Portfolio Choice with Sustainable Spending: A Model of Reaching for Yield

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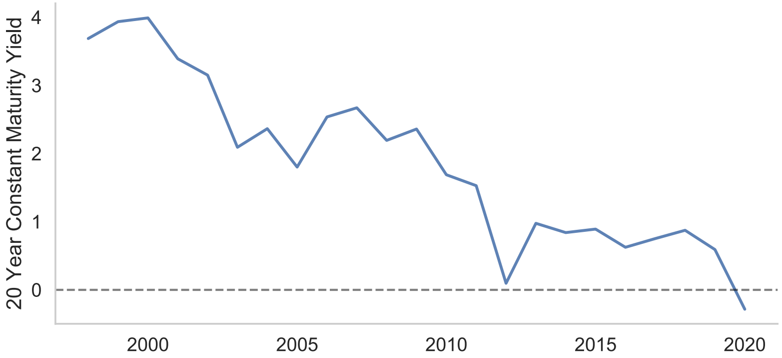
Reaching for Yield

- **Low for long:** Large and persistent decline in real interest rates during this century.
- **Reaching for yield (RFY):** The hypothesis that investors respond by taking more risk.
- Much discussed by central bankers, e.g. Stein (2013):

"A prolonged period of low interest rates, of the sort we are experiencing today, can create incentives for agents to take on greater duration or credit risks, or to employ additional financial leverage, in an effort to reach for yield."

Low for Long



20-year constant maturity TIPS yield, 1997-2020.

The Theory Puzzle

- Standard finance theory does **not** predict RFY: risktaking depends on risk premium, risk, and risk aversion but **not** the riskfree interest rate.
- Recent literature has proposed a variety of institutional explanations for RFY:
  - Fixed nominal return target (Rajan 2013), possibly related to zero lower bound for retail deposit rates (Di Maggio and Kacperczyk 2017).
  - Low rates lower the opportunity cost of holding liquid assets (e.g. reserves) which are needed for leveraged risktaking (Drechsler, Savov, and Schnabl (2018).
  - Low rates worsen the underfunding of pension plans, which react by gambling for resurrection (Andonov, Bauer, and Cremers 2017).
  - Low rates lengthen the duration of insurance company liabilities, which react by lengthening the duration and hence the yield of their assets (Ozdagli and Wang 2019).

Our Theory

- Start with a standard model of an infinitely lived investor with power utility (Merton 1969, 1971).
- Add a **sustainable spending constraint**, realistic for endowments and sovereign wealth funds (SWFs):
  - The investor must consume the expected real return on wealth each period.
  - This implies that wealth is expected to remain constant: the investor cannot plan to run down or accumulate wealth.
  - Two variants, arithmetic expected return vs. geometric expected return, differ in detail but the main results are the same.
- This one change to the standard model implies
  - RFY
  - Stronger RFY when the real interest rate is already low
  - Risktaking responds perversely to the risk premium when the real interest rate is low
  - In a nominal variant of the model, stronger RFY when inflation is low.

The Sustainable Spending Constraint (1)

- Tobin (1974):

  "The trustees of an endowed institution are the guardians of the future against the claims of the present. Their task is to preserve equity among generations. The trustees of an endowed university like my own assume the institution to be immortal. They want to know, therefore, the rate of consumption from endowment which can be sustained indefinitely."

