SOME ECONOMETRIC ISSUES THAT ARISE IN MACROECONOMETRIC MOELLING

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MODELS AND THEIR USE

- Increasingly I have the impression that DSGE models are rarely used for normal policy discussion or prediction, in the sense of using the model to query policy options for regular decisions
- DSGE models do however represent a potential macro economy
- Leads to the question of what sort of issues one might have if these models generated the data and we wanted to replicate the information in them using less structured models.
- There are problems with extracting the information that DSGE models contain
- Talk is based on conference paper plus one that appeared in the Economic Record last year - dealt with an Australian Multi-Sector(MSM) model

FOUR ISSUES DEALT WITH

- 1. Can SVARs give useful information about DSGE Impulse Responses?
- Chari et al say “no”, Christiano et al say “yes”. Look at the reasons and find that SVARs can be good. Whether data are I(0) or I(1) is an issue.
- 2. Have empirical DSGE model shocks got their claimed properties? Probably not and this may be due to over-identification.
- 3. Number of shocks and observed variables. Can’t estimate former if greater than the latter. Issue neglected but comes up in many contexts e.g. computation of output and credit gaps
- 4. Problems with way researchers treat measurement error.

CAN SVARS GIVE USEFUL INFORMATION ABOUT DSGE IMPULSES RESPONSES?

- We look at this under the assumption that DSGE is correct
- DSGE (linearized) is a SVAR or order p in all model variables
- Impulse responses to shocks in it are generated recursively as Cj = B1*C(j-1)+..+Bp*C(j-p) ; C0= contemporaneous effects. B1,..,Bp are the implied DSGE VAR coefficients
- Cj is j step ahead structural impulse response
- Approximating with SVAR of order q in observed variables has responses generated in same way but using C0* and B1*,…,Bq* from it
- We start by fixing the C0 from the SVAR equal to that from the DSGE model i.e.   C0=C0*
- Then any differences in SVAR and DSGE responses are just due to differences in Bj and Bj*. Call this a truncation error.

WHY MIGHT THE VAR COEFFICIENTS BE DIFFERENT?

- DSGE models are VARs in n core variables (e.g. SW n=10)
- Generally have p=2
- But we may only have m <=n observables (e.g. SW m=7) (Researchers don't treat all variables as observed)
- The n-m unobservables need to be recovered by essentially regressing them against observables and their lags
- If the number of lags needed to recover them is large then the approximating VAR order q will be much greater than p
- When is this likely to happen?
- The nature of data affects the answer

NATURE OF DATA

- First look at case where series are I(0) and then go on to I(1)
- Unobservables that may be hard to approximate with observables are flex price quantities and stocks
- Find in paper that there are truncation issues when small open economy stock of debt is not observed
- Suggests one should always try to measure stocks
- Otherwise problem does not seem great except in SW with flex price monetary rule. Replace with observables in the rule and o.k.
- Illustrate these features with some models in the paper

