Mind the gap! Stylized facts and structural models.

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Introduction

• Common in macroeconomics to compare dynamics induced by disturbances using SVAR and DSGE; see e.g. Gali (1999); Christiano et al. (2005); Iacoviello (2005), Basu and Bundik (2017), etc.

• Non-invertibility/truncation problems: Ravenna (2007), Fernandez et al. (2007), Giacomini (2013), Plagborg-Moller (2018), Pagan and Robinson (2018), Chahrour and Jurado (2018).

• Typically, if VAR has q shocks, use a theory with q or less disturbances. Does the DGP only has q disturbances? Which ones?

• Here DGP has q disturbances; empirical model $q_1 < q$ variables. **Identified shocks and identified dynamics become mongrels with little economic interpretation.**

• *Cross sectional deformation*:

- Identified shocks need not combine "types" of structural disturbances.

- Appropriate theoretical restrictions may be insufficient.

Difficult to match e.g, identified technology shocks to TFP disturbances.

• *Time deformation*:

- Identified shocks are, in general, linear combinations of *current and past* structural disturbances.

Perceived internal transmission stronger than in the DGP.

Punchlines

- VARs can not be too small: difficult to make sense of identified shocks.

- If VARs can not be sufficiently large, compare data VARs with the theory reduced to the same VAR observables. **Some structural disturbances may not be obtained from a given VAR**.

- (Corollary) VARs used to derive dynamic facts might change depending on the DGP and the disturbances of interest. **To identify monetary policy disturbances may need VARs with different variables if the DGP has financial disturbances or not**.

- Deformation vs. invertibility.

\- Problems distinct.

\- Long lags do not help to reduce cross sectional deformation.

- Early literature: Lutkepohl (1984), Hansen and Sargent (1991), Marcet (1991), Braun and Mittnik (1991), Faust and Leeper (1998), Forni and Lippi (1999).

- Related literature: Canova and Sahneh (2018), Wolf (2018).

Intuition

- Growth model with log preferences, full depreciation, iid shocks to TFP (Z_t), investment (V_t), preferences (B_t). Solution:

$$K_{t+1} = \alpha\beta V_t Z_t K_t^{\alpha} \quad (1)$$

$$C_t = (1-\alpha\beta) B_t Z_t K_t^{\alpha} \quad (2)$$

$$Y_t = Z_t K_t^{\alpha} \quad (3)$$

- System invertible if $0 \leq \alpha < 1$.

- Recursive system. All three shocks identifiable if VAR has three variables.

- System 1: $(\log K_{t+1}, \log Y_t)$

$$\begin{aligned} \log K_{t+1} &= \log(\alpha\beta) + \alpha \log k_t + u_{1t} \qquad (4) \\ \log Y_t &= \alpha \log k_t + u_{2t} \qquad (5) \end{aligned}$$

$u_{1t} = \log V_t + \log Z_t$, $u_{2t} = \log Z_t$. Cannot recover $B_t \rightarrow$ cross sectional deformation. System maintain recursivity: identification works for $\log V_t, \log Z_t$.

- System 2: $(\log K_{t+1}, \log C_t)$

$$\begin{aligned} \log K_{t+1} &= \log(\alpha\beta) + \alpha \log K_t + u_{1t} \qquad (6) \\ \log C_t &= \log(1 - \alpha\beta) + \alpha \log K_t + u_{2t} \qquad (7) \end{aligned}$$

$u_{1t} = \log V_t + \log Z_t$, $u_{2t} = \log B_t + \log Z_t$. u_t mix demand and supply disturbances. Cross sectional deformation. Recursivity lost; identification does not work.

- System 3: $(\log C_t, \log Y_t)$

$$\log C_t = \alpha \log(\alpha\beta) + \alpha \log C_{t-1} + u_{1t} \quad (8)$$
$$\log Y_t = \alpha \log(\alpha\beta) + \alpha \log Y_{t-1} + u_{2t} \quad (9)$$

- $u_{1t} = \log B_t - \alpha \log B_{t-1} + \log Z_t + \alpha \log V_{t-1}$.

- $u_{2t} = \log Z_t + \alpha \log V_{t-1}$.

- Time and cross sectional deformation.

- Impossible to go from $u_{jt}, j = 1, 2$ to demand and supply disturbances.

- Dynamics to identified u_{jt} shocks more persistent than dynamics to $\log B_t, \log V_t, \log Z_t$.

Relationship structural disturbances/empirical innovations

- (Log-) linear DGP:

$$x_t = A(\theta)x_{t-1} + B(\theta)e_t \quad (10)$$
$$y_t = C(\theta)x_{t-1} + D(\theta)e_t \quad (11)$$

x_t is $k \times 1$ vector of endogenous and exogenous states, $e_t \sim (0, \Sigma)$, Σ diagonal, is $q \times 1$ vector of disturbances, y_t is $m \times 1$ vector of endogenous controls. $A(\theta)$ is $k \times k$, $B(\theta)$ is $k \times q$, $C(\theta)$ is $m \times k$, $D(\theta)$ is $m \times q$, θ structural parameters.

- Observables $z_{it} = S_i[x_t, y_t]'$, S_i is $q_i \times q$ matrix.

Case 1: Empirical system eliminates some controls

- $S_1 = [I, S_{12}]$

- Innovations u_{1t} generated by

$$u_{1t} = z_{1t} - E[z_{1t}|\Omega_{1t-1}] \equiv z_{1t} - \tilde{F}_1 z_{1t-1} \tag{12}$$

Proposition 1
i) $u_{1t} = \lambda_1(\theta)e_t$, where $\lambda_1(\theta)$ is $q_i \times q$.
ii) A sufficient condition for the identification of e_j it is that the k-th row of $G_1(\theta) \equiv \begin{pmatrix} B(\theta) \\ S_{12}D(\theta) \end{pmatrix}$ has at most one non-zero element in the j-th position.

- Related to Faust and Leeper (1998).

- **Cases 2-3: The empirical system eliminates/repackages states**

- $S_2 = [S_{21}, S_{22}]; S_3 = [S_{31}, 0]$.

- Innovations $u_{it}, i = 2, 3$ generated by

$$u_{it} = z_{it} - E[z_{it}|\Omega_{it-1}] \equiv z_{it} - \tilde{F}_i z_{it-1} \tag{13}$$

Proposition 2

i) $u_{it} = \lambda_i(\theta, L)e_t$, λ_i is $q_i \times q$, each L, i=2,3.
ii) $u_{it} = \psi_i(\theta, L)u_{1t}$, $i = 2, 3$.

Dynamics

Proposition 3

i) If a shock can be identified from u_{1t} and if $\tilde{F}_1 = \begin{pmatrix} A(\theta) \\ S_{12}C(\theta) \end{pmatrix}$, structural dynamics in the empirical system proportional to those of the DGP.

ii) With $u_{it}, i = 2, 3$ responses to identified shocks distorted at all horizons.

- Braun and Mittnik (1991): expression for response biases in VARs.

