The Zero Lower Bound and Estimation Accuracy[1]

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[1] The views expressed in these slides are our own and do not necessarily reflect the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

MOTIVATION

- Estimating linear DSGE models is common
  - Fast and easy to implement
  - Used by many central banks
- Recent ZLB period calls into question linear methods
  - Creates a kink in the monetary policy rule
  - Linear methods ignore the effects of the ZLB
  - Leads to inaccurate estimates
  - Lower natural rate makes ZLB events more likely

Alternative Methods

1. Estimate fully nonlinear model (NL-PF)
    - Uses a projection method and particle filter
    - Most comprehensive treatment of the ZLB
    - Numerically very intensive
2. Estimate piecewise linear model (OB-IF)
    - Uses OccBin and an inversion filter
    - Almost as fast as linear methods
    - Captures the kink in the monetary policy rule
    - Ignores precautionary savings effects of the ZLB

CONTRIBUTION

- Compare the accuracy of the two methods
- Generate datasets from a medium-scale nonlinear model
- Generate many datasets with either:
  - No ZLB events
  - A single ZLB event with a fixed duration
- For each dataset, estimate a small-scale model
- Differences between the models creates misspecification
- Accounts for the reality that all models are misspecified

Related Literature

Key Findings

- NL-PF and OB-IF produce similar parameter estimates
- NL-PF predictions typically more accurate than OB-IF
  - Notional interest rate estimates
  - Expected ZLB duration
  - Probability of a $4+$ quarter ZLB event
  - Forecasts of the policy rate
- Increase in accuracy is often small because the precautionary savings effects of the ZLB and the effects of other nonlinearities are weak in canonical models

Data Generating Process

- Familiar medium-scale New Keynesian model
- One-period nominal bond
- Elastic labor supply and sticky wages
- Habit persistence and variable capital utilization
- Quadratic investment adjustment costs
- Monopolistically competitive intermediate firms
- Rotemberg quadratic price adjustment costs
- Occasionally binding ZLB constraint
- Risk premium, growth, and interest rate shocks

Estimation Methods

- Generate data by solving the nonlinear model ▸ Details
- Datasets: 50 for each ZLB duration, 120 quarters ▸ Details
- Estimated small-scale model is the DGP without:
  - Capital accumulation
  - Sticky wages
- Random walk Metropolis-Hastings algorithm:
  1. Mode Search ($5{,}000$ draws): initial covariance matrix
  2. Initial MH ($25{,}000$ draws): update covariance matrix
  3. Final MH ($50{,}000$ draws): calculate posterior mean
- Priors: Centered around truth ▸ Details
- Observables: Output growth, inflation rate, and nominal interest rate ▸ Details

Speed Tests

	NL-PF (16 Cores)	OB-IF (1 Core)	Lin-KF (1 Core)
	No ZLB Events		
Seconds per draw	6.7 $(6.1, 7.9)$	0.035 $(0.031, 0.040)$	0.002 $(0.002, 0.004)$
Hours per dataset	148.8 $(134.9, 176.5)$	0.781 $(0.689, 0.889)$	0.052 $(0.044, 0.089)$
	30 Quarter ZLB Events		
Seconds per draw	8.4 $(7.5, 9.5)$	0.096 $(0.051, 0.135)$	0.002 $(0.001, 0.003)$
Hours per dataset	186.4 $(167.6, 210.7)$	2.137 $(1.133, 3.000)$	0.049 $(0.022, 0.067)$

Parameter Estimates: No ZLB Events

Ptr	Truth	NL-PF-5%	OB-IF-0%	Lin-KF-5%
φ_p	100	151.1 (134.2, 165.8) [0.52]	142.6 (121.1, 157.3) [0.44]	151.4 (134.0, 165.7) [0.52]
h	0.8	0.66 (0.62, 0.70) [0.18]	0.64 (0.61, 0.67) [0.20]	0.66 (0.62, 0.69) [0.18]
ρ_s	0.8	0.76 (0.72, 0.80) [0.06]	0.76 (0.73, 0.81) [0.05]	0.76 (0.72, 0.80) [0.06]
ρ_i	0.8	0.79 (0.75, 0.82) [0.03]	0.76 (0.71, 0.79) [0.06]	0.79 (0.75, 0.82) [0.03]
σ_z	0.005	0.0032 (0.0023, 0.0039) [0.37]	0.0051 (0.0044, 0.0058) [0.09]	0.0032 (0.0023, 0.0039) [0.36]
σ_s	0.005	0.0052 (0.0040, 0.0066) [0.15]	0.0051 (0.0042, 0.0063) [0.13]	0.0053 (0.0040, 0.0067) [0.15]
σ_i	0.002	0.0017 (0.0014, 0.0020) [0.17]	0.0020 (0.0018, 0.0023) [0.08]	0.0017 (0.0015, 0.0020) [0.16]
ϕ_π	2.0	2.04 (1.88, 2.19) [0.06]	2.01 (1.84, 2.16) [0.06]	2.04 (1.88, 2.20) [0.06]
ϕ_y	0.5	0.35 (0.21, 0.54) [0.36]	0.32 (0.17, 0.48) [0.41]	0.35 (0.22, 0.54) [0.35]
Σ		[1.90]	[1.53]	[1.88]

