# Expectations-Driven Liquidity Traps: Implications for Monetary and Fiscal Policy*

Taisuke Nakata† Federal Reserve Board

Sebastian Schmidt‡ European Central Bank

First Draft: February 2019
This Draft: June 2019

**Abstract**

We study optimal monetary and fiscal policy in a New Keynesian model where occasional declines in agents' confidence can give rise to persistent liquidity trap episodes. Unlike in the case of fundamental-driven liquidity traps, there is no straightforward recipe for mitigating the welfare costs and the systematic inflation shortfall associated with expectations-driven liquidity traps. Raising the inflation target or appointing an inflation-conservative central banker improves inflation outcomes away from the lower bound but exacerbates the shortfall at the lower bound. Using government spending as an additional policy tool worsens stabilization outcomes both at and away from the lower bound. However, appointing a policymaker who is sufficiently less concerned with government spending stabilization than society can eliminate expectations-driven liquidity traps altogether.

*Keywords:* Effective Lower Bound, Sunspot Equilibria, Monetary Policy, Fiscal Policy, Discretion, Policy Delegation

*JEL-Codes:* E52, E61, E62

*We thank an anonymous referee of the ECB working paper series, Boragan Aruoba, Jean Barthélemy, Pablo Cuba-Borda, Alex Cukierman, Antoine Lepetit, Bartosz Maćkowiak, Morten Ravn, Yoichiro Tamanyu, and participants at various conferences and seminars for helpful comments. We would also like to thank Leonie Braeuer for her excellent research assistance. An earlier version of the paper circulated under the title "Simple Analytics of Expectations-Driven Liquidity Traps". The views expressed in this paper should be regarded as those solely of the authors, and are not necessarily those of the Federal Reserve Board of Governors, the Federal Reserve System, or the European Central Bank.

†Board of Governors of the Federal Reserve System, Division of Research and Statistics, 20th Street and Constitution Avenue N.W. Washington, D.C. 20551; Email: taisuke.nakata@frb.gov.

‡European Central Bank, Monetary Policy Research Division, 60640 Frankfurt, Germany; Email: sebastian.schmidt@ecb.int.

## 1 Introduction

The recent decade of low nominal interest rates and anemic inflation poses new challenges for monetary and fiscal policy. Current policy frameworks were predominantly designed at a time when the lower bound on nominal interest rates was not a major concern for central banks, and discretionary fiscal policy was not widely considered as an essential part of stabilization policies.[1]

This paper studies the implications of the lower bound on nominal interest rates for optimal monetary and fiscal policy. The analysis is based on a New Keynesian model that can be solved in closed form. The key difference to the existing literature on optimal policy with a lower bound is that we consider an equilibrium where liquidity trap episodes—i.e. periods where the lower bound constraint is binding—result from a decline in agents' confidence rather than from a deterioration of economic fundamentals. In this sunspot equilibrium, liquidity traps are rare but long-lasting events characterized by subdued economic activity and deflation that give rise to a systematic inflation shortfall in all states of the world.

We focus on two questions. First, is there a straightforward way to improve stabilization outcomes and welfare, taking as given the occasional occurrence of such expectations-driven liquidity traps? Second, is it possible to prevent the economy from falling into an expectations-driven liquidity trap? Following the policy delegation literature (e.g. Rogoff, 1985; Walsh, 1995; Svensson, 1997), we address these questions by assuming that society designs the policy framework and a discretionary policymaker sets the policy instruments in accordance with the assigned objective function.

We first study monetary policy in the absence of fiscal policy. In this case, the policymaker has only one instrument, the short-term nominal interest rate. The existing literature based on models with fundamental-driven liquidity traps suggests that society can improve stabilization outcomes and welfare by imposing a positive inflation target or by making inflation stabilization the primary policy objective (Nakata and Schmidt, 2019a).[2] The latter approach is usually referred to as inflation conservatism and goes back to Rogoff (1985).[3] These two approaches have in common that they raise inflation away from the lower bound. In models with fundamental-driven liquidity traps, higher inflation away from the lower bound mitigates the drop in output and inflation at the lower bound. However, in our model with expectations-driven liquidity traps, we find that increasing the inflation target or appointing an inflation-conservative central banker further reduces output and inflation in the state where confidence is low and the lower bound is binding. As a result, in the sunspot equilibrium, the welfare implications of these two policy delegation schemes are ambiguous. Indeed, we find that the optimal inflation target can be negative or positive. Likewise,

[1] At the time of writing, the U.S. Federal Reserve and the Bank of Canada are officially reviewing their monetary policy frameworks. Both central banks explicitly refer to the challenges for monetary policy associated with the lower bound (Wilkins, 2018; Clarida, 2019).

[2] Other monetary policy delegation schemes that are known to be desirable in the context of fundamental-driven liquidity traps are price level targeting, nominal GDP level targeting and interest rate gradualism. This paper focuses on frameworks that facilitate closed-form solutions of the model.

[3] Inflation conservatism was originally proposed as a remedy to the classic inflation bias problem. Formally, an inflation-conservative central banker puts less weight on output relative to inflation stabilization than society does.

we find that the optimal weight on inflation relative to output stabilization in the policymaker's objective function can be smaller or larger than the one in society's objective function.

Next, we turn to fiscal policy. Specifically, we allow the policymaker to use government spending as an additional policy tool.[4] As in models with fundamental-driven liquidity traps, the policymaker raises government spending whenever the lower bound constraint becomes binding, and she keeps government spending at an elevated level until confidence resumes and the lower bound constraint becomes slack. However, unlike in models with fundamental-driven liquidity traps—where fiscal policy improves allocations at and away from the lower bound—this fiscal policy intervention worsens stabilization outcomes both at and away from the lower bound. Taking as given the occasional occurrence of expectations-driven liquidity traps, it is therefore best for society to disincentivize the use of government spending as a stabilization tool. To do so, society has to assign a sufficiently high relative weight to government spending stabilization in the policymaker's objective function.[5]

Thus, the answer to our first question—is there a straightforward way to improve welfare of an economy that is subject to occasional expectations-driven liquidity traps?—is rather disappointing. None of the reviewed policy delegation schemes—a higher inflation target, inflation conservatism, and fiscal activism—unambiguously improves stabilization outcomes and welfare in our model. However, the answer to our second question—is it possible to prevent the economy from falling into an expectations-driven liquidity trap?—turns out to be more promising if government spending is part of the policymaker's toolkit. Specifically, we find that when society assigns a sufficiently low relative weight on government spending stabilization to the policymaker's objective function the sunspot equilibrium ceases to exist.[6] Conditional on the existence of the sunspot equilibrium, a marginal reduction in the policymaker's relative weight on government spending stabilization increases the spending stimulus at the lower bound and deteriorates stabilization outcomes both at and away from the lower bound. But when the relative weight is sufficiently small, the policymaker is willing to adjust government spending sufficiently elastically to deviations of inflation and the output gap from target that pessimistic expectations fail to be validated. In the remaining no-sunspot equilibrium, the sunspot shock does not affect private sector decisions and government spending stays constant. Nevertheless, we verify that the government spending expansion that would be implemented by a policymaker of this type if a decline in inflation and the output gap occurred and the lower bound became binding—maybe because of a fundamental shock—appears plausible from a quantitative perspective.

Our paper is related to a small but growing literature on equilibrium multiplicity and the lower bound on nominal interest rates. Benhabib et al. (2001) were the first to show that the lower bound constraint gives rise to two steady state equilibria in a model where monetary policy is governed by an interest-rate feedback rule. In one steady state the policy rate is strictly positive

[4] Like most of the related literature, we assume that the provision of public goods generates some household utility.

[5] In contrast, in models with fundamental-driven liquidity traps, society can further improve stabilization outcomes and welfare by appointing a policymaker who is less concerned with government spending stabilization than society as a whole. See Schmidt (2017).

[6] From an institutional perspective, this could be operationalized by the appointment of a decision-making fiscal council. Alternatively, one could think of society electing a policymaker with a certain type of preferences.

and inflation is at target, and in the other steady state the lower bound constraint is binding and inflation is below target.[7] Armenter (2018) and Nakata and Schmidt (2019a) show that the lower bound constraint can give rise to multiple Markov-perfect equilibria under optimal discretionary monetary policy. Moreover, Armenter (2018) shows that price-level targeting does not eliminate the equilibrium multiplicity.

This equilibrium multiplicity naturally opens the door for sunspot equilibria. Mertens and Ravn (2014) construct a sunspot equilibrium in a New Keynesian model with an interest-rate feedback rule and assess the effects of an exogenous increase in government spending when confidence is low and the lower bound is binding.[8] They find that a positive government spending shock is deflationary.[9] Bilbiie (2018) considers several other exogenous policy interventions such as an exogenous change in the policy rate path in an analytical model setup. Coyle and Nakata (2018) numerically solve a fully nonlinear New Keynesian model with an interest-rate rule allowing for both, fundamental-driven and expectations-driven liquidity traps, and find that the optimal inflation target in the policy rule is lower than in the model with fundamental-driven liquidity traps only.

A few papers have assessed the plausibility of expectations-driven liquidity traps empirically or used the concept for positive analysis of recent economic events. Aruoba et al. (2018) conduct a model-based empirical assessment to shed light on the type of liquidity trap events experienced by the U.S. economy and the Japanese economy, finding that Japan transitioned in the late 1990s to an expectations-driven liquidity trap state and that the U.S. had been in a fundamental-driven liquidity trap equilibrium.[10] Schmitt-Grohé and Uribe (2017) show that a model with downward nominal wage rigidities and a sunspot shock can mimic the economic dynamics of a recessionary lower bound episode that is followed by a jobless recovery. Lansing (2017) develops a New Keynesian model with a lower bound on nominal interest rates in which agents' beliefs about the steady state to which the economy converges in the long run depends on aggregate outcomes. This model with endogenous regime switches is applied to the U.S. economy. Jarociński and Maćkowiak (2018) use a sticky-price model with a sunspot shock to conduct counterfactual simulations of the euro area economic downturn in 2008-2015.

Our paper also makes contact with some existing studies on how to avoid expectations-driven liquidity traps and equilibrium multiplicity. Benhabib et al. (2002) and Woodford (2003) show how non-Ricardian fiscal policies that entail an off-equilibrium violation of the transversality condition can rule out perfect-foresight equilibria in which the economy converges to the steady state where

[7] Benhabib et al. (2001) also show that there usually exist an infinite number of perfect-foresight equilibria where the economy can originate arbitrarily close to the first steady state and converge to the second steady state.

[8] See also Boneva et al. (2016).

[9] Wieland (2018) shows that if one relaxes the assumption of Mertens and Ravn (2014) that the government spending shock is perfectly correlated with the sunspot shock, a sufficiently short-lived increase in government spending can be inflationary.

[10] Hirose (2018) estimates a DSGE model log-linearized around the deflationary steady state on Japanese data. Cuba-Borda and Singh (2019) compare a permanent expectations-driven liquidity trap to a permanent fundamental-driven liquidity trap in a model with government bonds in the utility and downward nominal wage rigidities. Their empirical analysis suggests that the permanent expectations-driven liquidity trap equilibrium fits Japanese data better than the permanent fundamental-driven liquidity trap equilibrium.

the lower bound constraint is binding. Sugo and Ueda (2008) and Schmitt-Grohé and Uribe (2014) consider alternative interest-rate feedback rules. Schmidt (2016) shows that it is possible to design Ricardian government spending rules that insulate the economy from expectations-driven liquidity traps. Tamanyu (2019) provides a similar analysis for the case of tax rules. Armenter (2018) shows that augmenting the objective function of a discretionary central bank with an objective for stabilizing the level of a long-run nominal interest rate can ensure the existence of a unique Markov-perfect equilibrium.

Finally, there is a rich literature on optimal monetary and fiscal policy in models with fundamental-driven liquidity traps. Studies on optimal monetary policy include Eggertsson and Woodford (2003), Jung et al. (2005), Adam and Billi (2006, 2007), Nakov (2008) and Nakata and Schmidt (2019a,b). Optimal fiscal policy is analyzed by e.g. Eggertsson and Woodford (2006), Eggertsson (2006), Schmidt (2013, 2017), Nakata (2016, 2017), Bilbiie et al. (2018) and Bouakez et al. (2016), among others.

The remainder of the paper is organized as follows. Section 2 presents the model, describing the private sector behavioral constraints, monetary policy and the shock structure, and defines the equilibria of interest. Section 3 presents results on equilibrium existence and stabilization outcomes. Section 4 assesses the desirability of a positive inflation target and inflation conservatism in the sunspot equilibrium. Section 5 extends the analysis to fiscal policy. Section 6 concludes.

# 2 Model

We use a standard infinite-horizon New Keynesian model formulated in discrete time. The economy is inhabited by identical households who consume and work, goods-producing firms that act under monopolistic competition and are subject to price rigidities, and a government. For now, we assume that the one-period nominal interest rate is the only policy instrument. In Section 5, the model is extended with government spending. More detailed descriptions of the model can be found in Woodford (2003) and Galí (2015). We work with a semi-loglinear version of the model that can be solved in closed form and allows us to derive analytical results.

## 2.1 Private sector behavior and welfare

Aggregate private sector behavior is described by a Phillips curve and a consumption Euler equation

$$\pi_t = \kappa y_t + \beta \mathrm{E}_t \pi_{t+1} \tag{1}$$

$$y_t = \mathrm{E}_t y_{t+1} - \sigma \left( i_t - \mathrm{E}_t \pi_{t+1} - r_t^n \right) \tag{2}$$

The private sector behavioral constraints have been (semi) log-linearized around the intended zero-inflation steady state. $\pi_t$ is the inflation rate between periods $t-1$ and $t$, $y_t$ denotes the output gap, $i_t$ is the *level* of the riskless nominal interest rate between periods $t$ and $t+1$, $r_t^n$ is the exogenous natural real rate of interest, and $\mathrm{E}_t$ is the rational expectations operator conditional on information available in period $t$. The parameters are defined as follows: $\beta \in (0,1)$ is the households' subjective discount factor, $\sigma > 0$ is the intertemporal elasticity of substitution in consumption, and $\kappa$ represents the slope of the Phillips curve.[11]

Households' welfare at time $t$ is given by the expected discounted sum of current and future utility flows. A second-order approximation to household preferences leads to

$$V_t = -\frac{1}{2}\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left[\pi_{t+j}^2 + \bar{\lambda} y_{t+j}^2\right], \tag{3}$$

where $\bar{\lambda} = \kappa/\theta$.[12]

## 2.2 Central bank

At the beginning of time, society delegates monetary policy to a central bank. The central bank does not have a commitment technology, that is, it acts under discretion. The monetary policy objective is given by

$$V_t^{CB} = -\frac{1}{2}\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left[(\pi_{t+j} - \pi^*)^2 + \lambda y_{t+j}^2\right], \tag{4}$$

where $\lambda \geq 0$ and $\pi^*$ are policy parameters to be set by society when designing the central bank's objective function. When $\lambda = \bar{\lambda}$ and $\pi^* = 0$, the central bank's objective function coincides with society's objective function (3).

The policy problem of a generic central bank is as follows. Each period $t$, it chooses the inflation rate, the output gap, and the nominal interest rate to maximize its objective function (4) subject to the behavioral constraints of the private sector (1)–(2), and the lower bound constraint $i_t \geq 0$, with the value and policy functions at time $t+1$ taken as given.

The first-order necessary conditions to this problem imply that interest rate policy is governed by the following targeting rule

$$[\kappa(\pi_t - \pi^*) + \lambda y_t]\, i_t = 0, \tag{5}$$

where $\kappa(\pi_t - \pi^*) + \lambda y_t = 0$ whenever $i_t > 0$ and $\kappa(\pi_t - \pi^*) + \lambda y_t < 0$ when the lower bound constraint is binding, $i_t = 0$. In words, each period the central bank aims to stabilize a weighted sum of current period's inflation rate (in deviation from target) and the output gap.

[11] $\kappa$ is itself a function of several structural parameters of the economy: $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\eta\theta)}(\sigma^{-1} + \eta)$, where $\alpha \in (0,1)$ denotes the share of firms that cannot reoptimize their price in a given period, $\eta > 0$ is the inverse of the labor-supply elasticity, and $\theta > 1$ denotes the price elasticity of demand for differentiated goods.

[12] See Woodford (2003). We assume that the steady state distortions arising from monopolistic competition are offset by a wage subsidy.

### 2.3 Sunspot shock

For the benchmark setup, we assume that there is no uncertainty regarding the economy's fundamentals. Specifically, $r_t^n = r^n = 1/\beta - 1$ for all $t$. However, agents expectations may be affected by a non-fundamental sunspot or 'confidence' shock $\xi_t$. The sunspot shock follows a two-state Markov process, $\xi_t \in (\xi_L, \xi_H)$. We refer to state $\xi_L$ as the low-confidence state and to state $\xi_H$ as the high-confidence state. The transition probabilities are given by

$$\text{Prob}\,(\xi_{t+1} = \xi_H | \xi_t = \xi_H) = p_H \tag{6}$$

$$\text{Prob}\,(\xi_{t+1} = \xi_L | \xi_t = \xi_L) = p_L \tag{7}$$

In words, $p_H \in (0, 1]$ is the probability of being in the high-confidence state in period $t+1$ conditional on being in the high-confidence state in period $t$, and can be interpreted as the persistence of high confidence. Note that while we allow the high-confidence state to be an absorbing state we do not restrict our analysis to this special case. $p_L \in (0, 1)$ is the probability of being in the low-confidence state in period $t + 1$ when the economy is in the low-confidence state in period $t$, and can be interpreted as the persistence of low confidence.[13]

Let $x_s$, $s \in \{L, H\}$ be the equilibrium value of some variable $x$ in state $\xi_s$. *Sunspots matter* if there is an equilibrium in which $\{\pi_L, y_L, i_L, V_L\} \neq \{\pi_H, y_H, i_H, V_H\}$. We are interested in a sunspot equilibrium where the economy is subject to recurring liquidity trap episodes. We associate the occurrence of these liquidity trap events with the low-confidence state.