An example

$$
\begin{aligned}
\chi_t &= \chi_{t+1} - \frac{1}{1-h} g_{t+1} + \frac{h}{1-h}\; g_t + r_t - \pi_{t+1} && (14)\\
\pi_t &= \pi_{t+1}\,\beta + k_p\,\left(\frac{h}{1-h} g_t + (1+\sigma_n)\; n_t\right) + k_p\,(\mu_t - \chi_t) && (15)\\
o_t &= \zeta_t + (1-\alpha)\; n_t && (16)\\
r_t &= \rho_r r_{t-1} + (1-\rho_r)\left(\phi_y\, g_t + \phi_p\, \pi_t\right) + \varepsilon_t && (17)\\
g_t &= a_t + o_t - o_{t-1} && (18)\\
\zeta_t &= \rho_z\,\zeta_{t-1} + \varepsilon_{zt} && (19)\\
a_t &= \rho_a\, a_{t-1} + \varepsilon_{at} && (20)\\
\chi_t &= \rho_\chi\,\chi_{t-1} + \varepsilon_{\chi_t} && (21)\\
\mu_t &= \rho_\mu\,\mu_{t-1} + \varepsilon_{\mu_t} && (22)\\
\epsilon_t &= \varepsilon_{mp_t} && (23)
\end{aligned}
$$

- Minimal state vector $x_{t-1} = [o_{t-1}, r_{t-1}, \zeta_{t-1}, a_{t-1}, \mu_{t-1}, \chi_{t-1}]'$ (6 × 1)

- Control vector $y_t = [g_t, o_t, \pi_t, n_t, r_t]'$ (5 × 1).

- Shock vector $e_t = [\varepsilon_{z_t}, \varepsilon_{a_t}, \varepsilon_{\chi_t}, \varepsilon_{\mu_t}, \varepsilon_{mp_t}]'$ (5 × 1)

- Set: $\alpha = 0.33; \beta = 0.99; \sigma_n = 1.5; h = 0.9; k_p = 0.05; \phi_y = 0.1; \phi_p = 1.5; \rho_r = 0.8; \rho_z = 0.5; \rho_a = 0.2; \rho_\chi = 0.5; \rho_\mu = 0.0$.

- **How would the shocks/dynamics of an empirical system with $q_1 \leq 4$ compare with the shocks/dynamics of the original model?**

- System with $z_t = (o_t, \pi_t, n_t, r_t)$.

$$\chi_t = \chi_{t+1} - \frac{1}{1-h}(a_{t+1} + o_{t+1} - o_t) + \frac{h}{1-h}(a_t + o_t - o_{t-1}) + r_t - \pi_{t+1} \quad (24)$$

$$\pi_t = \pi_{t+1}\beta + k_p \left(\frac{h}{1-h}(a_t + o_t - o_{t-1}) + (1+\sigma_n)\, n_t\right) + k_p\,(\mu_t - \chi_t) \quad (25)$$

$$o_t = \zeta_t + (1-\alpha)\, n_t \quad (26)$$

$$r_t = \rho_r\, r_{t-1} + (1-\rho_r)\, \left(\phi_y\,(a_t + o_t - o_{t-1}) + \phi_p \pi_t\right) + \varepsilon_{mp_t} \quad (27)$$

- State vector: $x_{t-1} = [o_{t-1}, r_{t-1}, \zeta_{t-1}, a_{t-1}, \mu_{t-1}, \chi_{t-1}]'$.

- Law of motion of the states (A, B matrices) unaltered.

- **Cross sectional deformation, no time deformation distortions**.

- System with $z_t = (o_t, \pi_t, n_t)$.

$$
\begin{aligned}
((1+\rho_r) - \rho_r L)\chi_t &= \chi_{t+1} - \frac{1}{1-h}(a_{t+1} + o_{t+1} - o_t) + (\frac{h+\rho_r}{1-h} + (1-\rho_r)\phi_y)(a_t + o_t - o_{t-1}) \\
&- (\frac{h\rho_r}{1-h})(a_{t-1} + o_{t-1} - o_{t-2}) + (\rho_r + (1-\rho_r)\phi_p)\pi_t + \varepsilon_{mp_t} - \pi_{t+1} \quad (28) \\
\pi_t &= \pi_{t+1}\beta + k_p\left(\frac{h}{1-h}(a_t + o_t - o_{t-1}) + (1+\sigma_n)\, n_t\right) + k_p(\mu_t - \chi_t) \quad (29) \\
o_t &= \zeta_t + (1-\alpha)n_t \quad (30)
\end{aligned}
$$

- State vector: $\hat{x}_{t-1} = [o_{t-1}, \mathbf{o_{t-2}}, \zeta_{t-1}, a_{t-1}, \mu_{t-1}, \chi_{t-1}]'$.

- Law of motion of the states (A, B matrices) altered.

- **Cross-sectional and time deformation distortions**.

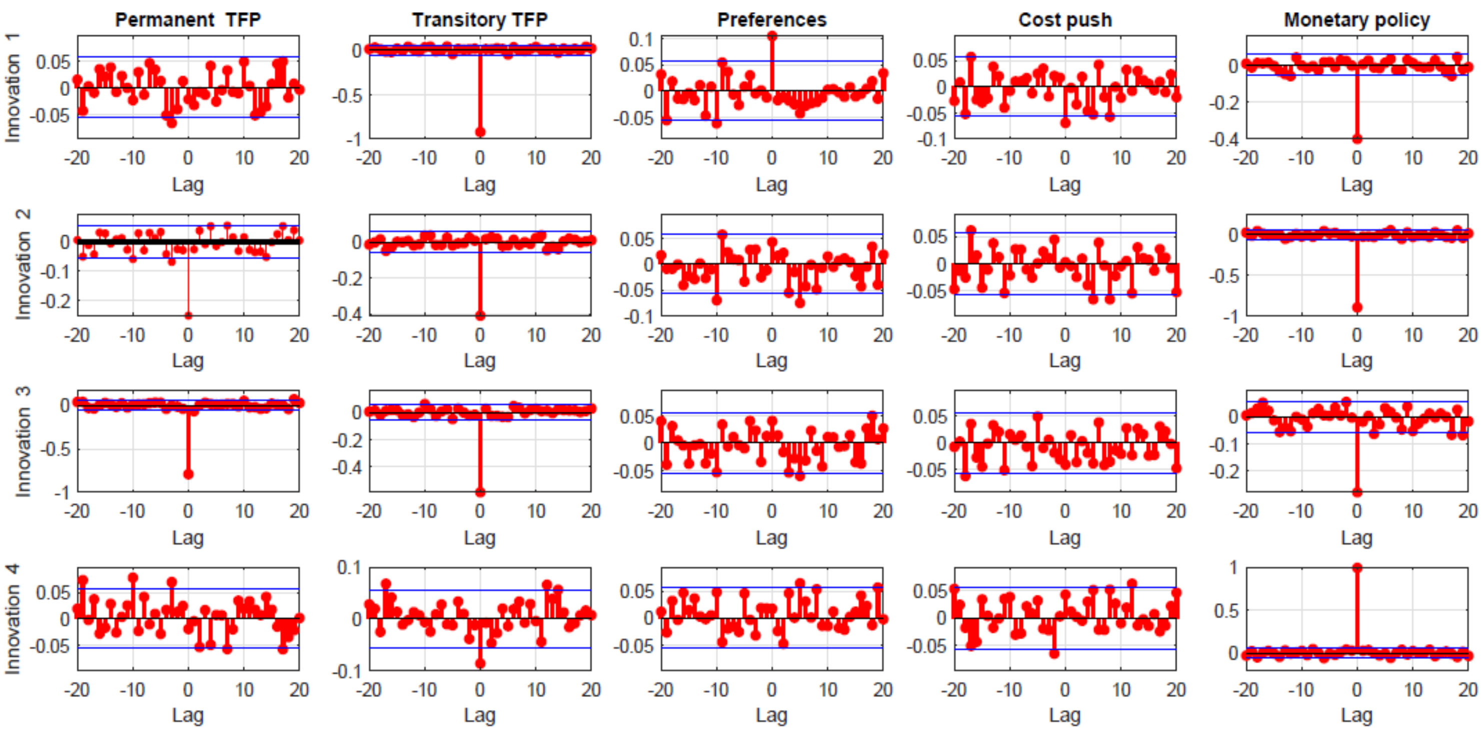
• System with $z_t = (\pi_t, n_t, r_t)$.

