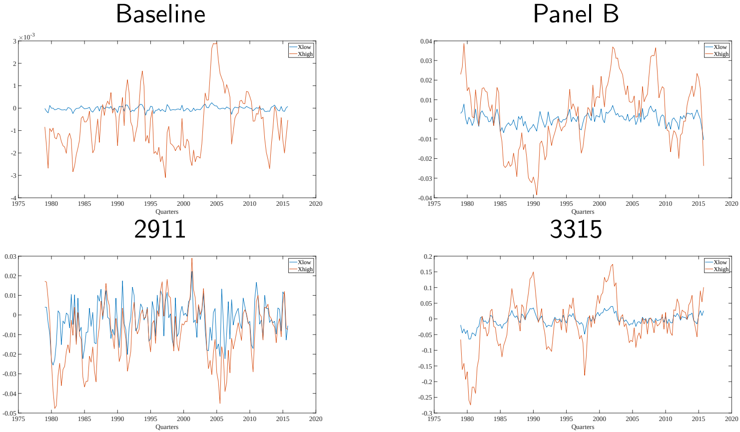
Under the assumption of known persistence ($\eta_t = 0$) for the growth rate of the TFP process and assuming that the distorted or "as if" posterior $\tilde{\mu}_t \equiv \xi_t(x_t) \otimes N(\hat{x}_t, \Omega)$, where the distortion arises from the role of ambiguity aversion, is a normal density with variance Ω but a different mean, $\tilde{x}_t$, we can derive log-linear approximation to the risk-free rate as

$$\begin{aligned} r^f = & -\log(\beta) - \left[\tilde{E}_t \left(\log(\lambda_{t+1}/\lambda_t) \right) + \frac{\widetilde{Var}_t \left(\log(\lambda_{t+1}/\lambda_t) \right)}{2} \right. \\ & + ((1-\gamma)\nu - 1)(\bar{g} + \rho \tilde{x}_t) + \frac{((1-\gamma)\nu - 1)^2}{2} (\sigma_x^2 + \sigma_A^2) \\ & \left. + \frac{((1-\gamma)\nu - 1)^2 \rho^2}{2} \widetilde{Var}_t(x_t) \right] + \text{extra variance terms} \end{aligned}$$

Simulation Results for the Risk-free Rate, Quarterly Rate

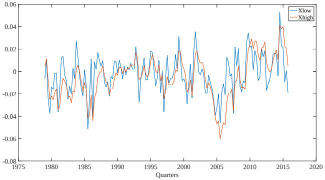
Model	Industry name	Risk-free rate r^f (in %)	$\tilde{X}$ (in %)	$SD(\tilde{X})$ (in %)	Uncertainty measure
Baseline model	–	1.55	-6.00e-05	0.063	–
Panel B	–	1.54	-2.43e-03	0.250	–
hline Industry 2421	Sawmills and planing mill, general	1.30	-2.96e-02	0.40	0.464
Industry 2869	Industrial organic chemicals	1.27	6.22e-03	0.23	0.529
Industry 2911	Petroleum mining	1.07	4.76e-04	0.41	0.650
Industry 3084	Plastic pipes	1.07	1.90e-03	0.31	0.424
Industry 3221	Glass containers	1.17	-6.07e-04	0.23	0.193
Industry 3315	Steel wiredrawing, steel nails and spikes	1.07	-8.00e-03	0.57	0.365
Industry 3448	Fabricated metal buildings and components	1.28	3.82e-03	0.23	0.395
Industry 3519	Internal combustion engines	1.05	4.21e-03	0.39	0.426
Industry 3621	Motors and generators	1.11	-4.47e-03	0.35	0.351
Industry 3731	Ship building and repairing	1.61	2.63e-03	0.28	0.407

Filtered beliefs underlying the risk-free rate values - Aggregate TFP and Industry Results

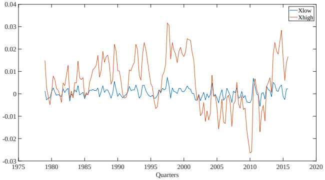


Filtered beliefs underlying the risk-free rate values - Aggregate TFP and Industry Results

3519



3731



Competitive Equilibrium (CE)

Show the equivalence between the solution to the social planner's problem and a competitive equilibrium (CE) in which

- households own shares in firms and hold their corporate debt;
- value-maximizing firms own the capital stock and make real investment decisions, which they finance through retained earning, equity or debt;
- the CE yields an MM Theorem regarding the equivalence of equity and debt finance;

Stochastic Discount Factor (SDF)

The CE also yields a stochastic discount factor (SDF) adjusted for ambiguity aversion which can be used to price all assets in equilibrium as

$$M_{t,t+1} = \zeta_t \exp((1-\gamma)g_{A,t+1}) \left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right)^{-\gamma},$$

where

$$\zeta_t = \frac{\left(E_{x_t}(\hat{v}(z_{t+1}, \hat{b}^d_{t+1}, \hat{\mu}_{t+1}) \exp((1-\gamma)g_{A,t+1}))\right)^{-\alpha}}{\left[E_{\hat{\mu}_t}\left(E_{x_t}(\hat{v}(z_{t+1}, \hat{b}^d_{t+1}, \hat{\mu}_{t+1}) \exp((1-\gamma)g_{A,t+1}))\right)^{1-\alpha}\right]^{\frac{\alpha}{1-\alpha}}}.$$

Recursive Contingent Claims Equilibrium (RCCE)

Also characterize a recursive (complete) contingent claims equilibrium (RCCE) in which

- households make consumption decisions and choose how much wealth to carry over to next period contingent on all possible realizations of the state next period, s_{t+1}, conditional on the history of the shocks up to time t, s^t;
- value-maximizing firms own the capital stock and real investment decisions, which they finance through the issuance of state-contingent securities to households.
- derive the CCE price used to price all securities in equilibrium as

$$p(s_{t+1}, s_t; \theta) = \beta \phi^{-1'} \left[\int_\Theta \phi \left(\int_{\Upsilon_{t+1}} \tilde{V}(a_{t+1}, s_{t+1}) d\pi_\theta(s_{t+1}, s_t) \right) d\mu(\theta | s_t) \right]$$

$$\int_\Theta \phi' \left(\int_{\Upsilon_{t+1}} \tilde{V}(a_{t+1}, s_{t+1}) d\pi_\theta(s_{t+1}, s_t) \right) d\mu(\theta | s_t) \frac{u'(c_{t+1})}{u'(c_t)} \pi_\theta(s_{t+1} | s_t) \mu(\theta | s_t),$$

Conclusions

- Uncertainty and ambiguity aversion are important factors deriving agents' decisions to work, to invest, and to consume.
- Possible extensions suggested by our work include
  - introducing market incompleteness
  - considering individual heterogeneity
  - allowing for models based sectoral measures of TFP shocks