**Definition 1** *The* **sunspot equilibrium** *with occasional liquidity traps is defined as a vector* $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ *that solves the following system of linear equations*

$$y_H = [p_H y_H + (1 - p_H) y_L] + \sigma [p_H \pi_H + (1 - p_H)\pi_L - i_H + r^n] \tag{8}$$

$$\pi_H = \kappa y_H + \beta [p_H \pi_H + (1 - p_H)\pi_L] \tag{9}$$

$$0 = \kappa(\pi_H - \pi^*) + \lambda y_H \tag{10}$$

$$y_L = [(1 - p_L) y_H + p_L y_L] + \sigma [(1 - p_L)\pi_H + p_L \pi_L - i_L + r^n] \tag{11}$$

$$\pi_L = \kappa y_L + \beta [(1 - p_L)\pi_H + p_L \pi_L] \tag{12}$$

$$i_L = 0, \tag{13}$$

[13] Mertens and Ravn (2014), Schmidt (2016), Aruoba et al. (2018) and Bilbiie (2018) also consider a sunspot shock that follows a two-state Markov process. However, Mertens and Ravn (2014), Schmidt (2016) and Bilbiie (2018) assume that the high-confidence state is an absorbing state, that is, $p_H = 1$. Aruoba et al. (2018) allow for recurring declines in confidence and assume that conditional on being in the high-confidence state agents attach a 1% probability to the possibility of ending up in the low-confidence state in the next period. Formally, in the context of our setup they impose $p_H = 0.99$.

*and satisfies the following two inequality constraints*

$$i_H > 0 \tag{14}$$

$$\kappa(\pi_L - \pi^*) + \lambda y_L < 0. \tag{15}$$

### 2.4 Alternative setup: Fundamental shock

Throughout the paper, we contrast results for the benchmark model—an economy that is subject to a sunspot shock—with those for an economy that is subject to a fundamental shock instead of a sunspot shock but is otherwise identical to the benchmark economy. In this alternative model, the natural real rate is assumed to be stochastic.

To keep the model setup as close as possible to the one with the sunspot shock, $r_t^n$ is assumed to follow a two-state Markov process. In the high-fundamental state, the natural real rate is strictly positive $r_H^n > 0$, and in the low-fundamental state it is strictly negative $r_L^n < 0$. The transition probabilities for the natural real rate shock are given by

$$\text{Prob}\left(r_{t+1}^n = r_H^n | r_t^n = r_H^n\right) = p_H^f \tag{16}$$

$$\text{Prob}\left(r_{t+1}^n = r_L^n | r_t^n = r_L^n\right) = p_L^f, \tag{17}$$

and are distinguished from the transition probabilities of the sunspot shock via the superscript $f$. The fundamental equilibrium in the model with the natural real rate shock is defined as follows.

**Definition 2** *The* **fundamental equilibrium** *with occasional liquidity traps is defined as a vector* $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ *that solves*

$$y_H = \left[p_H^f y_H + (1-p_H^f)y_L\right] + \sigma\left[p_H^f \pi_H + (1-p_H^f)\pi_L - i_H + r_H^n\right] \tag{18}$$

$$\pi_H = \kappa y_H + \beta\left[p_H^f \pi_H + (1-p_H^f)\pi_L\right] \tag{19}$$

$$y_L = \left[(1-p_L^f)y_H + p_L^f y_L\right] + \sigma\left[(1-p_L^f)\pi_H + p_L^f \pi_L - i_L + r_L^n\right] \tag{20}$$

$$\pi_L = \kappa y_L + \beta\left[(1-p_L^f)\pi_H + p_L^f \pi_L\right] \tag{21}$$

*as well as (10) and (13), and satisfies inequality constraints (14) and (15).*

This Markov-perfect equilibrium has been analyzed in detail in Nakata and Schmidt (2019a).[14] To keep the exposition parsimonious, we will refer to this paper for the proofs related to the fundamental equilibrium whenever applicable.[15]

[14] Nakata and Schmidt (2019a) analytically show that in this model with a two-state fundamental shock there exists another Markov-perfect equilibrium in which the lower bound constraint binds in the low and the high-fundamental state. Here, we do not consider this equilibrium.

[15] The notation used in Nakata and Schmidt (2019a) is slightly different from the one used here. They use $p_H$ to denote the probability that the economy is in the *low* state in the next period conditional on being in the high state today.

## 3 Basic properties of the sunspot equilibrium

This section presents conditions for existence of the sunspot equilibrium as well as equilibrium allocations and prices, and discusses how they compare to those of the fundamental equilibrium.

### 3.1 Equilibrium existence

The following proposition establishes the necessary and sufficient conditions for existence of the sunspot equilibrium.

**Proposition 1** *The sunspot equilibrium exists if and only if*

$$p_L - (1 - p_H) - \frac{1 - p_L + 1 - p_H}{\kappa\sigma} \left(1 - \beta p_L + \beta(1 - p_H)\right) > 0, \tag{22}$$

*and*

$$\pi^* > -\frac{\kappa^2 + \lambda(1 - \beta)}{\kappa^2} r^n. \tag{23}$$

**Proof:** See Appendix A.

Three observations are in order. First, for the sunspot equilibrium to exist, the two confidence states have to be sufficiently persistent. Second, prices have to be sufficiently flexible, i.e. $\kappa$ has to be sufficiently large. Third, the central bank's inflation target must be higher than some strictly negative lower bound. Note that conditional on the inflation target not being too low, equilibrium existence does not depend on the policy parameters $\lambda$ and $\pi^*$.

These conditions for existence of the sunspot equilibrium qualitatively differ from the conditions for existence of the fundamental equilibrium. In particular, for the fundamental equilibrium to exist, the low-fundamental state must not be too persistent (see Nakata and Schmidt, 2019a). Hence, the fundamental equilibrium stipulates an upper bound on the average duration of liquidity traps whereas the sunspot equilibrium stipulates a lower bound. When $p_H, p_H^f = 1$, the upper bound for the fundamental equilibrium to exist and the lower bound for the sunspot equilibrium to exist coincide.[16]

### 3.2 Allocations and prices

The allocations and prices in the sunspot equilibrium can be solved for in closed form. For now, we assume that the central bank has the same objective function as society as a whole. The signs of inflation and the output gap in the two confidence states are then unambiguously determined.

**Proposition 2** *Suppose $\lambda = \bar{\lambda}$ and $\pi^* = 0$. In the sunspot equilibrium, $\pi_L < 0$, $y_L < 0$, $\pi_H \leq 0$, $y_H \geq 0$. When $p_H < 1$, then $\pi_H < 0$, $y_H > 0$.*

[16] Appendix A provides a numerical illustration of the existence conditions for the sunspot equilibrium and for the fundamental equilibrium.

**Proof:** See Appendix A

When confidence is low, agents expect persistently low future income, and therefore increase desired saving at the expense of lower desired consumption. Due to the presence of price rigidities, prices do not fully adjust immediately and output falls. The central bank lowers the policy rate to equate desired saving to zero, but if agents are sufficiently pessimistic, the lower bound on the policy rate becomes binding. At the lower bound, to equate desired saving to zero, output has to fall, validating agents' pessimistic expectations. The lower bound is binding, and inflation and the output gap both settle below target.

When confidence is high, the policy rate is strictly positive but if $p_H < 1$ the risk of a future decline in confidence creates a monetary policy trade-off between inflation and output gap stabilization. Specifically, the possibility that confidence might fall in the future while the price set by a firm reoptimizing today is still in place provides an incentive for forward-looking firms to set a lower price than they would in the absence of any risk of a future drop in confidence. To counteract these deflationary forces, the central bank allows for a positive output gap, that is, it sets the policy rate in the high-confidence state such that the ex-ante real interest rate is below the constant natural real rate. In equilibrium, the high-confidence output gap is thus positive and inflation is below target.

The signs of output and inflation in the fundamental equilibrium are identical to those in the sunspot equilibrium. Output and inflation are negative in the low-fundamental state, and output (inflation) is positive (negative) in the high-fundamental state (see Nakata and Schmidt, 2019a). However, in the fundamental equilibrium, it is the temporarily negative natural real rate of interest in the low-fundamental state that leads to the decline in output and inflation in the low state.

## 3.3 Aggregate demand and aggregate supply schedules

In order to better understand equilibrium outcomes in the model with the sunspot shock and in the model with the fundamental shock, and how they are affected by the policy framework, we recast the models in terms of aggregate demand (AD) and aggregate supply (AS) curves. The AD curve is the set of pairs of inflation rates and output gaps consistent with Euler equation (2) where the policy rate is set in line with target criterion (5), and the AS curve is the set of pairs of inflation rates and output gaps consistent with Phillips curve (1). Specifically, we focus on the AD and AS schedules conditional on the economy being in the low state of the respective model. For the model with the sunspot shock the two curves in the low-confidence state are given by

$$\textbf{AD-sunspot:} \quad y_L = \min\left[\left(y_H + \sigma\pi_H + \frac{\sigma}{1-p_L}r^n\right) + \frac{\sigma p_L}{1-p_L}\pi_L, \frac{\kappa}{\lambda}(\pi^* - \pi_L)\right] \tag{24}$$

$$\textbf{AS-sunspot:} \quad y_L = -\frac{\beta(1-p_L)}{\kappa}\pi_H + \frac{1-\beta p_L}{\kappa}\pi_L, \tag{25}$$

where in each equation we distinguish between terms that are multiplied by $\pi_L$—the slope coefficient—and the other terms—the intercept. For the model with the fundamental shock, the two curves are given by

$$\textbf{AD-fundamental:}\quad y_L = \min\left[\left(y_H + \sigma\pi_H + \frac{\sigma}{1-p_L^f} r_L^n\right) + \frac{\sigma p_L^f}{1-p_L^f}\pi_L, \frac{\kappa}{\lambda}(\pi^* - \pi_L)\right] \tag{26}$$

$$\textbf{AS-fundamental:}\quad y_L = -\frac{\beta(1-p_L^f)}{\kappa}\pi_H + \frac{1-\beta p_L^f}{\kappa}\pi_L. \tag{27}$$

Figure 1 plots these AD-AS curves for the model with the sunspot shock (left panel) and for the model with the fundamental shock (right panel), assuming that the high state in both models is an absorbing state. One period corresponds to one quarter. We set $p_L = 0.9375$ in the model with the sunspot shock, implying an average duration of lower bound episodes of 4 years in the sunspot equilibrium. In the model with the fundamental shock, we set $p_L^f = 0.85$, implying an average duration of lower bound episodes of 1 1/2 years. The other parameter values are summarized in Table 1. For $\pi_H, y_H = 0$, the intercept terms in the AS curves are zero, whereas the intercept terms

Table 1: Parameter values for numerical example

| Parameter | Value | Economic interpretation |
|---|---|---|
| $\beta$ | 0.9975 | Subjective discount factor |
| $\sigma$ | 0.5 | Intertemporal elasticity of substitution in consumption |
| $\eta$ | 0.47 | Inverse labor supply elasticity |
| $\theta$ | 10 | Price elasticity of demand |
| $\alpha$ | 0.8106 | Share of firms per period keeping prices unchanged |
| $\lambda$ | $\bar{\lambda}$ | Policy parameter: Relative weight on output term |
| $\pi^*$ | 0 | Policy parameter: Inflation target |
| $r_H^n$ | $r^n$ | High-state natural real rate in model with fundamental shock |
| $r_L^n$ | -0.005 | Low-state natural real rate in model with fundamental shock |

Note: This parameterization implies $r^n = 0.0025$, $\kappa = 0.0194$, $\bar{\lambda} = 0.0019$.

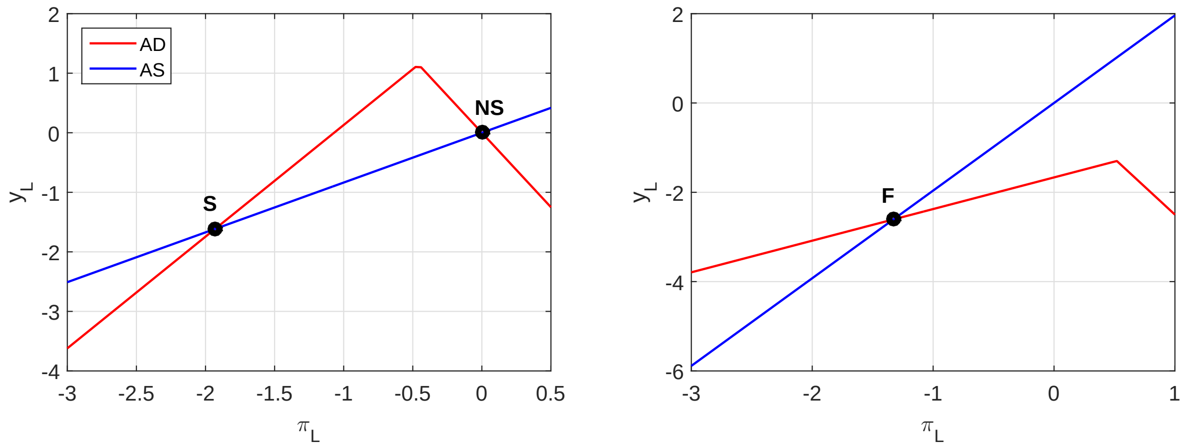
in the AD curves are positive (model with sunspot shock) and negative (model with fundamental shock), respectively.

The low-state AD-AS curves in the two models have several common features. First, due to the lower bound constraint, the AD curve has a kink. To the left of the kink, the lower bound constraint is binding and to the right of the kink the lower bound constraint is slack. Second, the AD curve is upward-sloping to the left of the kink—aggregate demand is increasing in inflation when the lower bound is binding because an increase in inflation lowers the ex-ante real interest rate—and downward-sloping to the right of the kink—aggregate demand is decreasing in inflation when the lower bound constraint is slack because the central bank raises the policy rate more than one-for-one with inflation. Third, the AS curve is monotonically upward-sloping—an increase in demand leads to an increase in inflation—and goes through the origin.

In the model with the sunspot shock, the AD curve is steeper than the AS curve. This is a necessary—and in case of $\pi^* = 0$ sufficient—condition for existence of the sunspot equilibrium.[17]

[17] See the condition for existence of the sunspot equilibrium (22) with $p_H = 1$.

Figure 1: Aggregate demand and aggregate supply in the low state




(a) Model with sunspot shock (b) Model with fundamental shock

Note: In the left panel, $S$ marks the sunspot equilibrium and $NS$ the no-sunspot equilibrium. In the right panel, $F$ marks the fundamental equilibrium. Inflation is expressed in annualized terms.

Intuitively, since the low-confidence state is highly persistent, households' desired consumption is very sensitive to changes in low-state inflation, i.e. the AD curve is relatively steep. At the same time, the high persistence of the low-confidence state makes firms' price setting very sensitive to changes in aggregate demand, i.e. the AS curve is relatively flat. Consistent with Proposition 2, when confidence is low, output and inflation are strictly negative in the sunspot equilibrium as represented by intersection point $S$. The panel also shows that besides the sunspot equilibrium, there is a second equilibrium—represented by intersection point $NS$—where the lower bound constraint on the policy rate is not binding and low-state output and inflation are at target. In this 'no-sunspot' equilibrium, the sunspot shock does not affect agents' behavior.

In the model with the fundamental shock, the AD curve is flatter than the AS curve, which is a necessary condition for the fundamental equilibrium to exist and reflects the relatively lower persistence of the low-fundamental state. In the fundamental equilibrium, marked by intersection point $F$ in the right panel, low-state output and inflation are negative, again in line with analytical results.

## 4 Monetary policy frameworks

Having shown that the sunspot equilibrium is associated with rare but long-lasting spells at the lower bound and chronic deflation, we now explore whether stabilization outcomes and welfare can be improved by assigning an objective function to the policymaker that differs from society's objective function. This section focuses on two monetary policy frameworks that are known to be desirable in models with fundamental-driven liquidity traps: a non-zero inflation target and inflation conservatism. The subsequent section extends the analysis to fiscal policy.

### 4.1 A non-zero inflation target

In the fundamental equilibrium, society's welfare can be improved by assigning a strictly positive inflation target to the central bank (Nakata and Schmidt, 2019a). This subsection explores the desirability of a non-zero inflation target in the sunspot equilibrium. Throughout this subsection, we assume $\lambda = \bar{\lambda}$.

While the signs of allocations and prices are sensitive to the quantitative value of the central bank's inflation target, the effects of a change in the target on allocations and prices are unambiguously determined.

**Proposition 3** *In the sunspot equilibrium,* $\frac{\partial \pi_L}{\partial \pi^*} < 0$*,* $\frac{\partial y_L}{\partial \pi^*} < 0$*,* $\frac{\partial \pi_H}{\partial \pi^*} > 0$*,* $\frac{\partial y_H}{\partial \pi^*} > 0$*.*

**Proof:** See Appendix A.

In the sunspot equilibrium, a marginal increase in the inflation target lowers output and inflation in the low-confidence state and raises output and inflation in the high-confidence state.[18] Consider first the high-confidence state. All else equal, if $\pi^*$ increases, the gap between the inflation target and actual inflation widens, and hence the central bank becomes more willing to tolerate a positive output gap to bring inflation again closer to its target. In equilibrium, an increase in $\pi^*$ therefore raises the output gap and inflation in the high-confidence state.

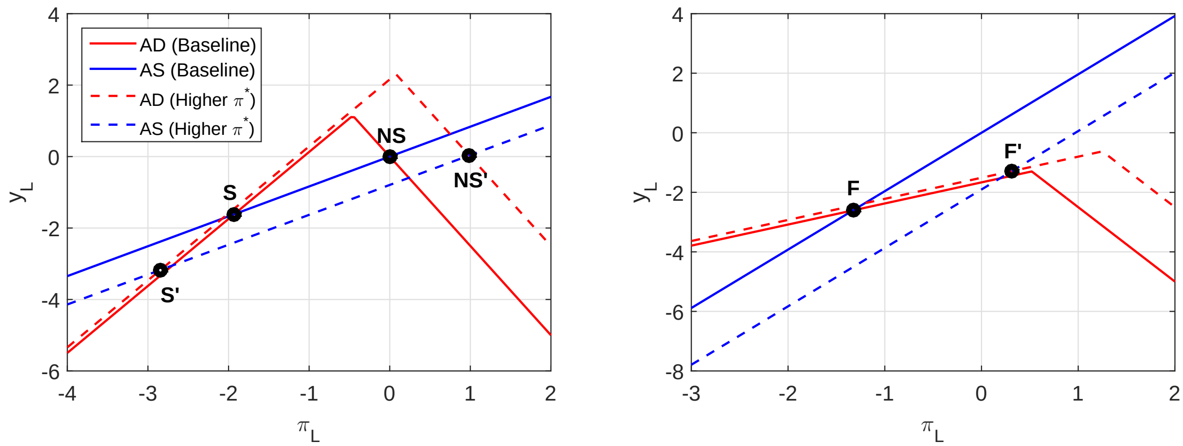
To understand why low-state output and inflation are increasing in $\pi^*$, we make use of the the AD-AS framework. The left panel of Figure 2 depicts how the low-confidence state AD and AS curves (24)–(25) are shifted in response to an increase in the central bank's inflation target, assuming that the high state is an absorbing state. An increase in the inflation target shifts the AD curve upwards, because, all else equal, agents increase their desired consumption given higher expected inflation. At the same time, the AS curve shifts downwards, as firms' desired price increases in light of higher expected inflation for given current demand. Hence, at the inflation rate consistent with the sunspot equilibrium in the baseline, marked by intersection point $S$, there is now excess demand. In the model with the sunspot shock, excess demand is increasing in the inflation rate as long as the lower bound is binding. To restore equilibrium, low-state inflation and output thus have to decline. The new intersection point $S'$ lies to the south-west of the baseline intersection point $S$.[19]

In the fundamental equilibrium, a marginal increase in the inflation target also raises high-state inflation. The effects on low-state outcomes, however, differ from those in the sunspot equilibrium. Higher inflation in the high-fundamental state lowers the conditional ex-ante real interest rate in the low-fundamental state. This stimulates aggregate demand and leads to an increase in low-state output and inflation (Nakata and Schmidt, 2019a). The right panel of Figure 2 depicts how in

[18] It can also be shown that $\frac{\partial \pi_H}{\partial \pi^*} < 1$. Together with Proposition 2, this implies that for any positive inflation target actual inflation in the high-confidence state is below target.