SMALL OPEN ECONOMY MODEL WITH DEBT

- Model is Justiniano and Preston (JIE, 2010))
- Has 34 Variables, 13 core variables and 12 observed. So only one unobserved – external debt
- All variables I(0) but external debt is very close to I(1)
- Next show some monetary and tech shock impulse responses. "All" means all 13 variables are used and "obs" is only 12 observed ones
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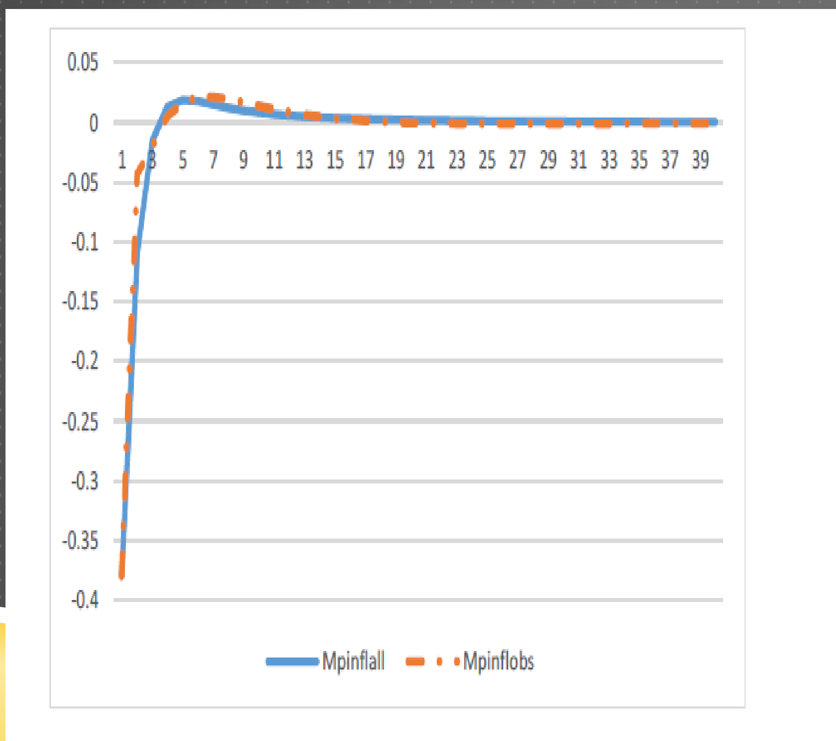
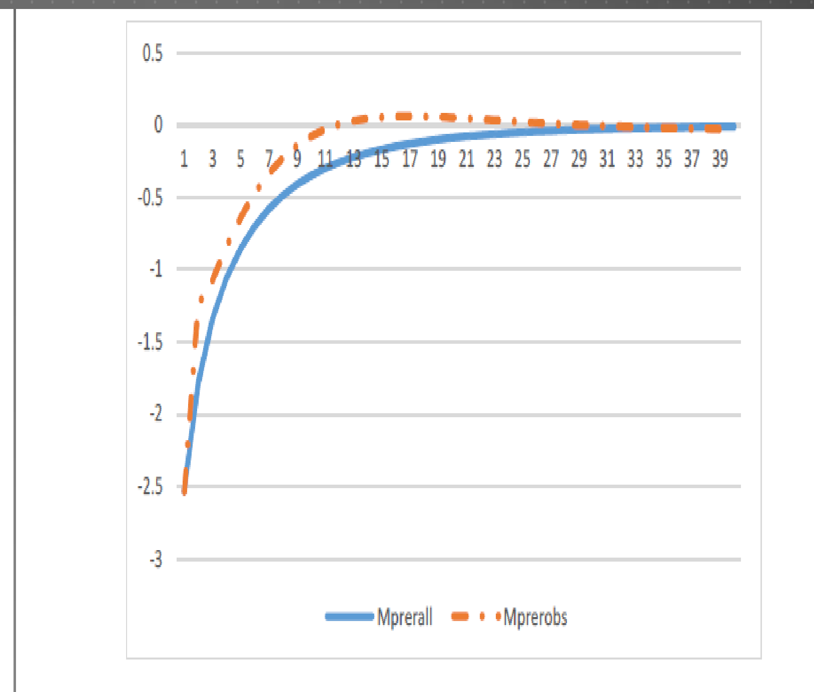
RESULTS FOR JUSTINIANO/PRESTON MODEL

Inflation/ Monetary Shock

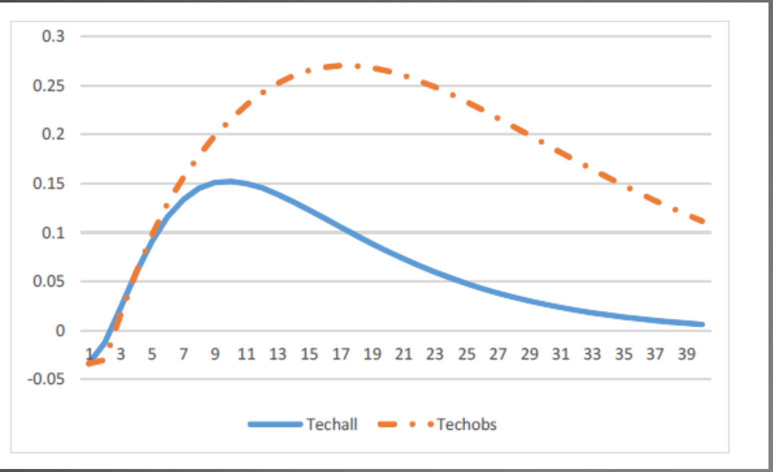


Real Ex Rate/ Monetary Shock



RESULTS FOR JUSTINIANO/PRESTON MODEL

Output/ Domestic Tech Shock



- Monetary shocks: SVAR captures DSGE quite well (at least up to 3 years)
- Real shocks (such as technology) DSGE returns to steady state much faster that $SVAR^{obs}$
- Accumulation of debt important for driving up risk premium in DSGE model
- $\therefore$ Stabilizing mechanism present in $SVAR^{all}$, only weakly present in $SVAR^{obs}$
- Omission of stock variables from SVAR is issue - Kapetanios et al.

