Macroeconomic Dynamics and Reallocation in a Pandemic

Dirk Krueger    Harald Uhlig    Taojun Xie

University of Pennsylvania, CEPR and NBER
University of Chicago, CEPR and NBER
National University of Singapore

Virtual Seminar

May 2020
Paper in a Nutshell

- **Question we ask:**
  - COVID 19 epidemic: how much government mitigation, when to open up again?
  - Key question here: how much will people do on their own?

- **What we do:**
  - Neoclassical growth cum SIR model.
  - Infection happens when people consume. (ERT: also when work, when interacting socially)
  - Key: **Heterogeneous consumption sectors** differ in infection risk.
  - Susceptible agents make conscious decisions. **Shift consumption** towards low-infection sectors.
  - Stylized model. Appropriate parameterization?

- **What we find:**
  - Output decline, infection rates **reduced by 80 percent** compared to homogeneous-sector economy.
  - Social planner stops even more drastic: **stops epidemic immediately**.
  - Too Panglossian?
The model: the macro part

- Living agents \( j \in [0, 1] \) or \( j \in \{s, i, r\} \) have utility

\[
U = \sum_{t=0}^{\infty} \beta^t u(c^j_t, n^j_t)
\]

where

\[
u(c, n) = \ln c - \theta \frac{n^2}{2}
\]

and

\[
c^j_t = \left( \int (c^j_{tk})^{1-1/\eta} dk \right)^{\eta/(\eta-1)} (1)
\]

- Dead agents: \( U = 0 \).
- Production and labor market: competitive, linear, frictionless. One unit of labor = 1 unit of any good.
- Budget constraint:

\[
\int c^j_{tk} dk = An^j_t (2)
\]

- Markets clear.
The model: the SIR part

- Agents can be susceptible, infected, recovered, or dead. Population fractions: $S_t, I_t, R_t$.
- Infection is transmitted, while consuming or autonomously.
  - Overall contagion parameters $\pi_s, \pi_a$.
  - Variety-specific relative contagiousness $\phi(k)$,

$$\int \phi(k) dk = 1$$  \hspace{1cm} (3)

  - Probability for a susceptible agent $s$ to become infected:

$$\tau_t = \pi_s I_t \int \phi(k)c^s_{tk} c^i_{tk} dk + \pi_a I_t,$$  \hspace{1cm} (4)

- Paper: similar mechanics in case of infection-via-workplace.
- Infection dynamics. Let $T_t$ denote newly infected.

$$T_t = \tau_t S_t$$  \hspace{1cm} (5)

$$S_{t+1} = S_t - T_t$$  \hspace{1cm} (6)

$$I_{t+1} = I_t + T_t - (\pi_r + \pi_d)I_t$$  \hspace{1cm} (7)

$$R_{t+1} = R_t + \pi_r I_t$$  \hspace{1cm} (8)
Analysis: Choices of Infected and Recovered Agents

\[ n_t^x = \frac{1}{\sqrt{\theta}}, \quad c_{tk}^x \equiv c_t^x = \frac{A}{\sqrt{\theta}} \]

for \( x \in \{i, r\} \).
Analysis: Choices of Susceptible Agents.

- Recall **infection constraint** (6):

\[
\tau_t = \pi_s I_t \int \phi(k) c_{tk}^s c_{tk}^i dk + \pi_a I_t,
\]

where \(c_{tk}^i \equiv c_t^i = \frac{A}{\sqrt{\theta}}\).

- Bellman equation:

\[
U^s_t = u(c^s_t, n^s_t) + \beta[(1 - \tau_t) U^s_{t+1} + \tau_t U^i_{t+1}]
\] (9)

- First-order condition wrt consumption of variety \(k\):

\[
u_1(c^s_t, n^s_t) \cdot \left(\frac{c^s_t}{c_{tk}^s}\right)^{1/\eta} = \lambda_{bt}^s + \lambda_{\tau_t}^s \pi_s \frac{A}{\sqrt{\theta}} I_t \phi(k)
\] (10)

where

- \(\lambda_{bt}^s\) is the Lagrange multiplier on the budget constraint
- \(\lambda_{\tau_t}^s\) is the Lagrange multiplier on the **infection constraint**.
Theoretical Results: $\eta = 0$ equal to “homogeneous”.

Proposition

Suppose that $\eta = 0$, i.e. that the consumption aggregation is Leontieff. In that case, the multisector economy is equivalent to a homogeneous-sector economy. Equation (6) becomes

$$\tau_t = \pi_s l_t c_t^s c_t^i + \pi_a l_t$$  \hspace{1cm} (11)

Recall: equation (6) was

$$\tau_t = \pi_s l_t \int \phi(k) c_{tk}^s c_{tk}^i dk + \pi_a l_t,$$
Theoretical Results: $\eta \to \infty$ implies low infection.

**Proposition**

Suppose that $\eta \to \infty$ (perfect substitutes in the limit). Let $k = \sup_k \{k \mid \phi(k) = \phi(0) \}$. Suppose $k > 0$. Then $c_{tk;\eta}^j \to c_{tk;\infty}^j$ for $j \in \{s, i, r\}$, where

$$c_{tk;\infty}^s = \begin{cases} 
    c_t^s/k & \text{for } k < k_0 \\
    0 & \text{for } k > k_0
\end{cases}$$

(12)

and

$$c_{tk}^j \equiv c_t^j \text{ for } j \in \{i, r\}$$

(13)

Equation (6) becomes

$$\tau_t = \phi(0)\pi_s l_t c_t^s c_t^i + \pi_a l_t$$

(14)

Note: size $k$ of low-infection sector does not matter.
Theoretical Results: $\eta = \infty$ implies a range.