[19] An increase in the inflation target also affects the no-sunspot equilibrium. With a non-zero inflation target, the central bank faces a trade-off between output stabilization and stabilization of inflation at target. In equilibrium, when the inflation target is positive, high-state inflation is slightly below target and the output gap is slightly positive.

Figure 2: The effect of increasing the central bank's inflation target




(a) Model with sunspot shock

(b) Model with fundamental shock

Note: Solid lines: $\pi^* = 0$; dashed lines: $\pi^* = 1/400$. In the left (right) panel, $S$ ($F$) marks the sunspot (fundamental) equilibrium in the baseline and $S'$ ($F'$) marks the sunspot (fundamental) equilibrium in the case of a higher $\pi^*$. $NS$ marks the no-sunspot equilibrium in the baseline, and $NS'$ marks the no-sunspot equilibrium in the case of a higher $\pi^*$. Inflation is expressed in annualized terms.

the model with the fundamental shock the low-state AD and AS curves (26)–(27) are shifted in response to an increase in the inflation target.

For the characterization of the welfare-maximizing inflation target in the model with the sunspot shock, it is also useful to show that there exists an inflation target such that inflation in the high-confidence state is stabilized at zero.

**Lemma 1** *There exists a $\pi^0 > 0$ such that in the sunspot equilibrium $\pi_H = 0$ if $\pi^* = \pi^0$.*

**Proof:** See Appendix A.

One can then establish the following result concerning the welfare-maximizing inflation target.

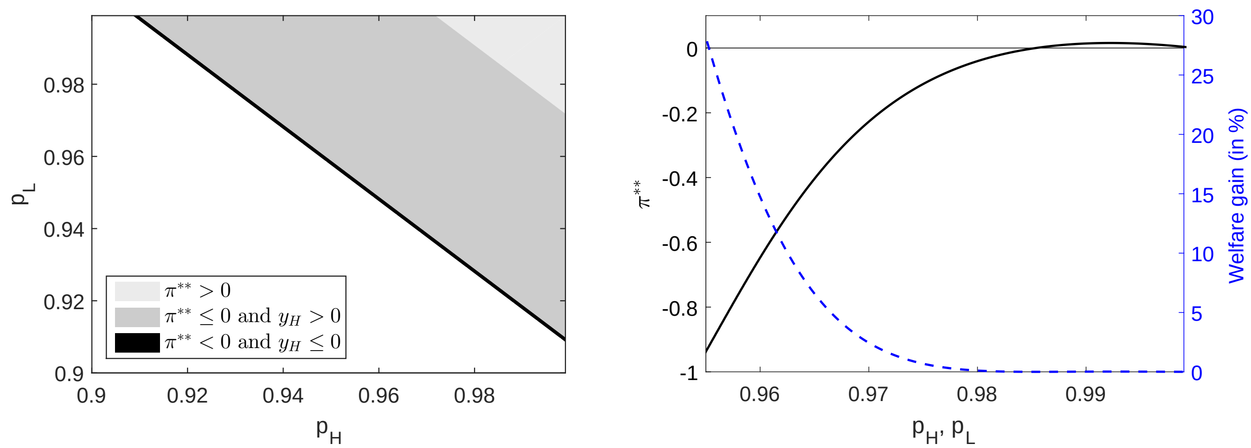
**Proposition 4** *Suppose $\lambda = \bar{\lambda}$ and $p_H < 1$. Let $\pi^{**}$ denote the value of $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2} r^n$ that maximizes households' unconditional welfare $EV_t$ where $V_t$ is defined in equation (3). In the sunspot equilibrium, $\pi^{**} < \pi^0$.*

**Proof:** See Appendix A.

Together with Proposition 3 and Lemma 1, this proposition means that the optimal inflation target can be negative or positive. However, this proposition also means that even if the optimal inflation target is positive, it will be below the level needed to engineer strictly positive inflation in the high-confidence state. The ambiguity concerning the sign of the optimal target can be understood from the fact that an increase in $\pi^*$ has a negative effect on low-state inflation (moving low-state inflation further into negative territory), and a positive effect on high-state inflation (moving high-state inflation closer to zero as long as $\pi^* < \pi^0$).[20] Only for the special case where the

[20] Appendix A provides a numerical example of how $\pi^*$ affects allocations and welfare in the sunspot equilibrium.

Figure 3: Optimal inflation target in model with sunspot shock




Note: In the right panel $p_H = p_L$. The optimal inflation target $\pi^{**}$ is expressed in annualized terms.

high-confidence state is an absorbing state, $p_H = 1$, the optimal inflation target is unambiguously negative.[21]

The left panel of Figure 3 shows how $\pi^{**}$ depends on $p_H$ and $p_L$, the persistence of the high and the low-confidence state, respectively.[22] The figure distinguishes three cases: i. $\pi^{**} > 0$ (light gray-shaded area), ii. $\pi^{**} \leq 0$ and $y_H > 0$ (gray-shaded area), and iii. $\pi^{**} < 0$ and $y_H \leq 0$ (black-shaded area). The white-shaded area represents pairs of $p_H$ and $p_L$ for which the sunspot equilibrium does not exist. When the two confidence states are highly persistent, the optimal inflation target is strictly positive. When the two states are less persistent, the optimal inflation target is negative. Most pairs $\{p_H, p_L\}$ that are consistent with equilibrium existence fall into this second category. If the pair of persistence parameters just marginally satisfies the conditions for equilibrium existence, the optimal inflation target is sufficiently negative to engineer a negative output gap in the high state.

The right panel of Figure 3 plots the optimal inflation target (left vertical axis, solid black line) and the welfare gain from assigning the optimal target to the central bank (right vertical axis, dashed blue line) as a function of the persistence of the two confidence states, assuming $p_H = p_L$. For sufficiently low values of $p_H$ and $p_L$, the optimal inflation target is negative and is increasing in the persistence parameters. When $p_H$ and $p_L$ are high enough, the optimal inflation target is slightly positive. The welfare gain of assigning an optimized inflation target to the central bank is most elevated when the persistence parameters take on the lowest possible values for which the sunspot equilibrium exists.

Next, we assess the desirability of inflation conservatism.

[21] This follows directly from Propositions 2 and 3.

[22] The values for the other model parameters are reported in Table 1.

### 4.2 Inflation conservatism

An inflation-conservative central banker is a policymaker who puts a higher relative weight on inflation stabilization than society as a whole ($\lambda < \bar{\lambda}$). In models with occasional fundamental-driven liquidity trap episodes, the appointment of an inflation-conservative policymaker improves welfare relative to the case where the policymaker has the same objective function as society as a whole (Nakata and Schmidt, 2019a). Specifically, if the only source of uncertainty is a natural real rate shock—as assumed for the model with the fundamental shock—then it is optimal to appoint a strictly inflation-conservative policymaker, i.e. $\lambda = 0$.

Let us now turn to the model with the sunspot shock. We first establish how a change in the central bank's relative weight on output stabilization $\lambda$ affects allocations and prices in the sunspot equilibrium and then explore the welfare implications. To focus on the role of inflation conservatism, we assume $\pi^* = 0$ throughout this subsection.

**Proposition 5** *Suppose $\pi^* = 0$ and $p_H < 1$. In the sunspot equilibrium, $\frac{\partial \pi_L}{\partial \lambda} > 0$, $\frac{\partial y_L}{\partial \lambda} > 0$, $\frac{\partial \pi_H}{\partial \lambda} < 0$, $\frac{\partial y_H}{\partial \lambda} < 0$.*

**Proof:** See Appendix A.

In the sunspot equilibrium, a marginal increase in the relative weight on output gap stabilization raises output and inflation in the low-confidence state and lowers output and inflation in the high-confidence state.[23] Qualitatively, the effects are thus the same as those of a marginal reduction in $\pi^*$ (see Proposition 3).

The next proposition focuses on the welfare implications of inflation conservatism.

**Proposition 6** *Suppose $\pi^* = 0$ and $p_H < 1$. Let $\lambda^*$ denote the value of $\lambda \in [0, \infty]$ that maximizes households' unconditional welfare $EV_t$ where $V_t$ is defined in equation (3). In the sunspot equilibrium, $\lambda^* > 0$.*

**Proof:** See Appendix A.

In words, strict inflation conservatism—the welfare-maximizing configuration in the fundamental equilibrium—is not desirable in the sunspot equilibrium. In the Appendix, we show that the optimal relative weight, $\lambda^*$, can be either smaller or bigger than households' relative weight on output gap stabilization $\bar{\lambda}$ and provide the corresponding necessary and sufficient conditions. The reason for this ambiguity with regard to the desirability of inflation conservatism is similar to why the optimal inflation target can be negative or positive.

Before turning to fiscal policy, it is useful to point out that there is a close relationship between inflation conservatism and a non-zero inflation target.

[23] If the high-confidence state was an absorbing state, $p_H = 1$, a change in $\lambda$ would not affect allocations, and, hence, welfare.

**Proposition 7** *Suppose $p_H < 1$. For any $\hat{\lambda} \geq 0$, there exists a $\hat{\pi}^*$ such that the sunspot equilibrium under optimal discretionary policy associated with the inflation conservatism regime satisfying $(\lambda = \hat{\lambda}, \pi^* = 0)$ is replicated by the inflation target regime satisfying $(\lambda = \bar{\lambda}, \pi^* = \hat{\pi}^*)$, where*

$$\hat{\pi}^* \equiv \frac{\beta(1-p_H)r^n}{\beta\hat{\lambda}(1-p_H) - (\kappa^2 + \hat{\lambda}(1-\beta))C}\left(\bar{\lambda} - \hat{\lambda}\right). \tag{28}$$

**Proof:** See Appendix A.

The reverse is not true, as a sufficiently negative inflation target results in a strictly negative high-state output gap, an allocation that is unattainable under inflation conservatism for any $\lambda \geq 0$.[24] An interesting implication of equation (28) is that if the allocation under the optimal inflation target is attainable under inflation conservatism, then the optimal inflation target $\pi^{**}$ is positive if and only if the optimal relative output weight $\lambda^*$ is smaller than society's weight $\bar{\lambda}$.[25]

In summary, it is not straightforward to improve the sunspot equilibrium by means of a simple modification of the central bank's objective function such as imposing a non-zero inflation target or a different relative weight on inflation stabilization than the one implied by households' preferences.

# 5 Fiscal policy

This section extends the analysis to fiscal policy. To do so, we explicitly model government spending, which can be used by the discretionary policymaker as an additional policy tool. We first show how the introduction of fiscal policy affects equilibrium existence and allocations, and then turn to the design of fiscal policy by asking how much relative weight should be put on government spending stabilization in the policymaker's objective function.

## 5.1 The model with fiscal policy

The aggregate private sector behavioral constraints in the model with government spending are

$$\pi_t = \kappa x_t + \beta \mathrm{E}_t \pi_{t+1} \tag{29}$$

$$x_t = (1-\Gamma)g_t + \mathrm{E}_t(x_{t+1} - (1-\Gamma)g_{t+1}) - \sigma\left(i_t - \mathrm{E}_t\pi_{t+1} - r_t^n\right), \tag{30}$$

where $g_t$ denotes government spending as a share of steady-state output, expressed in deviation from the steady-state ratio, $x_t \equiv y_t - \Gamma g_t$, with $\Gamma = \frac{\sigma^{-1}}{\sigma^{-1}+\eta}$, will be referred to as the modified output gap, and, in a slight abuse of notation, $\sigma$ now denotes the inverse of the elasticity of the marginal utility of private consumption with respect to *total output*.

[24] Likewise, a sufficiently positive inflation target results in a strictly positive high-state inflation rate, an allocation that is also unattainable under inflation conservatism for any $\lambda \geq 0$.

[25] To see this, note that $\beta\hat{\lambda}(1-p_H) - (\kappa^2 + \hat{\lambda}(1-\beta))C > 0$ in the sunspot equilibrium.

We assume that the provision of public goods provides utility to households and that utility is separable in private and public consumption. A second-order approximation to household preferences leads to[26]

$$V_t = -\frac{1}{2}\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left(\pi_{t+j}^2 + \bar{\lambda} x_{t+j}^2 + \bar{\lambda}_g g_{t+j}^2\right). \tag{31}$$

The relative weight on government spending stabilization satisfies $\bar{\lambda}_g = \bar{\lambda}\Gamma\left(1 - \Gamma + \frac{\sigma}{\nu}\right) > 0$, where $\nu$ denotes the inverse of the elasticity of the marginal utility of public consumption with respect to total output. As before, $\bar{\lambda} = \kappa/\theta$.

At the beginning of time, society delegates monetary and fiscal policy to a discretionary policymaker. The objective function of the policymaker is given by

$$V_t^{MF} = -\frac{1}{2}\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left(\pi_{t+j}^2 + \bar{\lambda} x_{t+j}^2 + \lambda_g g_{t+j}^2\right), \tag{32}$$

where $\lambda_g \geq 0$ is a policy parameter the value of which is chosen by society when designing the policymaker's objective function. When $\lambda_g = \bar{\lambda}_g$, the policymaker's objective function coincides with society's objective function. The policymaker's optimization problem and the first-order conditions are relegated to Appendix B.

As before, we focus on a sunspot equilibrium where the lower bound is binding in the low-confidence state and slack in the high-confidence state.

**Definition 3** *The sunspot equilibrium with fiscal policy and occasional liquidity traps is defined as a vector* $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$ *that solves the following system of linear equations*

$$x_H = p_H x_H + (1 - p_H)\left[x_L + (1 - \Gamma)(g_H - g_L)\right] + \sigma\left[p_H \pi_H + (1 - p_H)\pi_L - i_H + r^n\right] \tag{33}$$

$$\pi_H = \kappa x_H + \beta\left[p_H \pi_H + (1 - p_H)\pi_L\right] \tag{34}$$

$$\lambda_g g_H = -(1 - \Gamma)\left(\kappa \pi_H + \bar{\lambda} x_H\right) \tag{35}$$

$$0 = \kappa \pi_H + \bar{\lambda} x_H \tag{36}$$

$$x_L = p_L x_L + (1 - p_L)\left[x_H - (1 - \Gamma)(g_H - g_L)\right] + \sigma\left[(1 - p_L)\pi_H + p_L \pi_L - i_L + r^n\right] \tag{37}$$

$$\pi_L = \kappa x_L + \beta\left[(1 - p_L)\pi_H + p_L \pi_L\right] \tag{38}$$

$$\lambda_g g_L = -(1 - \Gamma)\left(\kappa \pi_L + \bar{\lambda} x_L\right) \tag{39}$$

$$i_L = 0, \tag{40}$$

[26] See Schmidt (2013) for details.

*and satisfies the following two inequality constraints*

$$i_H > 0 \tag{41}$$

$$\kappa\pi_L + \bar{\lambda}x_L < 0. \tag{42}$$

The sunspot equilibrium is compared to a fundamental equilibrium in a setup where the two-state sunspot shock is replaced with a two-state natural real rate shock. As before, we consider a Markov-perfect equilibrium where the lower bound constraint is slack in the high-fundamental state and binding in the low-fundamental state.

**Definition 4** *The fundamental equilibrium with fiscal policy and occasional liquidity traps is defined as a vector* $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$ *that solves the following system of linear equations*

$$x_H = p^f_H x_H + (1-p^f_H)\left[x_L + (1-\Gamma)(g_H - g_L)\right] + \sigma\left[p^f_H\pi_H + (1-p^f_H)\pi_L - i_H + r^n_H\right] \tag{43}$$

$$\pi_H = \kappa x_H + \beta\left[p^f_H\pi_H + (1-p^f_H)\pi_L\right] \tag{44}$$

$$x_L = p^f_L x_L + (1-p^f_L)\left[x_H - (1-\Gamma)(g_H - g_L)\right] + \sigma\left[(1-p^f_L)\pi_H + p^f_L\pi_L - i_L + r^n_L\right] \tag{45}$$

$$\pi_L = \kappa x_L + \beta\left[(1-p^f_L)\pi_H + p^f_L\pi_L\right] \tag{46}$$

*as well as (35), (36), (39) and (40), and satisfies the inequality constraints (41) and (42).*

## 5.2 Equilibrium existence and allocations

The following proposition establishes a necessary and sufficient condition for existence of the sunspot equilibrium in the model with fiscal policy. The condition for existence of the fundamental equilibrium in the model with the natural real rate shock is provided in Appendix D.

**Proposition 8** *The sunspot equilibrium exists if and only if*

$$\lambda_g\Omega(p_L, p_H, \kappa, \sigma, \beta) - (1-\Gamma)^2\frac{1-p_L+1-p_H}{\kappa\sigma}\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right] > 0, \tag{47}$$

*where* $\Omega(p_L, p_H, \kappa, \sigma, \beta) \equiv p_L - (1-p_H) - \frac{1-p_L+1-p_H}{\kappa\sigma}\left(1-\beta p_L + \beta(1-p_H)\right).$

**Proof:** See Appendix C.

From Proposition 1, we know that the sunspot equilibrium in the model without fiscal policy and a zero-inflation target exists if and only if $\Omega(\cdot) > 0$. In the model with fiscal policy, $\Omega(\cdot) > 0$ is a necessary but not a sufficient condition for existence of the sunspot equilibrium. Importantly, the condition for equilibrium existence depends on the policy parameter $\lambda_g$. Suppose $\Omega(\cdot) > 0$. Then, the sunspot equilibrium exists if and only if $\lambda_g > \frac{(1-\Gamma)^2}{\Omega(\cdot)}\frac{1-p_L+1-p_H}{\kappa\sigma}\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right] > 0$.

