A Structural Investigation of Quantitative Easing

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Note: All presented results are preliminary

Motivation

- **Zero-lower bound (ZLB) on nominal interest rates**
- Conventional monetary policy **ineffective**
- The effects of **unconventional monetary policy** at ZLB?

So far:

- VAR studies: QE affects financial variables
- DSGE studies: QE can affect real variables

Issue: Implied effects of QE depend on parameter choice

▶ **Bayesian estimation necessary**

Problem: ZLB is a strong nonlinearity

▶ **New methodology** (Boehl, 2019)

Motivation

- **Zero-lower bound (ZLB) on nominal interest rates**
- Conventional monetary policy **ineffective**
- The effects of **unconventional monetary policy** at ZLB?

This work:

- Use **estimated** DSGE model to quantify QE:

Smets & Wouters (2007)	+	Gertler & Karadi (2013)	=	
NK + bells + whistles	+	banks + QE		

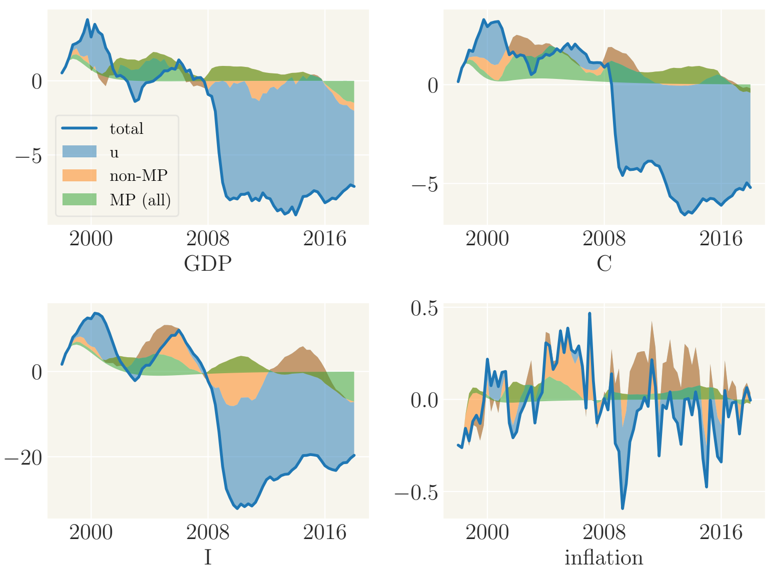
- New methodology: OBCs solution & nonlinear filtering (Boehl, 2019)

▶ Bayesian estimation with endogenous ZLB

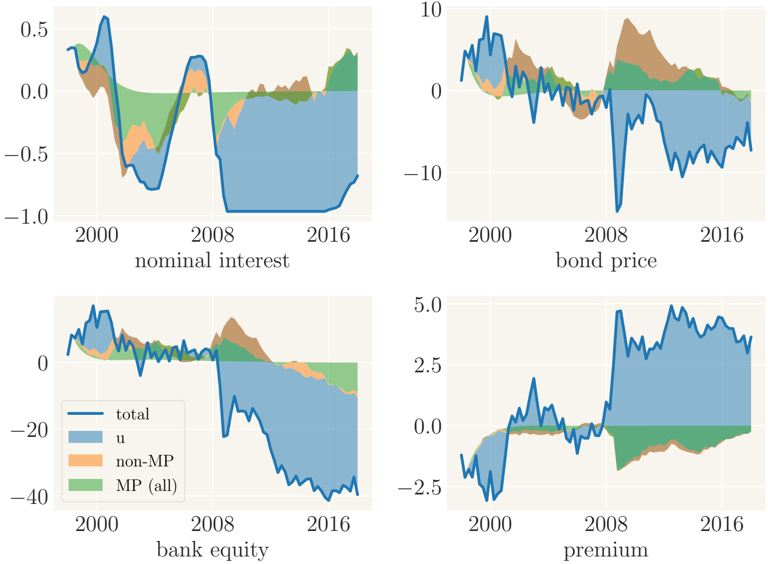
Key parameter estimates (incl. ZLB)

	distribution	mean	sd	mean	sd	hpd_2.5	hpd_97.5
ι_p	beta	0.500	0.15	0.340	0.084	0.174	0.492
ι_w	beta	0.500	0.15	0.445	0.142	0.167	0.733
ζ_p	beta	0.500	0.10	0.805	0.044	0.727	0.887
ζ_w	beta	0.500	0.10	0.680	0.058	0.555	0.783
Φ_p	normal	1.250	0.12	1.305	0.119	1.062	1.516
LEV	normal	3.000	1.00	1.802	0.457	1.131	2.616
θ	beta	0.950	0.05	0.908	0.074	0.762	0.994
λ_{cbl}	uniform	0.000	10.00	2.694	0.842	0.995	4.134
ρ	beta	0.700	0.20	0.784	0.040	0.697	0.849
ρ_u	beta	0.500	0.20	0.766	0.051	0.667	0.862
ρ_r	beta	0.700	0.20	0.488	0.092	0.332	0.683
ρ_g	beta	0.500	0.20	0.838	0.101	0.661	0.983
ρ_i	beta	0.500	0.20	0.816	0.069	0.692	0.944
ρ_z	beta	0.500	0.20	0.583	0.193	0.214	0.888
ρ_p	beta	0.700	0.20	0.260	0.053	0.158	0.363
ρ_w	beta	0.700	0.20	0.455	0.094	0.314	0.660
ρ_{cbl}	beta	0.500	0.20	0.555	0.064	0.439	0.670
ρ_{qe_b}	beta	0.500	0.20	0.863	0.041	0.778	0.945
ρ_{qe_k}	beta	0.500	0.20	0.921	0.033	0.857	0.980
...	...	...	...	...	...	...	...