Parameter Estimates: 30Q ZLB Events

Ptr	Truth	NL-PF-5%	OB-IF-0%	Lin-KF-5%
φ_p	100	188.4 (174.7, 202.7) [0.89]	183.4 (169.2, 198.5) [0.84]	191.6 (175.3, 204.1) [0.92]
h	0.8	0.68 (0.64, 0.71) [0.16]	0.63 (0.60, 0.67) [0.21]	0.67 (0.63, 0.70) [0.17]
ρ_s	0.8	0.81 (0.78, 0.84) [0.03]	0.82 (0.79, 0.86) [0.04]	0.82 (0.78, 0.86) [0.04]
ρ_i	0.8	0.80 (0.75, 0.84) [0.03]	0.77 (0.73, 0.81) [0.05]	0.84 (0.80, 0.88) [0.06]
σ_z	0.005	0.0040 (0.0030, 0.0052) [0.23]	0.0059 (0.0050, 0.0069) [0.22]	0.0043 (0.0030, 0.0057) [0.20]
σ_s	0.005	0.0050 (0.0039, 0.0062) [0.13]	0.0046 (0.0036, 0.0056) [0.15]	0.0047 (0.0037, 0.0061) [0.15]
σ_i	0.002	0.0015 (0.0013, 0.0019) [0.24]	0.0020 (0.0019, 0.0024) [0.09]	0.0016 (0.0014, 0.0019) [0.20]
ϕ_π	2.0	2.13 (1.94, 2.31) [0.09]	1.96 (1.77, 2.14) [0.06]	1.73 (1.52, 1.91) [0.15]
ϕ_y	0.5	0.42 (0.27, 0.62) [0.28]	0.44 (0.27, 0.61) [0.25]	0.32 (0.17, 0.47) [0.40]
Σ		[2.08]	[1.91]	[2.28]

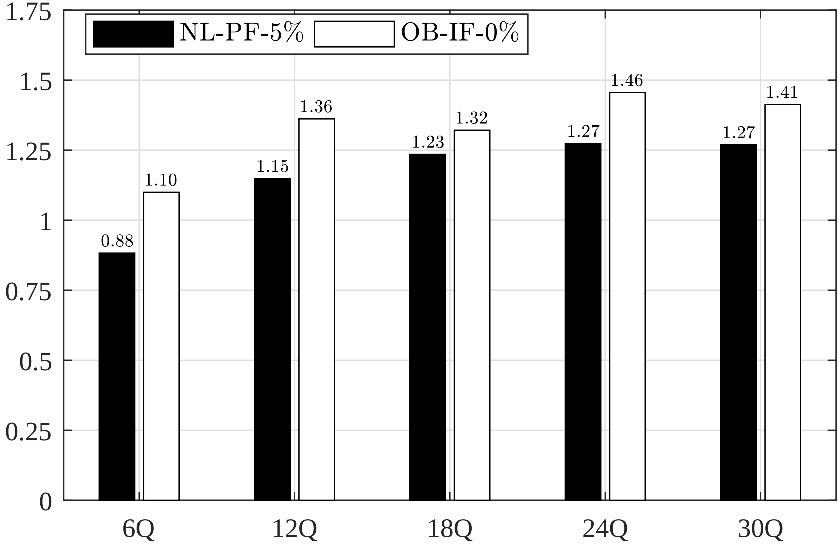
Lower Misspecification: No ZLB Events

Ptr	Truth	OB-IF-0%	OB-IF-0%-Sticky Wages	OB-IF-0%-DGP
φ_p	100	142.6 (121.1, 157.3) [0.44]	100.1 (76.9, 119.6) [0.13]	101.4 (80.1, 120.7) [0.12]
h	0.8	0.64 (0.61, 0.67) [0.20]	0.82 (0.78, 0.86) [0.04]	0.81 (0.75, 0.85) [0.04]
ρ_s	0.8	0.76 (0.73, 0.81) [0.05]	0.82 (0.76, 0.86) [0.04]	0.80 (0.76, 0.85) [0.03]
ρ_i	0.8	0.76 (0.71, 0.79) [0.06]	0.80 (0.77, 0.83) [0.02]	0.79 (0.75, 0.82) [0.03]
σ_z	0.005	0.0051 (0.0044, 0.0058) [0.09]	0.0038 (0.0031, 0.0044) [0.24]	0.0047 (0.0039, 0.0054) [0.11]
σ_s	0.005	0.0051 (0.0042, 0.0063) [0.13]	0.0085 (0.0056, 0.0134) [0.81]	0.0060 (0.0043, 0.0084) [0.30]
σ_i	0.002	0.0020 (0.0018, 0.0023) [0.08]	0.0020 (0.0018, 0.0022) [0.08]	0.0020 (0.0018, 0.0022) [0.08]
ϕ_π	2.0	2.01 (1.84, 2.16) [0.06]	1.91 (1.74, 2.04) [0.07]	1.92 (1.72, 2.08) [0.06]
ϕ_y	0.5	0.32 (0.17, 0.48) [0.41]	0.40 (0.24, 0.58) [0.28]	0.41 (0.24, 0.57) [0.26]
Σ		[1.53]	[1.71]	[1.03]