**Proposition**

Suppose that $\eta = \infty$, i.e. that the sector-level consumption goods are perfect substitutes. Let $\mu_t$ be any function of time satisfying

$$0 \leq \mu_t \leq \bar{\mu}$$

where $\bar{\mu}$ is defined as

$$\bar{\mu} = \frac{1}{\int \frac{1}{\phi(k)} dk}$$

and note that it satisfies

$$\phi(0) \leq \bar{\mu} \leq 1$$

Then there is an equilibrium with equation (6) replaced by

$$\tau_t = \mu_t \pi s I_t c_t^s c_t^i + \pi_a I_t$$
Numerical Results: Choice of Parameter Values

- Similar to Eichenbaum-Rebelo-Trabandt (2020).
- Set $\pi_a = 0$. Set $\pi_s$ to get 10% cons decline in homogeneous sector case.
- Mostly two equally-sized sectors: $\phi_1 = 0.2, \phi_2 = 1.8$.
- Why? In order to investigate the mechanism.
- $\eta = 10$. Also: $\eta = 3$.
- Variations:
  - $\pi_a > 0$ to obtain 50% susceptible in the limit.
  - 9 sectors.
  - Vary $\eta$.
  - Somewhat lower $\pi_s$. 
## Numerical Results: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\pi_a = 0$</th>
<th>$\pi_a \neq 0$</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_s$</td>
<td>$4.05 \times 10^{-7}$</td>
<td>$1.77 \times 10^{-7}$</td>
<td>infection from cons.</td>
</tr>
<tr>
<td>$\pi_r$</td>
<td>0.387</td>
<td>0.387</td>
<td>recovery</td>
</tr>
<tr>
<td>$\pi_d$</td>
<td>$1.944 \times 10^{-3}$</td>
<td>$1.944 \times 10^{-3}$</td>
<td>Death</td>
</tr>
<tr>
<td>$\pi_a$</td>
<td>0</td>
<td>0.34</td>
<td>autonomous infection</td>
</tr>
<tr>
<td>$\eta$</td>
<td>10</td>
<td>10</td>
<td>Elasticity of substitution</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$1.275 \times 10^{-3}$</td>
<td>$1.275 \times 10^{-3}$</td>
<td>Labor supply parameter</td>
</tr>
<tr>
<td>$A$</td>
<td>39.835</td>
<td>39.835</td>
<td>Productivity</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$0.96^{1/52}$</td>
<td>$0.96^{1/52}$</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.2</td>
<td>0.2</td>
<td>infection intensity, sect. 1</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>1.8</td>
<td>1.8</td>
<td>infection intensity, sect. 2</td>
</tr>
</tbody>
</table>
Numerical Results: Consumption Decline
Numerical Results: Consumption Decline, various $\phi_1$
Numerical Results: Deceased, various $\phi_1$, when $\pi_a = 0$. 

![Graph showing the percentage of heterogeneous-sector case and baseline case over weeks for various $\phi_1$ values ranging from 0.1 to 1.0.](image-url)
Numerical Results: Baseline Comparison

- **Infected**
  - Baseline
  - $\eta = 3$
  - $\phi = 1$

- **Susceptible**

- **Recovered**

- **Deceased**

- **Agg. consumption**

- **Agg. labor**
Numerical Results: with $\pi_a > 0$
Numerical Results: Deceased: $\pi_a = 0$ vs $\pi_a > 0$
Numerical Results: Variations in $\eta$, when $\pi_a = 0$. 
Numerical Results: Reversal for $\pi_s$ to 87%
Numerical Results: Sectoral Shifts

Low-infection sector

High-infection sector

Weeks
Numerical Results: Sectoral Shifts, 9 sectors
The chart shows the percentage change in spending from the beginning of the year. Each line is an average of the previous two weeks, which smooths out weekly anomalies. | Source: Earnest Research
Data: per NYT 2020-04-14, Leatherby-Gelles

Change in spending from 2019 for the week ending April 1. Bubbles are sized by industry sales.
A social planner solution

- Note: agents in the model know whether they are susceptible, infected or recovered (or dead).
- So let’s give the social planner the same knowledge:
  - Widespread testing.
  - Moral appeal.
- Intuition:
  - The social planner will seek to minimize the infection via infected agents ...
  - ... while still having to feed them.
- Note: no full separation is possible. Model too restrictive?
Numerical Results: Social Planner
Numerical Results: Social Planner

Per capita cons.

Low-infection

Baseline

$\eta = 3$

Constant $\phi$

High-infection

% of SS at competitive equilibrium

Weeks

% of SS at competitive equilibrium

Weeks

% of SS at competitive equilibrium

Weeks
Conclusions

- COVID 19 epidemic: re-opening debate.
- Key issue: how much will people do on their own?
- Model: Neoclassical growth cum SIR model. Infection happens, while consuming. **Sectoral/variety choices:** different good varieties differ in their infectiousness. Susceptible agents take this into account: reduce consumption and **shift towards low-infection varieties.**
- Result: output decline and infection rates reduced by 80 percent compared to homogeneous-sector version.
- Reversal rather than flattening of curve may be possible.
- Plus: an extreme social planner result.
- Too Panglossian? At least, this analysis offers some hope!