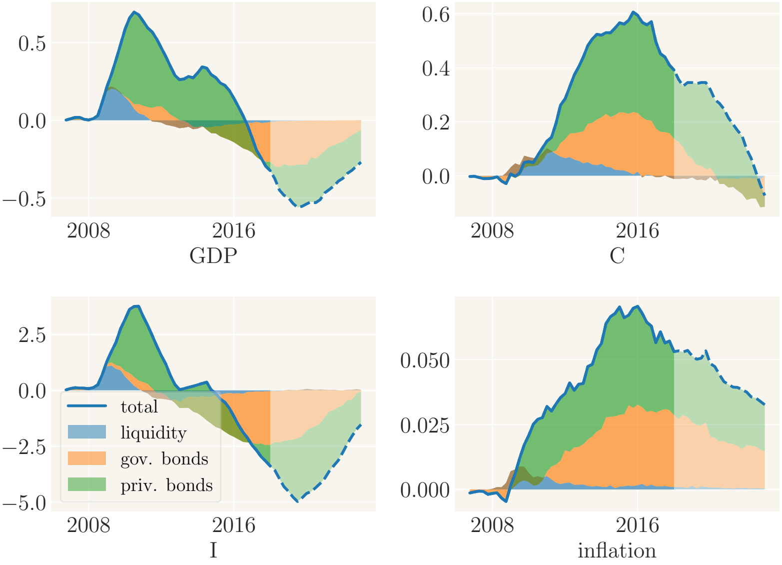
Historical decomposition I. more ZLB effects no banks



Historical decomposition II. more ZLB effects no banks



The net effects of QE Measures I IRFs QE vs. FG



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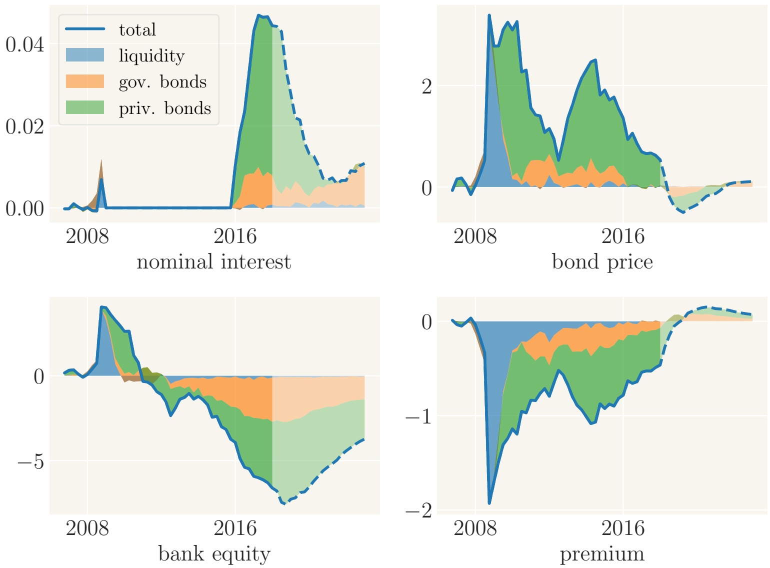
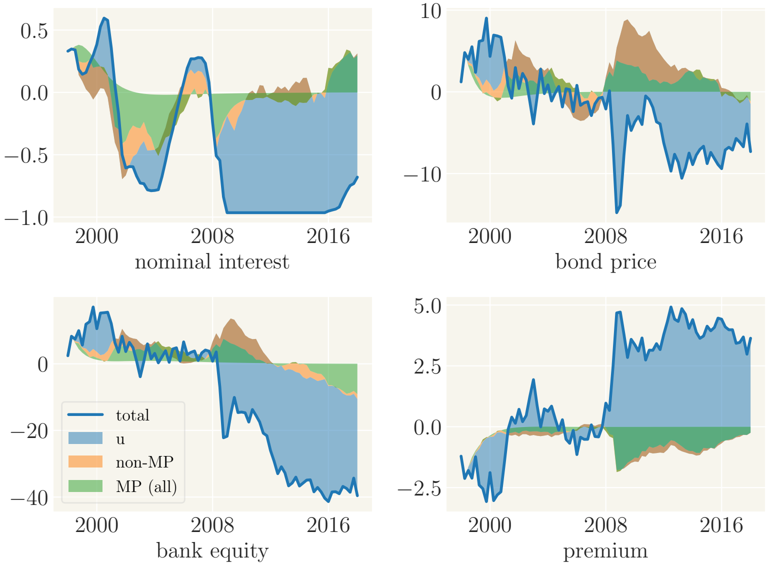


Thank you for your attention

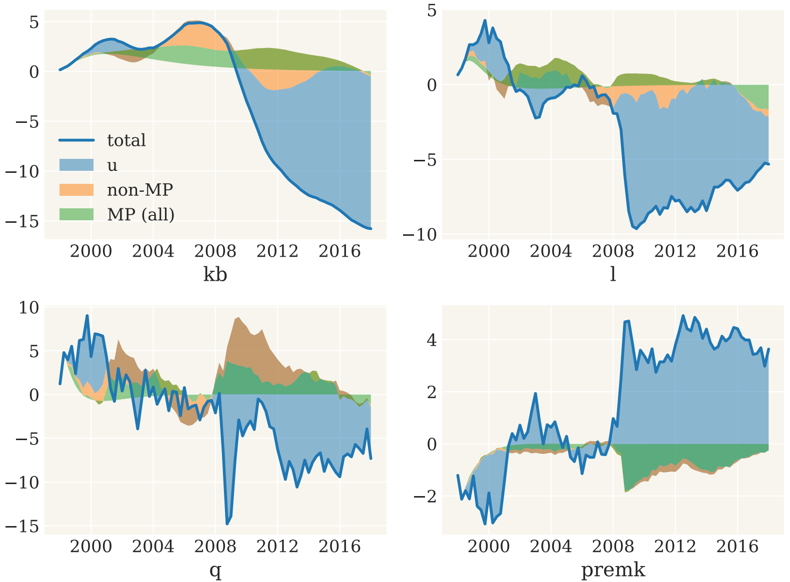
Key Contributions

- Short run: (moderate) stimulating effect of QE
- Long run: QE strong recessionary risk
- Private assets purchases more effective than gov. bond purchases
- **Technical challenge:** nonlinear estimation (ZLB) of large-scale model