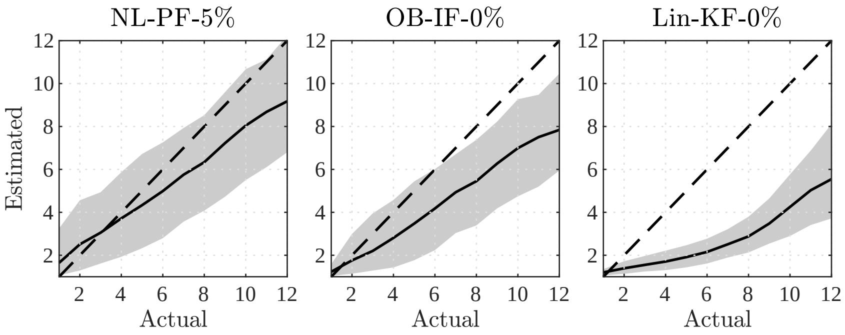
Lower Misspecification: 30Q ZLB Events

Ptr	Truth	OB-IF-0%	OB-IF-0%-Sticky Wages	OB-IF-0%-DGP
φ_p	100	183.4 (169.2, 198.5) [0.84]	129.8 (105.5, 152.3) [0.33]	128.4 (109.0, 148.1) [0.31]
h	0.8	0.63 (0.60, 0.67) [0.21]	0.80 (0.77, 0.85) [0.03]	0.77 (0.72, 0.84) [0.06]
ρ_s	0.8	0.82 (0.79, 0.86) [0.04]	0.84 (0.80, 0.88) [0.06]	0.82 (0.79, 0.86) [0.04]
ρ_i	0.8	0.77 (0.73, 0.81) [0.05]	0.80 (0.77, 0.84) [0.03]	0.79 (0.75, 0.83) [0.03]
σ_z	0.005	0.0059 (0.0050, 0.0069) [0.22]	0.0047 (0.0039, 0.0055) [0.12]	0.0055 (0.0047, 0.0066) [0.15]
σ_s	0.005	0.0046 (0.0036, 0.0056) [0.15]	0.0074 (0.0050, 0.0107) [0.60]	0.0051 (0.0039, 0.0068) [0.19]
σ_i	0.002	0.0020 (0.0019, 0.0024) [0.09]	0.0020 (0.0018, 0.0023) [0.08]	0.0020 (0.0018, 0.0024) [0.09]
ϕ_π	2.0	1.96 (1.77, 2.14) [0.06]	1.81 (1.63, 1.99) [0.11]	1.81 (1.62, 2.03) [0.11]
ϕ_y	0.5	0.44 (0.27, 0.61) [0.25]	0.50 (0.33, 0.73) [0.23]	0.50 (0.32, 0.74) [0.24]
Σ		[1.91]	[1.59]	[1.23]