$$
\begin{aligned}
\chi_t &= \chi_{t+1} - \frac{1}{1-h}\left(a_{t+1} + \zeta_{t+1} - \zeta_t + (1-\alpha)\,(n_{t+1} - n_t)\right) \\
&+ \frac{h}{1-h}\left(a_t + \zeta_t - \zeta_{t-1} + (1-\alpha)\,(n_t - n_{t-1})\right) + r_t - \pi_{t+1} && (31) \\
\pi_t &= \pi_{t+1}\,\beta + k_p\left(\frac{h}{1-h}\left(a_t + \zeta_t - \zeta_{t-1} + (1-\alpha)\,(n_t - n_{t-1})\right) + (1+\sigma_n)\,n_t\right) \\
&+ k_p\,(\mu_t - \chi_t) && (32) \\
r_t &= \rho\, r_{t-1} + (1-\rho)\,\left(\phi_y\left(a_t + \zeta_t - \zeta_{t-1} + (1-\alpha)\,(n_t - n_{t-1})\right) + \phi_p\,\pi_t\right) + \varepsilon_{mp,t} && (33)
\end{aligned}
$$

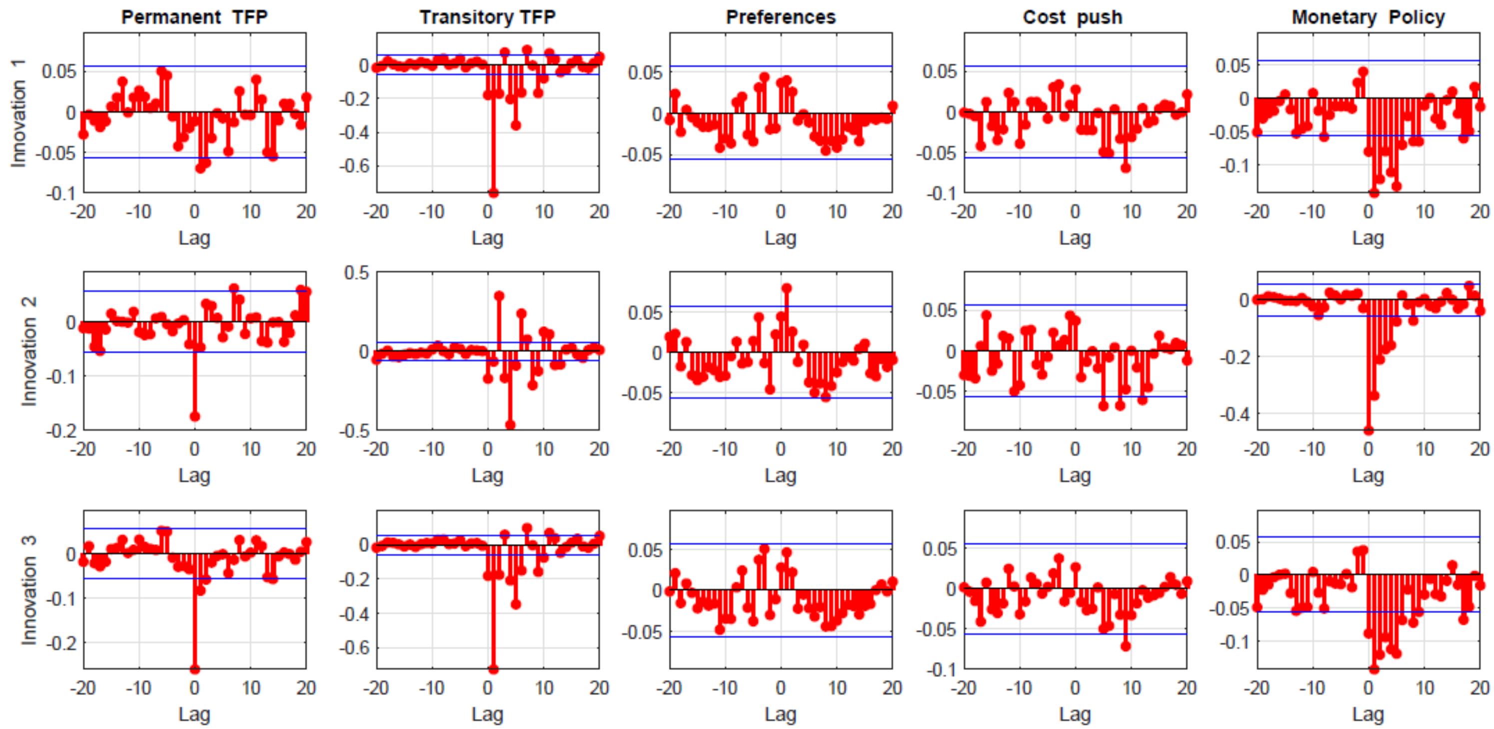
• State vector: $\hat{\mathrm{x}}_{t-1} = [\mathbf{n_{t-1}}, r_{t-1}, \zeta_{t-1}, a_{t-1}, \mu_{t-1}, \chi_{t-1}]'$.

• Law of motion of the states unchanged (given production function n_{t-1} proxies for o_{t-1}).