Historical decomposition back

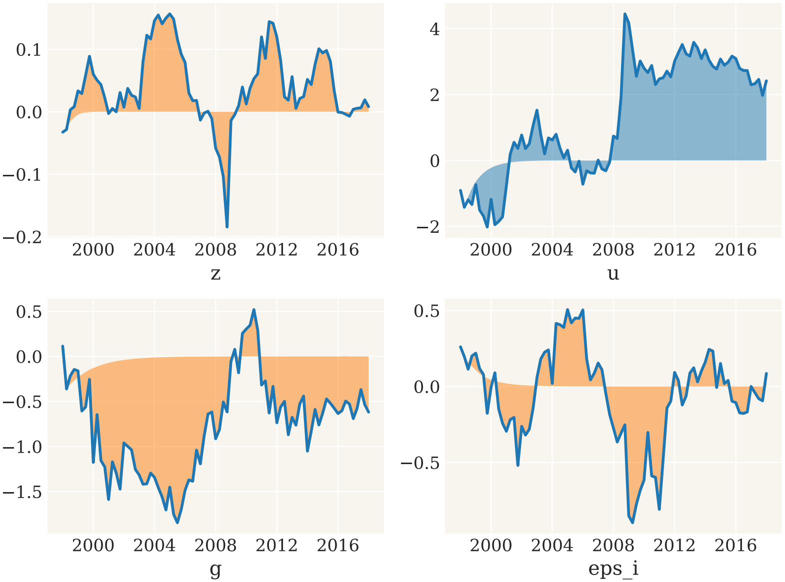


Historical decomposition back

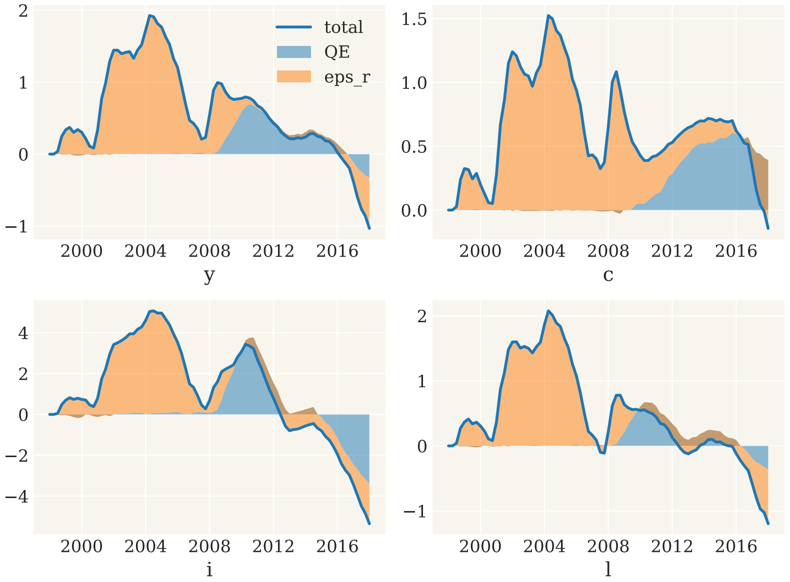


Historical decomposition

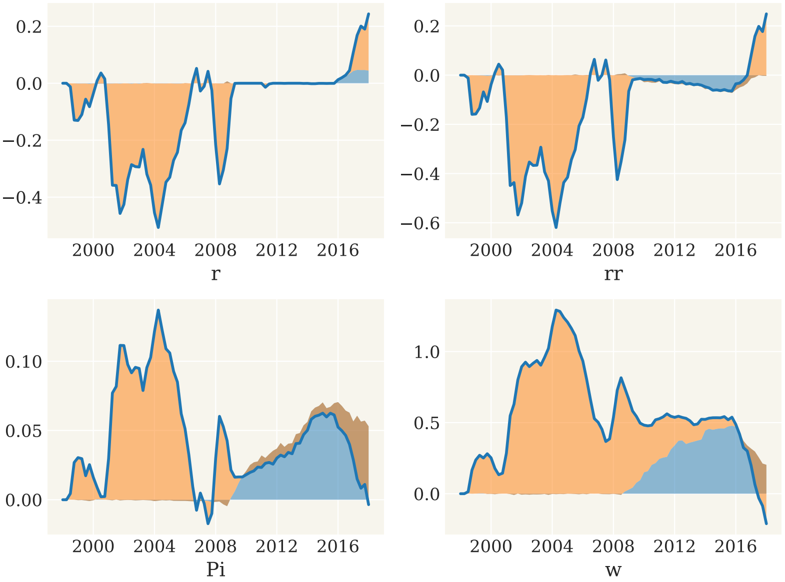
back



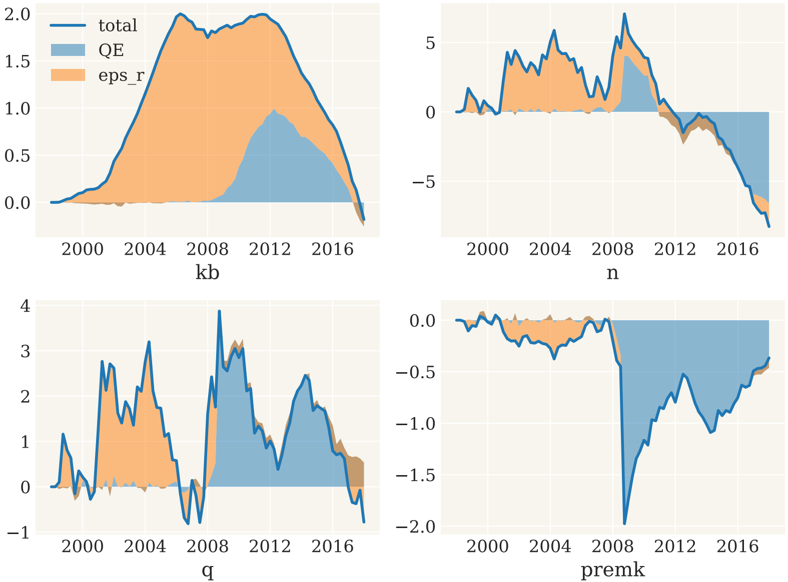
QE vs. FG I back



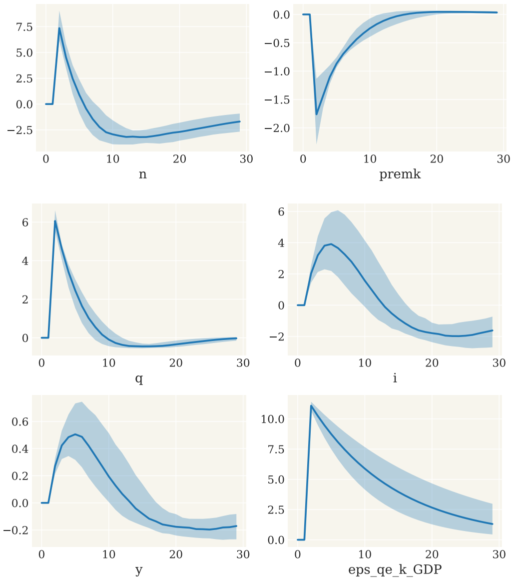
QE vs. FG II back



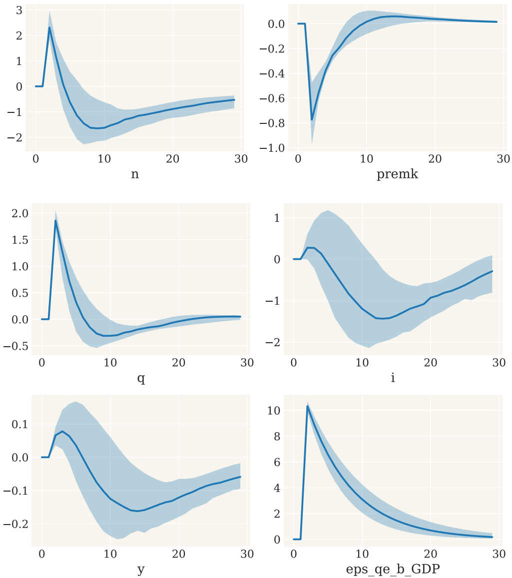
QE vs. FG III back



IRF to a shock to CB capital purchases back



IRF to a shock to CB gov. bond purchases back

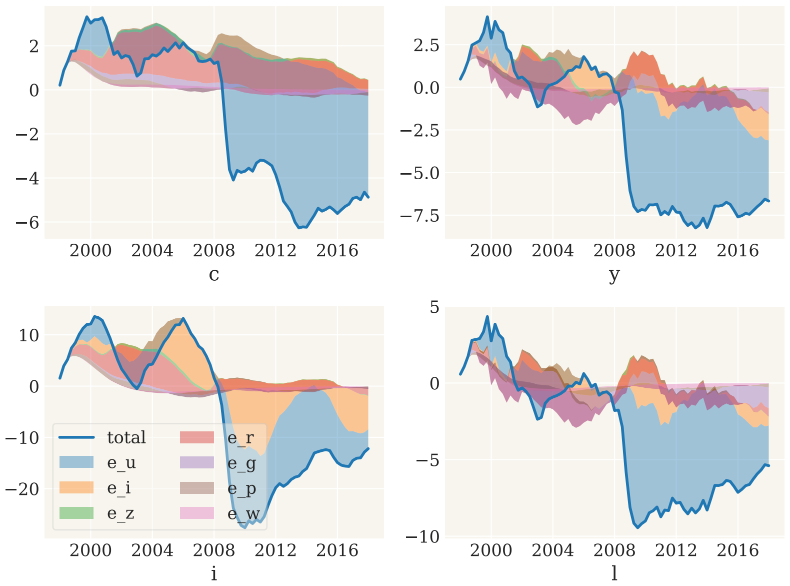


Parameter estimates SW07 model

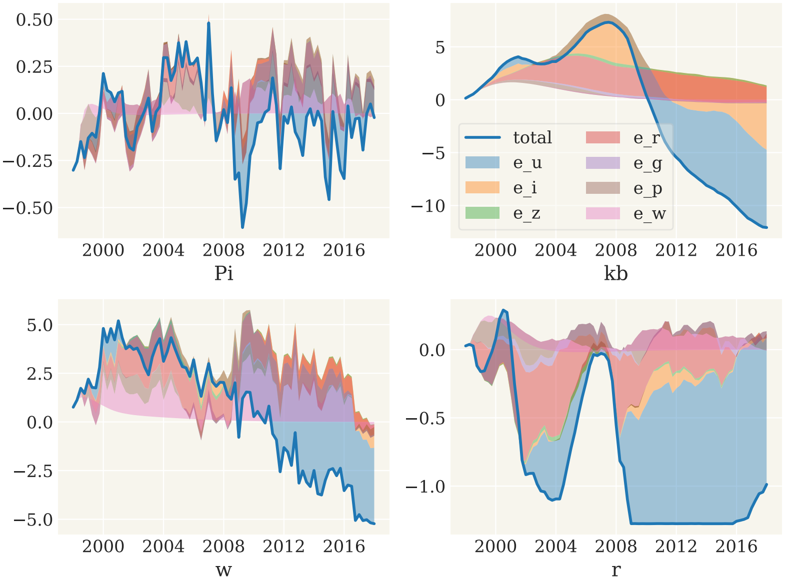
go back

	Prior		Posterior			
	mean	sd	mean	sd	hpd_2.5	hpd_97.5
ζ_p	0.500	0.10	0.758	0.044	0.664	0.835