THE MULTI-SECTOR MODEL OF REES ET AL (2016)

- The MSM is a small open economy DSGE model that has 3 producing sectors –non-tradeables, non-commodity exports and commodity exports.
- 77 variables but 23 core ones (others substitute out)
- There are 16 observable variables and 16 shocks- technology for all sectors plus foreign, monetary policy, cost shocks, preference shocks etc.
- 7 Unobserved variables
- Look first at truncation error

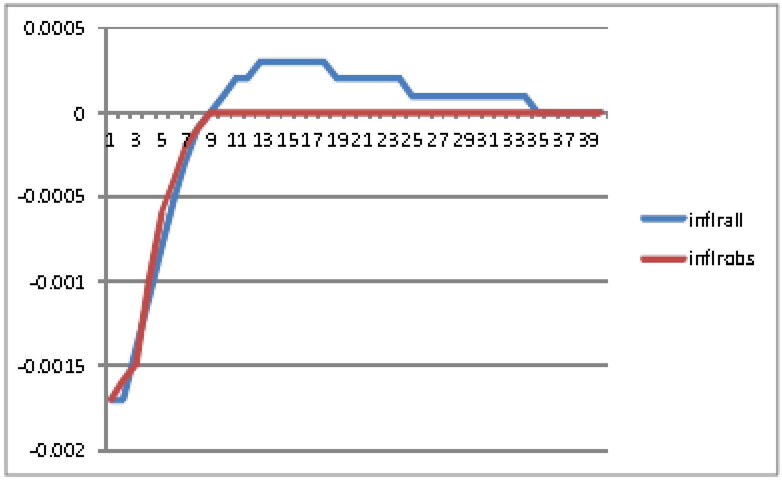


Figure 6: Impulse Response of Domestic Inflation to a Domestic Monetary Policy Shock, All and Just Observed Variables

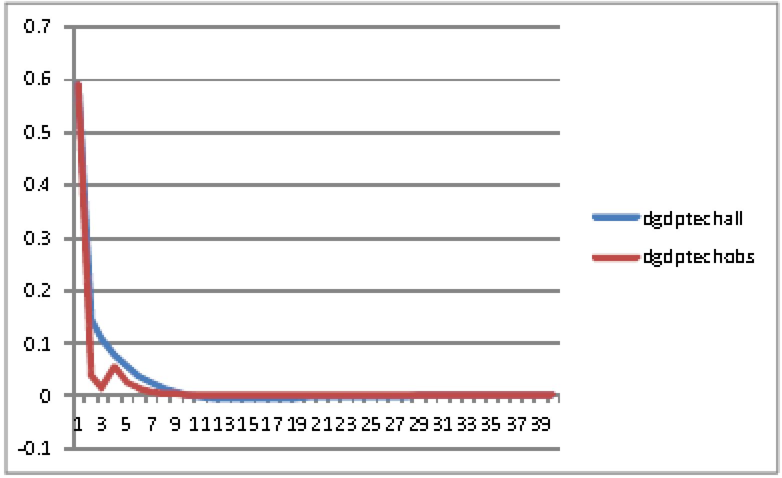


Figure 7: Impulse Response of Domestic GDP Growth to a Technology Shock, All and Just Observed Variables

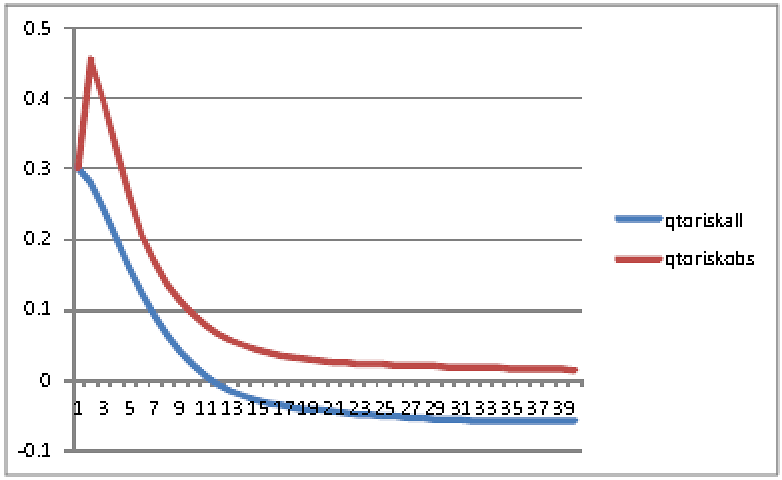


Figure 8: Impulse Response of the Real Exchange Rate to a Risk Premium Shock, All and Just Observed Variables