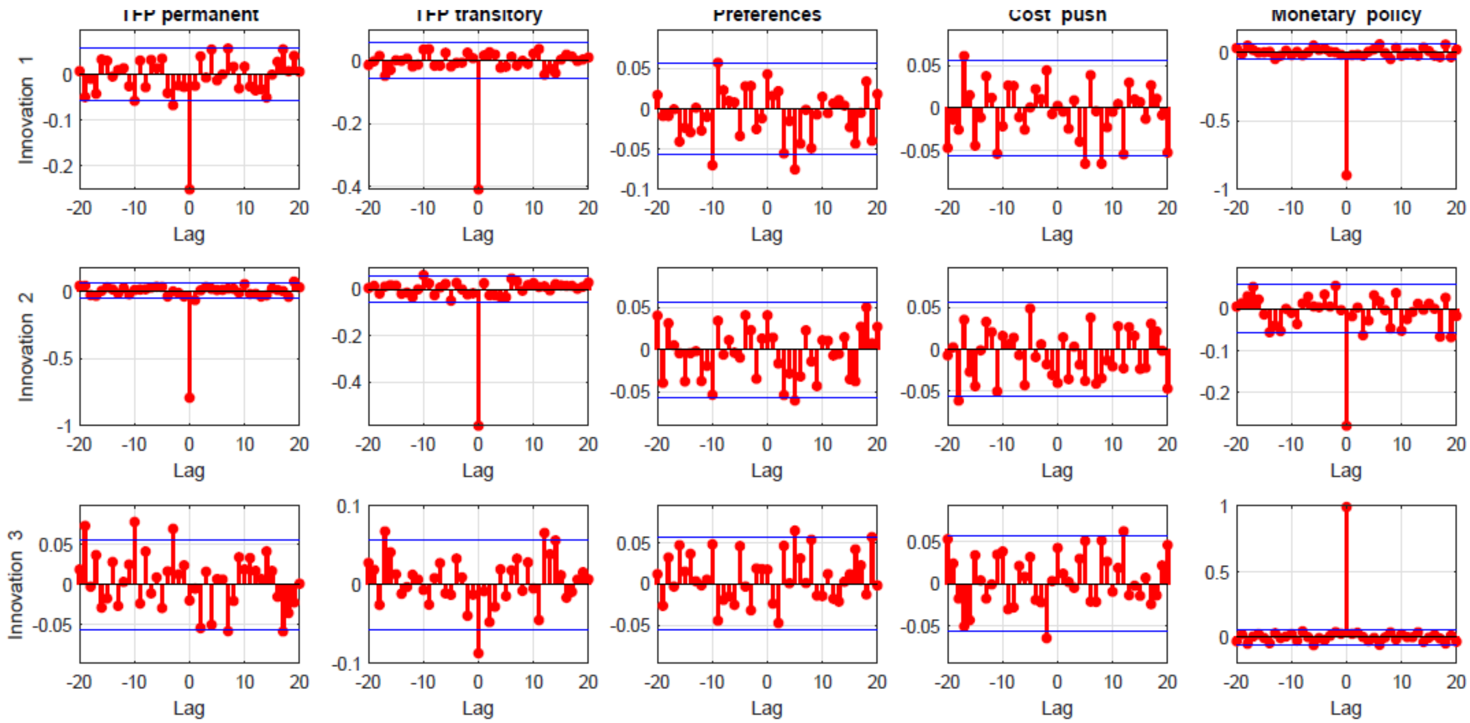
• **Cross-sectional deformation, limited time deformation distortions**.