The Sustainable Spending Constraint (2)

- Harvard website:

  "The University's spending practice has to balance two competing goals: the need to fund the operating budget with a stable and predictable distribution, and the obligation to **maintain the long-term value of endowment assets after accounting for inflation***."*

- Norges Bank Investment Management (NBIM) website:

  "So that the fund benefits as many people as possible in the future too, politicians have agreed on a fiscal rule which ensures that **we do not spend more than the expected return on the fund.***"*

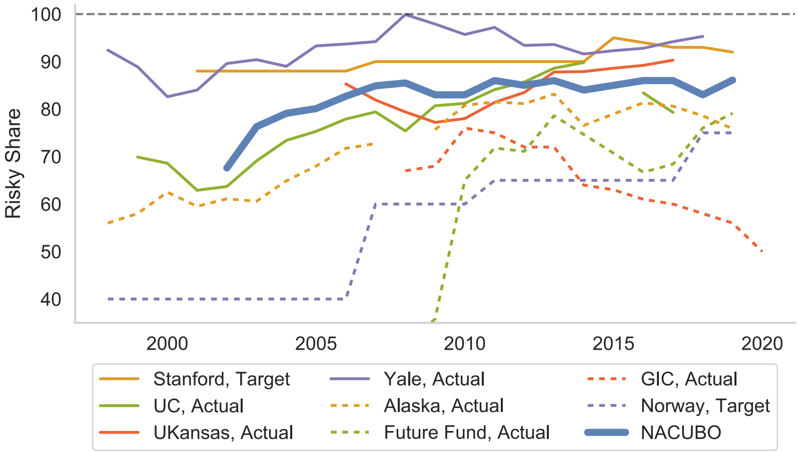
- These two formulations are equivalent.

An Example of RFY: NBIM

Bloomberg News 12/1/16:

> *Norway's $860 billion wealth fund recommended it add about $130 billion in stocks and sell off bonds as it presented a bleak view on the returns from its investments across the globe in the decades to come.*
>
> *The central bank's board, which oversees the fund, on Thursday recommended an increase in the equity share to 75% from 60%. That will raise the expected average annual real return to 2.5% percent over 10 years... compared with 2.1%... under the current setup.*
>
> *The world's largest sovereign wealth fund said that it expects an annual return of only 0.25% on bonds over the next decade and that the expected equity risk premium...will be just 3% in a cautious estimate.*

RFY by Endowments and SWFs



University endowments: solid lines. SWFs: dashed lines.

Broader Applicability

The sustainable spending constraint may be applicable to other investor types as well.

- Trusts with different income and principal beneficiaries, investing on behalf of the income beneficiary with a distant future date for principal disbursement.
  - Modern trust law interprets income as expected return (Sitkoff and Dukeminier 2017).
- Individuals with a behavioral reluctance to run down wealth by dissaving.
  - Our model complements the behavioral analysis by Lian, Ma, and Wang (2019) which emphasizes the role of reference rates.

Outline

- Review of the standard Merton model.
- Arithmetic and geometric sustainable spending constraints.
  - Reaching for yield, stronger when interest rates are low.
- Welfare cost of the constraints.
- Graphical analysis for intuition.
- Extensions:
  - A one-sided spending constraint.
  - Donations.
  - Inflation and a nominal spending constraint.
- Conclusion.

Merton Model Setup

- Choose consumption and asset allocation to maximize expected utility.
- Utility function has constant relative risk aversion γ and time preference rate ρ.
- The consumer lives off financial wealth w_t invested in two different assets.
- There is a constant riskfree interest rate r_f.
- The risky asset has risk premium μ and volatility σ.
- We write the risky portfolio share as α and the consumption-wealth ratio c_t/w_t as θ. (Both are constant.)

Merton Model Solution

- The risky share is a constant α given by the famous formula

$$\alpha = \frac{\mu}{\gamma\sigma^2}.$$

- The risky share depends on the reward for taking risk and on risk aversion, but **not** on the riskfree interest rate.
- Because the consumption-wealth ratio is a constant θ, consumption and wealth grow at the same rate.
- The expected (desired) growth rate of consumption and wealth is increasing in the riskfree rate r_f and decreasing in the rate of time preference ρ.
- But what if this is not possible?