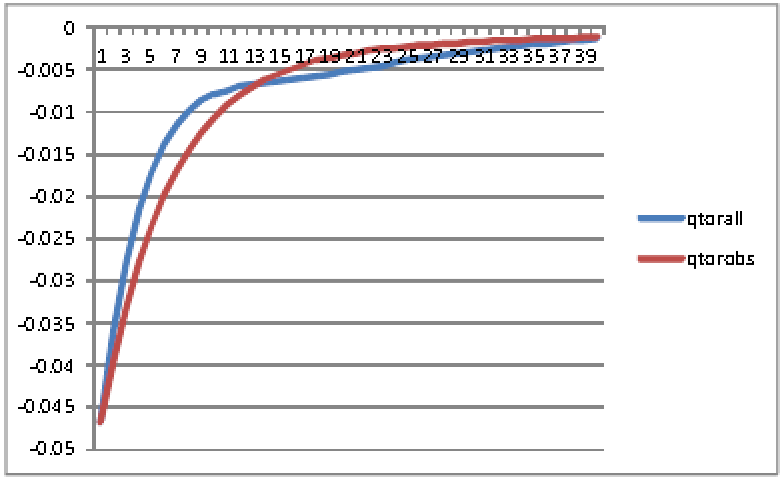


Figure 10: Impulse Response of the Real Exchange Rate to a Domestic Monetary Shock, All and Just Observed Variables

IMPLICATIONS OF THE APPROXIMATION

- Suggests that we can get reasonable estimates of the impulse responses using just observables by using an SVAR(2) when the MSM economy generates the data
- Points to the fact that one might use a model like FRB-US for modelling rather than a DSGE
- These are easier to build institutional features into
- Has been done in the Reserve Bank of Australia's MARTIN model (it is multi-sector)

LOOKING AT SHOCKS IN ESTIMATED DSGE MODELS

- In DSGE models shocks are assumed uncorrelated both with other shocks and across time when doing estimation
- An assumption does not make it true after estimation e.g. regression is often done assuming no serial correlation
- If the empirical shocks i.e. those constructed using the data, are contemporaneously correlated then
- (a) Can't do variance decompositions
- (b) Can't do variable decompositions
- Since can't vary one shock without affecting the other

HAVE SHOCKS GOT THE CLAIMED PROPERTIES?

- Why might this be a problem with DSGE models?
- Reason is that in an over-identified model i.e. more moments than parameters estimated, we can't satisfy **all** the moment conditions
- In MSM 45 parameters estimated and 600 moment conditions. So heavily over-identified
- There are 17 shocks so can't make all shocks uncorrelated with MLE or any Bayesian estimator
- The fact that you assume that in estimation doesn't make it hold.
- SVARs when exactly identified impose that assumption

THE MSM CORRELATIONS

Table 1 Correlations of Some Shocks from the Estimated MSM Model

$\mathrm{corr}(\varepsilon_{r,t}, \varepsilon_{y_t'})$		.67
$\mathrm{corr}(\varepsilon^*_{r,t}, \varepsilon_{\psi t})$		-.36
$\mathrm{corr}(\varepsilon_{\pi_t'}, \varepsilon_{\psi t})$		.44
$\mathrm{corr}(\varepsilon_{p't}, \varepsilon_{rt})$		.34
$\mathrm{corr}(\varepsilon_{gt}, \varepsilon_{\Upsilon_t})$		-.56
$\mathrm{corr}(\varepsilon^*_{r,t}, \mu_t)$		-.53
$\mathrm{corr}(\varepsilon_{ft}, \varepsilon_{nt})$		.78
$\mathrm{corr}(\varepsilon^*_{mt}, \varepsilon_{omt})$		.62
$\mathrm{corr}(\varepsilon_{mt}, \varepsilon_{nt})$		.32