Cross correlation function: $z_t = (o_t, \pi_t, n_t, r_t)$



Cross correlation function: $z_t = (o_t, \pi_t, n_t)$

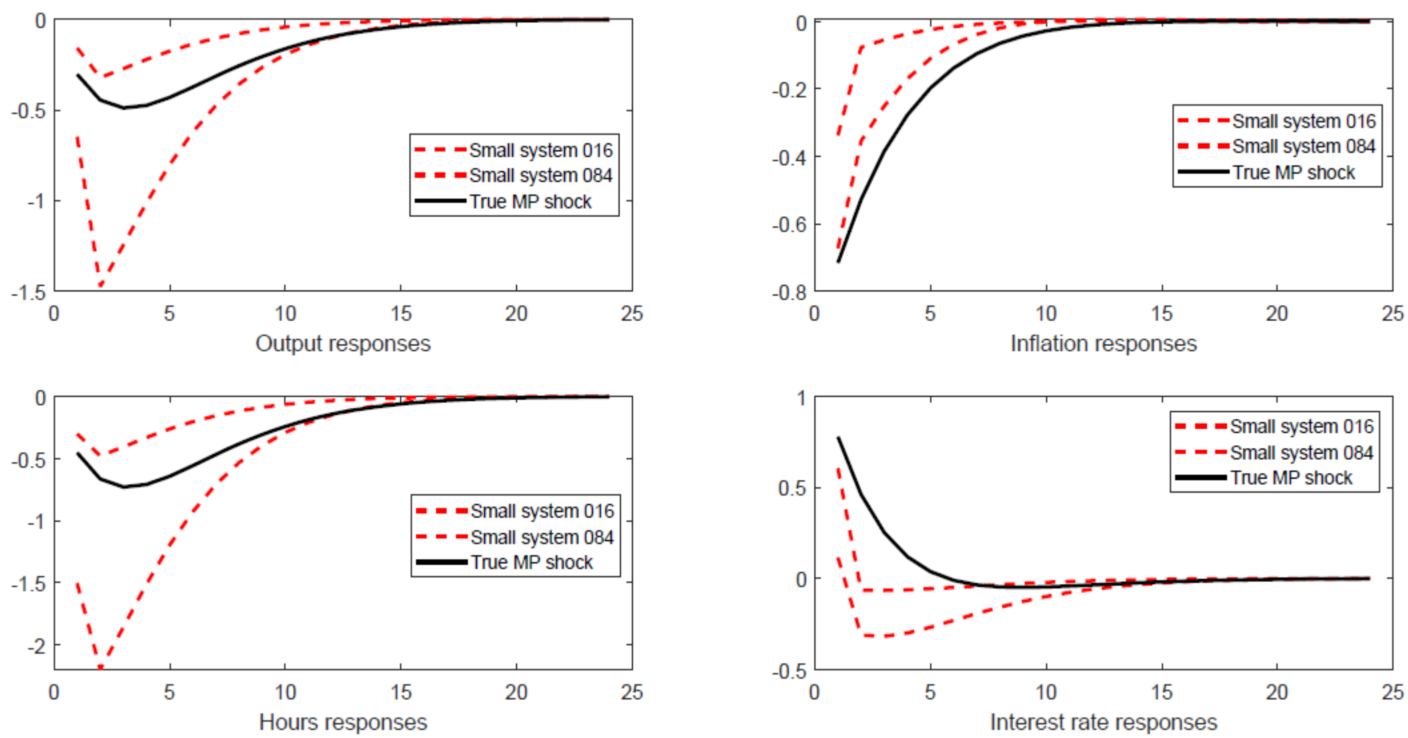


Cross correlation function: $z_t = (\pi_t, n_t, r_t)$

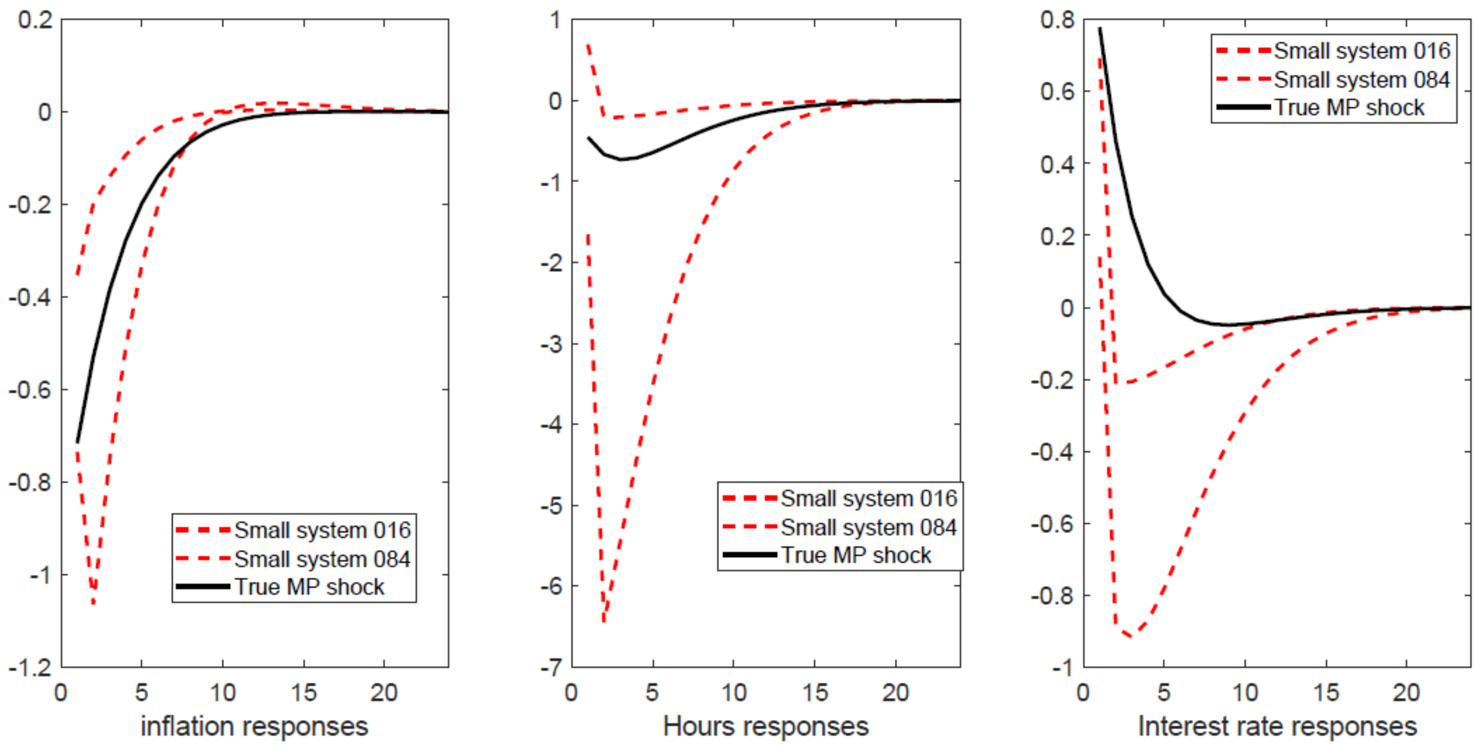
			Structural disturbances				
			a_t	ζ_t	χ_t	μ_t	ϵ_t
(y, π_t, n_t, r_t) innovations	λ_0	u_{1t}	0.018	-0.722	0.087	-0.005	-0.303
		u_{2t}	-0.158	-0.306	0.042	0.042	-0.716
		u_{3t}	-1.464	-1.078	0.131	-0.007	-0.452
		u_{4t}	-0.047	-0.086	0.014	0.012	0.778
(y_t, π_t, n_t) innovations	λ_0	u_{1t}	-0.05	0.71	0.11	0.03	-0.29
		u_{2t}	-0.19	-0.30	0.05	0.05	-0.70
		u_{3t}	-1.57	-1.06	-0.17	0.05	-0.43
	λ_1	u_{1t}	-0.07	-0.92	0.12	0.04	-0.41
		u_{2t}	-0.01	-0.28	0.03	0.01	-0.52
		u_{3t}	-0.25	-1.37	0.18	0.06	-0.61
	λ_2	u_{1t}	-0.05	-0.90	0.11	0.04	-0.46
		u_{2t}	-0.01	0.28	0.03	-0.01	-0.52
		u_{3t}	-0.09	-1.35	0.16	-0.07	-0.69

Wolf (2018): masquerading effects.