Definition of the Arithmetic Constraint

- **Consumption-wealth ratio equals the expected simple return** (aka arithmetic average return).

$$\theta = \frac{c_t}{w_t} = (r_f + \alpha\mu).$$

- From the budget constraint, then **the expected change in wealth is zero.**
  - Thus the two formulations of the constraint by Harvard and NBIM are equivalent.
  - Since c_t / w_t is constant, the expected change in consumption is also zero.
- The only choice variable in the problem is the risky portfolio share α.

Arithmetic Solution

- The risky share is

$$\alpha = \frac{-r_f + \sqrt{K}}{\mu(1+\gamma)},$$

  where

$$K = r_f^2 + 2\rho\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\mu}{\sigma}\right)^2.$$

- Standard properties:
  - Portfolio volatility $\alpha\sigma$ depends only on the Sharpe ratio μ/σ.
  - Risky share α is inversely related (although not inversely proportional) to σ^2 and γ.
- But there are nonstandard properties too!

Nonstandard Properties of the Arithmetic Model

Proposition

In the arithmetic model, the risky share α has the following properties.

1. *α is a decreasing and convex function of the riskfree rate r_f.*
2. *α is an increasing function of the rate of time preference ρ.*
3. *α is an increasing function of the risk premium μ when $r_f > 0$, and a decreasing function of μ when $r_f < 0$.*

Simple Intuition

- Lower riskfree rate or greater impatience lead the investor to want higher consumption (lower marginal utility) today relative to expected future consumption (marginal utility).
- In the standard model, this is achieved by dissaving.
- With a sustainable spending constraint, it is achieved by taking risk. This allows higher spending today, and the negative consequence (riskier consumption) is realized in the future.
- A lower risk premium has both a standard substitution effect (take less risk) and a nonstandard income effect similar to that of a lower riskfree rate.
- All the nonstandard effects get stronger as the riskfree rate declines.
  - Hence the interest in RFY today (this paper written in 2020 with 1970s technology!)
  - The nonstandard effect of the risk premium dominates when $r_f < 0$.

Definition of the Geometric Constraint

- A problem with the arithmetic model is that although the average of future wealth is equal to current wealth, future wealth is more often than not lower than current wealth.
  - There are a few ultra-rich scenarios that counterbalance many impoverished scenarios.
- To fix this problem, we can alternatively impose a geometric constraint in which **the consumption-wealth ratio equals the expected log return** (aka geometric average return):

$$\theta = \frac{c_t}{w_t} = (r_f + \alpha\mu - \frac{1}{2}\alpha^2\sigma^2).$$

- Then **the expected change in log wealth is zero**.
- This implies that current wealth is the median of future wealth: 50% of the time future wealth will be higher, 50% of the time it will be lower than today.
- The solution to the geometric model is more complex than the solution to the arithmetic model, but it has similar properties.

Nonstandard Properties of the Geometric Model

Proposition

In the geometric average model with $\gamma > 1$, the risky share α has the following properties:

1. *α is a decreasing and convex function of the riskfree rate r_f.*
2. *α is an increasing function of the rate of time preference ρ.*
3. *Define $r_f^* = -\rho/(\gamma^2 - 1)$. When $r_f > r_f^*$, α is an increasing function of the risk premium μ and when $r_f < r_f^*$, α is a decreasing function of μ.*
4. *The growth-optimal risky share μ/σ^2 is an upper bound on α.*