SUPPOSE DATA IS I(1) AND THERE ARE I(1) TECHNOLOGY SHOCKS

- Take a basic RBC model that has log consumption and log of output. Technology is $a(t)=a(t-1)+\varepsilon(t)$ and there is a stationary AR(1) preference shock.
- Assume observables are $\Delta y(t)$, $\Delta c(t)$
- The model is a VECM that is driven by 2 EC terms – $EC1(t) = y(t)-c(t)$ and $EC2(t)=y(t)-a(t)$
- Because EC2(t) is unobservable and is missing from the observables VECM it may be hard to capture with $\Delta c(t)$ and $\Delta y(t)$. In simple RBC models like this 50 lags are needed to even get an R2 of .7
- Chari et al was about this case. They left $y(t)-a(t)$ out of all the specifications. Christiano et al don't have it as all variables are I(0)
- Poskitt and Yao (JBES, 2017) work with I(1) variables and say SVR can't capture RBC impulse responses

IS THERE A WAY TO OVERCOME THE OMISSION?

- Looked at that using a latent variable VECM
- This requires one to specify a latent process such as for technology just as in DSGE models
- That is not from economics but purely a statistical assumption
- Poskitt and Yao found that the response of hours to technology in a small RBC model was poorly captured by a SVAR
- This SVAR omitted a latent EC term
- Following graph shows that allowing for a latent EC term works well in their impulse response function that was badly estimated

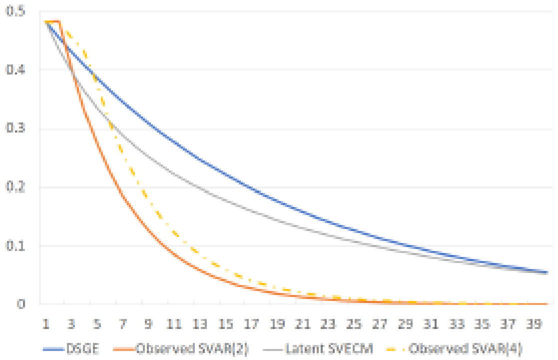


Figure 3: Impulse Response of Hours to a Technology Shock Using: (i) the RM of Poskitt and Yao (2017) (DSGE); (ii) a VAR(2) in the Observed Variables; (iii) a Latent-variable VECM and (iv) a VAR(4) in the Observed Variables

WHAT ABOUT C0 - THE CONTEMPORANEOUS MATRIX RESPONSES?

- How does the SVAR specification of C0 differ from a DSGE model?
- The problem is that DSGE structural equations generally have expectations in them. These are weighted averages of all the variables in the DSGE model
- This produces a SVAR equation but one couldn't estimate it as there are no excluded current endogenous variables or lags
- One might use a VAR to compute weights for the variables and so form the expectations
- If we don't then what do we do to estimate a C0 from the SVAR that is close to the DSGE one?

COMFAC RESTRICTIONS

- Suppose all variables observable
- Then the SVAR implied by most DSGE models is something like A0*z(t)=A1*z(t-1)+u(t). A0 will be functions of a smaller number of parameters than n^2 and there will be some zeroes in A1
- Suppose that none of these restrictions are known.
- Now DSGE models generally assume that u(t) follow univariate AR(1) processes with coefficients Φ and innovations ε(t)~N(0,I)
- This makes the implied SVAR a second order process but there are restrictions between the dynamics coming from the statistical assumption. These are COMFAC restrictions (Hendry and Mizon (1978))

COMFAC RESTRICTIONS

- In this SVAR there are 2*(n^2) parameters in A0 and A1 and n in Φ to estimate. There are 2*(n^2) parameters in VAR(2) and n(n+1)/2 in cov VAR residuals. Hence the model is over-identified by COMFAC restrictions
- Applying the instruments generated to the MSM external sector below we can estimate C0 very well
- Exact identification if Φ is triangular
- Next slides show COMFAC restrictions can estimate a standard NK model very well

Small New-Keynesian (NK) model

$$y_t = E_t(y_{t+1}) - (r_t - E_t(\pi_{t+1})) + u_{yt}$$

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa y_t + u_{\pi t}$$

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)(\gamma_y y_t + \gamma_\pi \pi_t) + \delta \Delta y_t + \varepsilon_{rt},$$

where u_{yt} and $u_{\pi t}$ follow AR(1) processes

$$u_{yt} = \rho_y u_{yt-1} + \varepsilon_{yt}$$

$$u_{\pi t} = \rho_\pi u_{\pi t-1} + \varepsilon_{\pi t},$$

$\varepsilon_{yt}, \varepsilon_{r\pi}, \varepsilon_{rt}$ uncorrelated with each other.