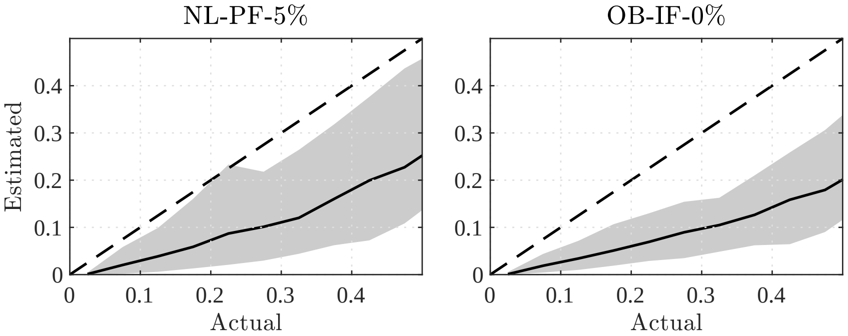
Notional Interest Rate Accuracy



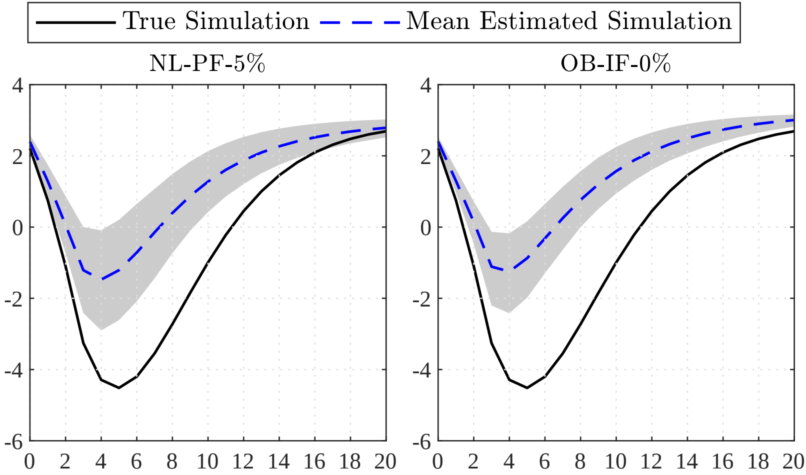
Expected ZLB Durations



4+ Quarter ZLB Event Probability

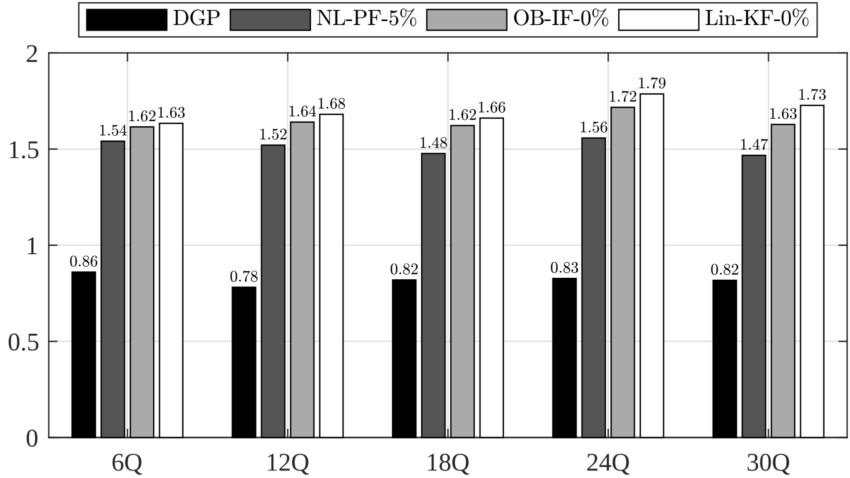


Notional Interest Rate Response



▸ Output Growth ▸ Inflation ▸ No Misspecification

Interest Rate Forecast Accuracy



Example Output Growth Inflation

Conclusion

- Two promising methods for dealing with ZLB:
  - Estimate the fully nonlinear model with a particle filter
  - Estimate the piecewise linear model with an inversion filter
- NL-PF is typically more accurate than OB-IF but the differences are often small
- Much larger gains in accuracy from estimating a richer, less misspecified piecewise linear model
- Important to examine whether findings are generalizable
- Nonlinear model is considerably more versatile

Additional Material

Related Literature

- Estimation accuracy using artificial datasets:
  - Fernandez-Villaverde and Rubio-Ramirez (2005): RBC model using linear and nonlinear methods
  - Hirose and Inoue (2016): New Keynesian model with a ZLB constraint using linear methods

- Estimates of global nonlinear models with actual data: (Gust et al., 2017; Iiboshi et al., 2018; Plante et al., 2018; Richter and Throckmorton, 2016)

- Effect of positive ME variances on parameter estimates: (Canova et al., 2014, Cuba-Borda et al., 2017, Herbst and Schorfheide, 2017)

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Adapted Particle Filter

1. Initialize the filter by drawing from the ergodic distribution.
2. For all particles $p \in \{1, \dots, N_p\}$ apply the following steps:
   2.1 Draw $\mathbf{e}_{t,p} \sim \mathbb{N}(\bar{\mathbf{e}}_t, I)$, where $\bar{\mathbf{e}}_t$ maximizes $p(\xi_t|\mathbf{z}_t)p(\mathbf{z}_t|\mathbf{z}_{t-1})$.
   2.2 Obtain $\mathbf{z}_{t,p}$ and the vector of variables, $\mathbf{w}_{t,p}$, given $\mathbf{z}_{t-1,p}$
   2.3 Calculate, $\xi_{t,p} = \hat{\mathbf{x}}_{t,p}^{model} - \hat{\mathbf{x}}_t^{data}$. The weight on particle p is

$$\omega_{t,p} = \frac{p(\xi_t|\mathbf{z}_{t,p})p(\mathbf{z}_{t,p}|\mathbf{z}_{t-1,p})}{g(\mathbf{z}_{t,p}|\mathbf{z}_{t-1,p}, \hat{\mathbf{x}}_t^{data})} \propto \frac{\exp(-\xi_{t,p}' H^{-1}\xi_{t,p}/2)\exp(-\mathbf{e}_{t,p}'\mathbf{e}_{t,p}/2)}{\exp(-(\mathbf{e}_{t,p} - \bar{\mathbf{e}}_t)'(\mathbf{e}_{t,p} - \bar{\mathbf{e}}_t)/2)}$$

   The model's likelihood at t is $\ell_t^{model} = \sum_{p=1}^{N_p} \omega_{t,p}/N_p$.

   2.4 Normalize the weights, $W_{t,p} = \omega_{t,p}/\sum_{p=1}^{N_p} \omega_{t,p}$. Then use systematic resampling with replacement from the particles.
3. Apply step 2 for $t \in \{1, \dots, T\}$. $\log \ell^{model} = \sum_{t=1}^{T} \log \ell_t^{model}$.