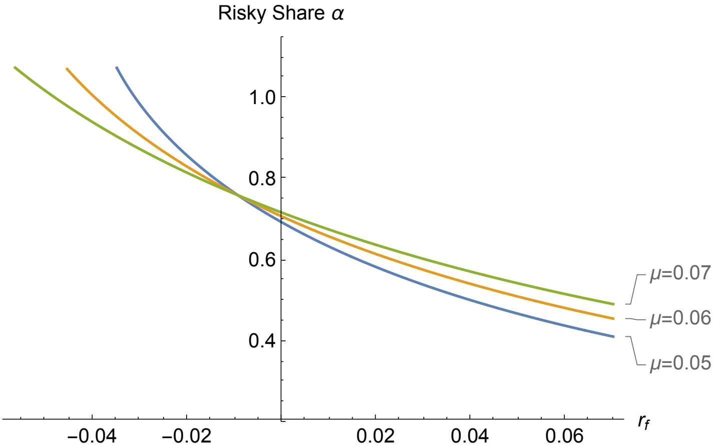
Interpretation

- The nonstandard effects of the riskfree rate and impatience have the same intuition as in the arithmetic model.
- The nonstandard effects get stronger as the riskfree rate declines.
- The nonstandard income effect of the risk premium dominates the standard substitution effect when the riskfree rate is sufficiently negative.
- The upper bound on risktaking is the growth-optimal portfolio with $\alpha = \mu/\sigma^2$, because this maximizes the expected log return and hence current consumption.

A Calibrated Example

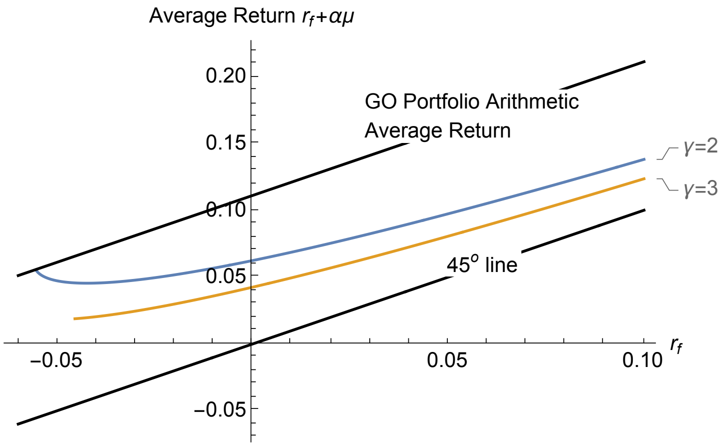
- We illustrate the model in a calibrated example with $\rho = 7.5\%$, $\gamma = 3$, $\sigma = 18\%$.
- Base case has $\mu = 6\%$, market Sharpe ratio $= 33\%$, Merton $\alpha = 62\%$.
  - This is close to the classic 60% rule of thumb for endowments' risky share.
- In the base case, the sustainable spending constraint is nonbinding at $r_f \approx 2\%$.
  - At this level of r_f, the geometric expected portfolio return (consumption-wealth ratio) $= 5.1\%$.
  - This is slightly above the typical historical distribution rates for endowments reported by Dahiya and Yermack (2018).
- The interest rate at which μ does not affect α is $r_f^* = -0.94\%$, and the corresponding $\alpha^* = 76\%$.

RFY and the Risk Premium



$$\rho = 0.075,\ \gamma = 3,\ \sigma = 0.18.$$

Expected Returns and the Riskfree Rate

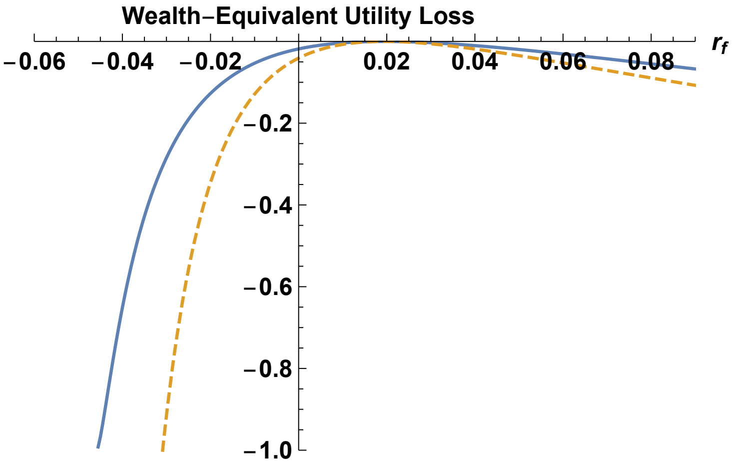


$\rho = 0.075,\ \mu = 0.06,\ \sigma = 0.18.$

Welfare Cost of Sustainable Spending Constraints

- A sustainable spending constraint forces the investor to deviate from the unconstrained Merton solution. How costly is this?
- We can solve for the fraction of wealth an investor would give up to escape the sustainable spending constraint.
- We plot this fraction both for the case where RFY occurs and where it is prohibited by a further constraint that the portfolio has to be invested with the Merton asset allocation.