Estimated MSM values $\kappa = .036$, $\rho_\pi = .31$

SVAR equation for inflation

$$\pi_t = a_{21}^0 y_t + a_{23}^0 r_t + a_{22}^1 \pi_{t-1} + a_{21}^1 y_{t-1} + a_{23}^1 r_{t-1} + \varepsilon_{\pi t}.$$

Five parameters to be estimated

Only three instruments $y_{t-1}, r_{t-1}, \pi_{t-1}$.

But COMFAC restrictions mean π_t equation is

$$\pi_t = a_{21}^0 y_t + a_{23}^0 r_t + \rho_\pi u_{\pi t-1} + \varepsilon_{\pi t}.$$

Instruments $\pi_{t-1}, y_{t-1}, r_{t-1}$

$$Est : \pi_t = .192 y_t + .164 r_t + .31 u_{\pi t-1} + \varepsilon_{\pi t},$$

$$True : \pi_t = .197 y_t + .169 r_t + .31 u_{\pi t-1} + \varepsilon_{\pi t},$$

Excellent match, can recover MSM shock $\varepsilon_{\pi t}$.

COMFAC assumption needs to be used to reproduce MSM impulses

COMFAC restriction does not come from the microeconomic foundations of the model.

Works for other equations as well

Final equation in the MSM external system

But here we also use restriction that shocks are uncorrelated

SVAR equation for r_t (MSM values)

$$r_t = .928 r_{t-1} + .154 y_t - .139 y_{t-1} + .107 \pi_t.$$

Using these instruments produces (10K simulated points)

$$r_t = .928 r_{t-1} + .15 y_t - .135 y_{t-1} + .105 \pi_t,$$

Even with 200 observations

$$r_t = .942 r_{t-1} + .14 y_t - .116 y_{t-1} + .078 \pi_t,$$

NUMBER OF SHOCKS AND OBSERVED VARIABLES

- One cannot estimate shocks if more of them than observables
- Simplest example is y(t)=a(t)+b(t) and a(t), b(t) are both N(0,1)
- Kalman filter says a(t)=.5*y(t) so combination of a(t) and b(t)
- Many cases where this happens – Measurement errors, TVP shocks, output gaps from components models
- You can estimate the impulse responses of y(t) to a(t) and b(t) if you can estimate the variances. These were given above
- Measurement errors raise other issues, particularly when apply to growth rates

WHAT TO MAKE OF MEASUREMENT ERROR?

- Increasingly common to write for variables *data=model* + *"measurement error"*
- In MSM lots of variables are observed in growth or difference form and this is the structure
- Δy(t)= $\Delta\, \boldsymbol{y^M(t)}$ + ζ (t) – **where** $\boldsymbol{y^M(t)}$ is the model variable
- The problem is what is the its nature of ζ (t)
- Mostly assumed to be equal to white noise ε(t)

WHAT TO MAKE OF "MEASUREMENT ERROR"?

- Because $\Delta y(t) = \Delta\, \boldsymbol{y^M(t)} + \zeta(t)$ this means that the difference between y(t) and $\boldsymbol{y^M(t)}$ is the cumulated sum of $\zeta(t)$
- If $\zeta(t)$ is white noise this implies no co-integration between y(t) and $\boldsymbol{y^M(t)}$ (data and model variables)
- If have $\Delta\, \boldsymbol{y^1(t)}$ and $\Delta\, \boldsymbol{y^2(t)}$ with the errors $\zeta 1(t)$ and $\zeta 2(t)$ then no co-integration between data $\boldsymbol{y^1(t)}$ and $\boldsymbol{y^2(t)}$
- Latter can be tested
- You can get co-integration between y(t) and $\boldsymbol{y^M(t)}$ by using $\zeta(t) = \Delta\varepsilon(t)$ not $\varepsilon(t)$
- More generally you need $\zeta(t)$ to follow an error correction process
- So you need to be careful with specifying measurement error

CONCLUSIONS

- Although we looked at an estimated DSGE Model (MSM) the problems apply to calibrated models as well
- Bayesian estimation creates extra difficulties. In an exactly identified SVAR ML produces uncorrelated shocks as it imposes all moment conditions. Since Bayesian estimates are combinations of MLE and the prior they don't produce uncorrelated shocks unless the prior is irrelevant
- DSGE models are very useful for training and for thinking about issues but I don't think they are sort of model one wants to use in regualar discussion of policy