Particle Adaption

1. Given $\mathbf{z}_{t-1}$ and a guess for $\bar{\mathrm{e}}_t$, obtain $\mathbf{z}_t$ and $\mathbf{w}_{t,p}$.
2. Calculate $\xi_t = \hat{\mathbf{x}}_t^{model} - \hat{\mathbf{x}}_t^{data}$, which is multivariate normal:

$$p(\xi_t|\mathbf{z}_t) = (2\pi)^{-3/2}|H|^{-1/2}\exp(-\xi_t' H^{-1}\xi_t/2)$$
$$p(\mathbf{z}_t|\mathbf{z}_{t-1}) = (2\pi)^{-3/2}\exp(-\bar{\mathbf{e}}_t'\bar{\mathbf{e}}_t/2)$$

   $H \equiv \mathrm{diag}(\sigma^2_{me,\hat{y}}, \sigma^2_{me,\pi}, \sigma^2_{me,i})$ is the ME covariance matrix.

3. Solve for the optimal $\bar{\mathrm{e}}_t$ to maximize

$$p(\xi_t|\mathbf{z}_t)p(\mathbf{z}_t|\mathbf{z}_{t-1}) \propto \exp(-\xi_t' H^{-1}\xi_t/2)\exp(-\bar{\mathbf{e}}_t'\bar{\mathbf{e}}_t/2)$$

   We converted MATLAB's `fminsearch` routine to Fortran.

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Nonlinear Solution Method

- Use linear solution as an initial conjecture: $\tilde{c}^A(\mathbf{z}_t)$, $\pi^A(\mathbf{z}_t)$
- For all nodes $d \in D$, implement the following steps:
  1. Solve for $\{\tilde{w}_t, \tilde{y}_t, i_t^n, i_t, \tilde{\lambda}_t\}$ given $\tilde{c}_{i-1}^A(\mathbf{z}_t^d)$ and $\pi_{i-1}^A(\mathbf{z}_t^d)$
  2. Use piecewise linear interpolation to solve for updated values of consumption and inflation, $\{\tilde{c}_{t+1}^m, \pi_{t+1}^m\}_{m=1}^M$, given each realization of the updated state vector, $\mathbf{z}_{t+1}$
  3. Given $\{\tilde{c}_{t+1}^m, \pi_{t+1}^m\}_{m=1}^M$, solve for future output, $\{\tilde{y}_{t+1}^m\}_{m=1}^M$, which enters expectations. Then numerically integrate.
  4. Use Chris Sims' `csolve` to determine the values of the policy functions that best satisfy the equilibrium system
- On iteration i, $\text{maxdist}_i \equiv \max\{|\tilde{c}_i^A - \tilde{c}_{i-1}^A|, |\pi_i^A - \pi_{i-1}^A|\}$. Continue iterating until $\text{maxdist}_i < 10^{-6}$ for all d

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Prior Distributions

Parameter		Dist.	Mean	SD
Rotemberg Price Adjustment Cost	φ	Norm	100.0	25.00
Inflation Gap Response	ϕ_π	Norm	2.000	0.250
Output Gap Response	ϕ_y	Norm	0.500	0.250
Habit Persistence	h	Beta	0.800	0.100
Risk Premium Shock Persistence	ρ_s	Beta	0.800	0.100
Notional Rate Persistence	ρ_i	Beta	0.800	0.100
Growth Rate Shock SD	σ_z	IGam	0.005	0.005
Risk Premium Shock SD	σ_s	IGam	0.005	0.005
Notional Rate Shock SD	σ_i	IGam	0.002	0.002

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State and Observation Equations

- Linear model

$$\hat{\mathbf{s}}_t = T(\vartheta)\hat{\mathbf{s}}_{t-1} + M(\vartheta)\varepsilon_t$$
$$\hat{\mathbf{x}}_t = H\hat{\mathbf{s}}_t + \xi_t$$

- Nonlinear Model

$$\mathbf{s}_t = \Psi(\vartheta, \mathbf{s}_{t-1}, \varepsilon_t)$$
$$\mathbf{x}_t = H\mathbf{s}_t + \xi_t$$

$\mathbf{x}_t = [y^g_t, \pi_t, i_t]$ (observables), $\varepsilon_t = [\varepsilon_{z,t}, \varepsilon_{s,t}, \varepsilon_{i,t}]$ (shocks), $\xi \sim \mathbb{N}(0, R)$ (measurement errors), ϑ (parameters), $\mathbf{s}_t = [\tilde{c}, n, \tilde{y}, \tilde{y}^{gdp}, y^g, \tilde{w}, \pi, i, i^n, mc, \tilde{\lambda}, z, s]$ (states)