Next, we characterize allocations and prices in the sunspot equilibrium.

**Proposition 9** *In the sunspot equilibrium, $\pi_L < 0$, $x_L < 0$, $g_L > 0$, $\pi_H \leq 0$, $x_H \geq 0$ and $g_H = 0$. When $p_H < 1$, then $\pi_H < 0$, $x_H > 0$.*

**Proof:** See Appendix C.

The policymaker increases government spending when the lower bound on nominal interest rates is binding, and keeps government spending at its steady state otherwise. The same holds true for the fundamental equilibrium. See Appendix D.

### 5.3 Welfare implications of fiscal policy

In the model with fundamental-driven liquidity traps, society can improve its welfare by appointing a "fiscally-activist" policymaker who puts less relative weight on government spending stabilization than society as a whole (Schmidt, 2017). To assess the welfare implications of fiscal policy in the sunspot equilibrium, we first establish how a marginal change in the policymaker's relative weight on government spending stabilization $\lambda_g$ affects allocations and prices.

**Proposition 10** *In the sunspot equilibrium, $\frac{\partial \pi_L}{\partial \lambda_g} > 0$, $\frac{\partial x_L}{\partial \lambda_g} > 0$, $\frac{\partial g_L}{\partial \lambda_g} < 0$, $\frac{\partial \pi_H}{\partial \lambda_g} \geq 0$, $\frac{\partial x_H}{\partial \lambda_g} \leq 0$. If $p_H < 1$, $\frac{\partial \pi_H}{\partial \lambda_g} > 0$, $\frac{\partial x_H}{\partial \lambda_g} < 0$.*

**Proof:** See Appendix C.

That is, the higher the relative weight on government spending stabilization in the policymaker's objective function, the smaller the fiscal stimulus in the low-confidence state. At the same time, an increase in $\lambda_g$ raises the inflation rate and the modified output gap in the low-confidence state. Finally, an increase in $\lambda_g$ raises the inflation rate and lowers the modified output gap in the high-confidence state. Thus, the higher $\lambda_g$ the closer to target is the economy.

In the fundamental equilibrium, an increase in $\lambda_g$ lowers government spending in the low state, as in the sunspot equilibrium. However, unlike in the sunspot equilibrium, an increase in $\lambda_g$ lowers the inflation rate and the modified output gap in the low state. Finally, it lowers inflation and raises the modified output gap in the high state. See Appendix D.

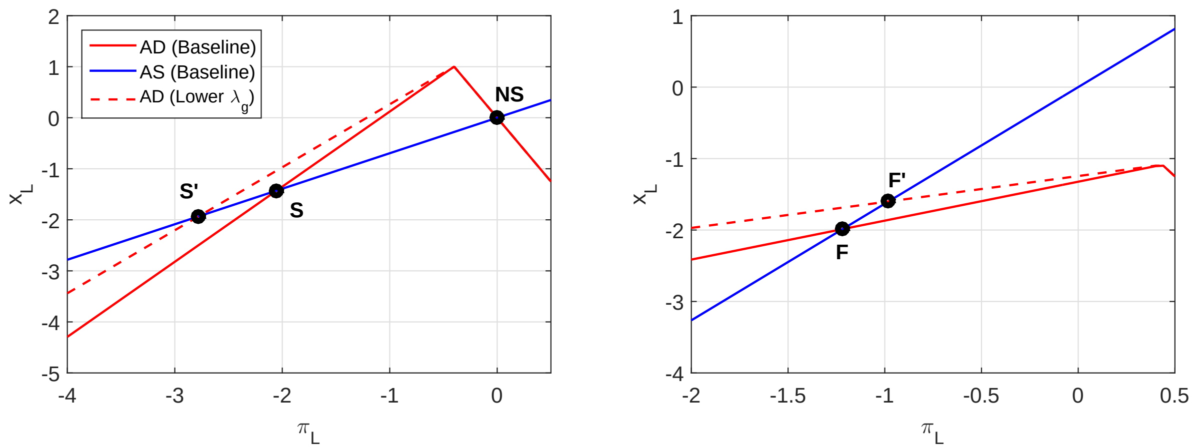
It is instructive to show how a change in $\lambda_g$ affects the low-state AD and AS curves in the two models. For both models, we assume that the high state is absorbing. The low-state AD and AS curves in the model with the sunspot shock and fiscal policy are then given by

$$\textbf{AD-sunspot:} \quad x_L = \min\left[\frac{1}{\lambda_g + (1-\Gamma)^2\bar{\lambda}}\left(\frac{\sigma\lambda_g}{1-p_L}r^n + \left(\frac{\sigma p_L \lambda_g}{1-p_L} - (1-\Gamma)^2\kappa\right)\pi_L\right), -\frac{\kappa}{\bar{\lambda}}\pi_L\right]$$

$$\textbf{AS-sunspot:} \quad x_L = \frac{1-\beta p_L}{\kappa}\pi_L,$$

where $\pi_H$ and $x_H$ have been set equal to zero. For the model with the fundamental shock and fiscal

Figure 4: The effect of reduction in $\lambda_g$ on low-state aggregate demand and supply




(a) Model with sunspot shock

(b) Model with fundamental shock

Note: Solid lines: $\lambda_g = \bar{\lambda}_g$; dashed lines: $\lambda_g = \bar{\lambda}_g/10$. In the left panel, $S$ marks the sunspot equilibrium in the baseline, $S'$ marks the sunspot equilibrium in case of a lower $\lambda_g$ and $NS$ marks the no-sunspot equilibrium. In the right panel, $F$ marks the fundamental equilibrium in the baseline and $F'$ marks the fundamental equilibrium in case of a lower $\lambda_g$. Inflation is expressed in annualized terms.

policy, the low-state AD and AS curves are given by

$$\textbf{AD-fundamental:} \quad x_L = \min\left[\frac{1}{\lambda_g + (1-\Gamma)^2\bar{\lambda}}\left(\frac{\sigma\lambda_g}{1-p_L^f}r_L^n + \left(\frac{\sigma p_L^f \lambda_g}{1-p_L^f} - (1-\Gamma)^2\kappa\right)\pi_L\right), -\frac{\kappa}{\bar{\lambda}}\pi_L\right]$$

$$\textbf{AS-fundamental:} \quad x_L = \frac{1-\beta p_L^f}{\kappa}\pi_L,$$

where again $\pi_H$ and $x_H$ have been set equal to zero.

Figure 4 depicts how the AD-AS curves are affected by a reduction in $\lambda_g$. The parameterization follows Table 1, except that we now account for a non-zero steady-state government spending to output ratio of 0.2. This ratio implies that the inverse of the elasticity of the marginal utility of private consumption with respect to output $\sigma$ becomes 0.4.[27] The inverse of the elasticity of the marginal utility of public consumption with respect to output $\nu$ is set to 0.1.[28] This implies $\bar{\lambda}_g = 0.0082$. As before, $p_L = 0.9375$ and $p_L^f = 0.85$. The intersection point $S$ in the left panel marks the sunspot equilibrium in the model with the sunspot shock for the baseline calibration, and the intersection point $NS$ marks the no-sunspot equilibrium. The intersection point $F$ in the right panel, in turn, marks the fundamental equilibrium in the model with the natural real rate shock for the baseline calibration. In both models, the AD curve becomes flatter to the left of the kink when $\lambda_g$ is lowered. Intuitively, when the policymaker adjusts government spending more aggressively to changes in inflation, aggregate demand, too, responds ceteris paribus more elastically to changes

[27] Assuming that the intertemporal elasticity of substitution in private consumption equals 0.5, as before, we have $\sigma = 0.5 \times 0.8 = 0.4$.

[28] This corresponds to the case in which the marginal utility of consumption of the public good decreases at the same rate as the marginal utility of consumption of the non-public good, i.e. $\nu = 0.5 \times 0.2 = 0.1$.

in inflation. In the model with the sunspot shock, the AD curve is steeper than the AS curve, and hence a flattening of the AD curve shifts the point at which the two curves intersect when the lower bound is binding to the south-west. In contrast, in the model with the fundamental shock, the AD curve is flatter than the AS curve, and hence a flattening of the AD curve shifts the point at which the two curves intersect to the north-east.

Propositions 9 and 10 together have a straightforward implication for the optimal value of $\lambda_g$ in the sunspot equilibrium.

**Proposition 11** *Let $\lambda_g^*$ denote the value of $\lambda_g$ that maximizes households' unconditional welfare $EV_t$ where $V_t$ is defined in equation (31). In the sunspot equilibrium, $\lambda_g^* \to \infty$.*

It is easy to show that as $\lambda_g \to \infty$, $g_L \to 0$. Intuitively, if it becomes infinitely costly for the policymaker to adjust government spending, she will not use it as a stabilization tool. This turns out to be the optimal configuration in the sunspot equilibrium. Put differently, introducing an additional policy tool in the form of government spending reduces welfare in the sunspot equilibrium. Conditional on the existence of the sunspot equilibrium, it is therefore optimal to make the use of the tool so expensive for the policymaker that she will refrain from using it.[29]

## 5.4 Why is government spending raised in the low-confidence state?

If an expansionary fiscal policy in the low-confidence state moves the economy further away from target in both confidence states, why does the policymaker not refrain from raising government spending in the low-confidence state for any $\lambda_g < \infty$? To shed light on this question consider the following thought experiment. Suppose, $\lambda_g \to \infty$, i.e. there is no systematic use of government spending for stabilization purposes in the low-confidence state. Consider some period $T \geq 0$ where the economy is in the low-confidence state and the lower bound is binding. For ease of exposition, let $p_H = 1$. The private sector behavioral constraints for period $T$ can then be written as

$$x_L^T = (1-\Gamma)g_L^T - p_L \frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L + \beta(1-p_H))\bar{\lambda} + \kappa\sigma\left(\kappa^2 + \bar{\lambda}(1-\beta p_H)\right)}{\kappa E} r^n + \sigma r^n$$
$$\pi_L^T = \kappa x_L^T - \beta p_L \frac{\kappa^2 + \bar{\lambda}(1-\beta p_H)}{E} r^n,$$

where $\pi_L^T, x_L^T, g_L^T$ are the inflation rate, the modified output gap and government spending in period $T$. Now suppose that in period $T$ there is an unexpected one-time increase in government spending. The marginal effect of this policy on the modified output gap and the inflation rate in period $T$ is $(\partial x_L^T / \partial g_L^T) = 1 - \Gamma > 0$ and $(\partial \pi_L^T / \partial g_L^T) = \kappa(1-\Gamma) > 0$. In words, the unexpected and temporary government spending stimulus raises the modified output gap and inflation in the low-confidence state.[30]

[29] Appendix C provides a numerical example of how $\lambda_g$ affects allocations and welfare in the sunspot equilibrium.
[30] This echoes the result by Wieland (2018) that it is the persistence of the fiscal policy intervention at the lower bound rather than the type of the liquidity trap that matters for the sign of government spending multipliers.

Hence, if expectations do not change, an increase in government spending is expansionary. A discretionary policymaker who raises government spending in the low-confidence state would like the private sector to expect the fiscal expansion to be temporary. However, if the economy continues to be in the low-confidence state in the next period, any discretionary policymaker with an objective function satisfying $\lambda_g < \infty$ will have an incentive to renege on her promise to undo the government spending expansion. A policy announcement of a one-time fiscal stimulus is therefore not credible. In equilibrium, agents anticipate that the discretionary policymaker will raise government spending whenever the economy transitions from the high-confidence state to the low-confidence state and that she will keep government spending at a higher level for as long as the economy remains in the low-confidence state. Since the low-confidence state is highly persistent, expansionary government spending at the lower bound is contractionary, as in Mertens and Ravn (2014).[31]

### 5.5 Avoiding the sunspot equilibrium

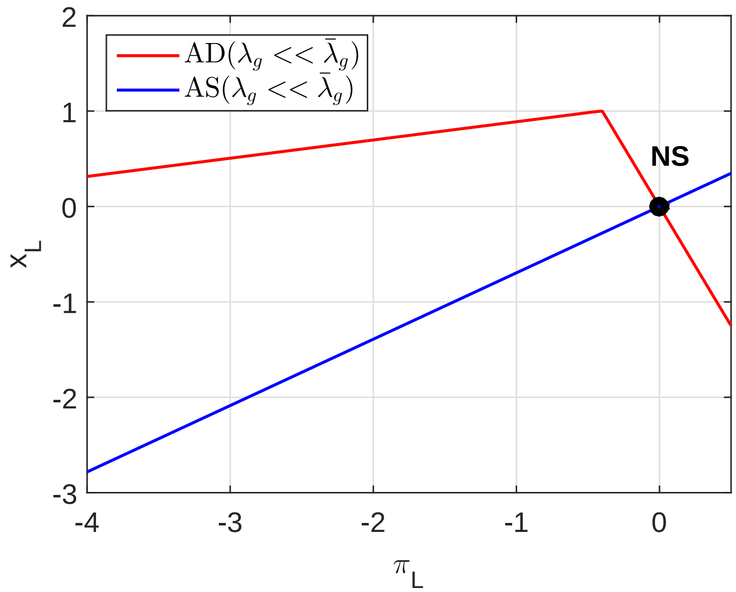
The results presented so far might appear disappointing from the perspective of policy design. Clearly, the sunspot equilibrium cannot be improved by allowing the discretionary policymaker to use government spending as an additional policy instrument. However, Proposition 8 implies that society may be able to eliminate the sunspot equilibrium and avoid expectations-driven liquidity traps altogether. To do so it has to make the relative weight on government spending stabilization in the policymaker's objective function sufficiently small.

Intuitively, when $\lambda_g \to 0$, the policymaker is willing to do "whatever it takes"—in terms of fiscal policy—to make sure that the weighted sum of inflation and the modified output gap are stabilized. Since the lower bound is not binding when this target criterion is met, $\lambda_g \to 0$ rules out the sunspot equilibrium. In this case, the only stationary equilibrium in the model with the sunspot shock is the no-sunspot equilibrium where the shock does not affect agents' behavior. In the no-sunspot equilibrium, all variables are at target in both confidence states. Figure 5 provides a graphical illustration. For a sufficiently low $\lambda_g$ the AD curve to the left of the kink becomes flatter than the AS curve and there is only one intersection point left, which is the one associated with the no-sunspot equilibrium.

From a practical perspective, an important question is whether a policymaker who puts a sufficiently small relative weight on government spending stabilization to rule out the sunspot equilibrium would be consistent with quantitatively plausible variations in government spending in the face of actual fluctuations in output and inflation. To shed light on this question, we conduct the following counterfactual experiment. We first calculate the annualized inflation rate and the output gap in the low state of the sunspot equilibrium when the policymaker has the same objective function as society ($\lambda_g = \bar{\lambda}_g$). Unlike for the numerical analysis based on the AD-AS curves, we do not have to assume that the high-confidence state is absorbing, and set $p_H = 0.98$. In this case,

[31] Appendix C provides a comparison of our setup where government spending is an endogenous variable set by an optimizing policymaker to the case where government spending is an exogenous variable, as in Mertens and Ravn (2014).

Figure 5: Avoiding the sunspot equilibrium




Note: $\lambda_g = 0.00012 << \bar{\lambda}_g$. NS marks the no-sunspot equilibrium. Inflation is expressed in annualized terms.

annualized inflation is $-2.5\%$ and the output gap is $-1.6\%$ in the low-confidence state. We then ask by how much a policymaker with a $\lambda_g$ low enough to rule out the sunspot equilibrium would raise government spending *taking as given* the above outcomes for inflation and output.

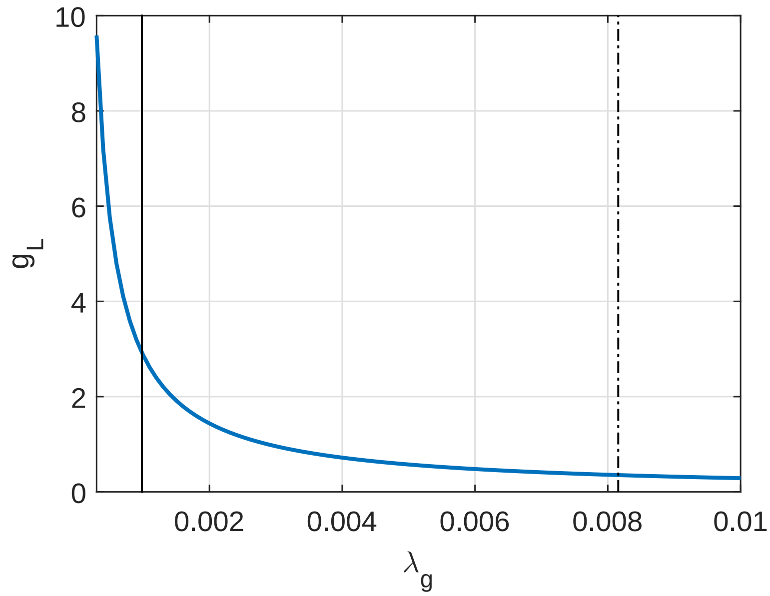
Figure 6 plots the counterfactual government spending response as a function of $\lambda_g$. A policymaker with a sufficiently small relative weight on government spending stabilization to rule out the sunspot equilibrium would raise government spending by at least 3% of total output, a quantitatively non-negligible but plausible number.

## 6 Conclusion

Expectations-driven liquidity traps differ from fundamental-driven liquidity traps in terms of their implications for the design of desirable monetary and fiscal stabilization policies. In particular, policy design becomes more complicated when liquidity trap episodes are caused by changes in agents' confidence than when they are caused by changes in the economy's fundamentals.

The occurrence of occasional fundamental-driven liquidity trap events makes it desirable for society to assign a strictly positive inflation target—high enough to generate positive inflation in the high state—or an inflation-conservative objective function to the central bank. No such clear-cut policy recommendations can be derived in case of expectations-driven liquidity trap events. The optimal inflation target may be negative or positive. Likewise, the optimal relative weight on inflation in the central bank's objective function may be smaller or larger than the weight that society puts on inflation stabilization, depending on parameter values. However, strict inflation conservatism or an inflation target high enough to generate positive inflation in the high state are

Figure 6: Counterfactual government spending response for alternative $\lambda_g$




Note: Counterfactual government spending is expressed as a share of steady state output in percentage point deviations from the steady state government spending to output ratio. The dash-dotted vertical line indicates the case where the policymaker has the same objective function as society as a whole, $\lambda_g = \bar{\lambda}_g$. For values of $\lambda_g$ to the left of the solid vertical line, the sunspot equilibrium does not exist.

never optimal in the sunspot equilibrium.