ζ_w	0.500	0.10	0.630	0.052	0.533	0.732
Φ	1.250	0.12	1.803	0.081	1.658	1.973
ϕ_π	1.700	0.25	1.517	0.257	1.033	2.018
ϕ_y	0.125	0.05	0.199	0.032	0.141	0.263
ϕ_{dy}	0.125	0.05	0.181	0.044	0.092	0.265
ρ	0.700	0.20	0.790	0.047	0.699	0.879
ρ_r	0.700	0.20	0.688	0.114	0.518	0.897
ρ_i	0.500	0.20	0.782	0.139	0.413	0.964
ρ_z	0.500	0.20	0.722	0.165	0.345	0.934
ρ_u	0.500	0.20	0.761	0.044	0.653	0.839
ρ_p	0.700	0.20	0.341	0.091	0.181	0.515
ρ_w	0.700	0.20	0.300	0.056	0.192	0.405
σ_u	0.100	2.00	1.769	0.429	0.925	2.620
σ_z	0.100	2.00	0.214	0.131	0.058	0.505
σ_r	0.100	2.00	0.150	0.078	0.076	0.245
σ_i	0.100	2.00	0.323	0.300	0.119	1.039
σ_p	0.100	2.00	0.284	0.104	0.112	0.496
σ_w	0.100	2.00	1.482	0.313	0.848	2.059