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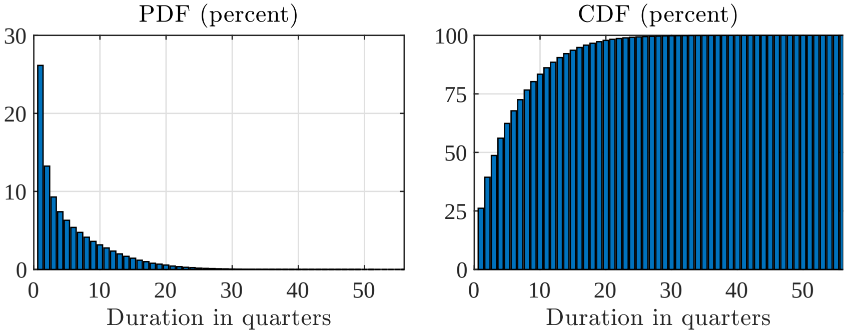
Dataset Statistics

	6Q	12Q	18Q	24Q	30Q
CDF of ZLB Durs	0.678	0.885	0.966	0.992	0.998
Sims to 50 Datasets	150,300	154,950	256,950	391,950	1,030,300



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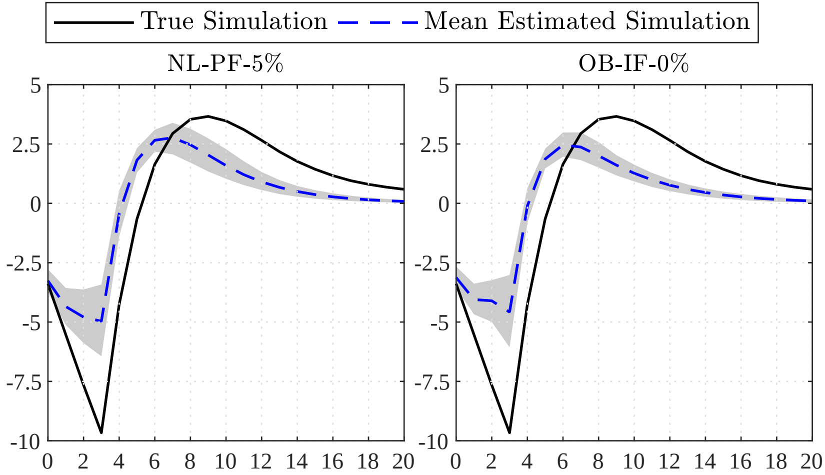
Output Growth Response



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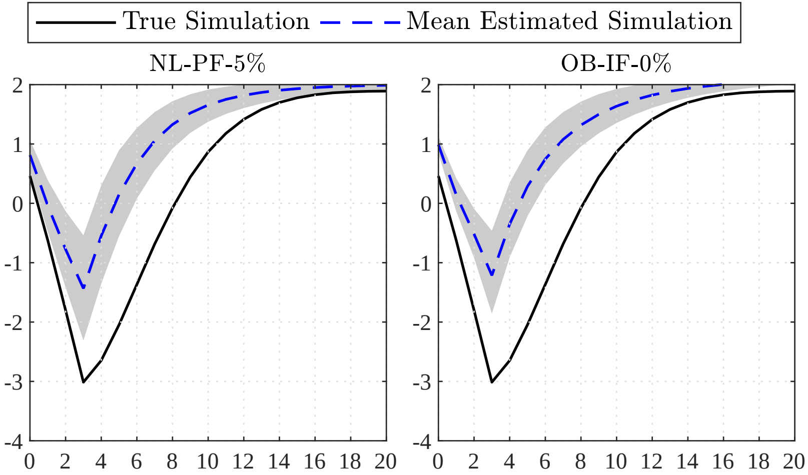
Inflation Rate Response



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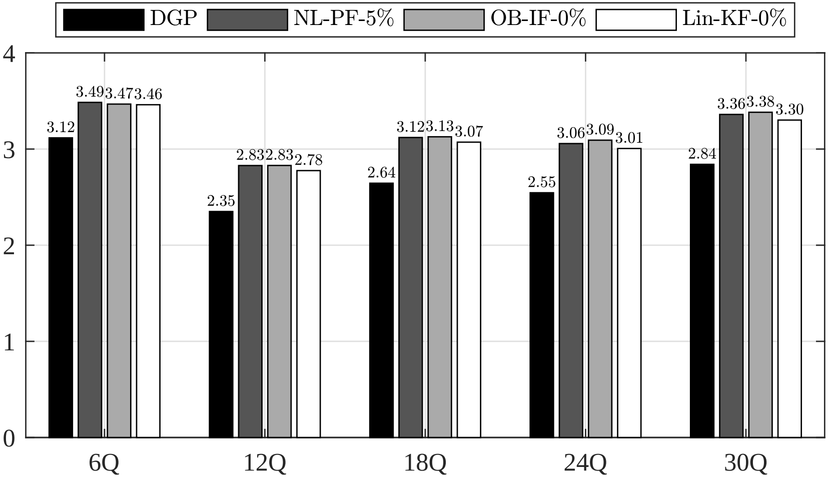
OUTPUT GROWTH FORECAST ACCURACY