Turning to fiscal policy, the use of government spending as an additional stabilization tool—welfare-improving in the case of fundamental-driven liquidity traps—is welfare-reducing in the case of expectations-driven liquidity traps. Nevertheless, it may be desirable to assign an explicit role to fiscal policy in an economy prone to the latter, for the appointment of a policymaker who puts a sufficiently small relative weight on government spending stabilization eliminates the sunspot equilibrium.

In this paper, we have focused on policy frameworks that allow for a closed-form solution. There are other frameworks that have featured prominently in the ongoing policy debate, such as price-level targeting and nominal-GDP targeting. The analysis of these frameworks in the model with expectations-driven liquidity traps is an interesting avenue for future research.

## References


Adam, K. and R. M. Billi (2006): "Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates," *Journal of Money, Credit and Banking*, 38, 1877–1905.

——— (2007): "Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates," *Journal of Monetary Economics*, 54, 728–752.

Armenter, R. (2018): "The Perils of Nominal Targets," *Review of Economic Studies*, 85, 50–86.

Aruoba, S. B., P. Cuba-Borda, and F. Schorfheide (2018): "Macroeconomic Dynamics Near the ZLB: A Tale of Two Countries," *Review of Economic Studies*, 85, 87–118.

Benhabib, J., S. Schmitt-Grohé, and M. Uribe (2001): "The Perils of Taylor Rules," *Journal of Economic Theory*, 96, 40–69.

——— (2002): "Avoiding Liquidity Traps," *Journal of Political Economy*, 110, 535–563.

Bilbiie, F. O. (2018): "Neo-Fisherian Policies and Liquidity Traps," CEPR Discussion Papers 13334, C.E.P.R. Discussion Papers.

Bilbiie, F. O., T. Monacelli, and R. Perotti (2018): "Is Government Spending at the Zero Lower Bound Desirable?" *American Economic Journal: Macroeconomics*, forthcoming.

Boneva, L. M., R. A. Braun, and Y. Waki (2016): "Some unpleasant properties of loglinearized solutions when the nominal rate is zero," *Journal of Monetary Economics*, 84, 216–232.

Bouakez, H., M. Guillard, and J. Roulleau-Pasdeloup (2016): "The Optimal Composition of Public Spending in a Deep Recession," Cahiers de Recherches Economiques du Département d'Econométrie et d'Economie politique (DEEP) 16.09, Université de Lausanne, Faculté des HEC, DEEP.

Clarida, R. H. (2019): "The Federal Reserve's Review of Its Monetary Policy Strategy, Tools, and Communication Practices," *2019 U.S. Monetary Policy Forum, sponsored by the Initiative on Global Markets at the University of Chicago Booth School of Business, New York, New York, February 22, 2019.*

Coyle, P. and T. Nakata (2018): "Optimal Inflation Target with Expectations-Driven Liquidity Traps," Manuscript.

Cuba-Borda, P. and S. R. Singh (2019): "Understanding Persistent Stagnation," International Finance Discussion Papers 1243, Board of Governors of the Federal Reserve System (U.S.).

Eggertsson, G. B. (2006): "The Deflation Bias and Committing to Being Irresponsible," *Journal of Money, Credit and Banking*, 38, 283–321.

Eggertsson, G. B. and M. Woodford (2003): "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 34, 139–235.

——— (2006): "Optimal Monetary and Fiscal Policy in a Liquidity Trap," in *NBER International Seminar on Macroeconomics 2004*, National Bureau of Economic Research, Inc, NBER Chapters, 75–144.

Galí, J. (2015): *Monetary Policy, Inflation, and the Business Cycle*, Princeton: Princeton University Press.

Hirose, Y. (2018): "An Estimated DSGE Model with a Deflation Steady State," *Macroeconomic Dynamics*, 1–35.

Jarociński, M. and B. Maćkowiak (2018): "Monetary-fiscal interactions and the euro area's malaise," *Journal of International Economics*, 112, 251 – 266.

Jung, T., Y. Teranishi, and T. Watanabe (2005): "Optimal Monetary Policy at the Zero-Interest-Rate Bound," *Journal of Money, Credit and Banking*, 37, 813–835.

Lansing, K. J. (2017): "Endogenous Regime Switching Near the Zero Lower Bound," Working Paper Series 2017-24, Federal Reserve Bank of San Francisco.

Mertens, K. and M. O. Ravn (2014): "Fiscal Policy in an Expectations-Driven Liquidity Trap," *The Review of Economic Studies*, 81, 1637–1667.

Nakata, T. (2016): "Optimal Fiscal and Monetary Policy with Occasionally Binding Zero Bound Constraints," *Journal of Economic Dynamics and Control*, 73, 220–240.

——— (2017): "Optimal government spending at the zero lower bound: A non-Ricardian analysis," *Review of Economic Dynamics*, 23, 150 – 169.

Nakata, T. and S. Schmidt (2019a): "Conservatism and liquidity traps," *Journal of Monetary Economics*, 104, 37 – 47.

——— (2019b): "Gradualism and Liquidity Traps," *Review of Economic Dynamics*, 31, 182 – 199.

Nakov, A. (2008): "Optimal and Simple Monetary Policy Rules with Zero Floor on the Nominal Interest Rate," *International Journal of Central Banking*, 4, 73–127.

Rogoff, K. (1985): "The Optimal Degree of Commitment to an Intermediate Monetary Target," *The Quarterly Journal of Economics*, 100, 1169–89.

Schmidt, S. (2013): "Optimal Monetary and Fiscal Policy with a Zero Bound on Nominal Interest Rates," *Journal of Money, Credit and Banking*, 45, 1335–1350.

——— (2016): "Lack of Confidence, the Zero Lower Bound, and the Virtue of Fiscal Rules," *Journal of Economic Dynamics and Control*, 70, 36–53.

——— (2017): "Fiscal Activism and the Zero Nominal Interest Rate Bound," *Journal of Money, Credit and Banking*, 49, 695–732.

Schmitt-Grohé, S. and M. Uribe (2014): "Liquidity Traps: An Interest-Rate-Based Exit Strategy," *The Manchester School*, 82, 1–14.

——— (2017): "Liquidity Traps and Jobless Recoveries," *American Economic Journal: Macroeconomics*, 9, 165–204.

Sugo, T. and K. Ueda (2008): "Eliminating a Deflationary Trap through Superinertial Interest Rate Rules," *Economics Letters*, 100, 119–122.

Svensson, L. E. O. (1997): "Optimal Inflation Targets, 'Conservative' Central Banks, and Linear Inflation Contracts," *American Economic Review*, 87, 98–114.

Tamanyu, Y. (2019): "Tax Rules to Prevent Expectations-driven Liquidity Trap," Keio-IES Discussion Paper Series 2019-005, Institute for Economics Studies, Keio University.

Walsh, C. E. (1995): "Optimal Contracts for Central Bankers," *American Economic Review*, 85, 150–67.

Wieland, J. (2018): "State-dependence of the Zero Lower Bound Government Spending Multiplier," Manuscript.

Wilkins, C. A. (2018): "Choosing the Best Monetary Policy Framework for Canada," *Remarks at the McGill University, Max Bell School of Public Policy, Montreal, Quebec, November 20, 2018.*

Woodford, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton: Princeton University Press.

# Appendix

## A Sunspot equilibrium in the model without fiscal policy

### A.1 Proof of Proposition 1

To proof Proposition 1 on the necessary and sufficient conditions for existence of the sunspot equilibrium, it is useful to proceed in four steps. Each step is associated with an auxiliary proposition.

Let

$$A := -\beta\lambda(1 - p_H), \tag{A.1}$$
$$B := \kappa^2 + \lambda(1 - \beta p_H), \tag{A.2}$$
$$C := \frac{(1 - p_L)}{\sigma\kappa}(1 - \beta p_L + \beta(1 - p_H)) - p_L, \tag{A.3}$$
$$D := -\frac{(1 - p_L)}{\sigma\kappa}(1 - \beta p_L + \beta(1 - p_H)) - (1 - p_L) = -1 - C, \tag{A.4}$$

and

$$E := AD - BC. \tag{A.5}$$

**Proposition A.1** *There exists a vector* $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ *that solves the system of linear equations (8)–(13).*

**Proof:** Rearranging the system of equations (8)–(13) and eliminating $y_H$ and $y_L$, we obtain two unknowns for $\pi_H$ and $\pi_L$ in two equations

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \begin{bmatrix} \kappa^2\pi^* \\ r^n \end{bmatrix}. \tag{A.6}$$

For what follows, it is useful to show that $E = 0$ is generically inconsistent with existence of the sunspot equilibrium. Since $B > 0$, we can always write $\pi_H = \kappa^2/B\pi^* - A/B\pi_L$. Plugging this into $C\pi_L + D\pi_H = r^n$ and multiplying both sides by $B$, we get $D\kappa^2\pi^* - E\pi_L = Br^n$. Since the right-hand side of this equation is strictly positive, $E = 0$ is inconsistent with the existence of the sunspot equilibrium for generic $\pi^*$.

Hence, we can invert the matrix on the left-hand-side of (A.6)

$$\begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \frac{1}{AD - BC} \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} \kappa^2\pi^* \\ r^n \end{bmatrix}. \tag{A.7}$$

Thus,

$$\pi_H = -\frac{C\kappa^2}{E}\pi^* + \frac{A}{E}r^n \tag{A.8}$$

and

$$\pi_L = \frac{D\kappa^2}{E}\pi^* - \frac{B}{E}r^n. \tag{A.9}$$

From the Phillips curves in both states, we obtain

$$y_H = \frac{\kappa\left(\beta(1-p_H) - (1-\beta)C\right)}{E}\pi^* + \frac{\beta\kappa(1-p_H)}{E}r^n \tag{A.10}$$

and

$$y_L = \frac{\kappa\left(\beta p_L - 1 - (1-\beta)C\right)}{E}\pi^* - \frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L + \beta(1-p_H))\lambda}{\kappa E}r^n. \tag{A.11}$$

**Proposition A.2** *Suppose equations (8)–(13) are satisfied. Then $\lambda y_L + (\kappa\pi_L - \pi^*) < 0$ if and only if (i) $E > 0$ and $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2}r^n$ or (ii) $E < 0$ and $\pi^* < -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2}r^n$.*

**Proof:** Using (A.9) and (A.11), we have

$$\lambda y_L + \kappa(\pi_L - \pi^*) = -\frac{\kappa^2 + \lambda\left(1-\beta p_L + \beta(1-p_H)\right)}{E}\kappa\left(\pi^* + \frac{\kappa^2+\lambda(1-\beta)}{\kappa^2}r^n\right). \tag{A.12}$$

Notice that $(\kappa^2 + \lambda\left(1-\beta p_L + \beta(1-p_H)\right))\kappa > 0$, and $\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2}r^n > 0$. Thus, if $E > 0$ and $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2}r^n$, then $\lambda y_L + \kappa(\pi_L - \pi^*) < 0$. Similarly, if $E < 0$ and $\pi^* < -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2}r^n$, then $\lambda y_L + \kappa(\pi_L - \pi^*) < 0$.

**Proposition A.3** *Suppose equations (8)–(13) are satisfied, $E > 0$ and $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2}r^n$. Then $i_H > 0$ if and only if $p_L - (1-p_H) - \frac{1-p_L+1-p_H}{\kappa\sigma}\left(1-\beta p_L + \beta(1-p_H)\right) > 0$.*

**Proof:** $i_H$ is given by

$$\begin{aligned} i_H =& \frac{1-p_H}{\sigma}\left(y_L - y_H\right) + p_H\pi_H + (1-p_H)\pi_L + r^n \\ =& \frac{\left(p_L - (1-p_H) - \frac{1-p_L+1-p_H}{\kappa\sigma}\left(1-\beta p_L + \beta(1-p_H)\right)\right)\kappa^2}{E}\left(\pi^* + \frac{\kappa^2+\lambda(1-\beta)}{\kappa^2}r^n\right), \end{aligned} \tag{A.13}$$

where in the second row we made use of (A.8)–(A.11).

**Proposition A.4** *Suppose equations (8)–(13) are satisfied, $E < 0$ and $\pi^* < -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2}r^n$. Then $i_H < 0$.*

**Proof:** First, substitute equations (A.1), (A.2), and (A.4) into equation (A.5) to obtain

$$E = \beta\lambda(1 - p_H) - \left(\kappa^2 + \lambda(1 - \beta)\right) C. \tag{A.14}$$

Hence, $E < 0$ implies $C > 0$.

**Corollary A.1** $C < 0$ *implies* $E > 0$.

Next, note that

$$p_L - (1 - p_H) - \frac{1 - p_L + 1 - p_H}{\kappa\sigma} (1 - \beta p_L + \beta(1 - p_H)) = -C - (1 - p_H) \frac{1 - \beta p_L + \beta(1 - p_H) + \kappa\sigma}{\kappa\sigma}.$$

Hence, $C > 0$ implies $p_L - (1 - p_H) - \frac{1-p_L+1-p_H}{\kappa\sigma} (1 - \beta p_L + \beta(1 - p_H)) < 0$.

**Corollary A.2** $p_L - (1 - p_H) - \frac{1-p_L+1-p_H}{\kappa\sigma} (1 - \beta p_L + \beta(1 - p_H)) > 0$ *implies* $C < 0$.

From equation (A.13), it follows that $p_L - (1 - p_H) - \frac{1-p_L+1-p_H}{\kappa\sigma} (1 - \beta p_L + \beta(1 - p_H)) < 0$, $E < 0$ and $\pi^* < -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2} r^n$ imply $i_H < 0$.

We are now ready to proof Proposition 1. For notational convenience, define

$$\Omega(p_L, p_H, \kappa, \sigma, \beta) \equiv p_L - (1 - p_H) - \frac{1 - p_L + 1 - p_H}{\kappa\sigma} (1 - \beta p_L + \beta(1 - p_H)). \tag{A.15}$$

**Proof of "if" part**: Suppose that $\Omega(\cdot) > 0$ and $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2} r^n$. According to Proposition A.1 there exists a vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ that solves equations (8)–(13). According to Corollary A.2, $\Omega(\cdot) > 0$ implies $C < 0$. According to Corollary A.1, $C < 0$ implies $E > 0$. According to Proposition A.2, $E > 0$ and $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2} r^n$ imply $\lambda y_L + \kappa(\pi_L - \pi^*) < 0$. According to Proposition A.3, given $E > 0$ and $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2} r^n$, $\Omega(\cdot) > 0$ implies $i_H > 0$.

**Proof of "only if" part**: Suppose that the vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ solves (8)–(13), and satisfies $\lambda y_L + \kappa(\pi_L - \pi^*) < 0$ and $i_H > 0$. According to Proposition A.2, $\lambda y_L + \kappa(\pi_L - \pi^*) < 0$ implies that either (i) $E > 0$ and $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2} r^n$ or (ii) $E < 0$ and $\pi^* < -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2} r^n$. According to Proposition A.4, (ii) is inconsistent with $i_H > 0$. Hence, $E > 0$ and $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2} r^n$. According to Proposition A.3, given $E > 0$ and $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2} r^n$, $i_H > 0$ implies $\Omega(\cdot) > 0$.

### A.2 Proof of Proposition 2

The allocations and prices in the sunspot equilibrium are given by

$$\begin{aligned}
\pi_L &= -\frac{(C+1)\kappa^2}{E}\pi^* - \frac{\kappa^2 + \lambda(1-\beta p_H)}{E} r^n \\
y_L &= \frac{\kappa\left(\beta p_L - 1 - (1-\beta)C\right)}{E}\pi^* - \frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L + \beta(1-p_H))\lambda}{\kappa E} r^n \\
\pi_H &= -\frac{C\kappa^2}{E}\pi^* - \frac{\beta\lambda(1-p_H)}{E} r^n \\
y_H &= \frac{\kappa\left(\beta(1-p_H) - (1-\beta)C\right)}{E}\pi^* + \frac{\beta\kappa(1-p_H)}{E} r^n
\end{aligned}$$

Assuming $\pi^* = 0$ and $\lambda > 0$, it holds

$$\begin{aligned}
\pi_L &= -\frac{\kappa^2 + \lambda(1-\beta p_H)}{E} r^n < 0 \\
y_L &= -\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L + \beta(1-p_H))\lambda}{\kappa E} r^n < 0 \\
\pi_H &= -\frac{\beta\lambda(1-p_H)}{E} r^n \leq 0 \\
y_H &= \frac{\beta\kappa(1-p_H)}{E} r^n \geq 0
\end{aligned}$$

When $p_H < 1$, $\pi_H < 0$ and $y_H > 0$.

### A.3 Proof of Proposition 3

Keeping in mind that $-1 < C < 0$ in the sunspot equilibrium, it holds,

$$\frac{\partial \pi_L}{\partial \pi^*} = -\frac{C+1}{E}\kappa^2 < 0 \tag{A.16}$$

$$\frac{\partial y_L}{\partial \pi^*} = -\frac{\beta(1-p_L) + (1-\beta)(C+1)}{E}\kappa < 0, \tag{A.17}$$

and

$$\frac{\partial \pi_H}{\partial \pi^*} = -\frac{C}{E}\kappa^2 > 0 \tag{A.18}$$

$$\frac{\partial y_H}{\partial \pi^*} = \frac{\beta(1-p_H) - (1-\beta)C}{E}\kappa > 0. \tag{A.19}$$

### A.4 Proof of Lemma 1

If $\pi^0$ exists, it holds $-\frac{C\kappa^2}{E}\pi^0 - \frac{\beta\lambda(1-p_H)}{E}r^n = 0$. Solving for $\pi^0$, one obtains

$$\pi^0 = -\frac{\beta\lambda(1-p_H)}{C\kappa^2} r^n, \tag{A.20}$$

where $C < 0$, and hence $\pi^0 > 0$.

### A.5 Proof of Proposition 4

Note first that

$$\mathrm{E}V = -\frac{1}{1-\beta}\frac{1}{2}\left[\frac{1-p_L}{1-p_L+1-p_H}\left(\pi_H^2+\bar{\lambda}y_H^2\right)+\frac{1-p_H}{1-p_L+1-p_H}\left(\pi_L^2+\bar{\lambda}y_L^2\right)\right], \tag{A.21}$$

where $V$ is defined in equation (3).