Welfare Cost in the Calibrated Example



$\rho = 0.075$, $\mu = 0.06$, $\sigma = 0.18$.

Solid blue line: RFY. Orange dashed line: Fixed portfolio allocation.

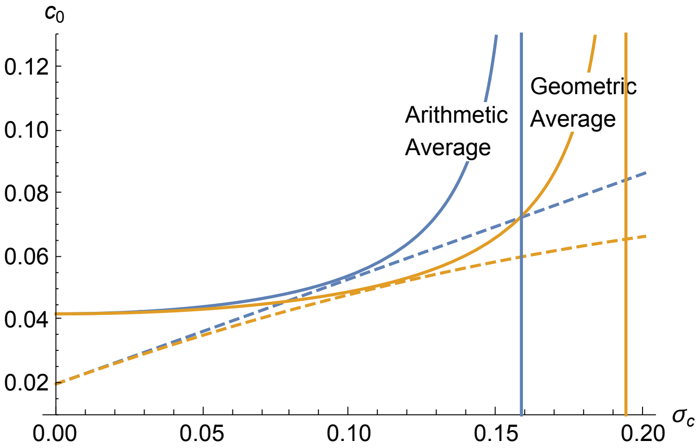
Lessons from the Example

- The example is calibrated so that the constraint does not bind when $r_f \approx 2\%$. Very low welfare costs for r_f in this neighborhood.
- Welfare costs increase as r_f moves away from 2%, particularly when r_f declines. Extreme values when r_f becomes negative.
- **Conjecture:** sustainable spending constraints were agreed with donors at a time when very low (negative) r_f was regarded as exceedingly unlikely.
- At very low levels of r_f, RFY greatly reduces the welfare cost of a sustainable spending contraint.
- **Conjecture:** endowments retained control of portfolio allocation, rather than promising donors to fix α, as a safety valve to mitigate the welfare cost of a sustainable spending constraint in the event that the economy enters a low-interest-rate regime.

Motivation

- To further develop intuition, we now rewrite the problem as one of choosing the initial level of consumption c_0, the mean consumption growth rate μ_c, and the volatility of consumption growth σ_c subject to constraints.
- We can draw indifference curves and constraints on a diagram for c_0 and σ_c, analogous to the classic mean-standard deviation diagram of static portfolio choice theory.
- Indifference curves slope up because people dislike volatility.
- The slope increases with the level of volatility, and there is an upper limit beyond which risk becomes unbearable.
- In the arithmetic model, the constraint is linear; in the geometric model, the constraint is concave.