Historical decomposition (no Banks) I back

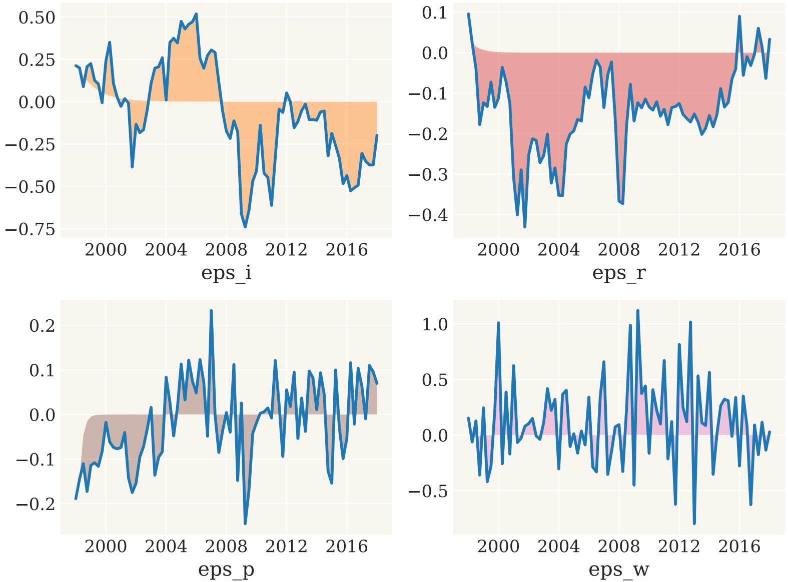


Historical decomposition (no Banks) II back



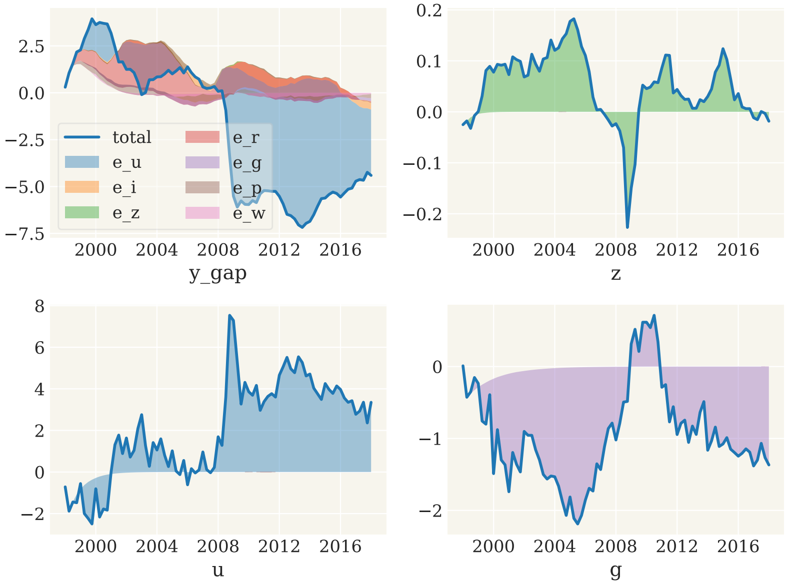
Historical decomposition (no Banks) III

back

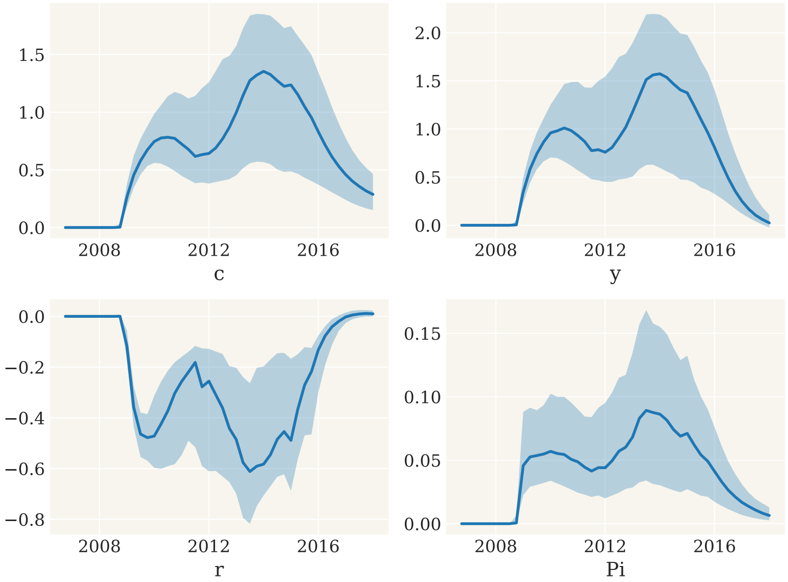


Historical decomposition (no Banks) IV

back

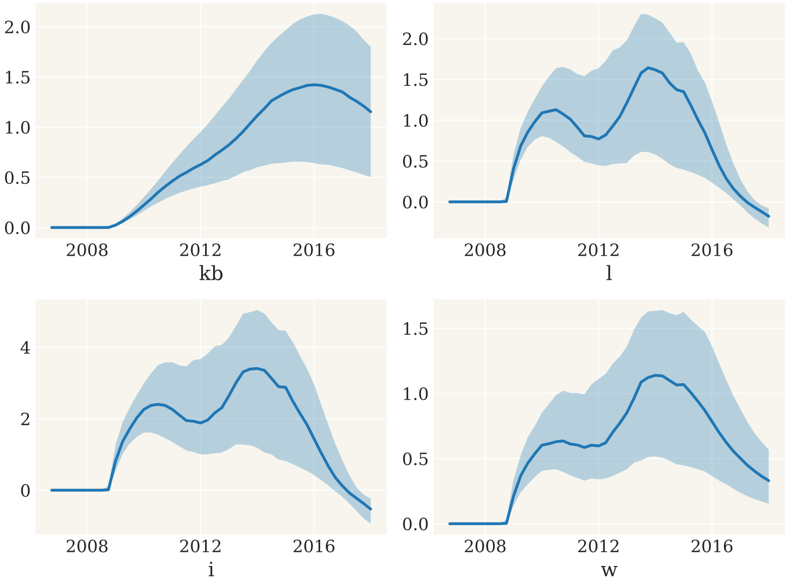


(Counterfactual) Effects of hitting the ZLB I back



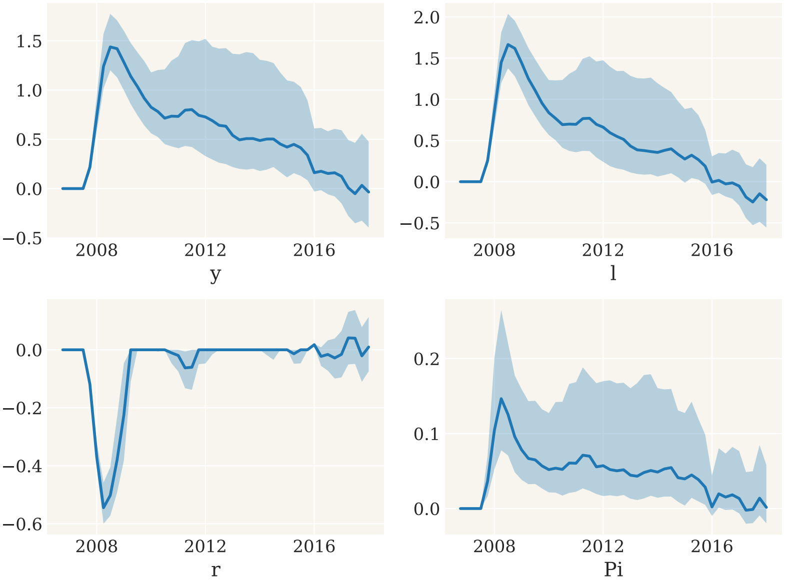
(Counterfactual) Effects of hitting the ZLB II

back



(Counterfactual) Effects of MP / Forward Guidance

back



Smoothing & Path-adjustment (Boehl, 2019) go back

- **Smoothing** (Transposed-Ensemble Rauch-Tung-Striebel smoother):

$$\mathbf{X}_{t|T} = \mathbf{X}_{t|t} + \bar{\mathbf{X}}_{t|t}\bar{\mathbf{X}}^{+}_{t+1|t}\left[\mathbf{X}_{t+1|T} - \mathbf{X}_{t+1|t}\right] \tag{1}$$