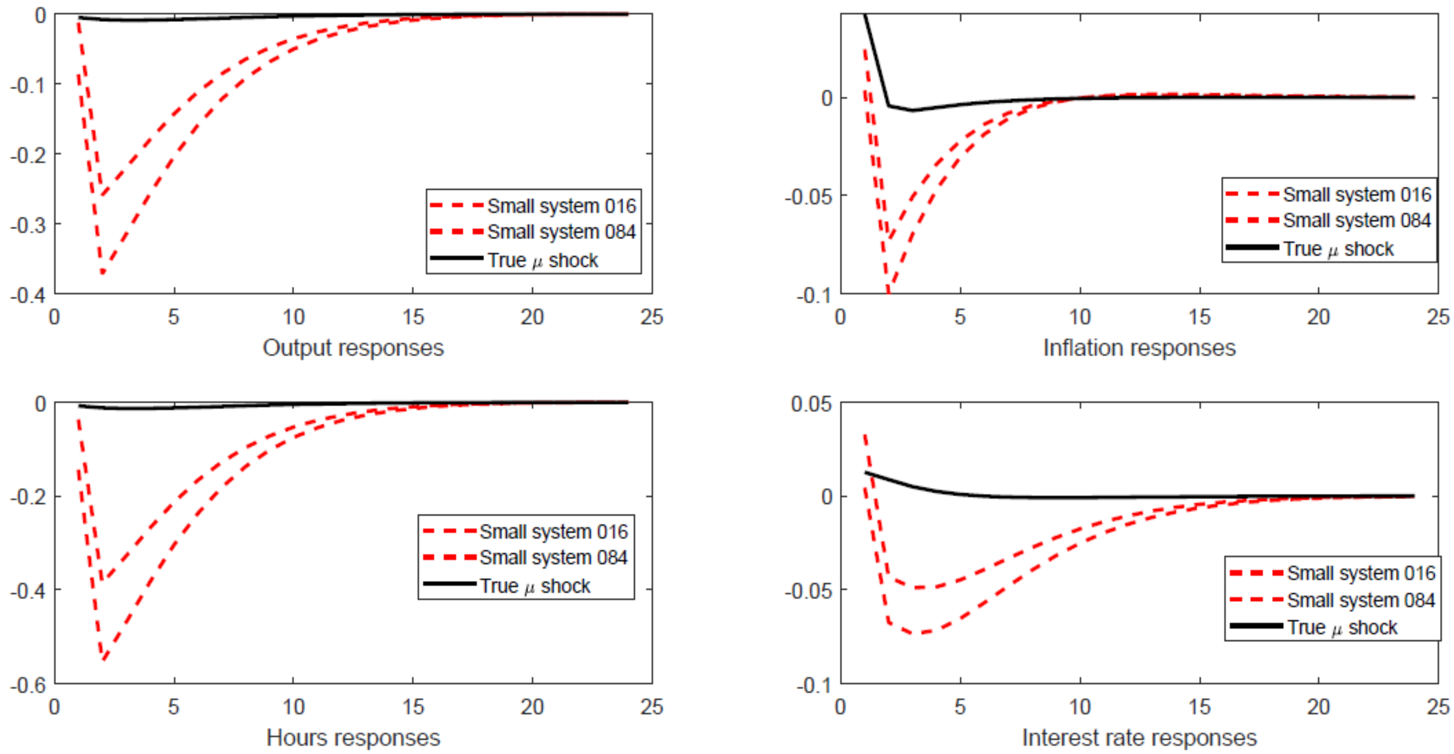
Impulse responses



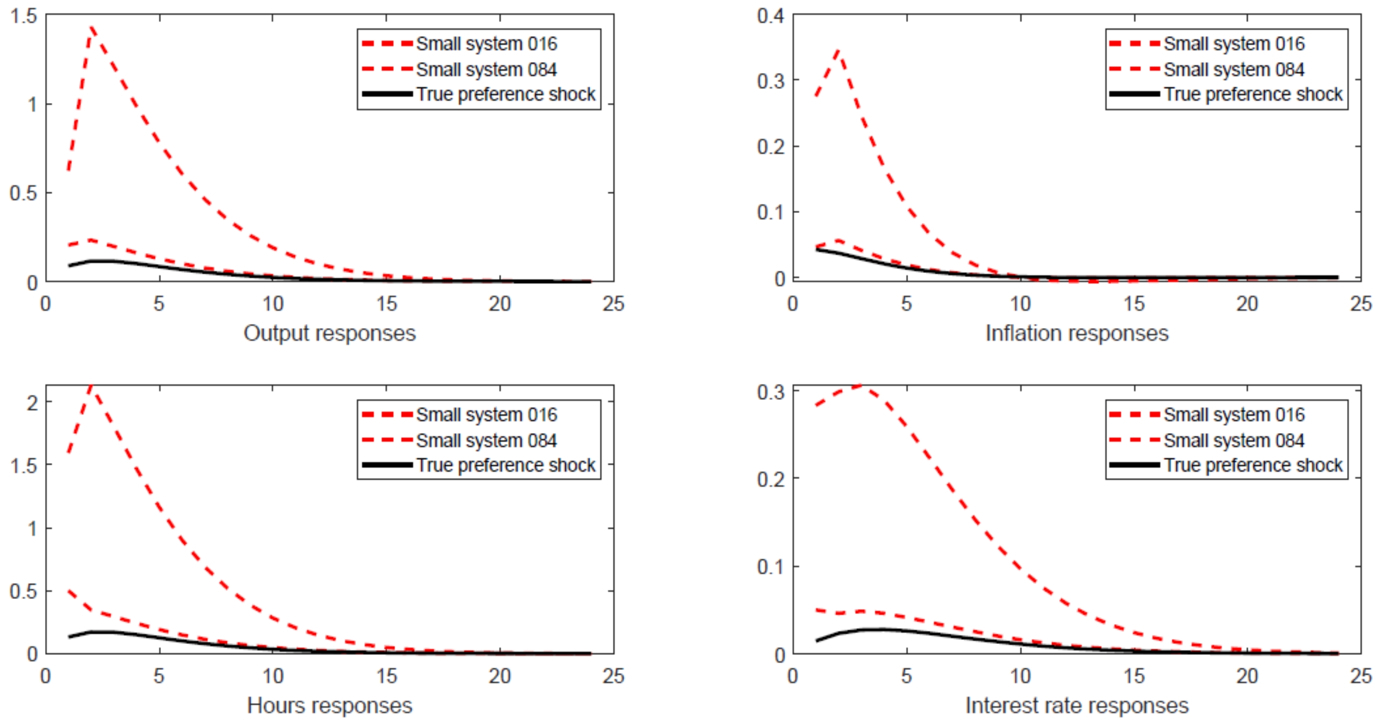
Monetary shocks, $z_t = (o_t, \pi_t, n_t, r_t)$



Monetary shocks, $z_t = (\pi_t, n_t, r_t)$.



Cost push shocks, $z_t = (o_t, \pi_t, n_t, r_t)$



Preference shocks, $z_t = (o_t, \pi_t, n_t, r_t)$.

Permanent TFP shocks: (g_t, n_t).

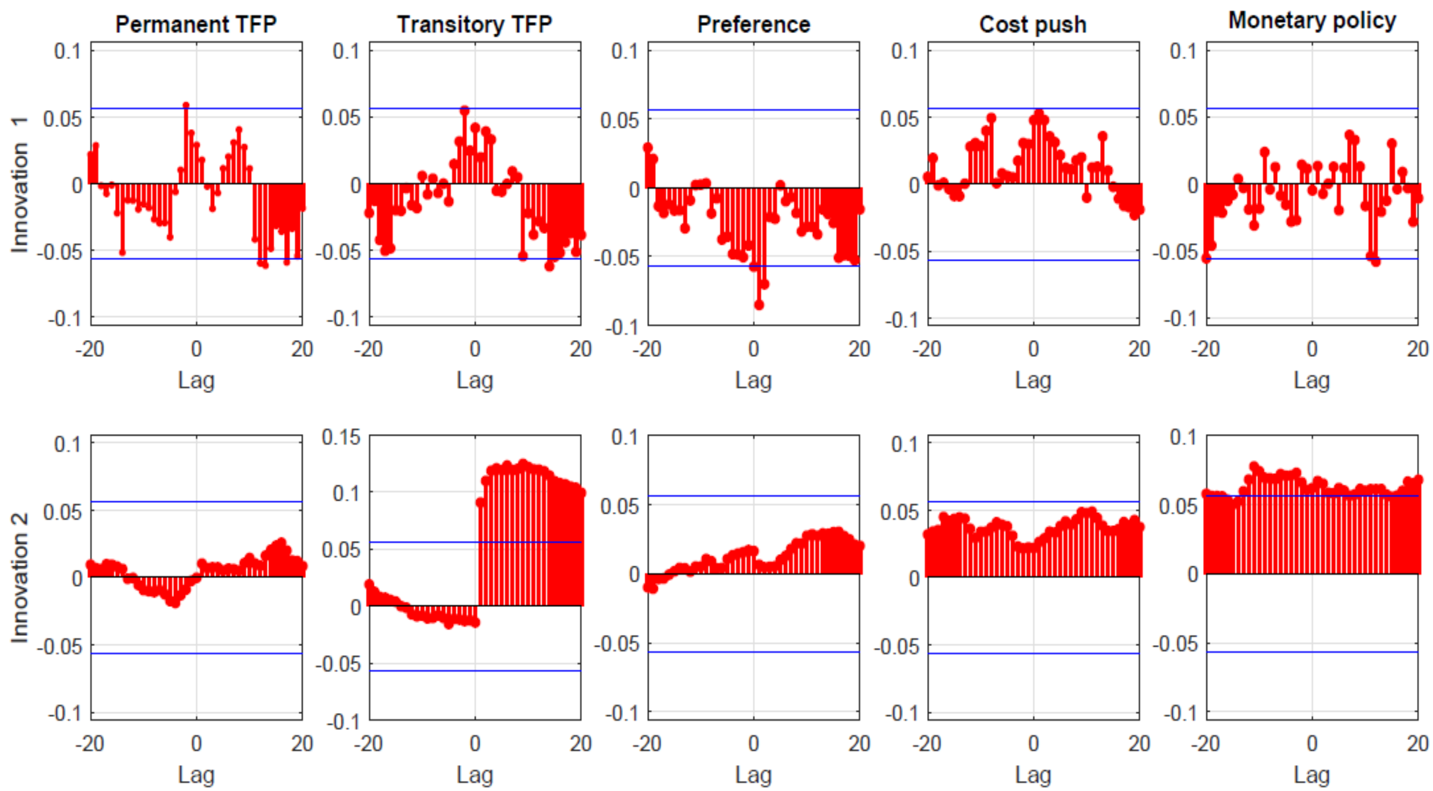
- Assume $\beta = (\rho_r + (1-\rho_r)\phi_p)^{-1}$.