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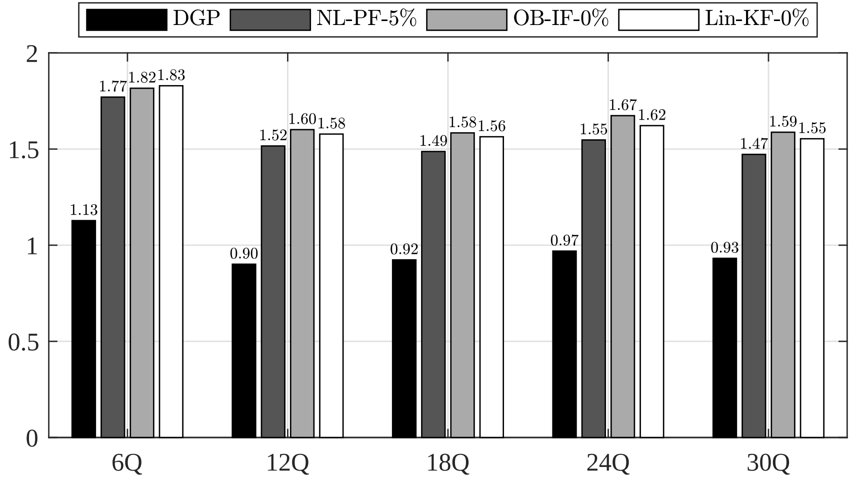
Inflation Rate Forecast Accuracy



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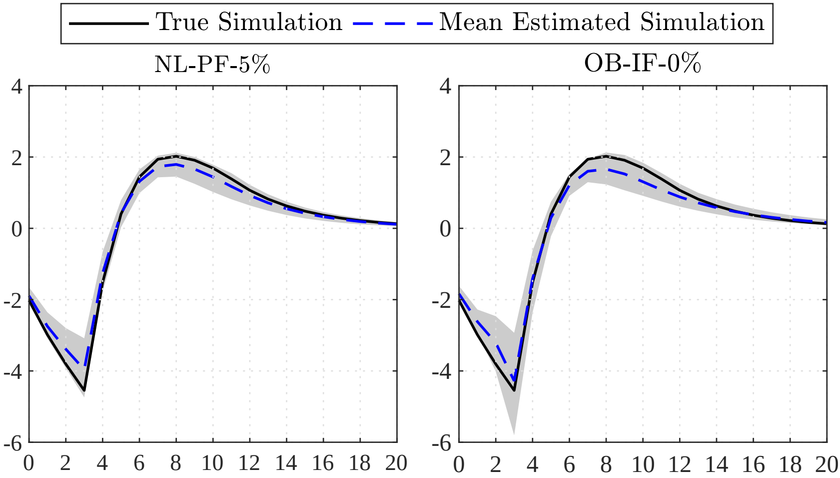
No Misspecification: No ZLB Events

Ptr	Truth	NL-PF-5%	OB-IF-0%	Lin-KF-5%
φ_p	100	96.8 (81.6, 109.9) [0.09]	94.3 (81.8, 108.3) [0.11]	103.7 (92.6, 118.4) [0.09]
h	0.8	0.79 (0.76, 0.82) [0.02]	0.79 (0.75, 0.82) [0.02]	0.80 (0.76, 0.83) [0.02]
ρ_s	0.8	0.80 (0.76, 0.83) [0.03]	0.81 (0.76, 0.85) [0.04]	0.82 (0.77, 0.86) [0.05]
ρ_i	0.8	0.82 (0.79, 0.84) [0.03]	0.79 (0.77, 0.82) [0.02]	0.82 (0.79, 0.84) [0.03]
σ_z	0.005	0.0037 (0.0029, 0.0046) [0.27]	0.0051 (0.0044, 0.0056) [0.08]	0.0038 (0.0029, 0.0046) [0.26]
σ_s	0.005	0.0047 (0.0035, 0.0058) [0.19]	0.0049 (0.0039, 0.0060) [0.16]	0.0047 (0.0034, 0.0059) [0.21]
σ_i	0.002	0.0016 (0.0013, 0.0020) [0.20]	0.0020 (0.0017, 0.0022) [0.07]	0.0016 (0.0013, 0.0019) [0.20]
ϕ_π	2.0	2.00 (1.81, 2.21) [0.06]	1.95 (1.74, 2.14) [0.06]	1.97 (1.76, 2.18) [0.07]
ϕ_y	0.5	0.45 (0.29, 0.61) [0.22]	0.46 (0.30, 0.63) [0.21]	0.46 (0.31, 0.63) [0.22]
Σ		[1.12]	[0.78]	[1.14]