- **Extraction** (Iterative path-adjustmend):
  - Fully reflects the nonlinearity of the transition function
  - Interested in shocks $\{\varepsilon_t\}_{t=0}^{T-1}$ that fully recover smoothened states (historical decomposition!)
  - Initialize $\hat{x}_0 = \mathrm{E}\mathbf{X}_{0|T}$, define $P_{t|T} = \mathrm{Cov}\{\mathbf{X}_{t|T}\}$.
  - For each t:

$$\hat{\varepsilon}_t = \arg\max_{\varepsilon} \left\{ \log f_N \left(g(\hat{x}_{t-1}, \varepsilon) | \bar{x}_{t|T}, P_{t|T} \right) \right\}, \tag{2}$$

$$\hat{x}_t = g(\hat{x}_{t-1}, \hat{\varepsilon}_t), \tag{3}$$

The (flat) Phillips Curve

	Prior		Posterior			
	mean	sd	mean	sd	hpd_2.5	hpd_97.5
ι_p	0.500	0.15	0.247	0.094	0.072	0.419
ι_w	0.500	0.15	0.452	0.143	0.196	0.723
ζ_p	0.500	0.10	0.758	0.044	0.664	0.835
Φ	1.250	0.12	1.803	0.081	1.658	1.973

$$\pi_t = \frac{\overline{\beta}}{1+\iota_p\overline{\beta}} E_t\pi_{t+1} + \kappa\hat{x}_t + \frac{\iota_p}{1+\iota_p\overline{\beta}}\pi_{t-1}$$

$$\kappa = \frac{(1-\zeta_p\overline{\beta})(1-\zeta_p)}{(1+\overline{\beta}\iota_p)\zeta_p(\epsilon_p(\Phi-1)+1)} \qquad \text{(slope of PC)}$$

$$\hat{x}_t = w_t - z_t + \alpha(l_t - k_t) \qquad \text{(marginal costs)}$$

- SW07: $\kappa \approx 0.02$
- here: $\kappa \approx 0.007$!
- Key-incredient of NK model?

Equilibrium conditions

Definition (transition equilibrium)

A rational expectation solution $S(l^*, k^*)$ is a rational expectations *equilibrium* iff

$$\mathbf{b}L_s(l^*, k^*) \geq \bar{r} \quad \forall s < l^* \wedge s \geq k^* + l^* \tag{4}$$

and

$$\mathbf{b}L_s(l^*, k^*) < \bar{r} \quad \forall l^* \leq s < k^* + l^*. \tag{5}$$

go back

Linearized equilibrium back

$$
\begin{aligned}
c_t =& \frac{1/\gamma}{(1+1/\gamma)} c_{t-1} + \frac{1}{1+1/\gamma} E_t[c_{t+1}] \\
&- \frac{(1-1/\gamma)}{(1+1/\gamma)\sigma_c} (r_t - E_t[\pi_{t+1}] + v_{d,t}) \\
i_t =& \frac{1}{1+\overline{\beta}} [i_{t-1}] + \frac{\overline{\beta}}{1+\overline{\beta}} E_t[i_{t+1}] + \frac{1}{(1+\overline{\beta})\gamma^2 S''} q_t^k \\
\overline{k}_t =& (1-\delta)/\gamma \overline{k}_{t-1} + (1-(1-\delta)/\gamma)\hat{i}_t \\
&+ (1-(1-\delta)/\gamma)(1+\overline{\beta})\gamma^2 S'' v_{i,t} \\
R_t - E_t[\pi_{t+1}] + v_{d,t} =& \frac{R^k}{R^k + (1-\delta)} E_t[r_{t+1}^k] + \frac{(1-\delta)}{R^k + (1-\delta)} E_t[q_{t+1}^k] - q_t^k \\
k_t =& \frac{1-\psi}{\psi} r_t^k + \overline{k}_{t-1} \\
k_t =& w_t - r_t^k + l_t \\
y_t =& \Phi(\alpha k_t + (1-\alpha) l_t + z_t)
\end{aligned}
$$

Linearized equilibrium II back

$$y_t = \frac{G}{Y} g_t + \frac{C}{Y} c_t + \frac{I}{Y} i_t + \frac{R^k K}{Y} \frac{1-\psi}{\psi} r_t^k$$

$$\pi_t = \frac{\overline{\beta}}{1+\iota_p\overline{\beta}} E_t\pi_{t+1} + \frac{\iota_p}{1+\iota_p\overline{\beta}}\pi_{t-1} + \frac{(1-\zeta_p\overline{\beta})(1-\zeta_p)}{(1+\overline{\beta}\iota_p)\zeta_p((\Phi-1)\epsilon_p+1)}(w_t - z_t + \alpha l_t - \alpha k_t)$$

$$w_t = \frac{1}{1+\overline{\beta}\gamma}(w_{t-1} + \iota_w\pi_{t-1}) + \frac{\overline{\beta}\gamma}{1+\overline{\beta}\gamma} E_t[w_{t+1} + \pi_{t+1}] - \frac{1+\iota_w\overline{\beta}\gamma}{1+\overline{\beta}\gamma}\pi_t + \frac{(1-\zeta_w\overline{\beta}\gamma)(1-\zeta_w)}{(1+\overline{\beta}\gamma)\zeta_w((\lambda_w-1)\epsilon_w+1)}(w_t^h - w_t)$$

$$w_t^h = \frac{\sigma_c}{(1-h)}(c_t - hc_{t-1}) + \frac{L}{1-L} l_t$$

$$r_t = max\{0, \rho r_{t-1} + (1-\rho)(\phi_\pi \pi_t + \phi_y \widetilde{y}_t + \phi_{dy}(\widetilde{y}_t - \widetilde{y}_{t-1})) + v_{r_t}\}$$

Linearized equilibrium III

back

$$
\begin{aligned}
v_{d,t} =& \rho_d v_{d,t-1} + \epsilon_t^d, \\
z_t =& \rho_z z_{t-1} + \epsilon_t^z, \\
g_t =& \rho_g g_{t-1} + \epsilon_t^g, \\
v_{r,t} =& \rho_r v_{r,t-1} + \epsilon_t^r, \\
v_{i,t} =& \rho_i v_{i,t-1} + \epsilon_t^i, \\
v_{p,t} =& \rho_p v_{p,t-1} + \epsilon_t^p, \\
v_{w,t} =& \rho_w v_{w,t-1} + \epsilon_t^w,
\end{aligned}
$$

Households

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - hC_{t-1})^{1-\sigma_c} - 1}{1-\sigma_c} - \nu\, log(1 - L_t) \right)$$

$$P_t C_t + \frac{D_t}{v_{d,t} R_t} = D_{t-1} + W_t L_t - T_t + \mathfrak{P}_t.$$