$$\begin{aligned}(1+\rho_r)\chi_t &= \rho_r\chi_{t-1} + \chi_{t+1} + \frac{1}{1-h}g_{t+1} + (\frac{\rho_r + h}{1-h} + (1-\rho_r)\phi_y)g_t \\ &- \frac{h\rho_r}{1-h}g_{t-1} + \epsilon_{mp_t} + \kappa_p(\frac{h}{1-h}g_t + (1+\sigma_n)n_t) + \kappa_p(\mu_t - \chi_t) \end{aligned} \quad (34)$$

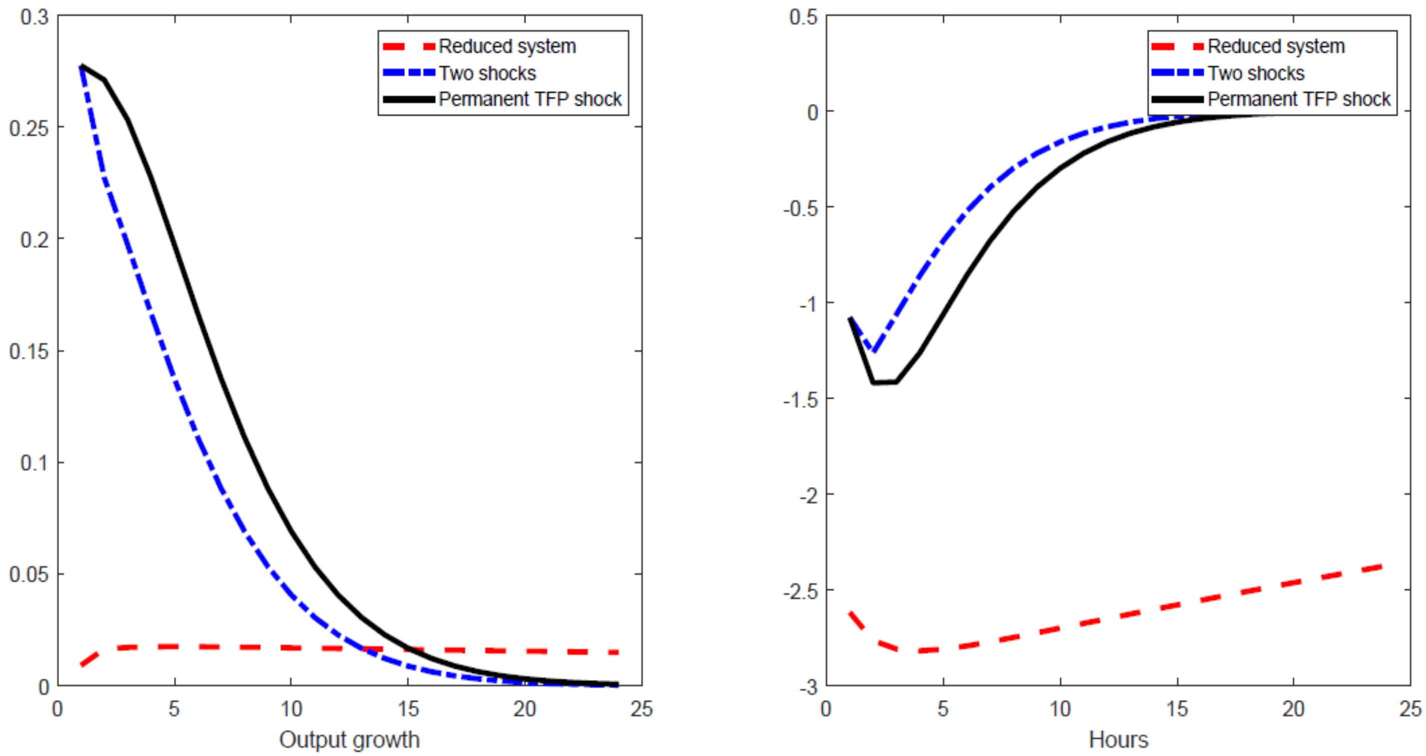
$$g_t = a_t + \zeta_t + (1-\alpha)n_t - \zeta_{t-1} - (1-\alpha)n_{t-1} \quad (35)$$

- $\hat{\mathrm{x}}_{t-1} = [\mathbf{g_{t-1}}, \mathbf{n_{t-1}}, \zeta_{t-1}, a_{t-1}, \mu_{t-1}, \chi_{t-1}]'$.

- Identify permanent TFP shocks via long run restrictions.



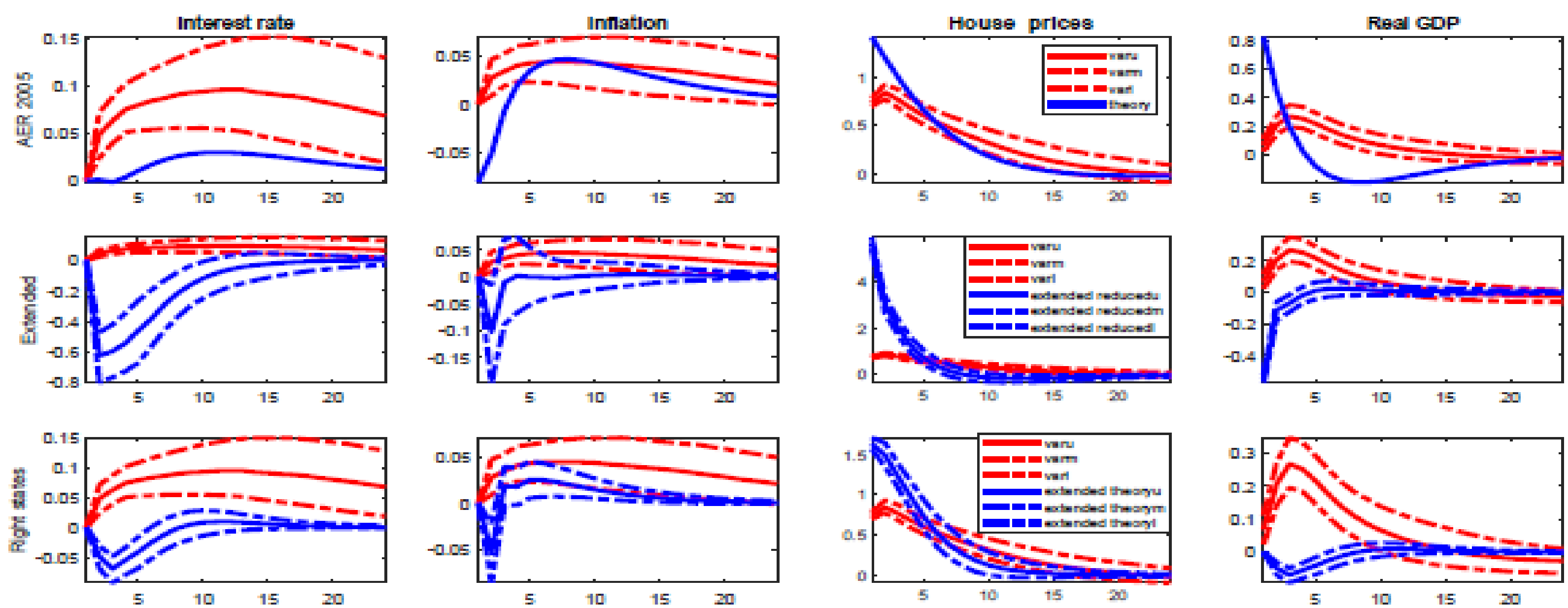
Permanent TFP
Transitory TFP
Preference
Cost push
Monetary policy
Innovation 1
Innovation 2
Lag
0.1
0.05
0
-0.05
-0.1
0.15
-20
20



- Hours fall; but impact and propagation is wrong if DGP has 5 shocks.