Assuming $\lambda = \bar{\lambda}$, the partial derivative of $\mathrm{E}V$ with respect to $\pi^*$ is

$$\begin{aligned}\frac{\partial \mathrm{E}V}{\partial \pi^*} = -&\frac{1}{(1-\beta)(1-p_L+1-p_H)E^2}\Bigg\{\Big[\left(\kappa^2+\bar{\lambda}(1-\beta)^2\right)\left((1-p_H)(C+1)^2+(1-p_L)C^2\right)\\ &+\bar{\lambda}\beta(1-p_H)(1-p_L)(1-\beta p_L+1-\beta p_H)\Big]\kappa^2\pi^*+\Big[\bar{\lambda}\left(\kappa^2+\bar{\lambda}(1-\beta)\right)(1-\beta p_L+\beta(1-p_H))\\ &(\beta(1-p_L)+(1-\beta)(C+1))+\left(\kappa^2+\bar{\lambda}(1-\beta+\beta^2(1-p_L+1-p_H))\right)\kappa^2(C+1)\\ &-(\beta\kappa)^2\bar{\lambda}(1-p_L)\Big](1-p_H)r^n\Bigg\}.\end{aligned}$$

Note that all terms in the square brackets which are multiplied by $\pi^*$ are positive. In the square brackets which are multiplied by $r^n$ all terms are positive except for the last one, $-(\beta\kappa)^2\bar{\lambda}(1-p_L) < 0$.

The first-order necessary condition for the welfare-maximizing inflation target is $\frac{\partial \mathrm{E}V}{\partial \pi^*} = 0$. Solving for $\pi^*$, one obtains

$$\pi^{**} = -\frac{1-p_H}{\kappa^2}\frac{\bar{\lambda}\left(\kappa^2+\bar{\lambda}(1-\beta)\right)(1-\beta p_L+\beta(1-p_H))(\beta(1-p_L)+(1-\beta)(C+1))+\left(\kappa^2+\bar{\lambda}(1-\beta+\beta^2(1-p_L+1-p_H))\right)\kappa^2(C+1)-(\beta\kappa)^2\bar{\lambda}(1-p_L)}{\left(\kappa^2+\bar{\lambda}(1-\beta)^2\right)\left((1-p_H)(C+1)^2+(1-p_L)C^2\right)+\bar{\lambda}\beta(1-p_H)(1-p_L)(1-\beta p_L+1-\beta p_H)}r^n$$

Note that $\pi^{**} > -\frac{\kappa^2+\bar{\lambda}(1-\beta)}{\kappa^2}r^n$ whenever existence condition (22) is satisfied. Specifically, $\pi^{**} > -\frac{\kappa^2+\bar{\lambda}(1-\beta)}{\kappa^2}r^n$ if and only if

$$\begin{aligned}&\left(\kappa^2+\bar{\lambda}(1-\beta)\right)\left\{(\kappa^2+\bar{\lambda}(1-\beta)^2)C\left[(1-p_L+1-p_H)C+1-p_H\right]\right\}\\ &>\left[\left(\kappa^2+\bar{\lambda}(1-\beta)\right)\bar{\lambda}\beta(1-\beta)(1-p_H)+(\beta\kappa)^2\bar{\lambda}(1-p_H)\right]\left[(1-p_L+1-p_H)C+1-p_H\right],\end{aligned}$$

where $(1-p_L+1-p_H)C+1-p_H = -(1-p_L)\Omega(p_L,p_H,\kappa,\sigma,\beta) < 0$. Hence, the left-hand side of the inequality is positive and the right-hand side is negative, so that the inequality is satisfied.

Next, we show that $\pi^{**} < \pi^0$. This requires

$$-\frac{\beta\bar{\lambda}}{C} > -\frac{\bar{\lambda}\left(\kappa^2+\bar{\lambda}(1-\beta)\right)(1-\beta p_L+\beta(1-p_H))\left(\beta(1-p_L)+(1-\beta)(C+1)\right)+\left(\kappa^2+\bar{\lambda}(1-\beta+\beta^2(1-p_L+1-p_H))\right)\kappa^2(C+1)-(\beta\kappa)^2\bar{\lambda}(1-p_L)}{(\kappa^2+\bar{\lambda}(1-\beta)^2)\left((1-p_H)(C+1)^2+(1-p_L)C^2\right)+\bar{\lambda}\beta(1-p_H)(1-p_L)(1-\beta p_L+1-\beta p_H)},$$

which can be rewritten as

$$\begin{aligned}&\beta\bar{\lambda}\kappa^2(1-p_L)(1-\beta)C^2+\beta\bar{\lambda}^2(1-\beta)^2(1-p_L)C^2+\beta\bar{\lambda}\left(\kappa^2+\bar{\lambda}(1-\beta)^2\right)(1-p_H)(C+1)^2+(\beta\bar{\lambda})^2(1-p_L)(1-p_H)(1-\beta p_L+1-\beta p_H)\\&>(\beta\kappa)^2\bar{\lambda}(1-p_L)(1-p_H)C+\kappa^2\left(\kappa^2+\bar{\lambda}(1-\beta p_H)\right)C+\left[\kappa^2(1-\beta p_L)+\bar{\lambda}(1-\beta)(1-\beta p_L+1-\beta p_H)\right]\left[\beta(1-p_L)+(1-\beta)(C+1)\right]\bar{\lambda}C.\end{aligned}$$

Note that all terms on the left-hand side of the inequality sign are strictly positive and all terms on the right-hand side are strictly negative. This completes the proof.

## A.6 Proof of Proposition 5

Suppose $\pi^* = 0$ and $p_H < 1$. It holds

$$\begin{aligned}\frac{\partial\pi_L}{\lambda} &= \frac{\beta\kappa^2(1-p_H)(1-p_L)}{E^2}\frac{\kappa\sigma+(1-\beta p_L+\beta(1-p_H))}{\kappa\sigma}r^n > 0\\ \frac{\partial y_L}{\partial\lambda} &= \frac{\beta\kappa(1-p_H)(1-p_L)}{E^2}\frac{\kappa\sigma+(1-\beta)(1-\beta p_L+\beta(1-p_H))}{\kappa\sigma}r^n > 0\\ \frac{\partial\pi_H}{\partial\lambda} &= -\frac{\beta\kappa^2(1-p_H)}{E^2}\left[\Omega(p_L,p_H,\kappa,\sigma,\beta)+(1-p_H)\frac{\kappa\sigma+(1-\beta p_L+\beta(1-p_H))}{\kappa\sigma}\right]r^n < 0\\ \frac{\partial y_H}{\partial\lambda} &= -\frac{\beta\kappa(1-p_H)}{E^2}\left[(1-\beta)\Omega(p_L,p_H,\kappa,\sigma,\beta)+(1-p_H)\frac{\kappa\sigma+(1-\beta)(1-\beta p_L+\beta(1-p_H))}{\kappa\sigma}\right]r^n < 0\end{aligned}$$

## A.7 Proof of Proposition 6

Note first that

$$\mathrm{E}V = -\frac{1}{1-\beta}\frac{1}{2}\left[\frac{1-p_L}{1-p_L+1-p_H}\left(\pi_H^2+\bar{\lambda}y_H^2\right)+\frac{1-p_H}{1-p_L+1-p_H}\left(\pi_L^2+\bar{\lambda}y_L^2\right)\right], \tag{A.22}$$

where $V$ is defined in equation (3).

Assuming $\pi^* = 0$, the partial derivative of $\mathrm{E}V$ with respect to $\lambda$ is

$$
\begin{aligned}
\frac{\partial \mathrm{EV}}{\partial \lambda} = & \frac{\beta\left((1-p_H)r^n\right)^2}{(1-\beta)(1-p_L+1-p_H)E^3}\Bigg\{\Big[\beta\kappa^2(1-p_L)C+\kappa^2(1-\beta p_H)(C+1) \\
& +\bar{\lambda}(1-\beta)(1-\beta p_L+\beta(1-p_H))\left((1-\beta)(C+1)+\beta(1-p_L)\right)\Big]\lambda \\
& +\beta\kappa^2\left[(1-p_L)(1-p_H)\beta\bar{\lambda}-(1-p_L)(1-\beta)C\bar{\lambda}\right]+\kappa^4(C+1) \\
& +\bar{\lambda}(1-\beta p_L)\kappa^2\left((1-\beta)(C+1)+\beta(1-p_L)\right)\Bigg\}.
\end{aligned}
$$

Note that since $(C+1) > 0$ and $C < 0$, all terms in curly brackets are positive except for the very first one, $\beta\kappa^2(1-p_L)C < 0$. Also note that since in the sunspot equilibrium $E > 0$, the term in front of the curly brackets is positive for any $\lambda \geq 0$. Since the only negative term in curly brackets is multiplied by $\lambda$, $\frac{\partial \mathrm{EV}}{\partial \lambda}_{|\lambda=0} > 0$, and therefore $\lambda^* > 0$.

Furthermore, if

$$
\kappa^2\beta(1-p_L)C+\kappa^2(1-\beta p_H)(C+1)+\bar{\lambda}(1-\beta)\left(1-\beta p_L+\beta(1-p_H)\right)\left((C+1)(1-\beta)+\beta(1-p_L)\right) \geq 0,
$$

then $\frac{\partial \mathrm{EV}}{\partial \lambda} > 0$ for all $\lambda \geq 0$. Hence, in this case no interior solution for $\lambda^*$ exists and $\lambda^* = \infty$.

If instead

$$
\kappa^2\beta(1-p_L)C+\kappa^2(1-\beta p_H)(C+1)+\bar{\lambda}(1-\beta)\left(1-\beta p_L+\beta(1-p_H)\right)\left((C+1)(1-\beta)+\beta(1-p_L)\right) < 0,
$$

then

$$
\lambda^* = -\frac{\beta\kappa^2\left[(1-p_L)(1-p_H)\beta\bar{\lambda}-(1-p_L)(1-\beta)C\bar{\lambda}\right]+\kappa^4(C+1)+\bar{\lambda}(1-\beta p_L)\kappa^2\left((1-\beta)(C+1)+\beta(1-p_L)\right)}{\kappa^2\beta(1-p_L)C+\kappa^2(1-\beta p_H)(C+1)+\bar{\lambda}(1-\beta)\left(1-\beta p_L+\beta(1-p_H)\right)\left((C+1)(1-\beta)+\beta(1-p_L)\right)}
$$

In this case, $\lambda^* > \bar{\lambda}$ if

$$
(\beta\kappa)^2(1-p_L)\bar{\lambda}\underbrace{(C+1-p_H)}_{<0}+\kappa^2\left(\kappa^2+(1-\beta p_H)\bar{\lambda}\right)(C+1)+\left(\kappa^2(1-\beta p_L)+(1-\beta)\bar{\lambda}(1-\beta p_L+\beta(1-p_H))\right)\left(\beta(1-p_L)+(1-\beta)(C+1)\right)\bar{\lambda} > 0
$$

## A.8 Proof of Proposition 7

Let $X_{S|\lambda=\bar{\lambda},\pi^*=\hat{\pi}^*}$ denote the outcome of variable $X \in \{\pi, y\}$ in state $S \in \{H, L\}$ of the sunspot equilibrium when $\lambda = \bar{\lambda}$ and $\pi^* = \hat{\pi}^*$, and $X_{S|\lambda=\hat{\lambda},\pi^*=0}$ when $\lambda = \hat{\lambda}$ and $\pi^* = 0$. We need to show that $X_{S|\lambda=\bar{\lambda},\pi^*=\hat{\pi}^*} = X_{S|\lambda=\hat{\lambda},\pi^*=0}$ for all $X \times S$ and any $\hat{\lambda} \geq 0$.

High-state inflation:

$$\begin{aligned}\pi_{H|\lambda=\bar{\lambda},\pi^*=\hat{\pi}^*} =& -\frac{C\kappa^2}{[\beta\bar{\lambda}(1-p_H)-(\kappa^2+\bar{\lambda}(1-\beta))C]}\frac{\beta(1-p_H)(\bar{\lambda}-\hat{\lambda})}{[\beta\hat{\lambda}(1-p_H)-(\kappa^2+\hat{\lambda}(1-\beta))C]}r^n\\ &-\frac{\beta\bar{\lambda}(1-p_H)}{\beta\bar{\lambda}(1-p_H)-(\kappa^2+\bar{\lambda}(1-\beta))C}r^n\\ =& -\frac{\beta\hat{\lambda}(1-p_H)}{\beta\hat{\lambda}(1-p_H)-(\kappa^2+\hat{\lambda}(1-\beta))C}r^n\\ =&\pi_{H|\lambda=\hat{\lambda},\pi^*=0}\end{aligned}$$

High-state output:

$$\begin{aligned}y_{H|\lambda=\bar{\lambda},\pi^*=\hat{\pi}^*} =& \frac{\kappa\left(\beta(1-p_H)-(1-\beta)C\right)}{[\beta\bar{\lambda}(1-p_H)-(\kappa^2+\bar{\lambda}(1-\beta))C]}\frac{\beta(1-p_H)(\bar{\lambda}-\hat{\lambda})}{[\beta\hat{\lambda}(1-p_H)-(\kappa^2+\hat{\lambda}(1-\beta))C]}r^n\\ &+\frac{\beta\kappa(1-p_H)}{\beta\bar{\lambda}(1-p_H)-(\kappa^2+\bar{\lambda}(1-\beta))C}r^n\\ =&\frac{\beta\kappa(1-p_H)}{\beta\hat{\lambda}(1-p_H)-(\kappa^2+\hat{\lambda}(1-\beta))C}r^n\\ =&y_{H|\lambda=\hat{\lambda},\pi^*=0}\end{aligned}$$

Low-state inflation:

$$\begin{aligned}\pi_{L|\lambda=\bar{\lambda},\pi^*=\hat{\pi}^*} =& \frac{D\kappa^2}{[\beta\bar{\lambda}(1-p_H)-(\kappa^2+\bar{\lambda}(1-\beta))C]}\frac{\beta(1-p_H)(\bar{\lambda}-\hat{\lambda})}{[\beta\hat{\lambda}(1-p_H)-(\kappa^2+\hat{\lambda}(1-\beta))C]}r^n\\ &-\frac{\kappa^2+\bar{\lambda}(1-\beta p_H)}{\beta\bar{\lambda}(1-p_H)-(\kappa^2+\bar{\lambda}(1-\beta))C}r^n\\ =& -\frac{\kappa^2+\hat{\lambda}(1-\beta p_H)}{\beta\hat{\lambda}(1-p_H)-(\kappa^2+\hat{\lambda}(1-\beta))C}r^n\\ =&\pi_{L|\lambda=\hat{\lambda},\pi^*=0}\end{aligned}$$

Low-state output:

$$\begin{aligned}y_{L|\lambda=\bar{\lambda},\pi^*=\hat{\pi}^*} =& \frac{\kappa(\beta p_L-1-(1-\beta)C)}{[\beta\bar{\lambda}(1-p_H)-(\kappa^2+\bar{\lambda}(1-\beta))C]}\frac{\beta(1-p_H)(\bar{\lambda}-\hat{\lambda})}{[\beta\hat{\lambda}(1-p_H)-(\kappa^2+\hat{\lambda}(1-\beta))C]}r^n\\ &-\frac{\kappa^2(1-\beta p_L)+\bar{\lambda}(1-\beta)\left(1-\beta p_L+\beta(1-p_H)\right)}{\beta\bar{\lambda}(1-p_H)-(\kappa^2+\bar{\lambda}(1-\beta))C}r^n\\ =& -\frac{\kappa^2(1-\beta p_L)+\hat{\lambda}(1-\beta)\left(1-\beta p_L+\beta(1-p_H)\right)}{\beta\hat{\lambda}(1-p_H)-(\kappa^2+\hat{\lambda}(1-\beta))C}r^n\\ =&y_{L|\lambda=\hat{\lambda},\pi^*=0}\end{aligned}$$

### A.9 Numerical example

This subsection provides a numerical example of the sunspot equilibrium in the model without fiscal policy. One period is assumed to correspond to one quarter, and the parameterisation follows Table 1.

Figure A.1 plots the region of existence for the sunspot equilibrium in the $(p_H, p_L)$ space (black area), and the region of existence for the fundamental equilibrium in the $(p_H^f, p_L^f)$ space (gray area).[32]

Figure A.1: Existence regions for sunspot equilibrium and fundamental equilibrium

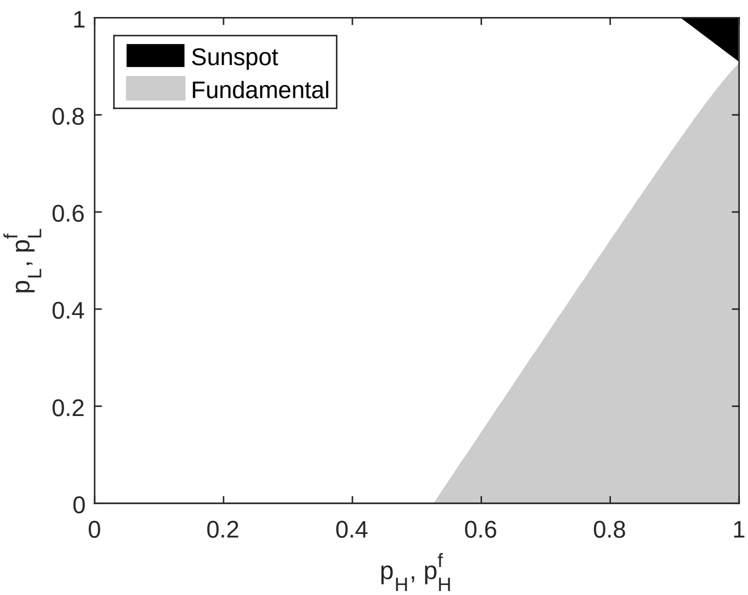



Figure A.2 shows how allocations and welfare in the sunspot equilibrium depend on the central bank's inflation target $\pi^*$. We set $p_L = 0.9375$ and $p_H = 0.98$. In this particular example, the optimal inflation target is negative.