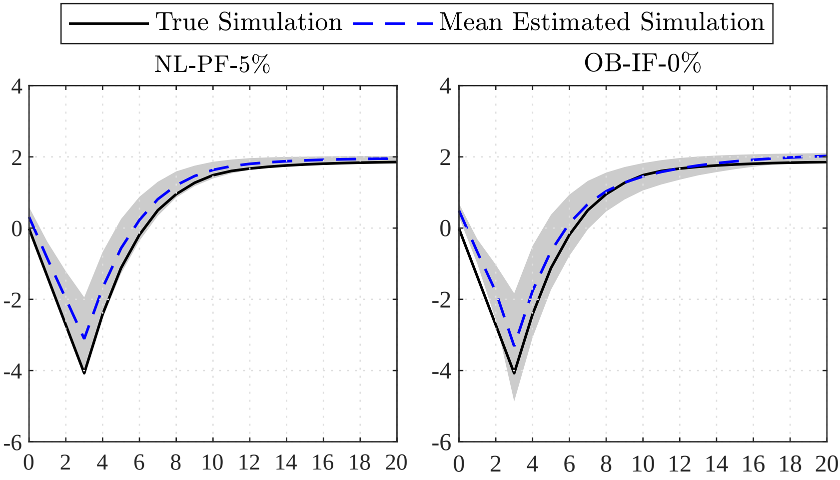
No Misspecification: 30Q ZLB Events

Ptr	Truth	NL-PF-5%	OB-IF-0%	Lin-KF-5%
φ_p	100	109.8 (89.5, 130.3) [0.15]	110.6 (95.3, 125.1) [0.15]	128.5 (111.2, 145.3) [0.30]
h	0.8	0.79 (0.77, 0.82) [0.02]	0.79 (0.77, 0.82) [0.02]	0.79 (0.76, 0.82) [0.03]
ρ_s	0.8	0.83 (0.78, 0.86) [0.04]	0.84 (0.80, 0.87) [0.06]	0.87 (0.83, 0.91) [0.10]
ρ_i	0.8	0.82 (0.78, 0.85) [0.03]	0.79 (0.74, 0.82) [0.03]	0.86 (0.83, 0.88) [0.08]
σ_z	0.005	0.0035 (0.0025, 0.0045) [0.33]	0.0052 (0.0043, 0.0061) [0.11]	0.0034 (0.0026, 0.0044) [0.33]
σ_s	0.005	0.0043 (0.0032, 0.0058) [0.22]	0.0046 (0.0034, 0.0057) [0.17]	0.0036 (0.0027, 0.0046) [0.32]
σ_i	0.002	0.0014 (0.0010, 0.0018) [0.31]	0.0019 (0.0016, 0.0022) [0.10]	0.0015 (0.0012, 0.0017) [0.27]
ϕ_π	2.0	2.01 (1.82, 2.20) [0.06]	1.80 (1.58, 2.06) [0.12]	1.62 (1.42, 1.86) [0.20]
ϕ_y	0.5	0.48 (0.28, 0.61) [0.18]	0.52 (0.32, 0.73) [0.23]	0.50 (0.34, 0.66) [0.19]
Σ		[1.35]	[0.99]	[1.82]

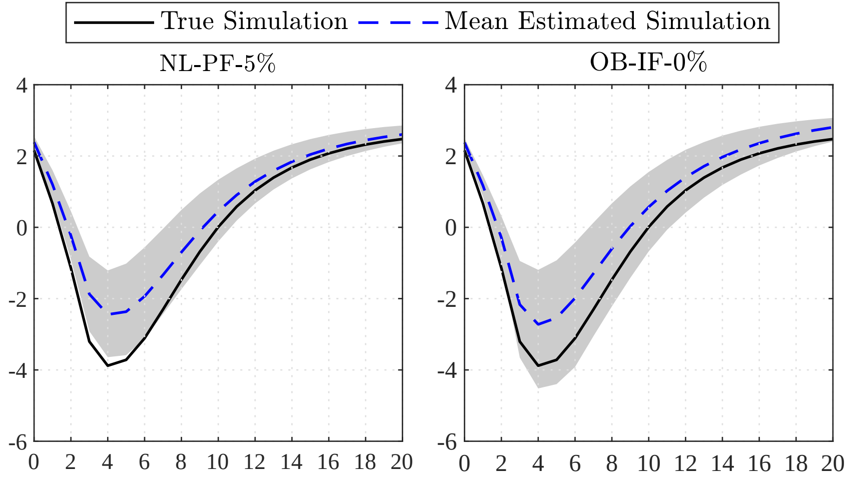
No Misspecification: Output Growth



No Misspecification: Inflation Rate



No Misspecification: Notional Rate



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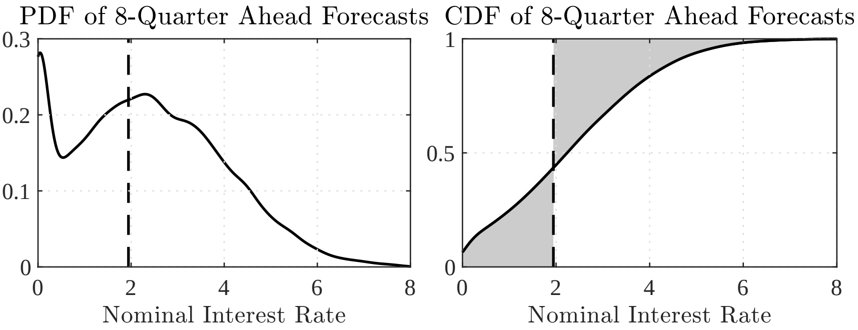
Forecast Accuracy Example



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