Transmission of house price shocks: Iacoviello (2005)

• Theory has preferences, technology, cost push, monetary policy disturbances. Only 4 disturbances in DGP? Add LTV constraint and impatient consumers wealth shocks (Rabanal,2018, Linde',2018).



Innovations	Disturbances						
	e_{Rt}	e_{jt}	e_{ut}	e_{at}	e_{hast}	e_{i_1t}	e_{i_2t}
R_t	1.0	0	0	0	0	0	0
π_t	-0.53	-0.003	1.43	-0.11	-0.13	0.18	0.24
q_t	**-1.83**	**0.05**	-0.80	0.13	0.33	**-1.27**	-0.51
y_t	-3.92	0.03	-1.14	-0.02	-0.09	2.46	0.92

• Cross correlation coefficients:

- q_t innovations - preference shocks e_{jt}: 0.67 (0.60,0.72).

- q_t innovations - preference shocks e_{i_1t}:-0.63 (-0.67,-0.59).

- q_t innovations - preference shocks e_{jt}: 0.92 (0.88, 0.96). (DGP with 4 disturbances)

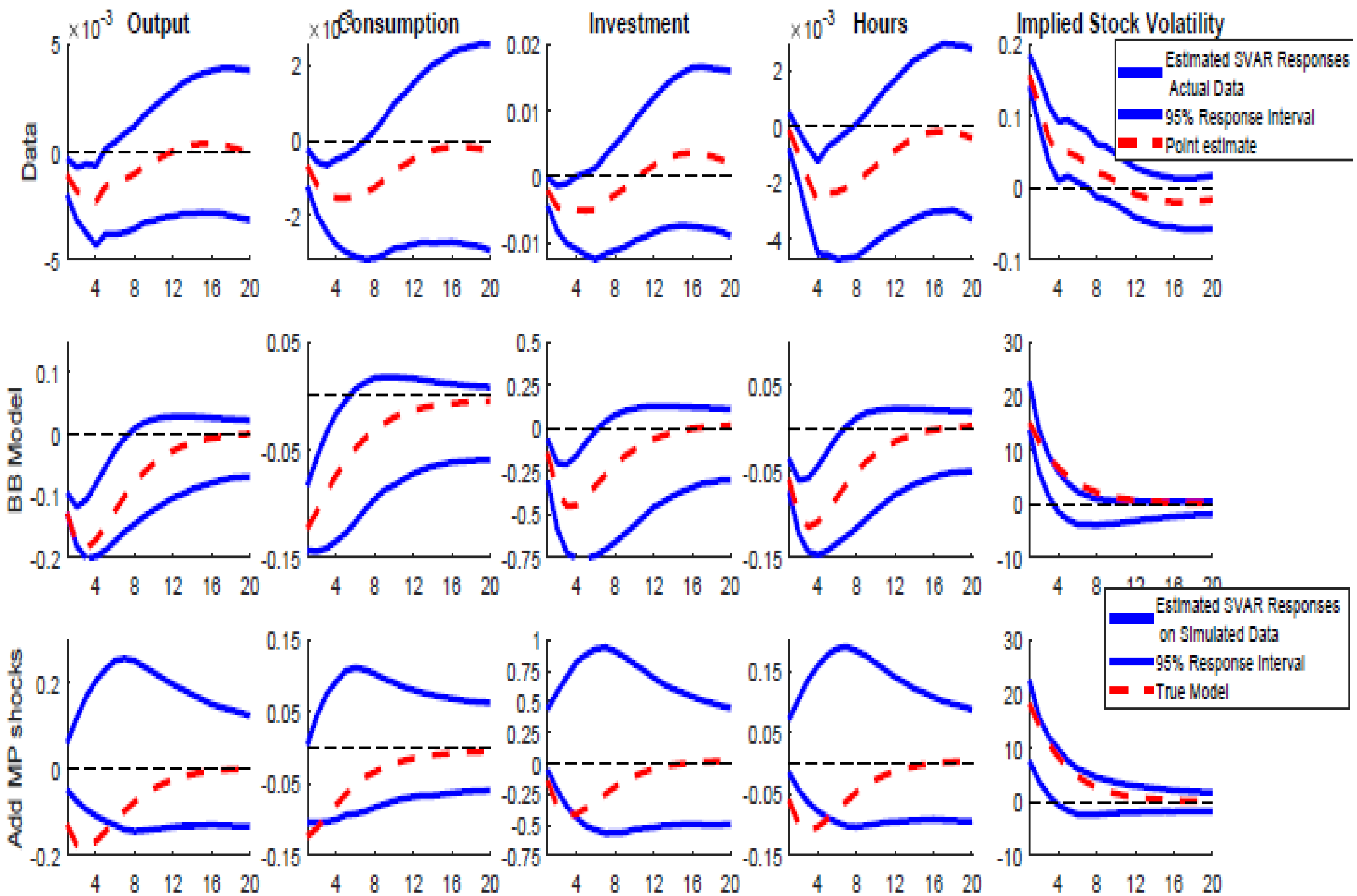
Transmission of uncertainty shocks: Basu and Bundick (2017)

• Model with TFP, preference and preference volatility disturbances. Solved with third order perturbation.

• Pruned solution linear but state and shock vector huge (432 states, 1112 shocks).

• Use a linear VAR with 8 variables for the data. Potentially huge deformation distortions.

• Monetary policy shock is missing. Nominal rate included in the VAR. What happens to the theory-data match if it is included?



Output
Consumption
Investment
Hours
Implied Stock Volatility
Data
BB Model
Add MP shocks
Estimated SVAR Responses
Actual Data
95% Response Interval
Point estimate
Estimated SVAR Responses
on Simulated Data
95% Response Interval
True Model

- Cross correlation coefficients:

- VIX innovations - uncertainty disturbances: 0.63 (0.50, 0.74).

- VIX innovations - monetary disturbances: -0.46 (-0.50, -0.41).

- VIX innovations - uncertainty disturbances: 0.77 (0.68, 0.86). (DGP with 3 disturbances)

Conclusions

- Deformation may make identified shocks and dynamics mongrels with little economic interpretation. Magnitude and sign distortions.

- Recovered shocks do not necessarily aggregate only structural disturbances of the same type (cross sectional deformation). Sound theory restrictions insufficient.

- Recovered shocks are linear combinations of current and past structural disturbances (time deformation).

- Empirical model used to derive dynamic facts might change depending on the theoretical model and the disturbances of interest.