# B Policy problem in the model with fiscal policy

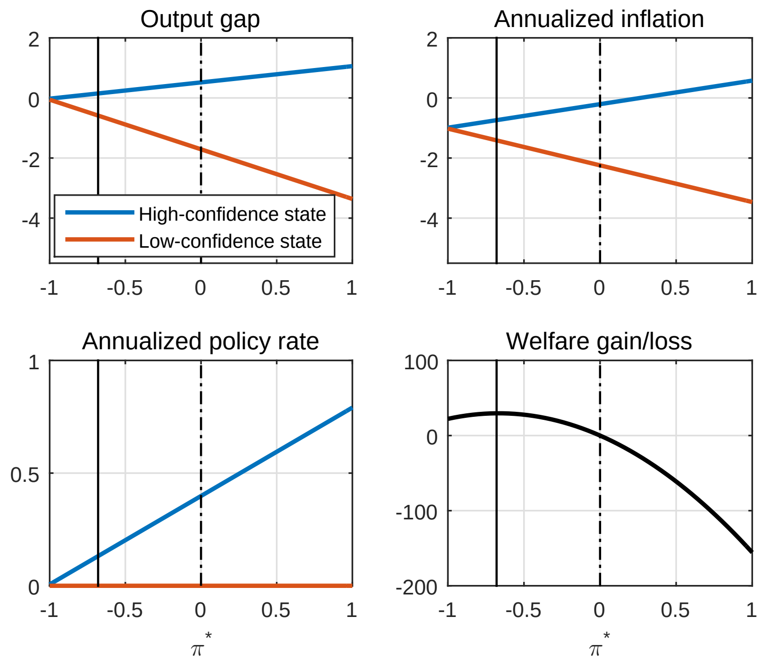
At the beginning of time, society delegates monetary and fiscal policy to a discretionary policy-maker. The objective function of the policymaker is given by

$$V_t^{MF} = -\frac{1}{2}\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left(\pi_{t+j}^2 + \bar{\lambda} x_{t+j}^2 + \lambda_g g_{t+j}^2\right), \tag{B.1}$$

where for $\lambda_g = \bar{\lambda}_g$, the policymaker's objective function coincides with society's objective function.

[32] In case of the fundamental equilibrium, the condition for equilibrium existence depends on the value of the natural real rate in the low-fundamental state, $r_L^n$. The region of existence is shrinking in the absolute value of $r_L^n$.

Figure A.2: Allocations and welfare as a function of $\pi^*$




Note: Dash-dotted vertical lines indicate the case where the central bank has the same objective function as society as a whole, i.e. $\pi^* = 0$. Solid vertical lines indicate the welfare-maximizing inflation target. The welfare gain/loss is expressed relative to the welfare level achieved when the inflation target is zero (in percent).

The optimization problem of a generic policymaker acting under discretion is as follows. Each period $t$, she chooses the inflation rate, the modified output gap, government spending, and the nominal interest rate to maximize its objective function (B.1) subject to the behavioral constraints of the private sector and the lower bound constraint, with the policy functions at time $t+1$ taken as given. Since the model features no endogenous state variable, the central bank solves a sequence of static optimization problems

$$\max_{\pi_t, x_t, g_t, i_t} -\frac{1}{2}\left(\pi_t^2 + \bar{\lambda} x_t^2 + \lambda_g g_t^2\right) \tag{B.2}$$

subject to

$$\pi_t = \kappa x_t + \beta \mathrm{E}_t \pi_{t+1} \tag{B.3}$$

$$x_t = \mathrm{E}_t x_{t+1} + (1-\Gamma)(g_t - \mathrm{g}_{t+1}) - \sigma\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^n\right) \tag{B.4}$$

$$i_t \geq 0 \tag{B.5}$$

The consolidated first order conditions are

$$(\kappa\pi_t + \bar{\lambda}x_t)i_t = 0 \tag{B.6}$$

$$\kappa\pi_t + \bar{\lambda}x_t \leq 0 \tag{B.7}$$

$$i_t \geq 0 \tag{B.8}$$

$$\lambda_g g_t + (1-\Gamma)(\kappa\pi_t + \bar{\lambda}x_t) = 0 \tag{B.9}$$

together with the private sector behavioral constraints.

# C Sunspot equilibrium in the model with fiscal policy

## C.1 Proof of Proposition 8

To proof Proposition 8 on the necessary and sufficient condition for existence of the sunspot equilibrium, it is useful to proceed in three steps. Each step is associated with an auxiliary proposition.

Let

$$\tilde{C} := \lambda_g C + \left(\kappa^2 + \bar{\lambda}(1-\beta p_L)\right)\frac{(1-\Gamma)^2}{\kappa\sigma}(1-p_L), \tag{C.1}$$

$$\tilde{D} := \lambda_g D - \beta\bar{\lambda}\frac{(1-\Gamma)^2}{\kappa\sigma}(1-p_L)^2, \tag{C.2}$$

and

$$\begin{aligned}\tilde{E} :=& A\tilde{D} - B\tilde{C}\\ =& \lambda_g E - \frac{(1-\Gamma)^2(1-p_L)}{\kappa\sigma}\left(\kappa^2 + \bar{\lambda}(1-\beta)\right)\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right],\end{aligned} \tag{C.3}$$

where $A, B, C, D$ and $E$ are defined in (A.1)–(A.5).

**Proposition C.1** *There exists a vector $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$ that solves the system of linear equations (33)–(40).*

**Proof:** Rearranging the system of equations (33)–(40) and eliminating $x_H$, $i_H$, $g_H$, $x_L$, $i_L$ and $g_L$, we obtain two unknowns for $\pi_H$ and $\pi_L$ in two equations

$$\begin{bmatrix} A & B \\ \tilde{C} & \tilde{D} \end{bmatrix}\begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda_g r^n \end{bmatrix}. \tag{C.4}$$

For what follows, it is useful to show that $\tilde{E} = 0$ is inconsistent with existence of the sunspot equilibrium. Since $B > 0$, we can always write $\pi_H = -A/B\pi_L$. Plugging this into $\tilde{C}\pi_L + \tilde{D}\pi_H = \lambda_g r^n$ and multiplying both sides by $B$, we get $-\tilde{E}\pi_L = B\lambda_g r^n$. Since the right-hand side of this equation is strictly positive for $\lambda_g > 0$, $\tilde{E} = 0$ is inconsistent with the existence of the sunspot equilibrium. Hence, we can invert the matrix on the left-hand-side of (C.4)

$$\begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \frac{1}{A\tilde{D} - B\tilde{C}} \begin{bmatrix} \tilde{D} & -B \\ -\tilde{C} & A \end{bmatrix} \begin{bmatrix} 0 \\ \lambda_g r^n \end{bmatrix}. \tag{C.5}$$

Thus,

$$\pi_H = \frac{A}{\tilde{E}} \lambda_g r^n \tag{C.6}$$

and

$$\pi_L = \frac{-B}{\tilde{E}} \lambda_g r^n. \tag{C.7}$$

From the Phillips curves in both states, we obtain

$$x_H = \frac{\beta\kappa(1 - p_H)}{\tilde{E}} \lambda_g r^n \tag{C.8}$$

and

$$x_L = -\frac{(1 - \beta p_L)\kappa^2 + (1 - \beta)(1 - \beta p_L + \beta(1 - p_H))\bar{\lambda}}{\kappa\tilde{E}} \lambda_g r^n. \tag{C.9}$$

Using the target criterion for fiscal policy in the low-confidence state (39), we obtain

$$g_L = \frac{(1 - \Gamma)\left(\kappa^2 + \bar{\lambda}(1 - \beta)\right)\left(\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H))\right)}{\kappa\tilde{E}} r^n. \tag{C.10}$$

Using the consumption Euler equation in the high-confidence state (33), we obtain

$$\begin{aligned} i_H = \Bigg[ 1 - \frac{1 - p_H}{\tilde{E}} \Bigg( & \lambda_g \left( \kappa^2 + \bar{\lambda} + (\kappa^2 + \bar{\lambda}(1 - \beta)) \frac{1 - \beta p_L + \beta(1 - p_H)}{\kappa\sigma} \right) \\ & + \frac{(1 - \Gamma)^2}{\kappa\sigma} (\kappa^2 + \bar{\lambda}(1 - \beta)) \left( \kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H)) \right) \Bigg) \Bigg] r^n. \end{aligned} \tag{C.11}$$

Finally, from equations (35) and (40), we have $g_H = 0$, and $i_L = 0$.

**Proposition C.2** *Suppose equations (33)–(40) are satisfied. Then* $\bar{\lambda} x_L + \kappa \pi_L < 0$ *if and only if* $\tilde{E} > 0$.

**Proof:** Using (C.7) and (C.9), we have

$$\bar{\lambda} x_L + \kappa \pi_L = -\frac{\left(\kappa^2 + \bar{\lambda}(1 - \beta)\right)\left(\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H))\right)}{\kappa\tilde{E}} \lambda_g r^n \tag{C.12}$$

Notice that $\lambda_g r^n > 0$ and $\left(\kappa^2 + \bar{\lambda}(1-\beta)\right)\left(\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right) > 0$. Thus, if $\bar{\lambda}x_L + \kappa\pi_L < 0$, then $\tilde{E} > 0$. Similarly, if $\tilde{E} > 0$, then $\bar{\lambda}x_L + \kappa\pi_L < 0$.

**Proposition C.3** *Suppose equations (33)–(40) are satisfied and $\tilde{E} > 0$. Then $i_H > 0$ if and only if $\lambda_g\Omega(p_L, p_H, \kappa, \sigma, \beta) - (1-\Gamma)^2\frac{1-p_L+1-p_H}{\kappa\sigma}\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right] > 0$, where $\Omega(\cdot)$ is defined in (A.15).*

**Proof:** First, notice that $i_H$ is given by

$$\begin{aligned} i_H =& \frac{1-p_H}{\sigma}\left(x_L - x_H + (1-\Gamma)(g_H - g_L)\right) + p_H\pi_H + (1-p_H)\pi_L + r^n \\ =& \frac{\left(\kappa^2 + \bar{\lambda}(1-\beta)\right)r^n}{\tilde{E}}\left[\lambda_g\Omega(p_L, p_H, \kappa, \sigma, \beta) - (1-\Gamma)^2\frac{1-p_L+1-p_L}{\kappa\sigma}\left(\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right)\right], \end{aligned} \tag{C.13}$$

where in the second row we made use of (C.6)–(C.10). Notice also that $\frac{\left(\kappa^2+\bar{\lambda}(1-\beta)\right)r^n}{\tilde{E}} > 0$. Thus, if $\lambda_g\Omega(p_L, p_H, \kappa, \sigma, \beta) - (1-\Gamma)^2\frac{1-p_L+1-p_H}{\kappa\sigma}\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right] > 0$ then $i_H > 0$. Similarly, if $i_H > 0$ then $\lambda_g\Omega(p_L, p_H, \kappa, \sigma, \beta) - (1-\Gamma)^2\frac{1-p_L+1-p_H}{\kappa\sigma}\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right] > 0$.

We are now ready to proof Proposition 8. For notational convenience, define

$$\tilde{\Omega}(p_L, p_H, \kappa, \sigma, \beta, \Gamma, \lambda_g) = \lambda_g\Omega(p_L, p_H, \kappa, \sigma, \beta) - (1-\Gamma)^2\frac{1-p_L+1-p_H}{\kappa\sigma}\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right] \tag{C.14}$$

**Proof of "if" part**: Suppose that $\tilde{\Omega}(\cdot) > 0$. According to Proposition C.1 there exists a vector $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$ that solves equations (33)–(40). Notice that

$$\begin{aligned} (\kappa^2 + \bar{\lambda}(1-\beta))\tilde{\Omega}(\cdot) =& \tilde{E} - (1-p_H)\left[\lambda_g\left(\kappa^2 + \bar{\lambda} + (\kappa^2 + \bar{\lambda}(1-\beta))\frac{1-\beta p_L + \beta(1-p_H)}{\kappa\sigma}\right)\right. \\ & \left. + \frac{(1-\Gamma)^2}{\kappa\sigma}(\kappa^2 + \lambda(1-\beta))\left(\kappa^2 + \lambda(1-\beta p_L + \beta(1-p_H))\right)\right]. \end{aligned}$$

Hence, $\tilde{\Omega}(\cdot) > 0$ implies $\tilde{E} > 0$. According to Proposition C.2, $\tilde{E} > 0$ implies $\bar{\lambda}x_L + \kappa\pi_L < 0$. According to Proposition C.3, given $\tilde{E} > 0$, $\tilde{\Omega}(\cdot) > 0$ implies $i_H > 0$.

**Proof of "only if" part**: Suppose that the vector $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$ solves (33)–(40), and satisfies $\bar{\lambda}x_L + \kappa\pi_L < 0$ and $i_H > 0$. According to Proposition C.2, $\bar{\lambda}x_L + \kappa\pi_L < 0$ implies $\tilde{E} > 0$. According to Proposition C.3, $\tilde{E} > 0$ and $i_H > 0$ imply $\tilde{\Omega}(\cdot) > 0$.

## C.2 Proof of Proposition 9

In the sunspot equilibrium, allocations and prices are given by

$$\pi_L = -\frac{\kappa^2 + \bar{\lambda}(1-\beta p_H)}{\tilde{E}}\lambda_g r^n < 0 \tag{C.15}$$

$$x_L = -\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L + \beta(1-p_H))\bar{\lambda}}{\kappa\tilde{E}}\lambda_g r^n < 0 \tag{C.16}$$

$$g_L = \frac{(1-\Gamma)\left(\kappa^2 + \bar{\lambda}(1-\beta)\right)\left(\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right)}{\kappa\tilde{E}} r^n > 0 \tag{C.17}$$

$$\pi_H = -\frac{\beta\bar{\lambda}(1-p_H)}{\tilde{E}}\lambda_g r^n \leq 0 \tag{C.18}$$

$$x_H = \frac{\beta\kappa(1-p_H)}{\tilde{E}}\lambda_g r^n \geq 0 \tag{C.19}$$

$$g_H = 0, \tag{C.20}$$

where $\tilde{E} > 0$ is defined in equation (C.3). When $p_H < 0$, $\pi_H < 0$ and $x_H > 0$.

## C.3 Proof of Proposition 10

In the sunspot equilibrium, it holds

$$\frac{\partial \pi_L}{\partial \lambda_g} = \frac{(\kappa^2 + \bar{\lambda}(1-\beta p_H))(1-\Gamma)^2(\kappa\sigma)^{-1}(1-p_L)(\kappa^2+\bar{\lambda}(1-\beta))\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right]}{\tilde{E}^2} r^n > 0$$

$$\frac{\partial x_L}{\partial \lambda_g} = \left[\kappa^2(1-\beta p_L) + \bar{\lambda}(1-\beta)(1-\beta p_L + \beta(1-p_H))\right]$$
$$\times \frac{(1-\Gamma)^2(\kappa\sigma)^{-1}(1-p_L)(\kappa^2+\bar{\lambda}(1-\beta))\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right]}{\kappa\tilde{E}^2} r^n > 0$$

$$\frac{\partial g_L}{\lambda_g} = -\frac{(1-\Gamma)\left(\kappa^2 + \bar{\lambda}(1-\beta)\right)\left(\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right)}{\kappa\tilde{E}^2} E r^n < 0$$

and

$$\frac{\partial \pi_H}{\partial \lambda_g} = \frac{\beta\bar{\lambda}(1-p_H)(1-\Gamma)^2(\kappa\sigma)^{-1}(1-p_L)(\kappa^2+\bar{\lambda}(1-\beta))\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right]}{\tilde{E}^2} r^n \geq 0$$

$$\frac{\partial x_H}{\partial \lambda_g} = -\frac{\beta\kappa(1-p_H)(1-\Gamma)^2(\kappa\sigma)^{-1}(1-p_L)(\kappa^2+\bar{\lambda}(1-\beta))\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right]}{\tilde{E}^2} r^n \leq 0.$$

When $p_H < 1$, $\frac{\partial \pi_H}{\partial \lambda_g} > 0$ and $\frac{\partial x_H}{\partial \lambda_g} < 0$.

## C.4 Comparison with an exogenous increase in government spending

In our analysis of fiscal policy, government spending is an endogenous variable set by an optimizing policymaker. A more common approach in the literature on fiscal policy in expectations-driven liquidity traps is to treat the fiscal policy instrument as an exogenous variable (e.g. Mertens and Ravn, 2014; Bilbiie, 2018). We therefore provide a brief comparison of these two approaches.

Suppose that government spending follows an exogenous process that is perfectly correlated with the sunspot shock, as commonly assumed in the literature, i.e. $g_t = g_L$ if $\xi_t = \xi_L$ and $g_t = g_H$ if $\xi_t = \xi_H$, where $g_L > g_H = 0$. For this case, the definition of the sunspot equilibrium has to be slightly modified.

**Definition 5** *The sunspot equilibrium in the model with the sunspot shock and* **exogenous** *fiscal policy is given by a vector* $\{x_H, \pi_H, i_H, x_L, \pi_L, i_L\}$ *that solves the system of linear equations (33), (34), (36), (37), (38), (40), and satisfies the inequality constraints (41) and (42).*

Assuming that the high-confidence state is absorbing ($p_H = 1$), the low-confidence-state AD and AS curves in the model with exogenous fiscal policy are given by

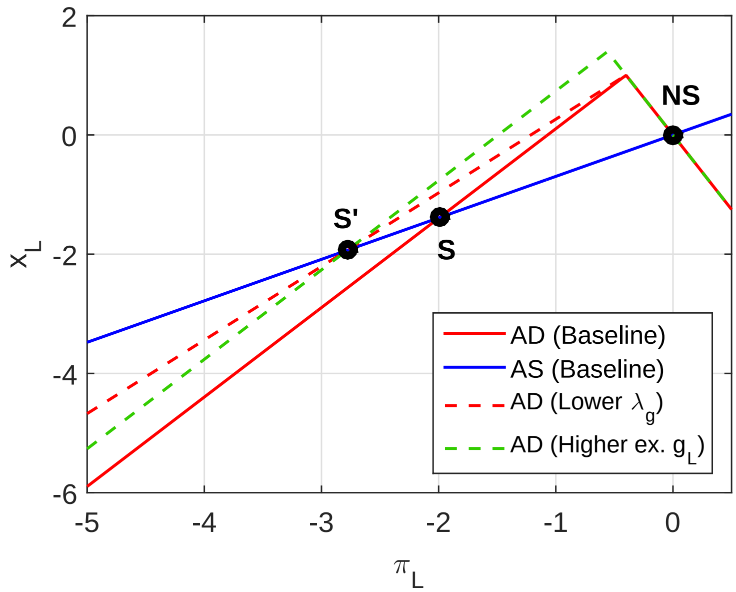
$$\textbf{AD-sunspot g-ex:} \quad x_L = \min\left[\left(\frac{\sigma}{1-p_L}r^n + (1-\Gamma)g_L\right) + \frac{\sigma p_L}{1-p_L}\pi_L, -\frac{\kappa}{\lambda}\pi_L\right] \tag{C.21}$$

$$\textbf{AS-sunspot g-ex:} \quad x_L = \frac{1-\beta p_L}{\kappa}\pi_L \tag{C.22}$$

Figure C.1 compares the effects of a reduction in $\lambda_g$—which in equilibrium results in an increase in $g_L$—on the AD-AS curves in the model with endogenous fiscal policy to those of an increase in $g_L$ in the model with exogenous fiscal policy. For the baseline, it is assumed that $\lambda_g = \infty$ in the model with endogenous fiscal policy and $g_L = 0$ in the model with exogenous fiscal policy. Hence, in the baseline, the low-state AD curve is the same whether fiscal policy is endogenous or exogenous. The sunspot equilibrium in the baseline is represented by the intersection of the AD curve (red solid line) with the AS curve (blue solid line), marked by point $S$. When considering an increase in low-state government spending in the model with exogenous fiscal policy, we calibrate the stimulus to be of the same size as the equilibrium increase in government spending that occurs in the model with endogenous fiscal policy in response to the reduction in $\lambda_g$.