- C_t - consumption
- L_t - labor
- W_t - real wage
- D_t - bank deposits
- $v_{d,t}$ - risk premium shock
- R_t - real rate on deposits
- T_t - lump sum taxes
- $\mathfrak{P}_t$- profits from firms and banks

back

Unions

$$\max_{W_t(i)} E_t \sum_{s=0}^{\infty} (\beta\zeta_w)^s \frac{\Lambda_{t,t+s}}{\Pi_{t,t+s}} \left[W_t(i)\Pi_{l=1}^{s}(\Pi_{t+l-1}^{\imath_w}\Pi^{1-\imath_w}) - MRS_{t+s} \right] L_{t+s}(i)$$

$$s.t. \frac{L_{t+s}(i)}{L_{t+s}} = G_w^{'-1} \left(\frac{W_t(i)\Pi_{l=1}^{s}(\Pi_{t+l-1}^{\imath_w}\Pi^{1-\imath_w})}{W_{t+s}} \tau_{t+s}^{w} \right).$$

$W_t(i)$ - wage set by union i; ζ_w - Calvo parameter; $\Lambda_{t,t+s}$ - SDF; $\imath_w$ - the degree of wage indexation; MRS_t - mraginal rate of substitution; G_w - Kimball aggregator.

$$W_t = [(1-\zeta_w)(W_t^*)G_w^{'-1} \left[\frac{W_t^* \tau_t^w}{W_t} \right] +$$

$$\zeta_w \Pi_{t-1}^{\imath_w} \Pi^{(1-\imath_w)}) W_{t-1} G_w^{'-1} \left[\frac{\Pi_{t-1}^{\imath_w}\Pi^{(1-\imath_w)}) W_{t-1} \tau_t^w}{W_t} \right],$$

W_t^* - optimal wage

back

Firms

- Intermediate good producers details
  - Cobb Douglas production function; employ labor and capital
  - perfect competition
  - buy and re-sell entire capital stock each period
  - capital purchases are financed with bank loans
- Capital good producers details
  - perfect competition
  - buys and re-sells capital to intermediate good producer
  - repairs used capital, invests in new capital
  - subject to investment adjustment costs.
- Retailers details
  - monopolistic competition, Calvo pricing

Intermediate goods producers

$$\max_{K_t, L_t, U_t} E_t[\beta\Lambda_{t,t+1}(-R_{k,t+1}Q_t\overline{K}_t(i) + P_{m,t+1}(i)Y_{m,t+1}(i) - W_{t+1}L_{t+1}(i)...$$

$$... - a(U_t)K_t(i) + (1-\delta)Q_{t+1}\overline{K}_t(i))]$$

$$s.t. \qquad Y_{m,t}(i) = e^{z_t}K_t(i)^{\alpha}\left(\gamma^t L_t(i)\right)^{1-\alpha} - \gamma^t\Phi, \tag{6}$$

Y_{mt} - intermediate good; P_{mt} - price of intermediate good; z_t - technology shock; δ_t - depreciation rate; $\overline{K}_t$ - physical capital stock; K_t - effective capital; U_t - utilization rate; Q_t - price of capital; $R_{k,t+1}$ - real return of capital; Φ - fixed cost; γ - growth trend;

back

Capital Goods Producers

Capital accumulation

$$\overline{K}_t = (1-\delta)\overline{K}_{t-1} + v_{i,t}\left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t,$$

Objective of Capital Good producer

$$\max_{I_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_{0,t} \left\{ Q_t \left(1 - S\left(\frac{I_t(k)}{I_{t-1}(k)}\right)\right) v_{i,t} - 1 \right\} I_t.$$

First-order condition for optimal investment:

$$\begin{aligned} 1 \quad = \quad & Q_t v_{i,t} \left(1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right) \\ & + E_t \left\{ Q_{t+1} v_{i,t+1} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \right\} \end{aligned}$$

- I_t - Investment; $v_{i,t}$ - investment specific technology shock

Retailers

$$\max_{P_t(i)} E_t \sum_{s=0}^{\infty} (\beta\zeta_p)^s \frac{\Lambda_{t,t+s}}{\Pi_{t,t+s}} \left[P_t(i)\Pi_{l=1}^{s}(\Pi_{t+l-1}^{\imath_p}\Pi^{1-\imath_p}) - MC_{t+s}\right] Y_{t+s}(i)$$

$$s.t. \quad \frac{Y_{t+s}(i)}{Y_{t+s}} = G^{'-1}\left(\frac{P_t(i)\Pi_{l=1}^{s}(\Pi_{t+l-1}^{\imath_p}\Pi^{1-\imath_p})}{P_{t+s}}\tau_{t+s}\right).$$

Aggregate price index

$$P_t = [(1-\zeta_p)(P_t^*)G^{'-1}\left[\frac{P_t^*\tau_t}{P_t}\right] + \zeta_p\Pi_{t-1}^{\imath_p}\Pi^{(1-\imath_p)}P_{t-1}G^{'-1}\left[\frac{\Pi_{t-1}^{\imath_p}\Pi^{(1-\imath_p)})P_{t-1}\tau_t}{P_t}\right]$$

$P_t(i)$ - price set by firm i; $\Pi_{t,t+s}$ - gross inflation, $\Lambda_{t,t+s}$ -SDF, Y_t - demand for intermediate goods; MC_t - marginal cost; $\imath_p$ - degree of price indexation; G - Kimball aggregator; P_t^* - optimal price;

back

Banks

Banks' balance sheet

$$Q_t K_{b,t} + Q_t^b B_{b,t} = D_t + N_t$$

Law of motion of net worth

$$N_t = R_{k,t} Q_{t-1} K_{b,t-1} + R_{b,t} Q_{t-1}^b B_{b,t-1} - v_{d,t-1} R_{t-1} D_{t-1}$$

- Q_t, Q_t^b - prices of capital asset and government bonds
- $K_{b,t}$ - claims on capital stock held by banks
- $B_{b,t}$ - government bond held by banks
- D_t deposits
- N_t - net worth
- $R_{k,t}, R_{b,t}, R_t$ - interest rates on capital, bonds and deposits
- $v_{d,t}$ - risk premium shock (AR(1)-process)

back

Banks

Each period a fraction of bankers, (1-θ), exits the business with a fixed probability. When they exit they consume their accumulated net worth. Hence, bankers maximize the terminal value of their net worth

$$V_t = \max_{\{K_{b,t}\},\{B_{b,t}\},\{D_t\}} \quad E_t \Lambda_{t,t+1}[(1-\theta)N_{t+1} + \theta V_{t+1}],$$

subject to an incentive constraint.