In the model with endogenous fiscal policy a change in $\lambda_g$ affects the slope of the AD curve to the left of the kink. A reduction in $\lambda_g$ makes the AD curve flatter (red dashed line). In the model with exogenous fiscal policy, a change in low-state government spending instead affects the intercept term in the AD curve and results in a level shift to the left of the kink. An increase in low-state government spending shifts the AD curve upwards (green dashed line). While the sunspot equilibria in the two models are observationally equivalent by construction (see point $S'$), the two AD curves are not observationally equivalent. Since an exogenous increase in low-state government spending does not affect the slope of the AD curve, a policy intervention of this type is in general unsuited to eliminate the sunspot equilibrium.

Figure C.1: Low-confidence state AD-AS curves: Endogenous vs exogenous fiscal policy




Note: Solid lines: $\lambda_g = \infty$ (fiscal policy endogenous), $g_L = 0$ (fiscal policy exogenous); red dashed line: $\lambda_g = \bar{\lambda}_g/10$ (fiscal policy endogenous); green dashed line: $g_L = 4$ (fiscal policy exogenous). Inflation is expressed in annualized terms.

### C.5 Numerical example

This subsection provides a numerical example of how allocations and welfare depend on the relative weight that the policymaker's objective function puts on government spending stabilization $\lambda_g$. The parameterisation follows Table 1 except that we account for a non-zero steady-state government spending to output ratio of 0.2, which implies that the inverse of the elasticity of the marginal utility of private consumption with respect to output $\sigma$ becomes 0.4. The inverse of the elasticity of the marginal utility of public consumption with respect to output $\nu$ is set to 0.1, as in Section 5. This implies $\bar{\lambda}_g = 0.0082$. In addition, $p_L = 0.9375$ and $p_H = 0.98$.

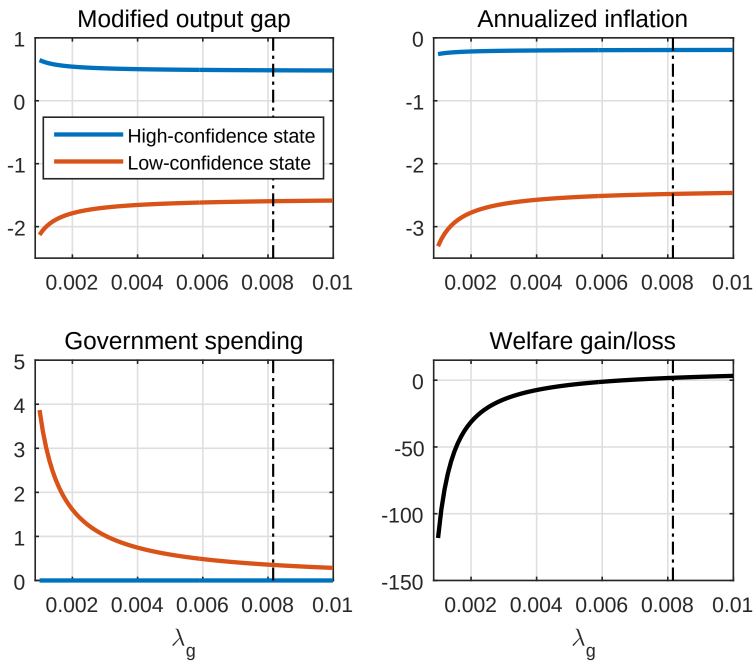
## D Fundamental equilibrium in the model with fiscal policy

### D.1 Existence of the fundamental equilibrium

**Proposition 12** *The fundamental equilibrium in the model with government consumption and a two-state natural real rate shock exists if and only if*

$$\begin{aligned}\tilde{E}^f <& (1-p_H^f)\frac{r_L^n}{r_H^n}\Bigg[\lambda_g\left(\kappa^2+\bar{\lambda}+(\kappa^2+\bar{\lambda}(1-\beta))\frac{1-\beta p_L^f+\beta(1-p_H^f)}{\kappa\sigma}\right) \\ &+\frac{(1-\Gamma)^2}{\kappa\sigma}(\kappa^2+\bar{\lambda}(1-\beta))\left(\kappa^2+\bar{\lambda}(1-\beta p_L^f+\beta(1-p_H^f))\right)\Bigg]\end{aligned} \quad \text{(D.1)}$$

Figure C.2: Allocations and welfare as a function of $\lambda_g$




Note: Dash-dotted vertical lines indicate the case where the policymaker has the same objective function as society as a whole, i.e. $\lambda_g = \bar{\lambda}_g$. The welfare gain/loss is expressed relative to the welfare level achieved when $\lambda_g = \bar{\lambda}_g$ (in percent).

*where* $\tilde{E}^f \equiv \lambda_g E^f - \frac{(1-\Gamma)^2(1-p_L^f)}{\kappa\sigma}\left(\kappa^2 + \bar{\lambda}(1-\beta)\right)\left[\kappa^2 + \bar{\lambda}(1-\beta p_L^f + \beta(1-p_H^f))\right]$.

To proof Proposition 12, we proceed again in three steps. Each step is associated with an auxiliary proposition.

**Proposition D.1** *There exists a vector* $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$ *that solves the system of linear equations (35), (36), (39), (40), and (43)–(46).*

**Proof:** Let

$$A^f := -\beta\bar{\lambda}(1-p_H^f), \tag{D.2}$$

$$B^f := \kappa^2 + \bar{\lambda}(1-\beta p_H^f), \tag{D.3}$$

$$C^f := \frac{(1-p_L^f)}{\sigma\kappa}(1-\beta p_L^f + \beta(1-p_H^f)) - p_L^f, \tag{D.4}$$

$$D^f := -\frac{(1-p_L^f)}{\sigma\kappa}(1-\beta p_L^f + \beta(1-p_H^f)) - (1-p_L^f) = -1 - C^f, \tag{D.5}$$

and

$$E^f := A^f D^f - B^f C^f. \tag{D.6}$$

Rearranging the system of equations and eliminating $x_H$, $i_H$, $g_H$, $x_L$, $i_L$ and $g_L$, we obtain two unknowns for $\pi_H$ and $\pi_L$ in two equations

$$\begin{bmatrix} A^f & B^f \\ \tilde{C}^f & \tilde{D}^f \end{bmatrix} \begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda_g r_L^n \end{bmatrix}, \tag{D.7}$$

where

$$\tilde{C}^f := \lambda_g C^f + \left(\kappa^2 + \bar{\lambda}(1-\beta p_L^f)\right) \frac{(1-\Gamma)^2}{\kappa\sigma}(1-p_L^f), \tag{D.8}$$

$$\tilde{D}^f := \lambda_g D^f - \beta\bar{\lambda}\frac{(1-\Gamma)^2}{\kappa\sigma}(1-p_L^f)^2. \tag{D.9}$$

Define $\tilde{E}^f := A^f\tilde{D}^f - B^f\tilde{C}^f$. For what follows, it is useful to show that $\tilde{E}^f = 0$ is inconsistent with existence of the fundamental equilibrium. Since $B > 0$, we can always write $\pi_H = -A^f/B^f\pi_L$. Plugging this into $\tilde{C}^f\pi_L + \tilde{D}^f\pi_H = \lambda_g r_L^n$ and multiplying both sides by $B^f$, we get $-\tilde{E}^f\pi_L = B^f\lambda_g r_L^n$. Since the right-hand side of this equation is strictly negative for $\lambda_g > 0$, $\tilde{E}^f = 0$ is inconsistent with the existence of the fundamental equilibrium. Hence, we can invert the matrix on the left-hand-side of (D.7)

$$\begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \frac{1}{A^f\tilde{D}^f - B^f\tilde{C}^f} \begin{bmatrix} \tilde{D}^f & -B^f \\ -\tilde{C}^f & A^f \end{bmatrix} \begin{bmatrix} 0 \\ \lambda_g r_L^n \end{bmatrix}. \tag{D.10}$$

Thus,

$$\pi_H = \frac{A^f}{\tilde{E}^f}\lambda_g r_L^n \tag{D.11}$$

and

$$\pi_L = \frac{-B^f}{\tilde{E}^f}\lambda_g r_L^n. \tag{D.12}$$

From the Phillips curves in both states, we obtain

$$x_H = \frac{\beta\kappa(1-p_H^f)}{\tilde{E}^f}\lambda_g r_L^n \tag{D.13}$$

and

$$x_L = -\frac{(1-\beta p_L^f)\kappa^2 + (1-\beta)(1-\beta p_L^f + \beta(1-p_H^f))\bar{\lambda}}{\kappa\tilde{E}^f}\lambda_g r_L^n. \tag{D.14}$$

Using the target criterion for fiscal policy in the low-confidence state (39), we obtain

$$g_L = \frac{(1-\Gamma)\left(\kappa^2 + \bar{\lambda}(1-\beta)\right)\left(\kappa^2 + \bar{\lambda}(1-\beta p_L^f + \beta(1-p_H^f))\right)}{\kappa\tilde{E}^f} r_L^n. \tag{D.15}$$

Using the consumption Euler equation in the high-confidence state (43), we obtain

$$
\begin{aligned}
i_H = & r_H^n - \frac{1-p_H^f}{\tilde{E}^f}\left(\lambda_g\left(\kappa^2+\bar{\lambda}+(\kappa^2+\bar{\lambda}(1-\beta))\frac{1-\beta p_L^f+\beta(1-p_H^f)}{\kappa\sigma}\right)\right. \\
& \left. + \frac{(1-\Gamma)^2}{\kappa\sigma}(\kappa^2+\bar{\lambda}(1-\beta))\left(\kappa^2+\bar{\lambda}(1-\beta p_L^f+\beta(1-p_H^f))\right)\right) r_L^n. \qquad \text{(D.16)}
\end{aligned}
$$

Finally, from equations (35) and (40), we have $g_H = 0$, and $i_L = 0$.

**Proposition D.2** *Suppose equations (35), (36), (39), (40), and (43)–(46) are satisfied. Then* $\bar{\lambda}x_L + \kappa\pi_L < 0$ *if and only if* $\tilde{E}^f < 0$.

**Proof:** Using (D.12) and (D.14), we have

$$
\bar{\lambda}x_L + \kappa\pi_L = -\frac{(\kappa^2+\bar{\lambda}(1-\beta))\left(\kappa^2+\bar{\lambda}(1-\beta p_L^f+\beta(1-p_H^f))\right)}{\kappa\tilde{E}^f}\lambda_g r_L^n \qquad \text{(D.17)}
$$

Notice that $\lambda_g r_L^n < 0$ and $(\kappa^2+\bar{\lambda}(1-\beta))\left(\kappa^2+\bar{\lambda}(1-\beta p_L^f+\beta(1-p_H^f))\right) > 0$. Thus, if $\bar{\lambda}x_L + \kappa\pi_L < 0$, then $\tilde{E}^f < 0$. Similarly, if $\tilde{E}^f < 0$, then $\bar{\lambda}x_L + \kappa\pi_L < 0$.

**Proposition D.3** *Suppose equations (35), (36), (39), (40), and (43)–(46) are satisfied and* $\tilde{E}^f < 0$. *Then* $i_H > 0$ *if and only if* $\tilde{E}^f < \underline{\tilde{E}}^f$,
*where* $\underline{\tilde{E}}^f \equiv (1-p_H^f)\frac{r_L^n}{r_H^n}\left[\lambda_g\left(\kappa^2+\bar{\lambda}+(\kappa^2+\bar{\lambda}(1-\beta))\frac{1-\beta p_L^f+\beta(1-p_H^f)}{\kappa\sigma}\right)+\frac{(1-\Gamma)^2}{\kappa\sigma}(\kappa^2+\bar{\lambda}(1-\beta))\left(\kappa^2+\bar{\lambda}(1-\beta p_L^f+\beta(1-p_H^f))\right)\right]$

**Proof:** First, notice that $i_H$ is given by

$$
\begin{aligned}
i_H = & \frac{1-p_H^f}{\sigma}(x_L - x_H + (1-\Gamma)(g_H - g_L)) + p_H^f\pi_H + (1-p_H^f)\pi_L + r_H^n \\
= & r_H^n - \frac{1-p_H^f}{\tilde{E}^f}\left[\lambda_g\left(\kappa^2+\bar{\lambda}+(\kappa^2+\bar{\lambda}(1-\beta))\frac{1-\beta p_L^f+\beta(1-p_H^f)}{\kappa\sigma}\right)\right. \\
& \left. + \frac{(1-\Gamma)^2}{\kappa\sigma}(\kappa^2+\bar{\lambda}(1-\beta))\left(\kappa^2+\bar{\lambda}(1-\beta p_L^f+\beta(1-p_H^f))\right)\right] r_L^n, \qquad \text{(D.18)}
\end{aligned}
$$

The term in square brackets is strictly positive, $r_H^n > 0$, $r_L^n < 0$ and $\tilde{E}^f < 0$. Thus, if $\tilde{E}^f < \underline{\tilde{E}}^f$ then $i_H > 0$. Similarly, if $i_H > 0$ then $\tilde{E}^f < \underline{\tilde{E}}^f$.

We are now ready to proof Proposition 12.

**Proof of "if" part**: Suppose that $\tilde{E}^f < \underline{\tilde{E}}^f$. According to Proposition D.1 there exists a vector $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$ that solves equations (35), (36), (39), (40), and (43)–(46). Notice that $\underline{\tilde{E}}^f < 0$. Hence, $\tilde{E}^f < \underline{\tilde{E}}^f$ implies $\tilde{E}^f < 0$. According to Proposition D.2, $\tilde{E}^f < 0$ implies $\bar{\lambda}x_L + \kappa\pi_L < 0$. According to Proposition D.3, given $\tilde{E}^f < 0$, $\tilde{E}^f < \underline{\tilde{E}}^f$ implies $i_H > 0$.

**Proof of "only if" part**: Suppose that the vector $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$ solves (35), (36), (39), (40), (43)–(46), and satisfies $\bar{\lambda}x_L + \kappa\pi_L < 0$ and $i_H > 0$. According to Proposition D.2, $\bar{\lambda}x_L + \kappa\pi_L < 0$ implies $\tilde{E}^f < 0$. According to Proposition D.3, $\tilde{E}^f < 0$ and $i_H > 0$ imply $\tilde{E}^f < \underline{\tilde{E}}^f$.

## D.2 Allocations and prices

In the fundamental equilibrium, allocations and prices are given by:

$$\pi_L = -\frac{\kappa^2 + \bar{\lambda}(1-\beta p_H^f)}{\tilde{E}^f}\lambda_g r_L^n < 0 \tag{D.19}$$

$$x_L = -\frac{(1-\beta p_L^f)\kappa^2 + (1-\beta)(1-\beta p_L^f + \beta(1-p_H^f))\bar{\lambda}}{\kappa\tilde{E}^f}\lambda_g r_L^n < 0 \tag{D.20}$$

$$g_L = \frac{(1-\Gamma)\left(\kappa^2 + \bar{\lambda}(1-\beta)\right)\left(\kappa^2 + \bar{\lambda}(1-\beta p_L^f + \beta(1-p_H^f))\right)}{\kappa\tilde{E}^f} r_L^n > 0 \tag{D.21}$$

$$\pi_H = -\frac{\beta\bar{\lambda}(1-p_H^f)}{\tilde{E}^f}\lambda_g r_L^n \leq 0 \tag{D.22}$$

$$x_H = \frac{\beta\kappa(1-p_H^f)}{\tilde{E}^f}\lambda_g r_L^n \geq 0 \tag{D.23}$$

$$g_H = 0. \tag{D.24}$$

When $p_H^f < 1$, $\pi_H < 0$ and $x_H > 0$.

## D.3 Effects of a marginal change in $\lambda_g$

The partial derivatives of the policy functions with respect to $\lambda_g$ are

$$\frac{\partial \pi_L}{\partial \lambda_g} = \frac{(\kappa^2 + \bar{\lambda}(1-\beta p_H^f))(1-\Gamma)^2(\kappa\sigma)^{-1}(1-p_L^f)(\kappa^2+\bar{\lambda}(1-\beta))\left[\kappa^2 + \bar{\lambda}(1-\beta p_L^f + \beta(1-p_H^f))\right]}{(\tilde{E}^f)^2} r_L^n < 0$$

$$\frac{\partial x_L}{\partial \lambda_g} = \left[\kappa^2(1-\beta p_L^f) + \bar{\lambda}(1-\beta)(1-\beta p_L^f + \beta(1-p_H^f))\right]$$

$$\times \frac{(1-\Gamma)^2(\kappa\sigma)^{-1}(1-p_L^f)(\kappa^2+\bar{\lambda}(1-\beta))\left[\kappa^2 + \bar{\lambda}(1-\beta p_L^f + \beta(1-p_H^f))\right]}{\kappa(\tilde{E}^f)^2} r_L^n < 0$$

$$\frac{\partial g_L}{\lambda_g} = -\frac{(1-\Gamma)\left(\kappa^2 + \bar{\lambda}(1-\beta)\right)\left(\kappa^2 + \bar{\lambda}(1-\beta p_L^f + \beta(1-p_H^f))\right)}{\kappa(\tilde{E}^f)^2} E^f r_L^n,$$

and

$$\frac{\partial \pi_H}{\partial \lambda_g} = \frac{\beta\bar{\lambda}(1-p_H^f)(1-\Gamma)^2(\kappa\sigma)^{-1}(1-p_L^f)(\kappa^2+\bar{\lambda}(1-\beta))\left[\kappa^2 + \bar{\lambda}(1-\beta p_L^f + \beta(1-p_H^f))\right]}{(\tilde{E}^f)^2} r_L^n \leq 0$$

$$\frac{\partial x_H}{\partial \lambda_g} = -\frac{\beta\kappa(1-p_H^f)(1-\Gamma)^2(\kappa\sigma)^{-1}(1-p_L)(\kappa^2+\bar{\lambda}(1-\beta))\left[\kappa^2 + \bar{\lambda}(1-\beta p_L^f + \beta(1-p_H^f))\right]}{(\tilde{E}^f)^2} r_L^n \geq 0.$$

When $p_H^f < 1$, $\frac{\partial \pi_H}{\partial \lambda_g} < 0$ and $\frac{\partial x_H}{\partial \lambda_g} > 0$.