$$V_t \geq \lambda Q_t K_{b,t} + \lambda_b Q_t^b B_{b,t}.$$

- λ, λ_b - fraction of assets that banker can divert

Assumption: Incentive constraint is always binding

back

Banks - First Order Conditions

Guess: value function is linear in loans, government bonds and net worth:

$$V_t = \nu_{kt} Q_t K_t + \nu_{bt} Q_t^b B_t + \nu_{nt} N_t$$

FOC for K_t, B_t. μ_t:

$$\nu_{kt} = \lambda \frac{\mu_t}{1+\mu_t} \tag{7}$$

$$\nu_{bt} = \lambda_b \frac{\mu_t}{1+\mu_t} \tag{8}$$

$$Q_t K_t = \frac{\nu_{bt} - \lambda_b}{(\lambda - \nu_{kt})} Q_t^b B_t + \frac{\nu_{nt}}{\lambda - \nu_{kt}} N_t \tag{9}$$

- μ_t - Lagrange Multiplier of the incentive constraint

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Solution to the Bank's Problem

Guess: value function is linear in loans, government bonds and net worth:

$$V_t = \nu_{kt} Q_t K_t + \nu_{bt} Q_t^b B_t + \nu_{nt} N_t$$

solution for the coefficients:

$$\nu_{k,t} = \beta E_t \Omega_{t+1} (R_{k,t+1} - v_{d,t} R_t), \quad (10)$$

$$\nu_{b,t} = \beta E_t \Omega_{t+1} (R_{b,t+1} - v_{d,t} R_t), \quad (11)$$

$$\nu_{n,t} = \beta E_t \Omega_{t+1} v_{d,t} R_t. \quad (12)$$

where the stochastic discount factor of the banker is defined as:

$$\Omega_t \equiv \Lambda_{t-1,t} [(1-\theta) + \theta (\nu_{nt} (1 + \mu_t))] \quad (13)$$

with $\Lambda_{t-1,t}$ being the SDF of the household

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Banks - Aggregation

Each period a fraction of bankers, (1-θ), exits the business with a fixed probability, and is replaced by new bankers, which are given a fraction ω of the total assets.
net worth by existing and new bankers:

$$N_t = N_{nt} + N_{et} \tag{14}$$

$$N_{et} = \theta[(R_{kt}Q_{t-1}K_{t-1} + R_{bt}Q^b_{t-1}B_{t-1} - v_{d,t-1}R_{t-1}D_{t-1}] \tag{15}$$

$$N_{nt} = \omega[(R_{kt}Q_{t-1}K_{t-1} + R_{bt}Q^b_{t-1}B_{t-1}] \tag{16}$$

aggregate balance sheet

$$Q_tK_t + Q^b_tB_t = D_t + N_t \tag{17}$$

leverage ratio:

$$\phi_t = (Q_tK_t + Q^b_tB_t)/N_t \tag{18}$$

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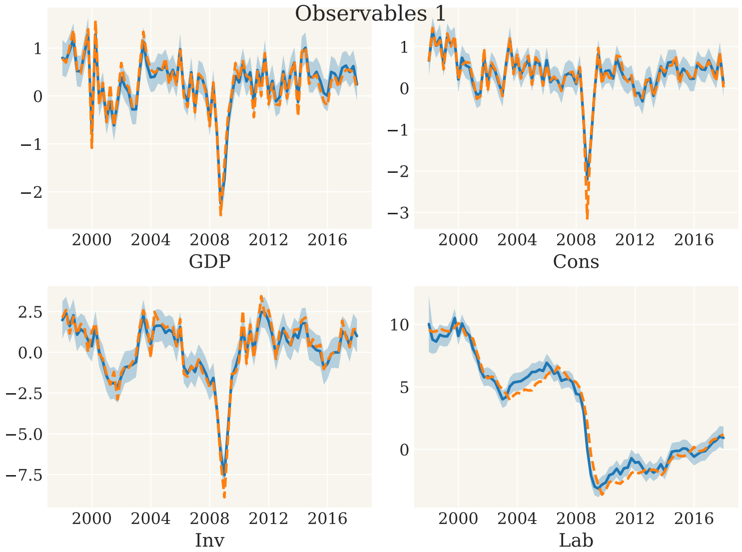
Fiscal Policy

budget constraint	$G_t + R_{b,t} Q^b_{t-1} B_{t-1} = Q^b_t B_t + T_t$
government spending	$G_t = G \cdot e^{g_t}$
government spending	$g_t = \rho_g g_{t-1} + \varepsilon^g_t$
tax revenues	$T_t = T + \kappa_b (B_{t-1} - B)$
return on bonds	$R_{bt} = \frac{r_c + \rho_c Q^b_t}{Q^b_{t-1}}$

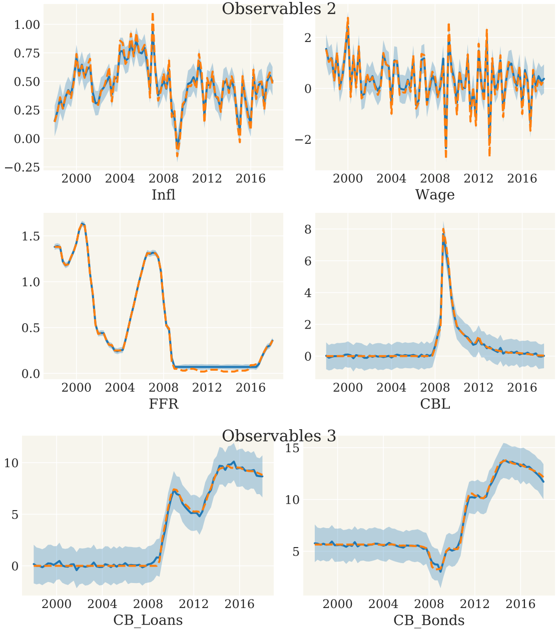
G_t - government spending;
T_t - tax revenues;
g_t - government spending shock;
r_c - coupon on bond;
ρ_c - decay rate of consol;

back

Data and Filtered series I go back



Data and Filtered series II go back



Calibrated parameter

$trend$	0.344	pre-crisis average
$mean_L$	6.5415	pre-crisis average
$mean_\Pi$	0.5	2% inflation target
$mean_{LSAP,B}$	5.65	pre-crisis average
$mean_{LSAP,K}$	0	pre-crisis average
$mean_{CBL}$	0	pre-crisis average
λ_w	1.1	10% markup in labor market
ϵ	10	as in SW (2007)
h	0.72	as in SW (2007)
α	0.19	as in SW (2007)
ψ	0.79	mean value for trial estimations

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