Arithmetic and Geometric Constraints

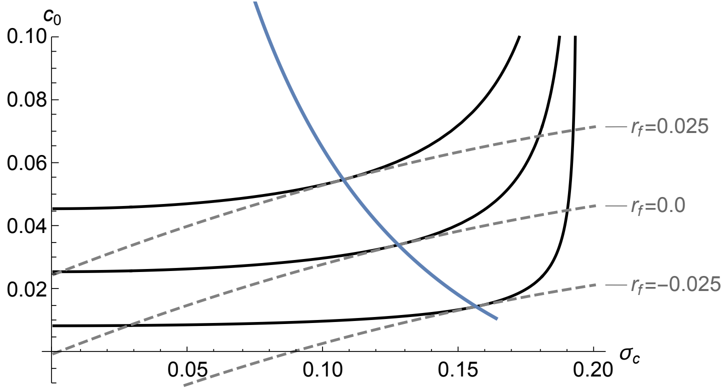


$$\rho = 0.075,\ \gamma = 3,\ r_f = 0.02,\ \mu = 0.06,\ \sigma = 0.18.$$

Graphical Intuition for RFY

- As value decreases, both the level and slope of the indifference curves decrease at any given σ_c.
- A decrease in the riskfree rate is a parallel shift down in the constraint so it decreases value.
- But then, at the original σ_c there is no longer a tangency. The new tangency has a higher σ_c where the indifference curve is steeper and the geometric constraint is flatter.
- These effects are stronger when the riskfree rate is low, until the point is reached where the problem no longer has a solution.

Reaching for Yield

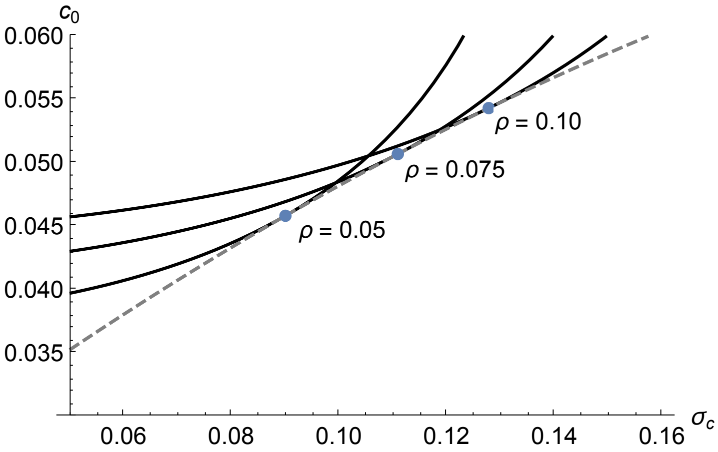


$$\rho = 0.075,\ \gamma = 3,\ \mu = 0.06,\ \sigma = 0.18.$$

Graphical Intuition for Impatience

- As the investor becomes more impatient (has a higher rate of time preference), the indifference curves become flatter at any given σ_c.
- Hence, the tangency point shifts to the right implying a higher σ_c.
- In the limit as time preference increases, the indifference curves are horizontal and the investor chooses the growth-optimal portfolio where the constraint is also horizontal.

Impatience



$$\gamma = 3,\ r_f = 0.02,\ \mu = 0.06,\ \sigma = 0.18.$$

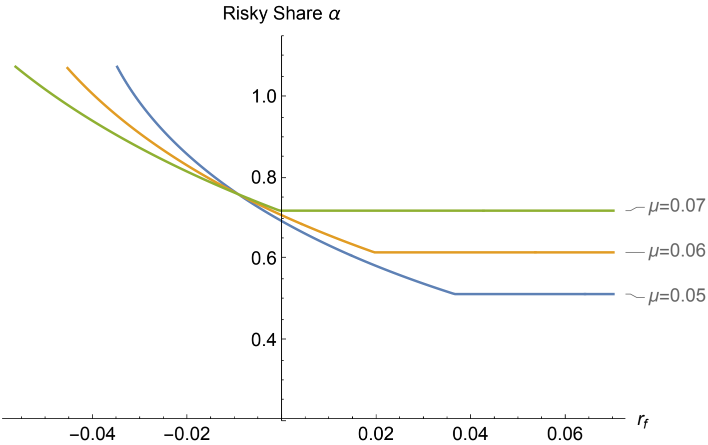
Graphical Intuition for the Risk Premium Effect

- An increase in the risk premium makes the spending constraint steeper but also raises it whenever the portfolio initially takes risk.
- The former substitution effect increases risktaking, but the latter income effect reduces it.
- The income effect is stronger whenever initial risktaking is greater, and hence is stronger at low levels of the riskfree interest rate.
- If the riskfree rate is sufficiently negative, the income effect dominates.

Extensions of the Static Model

- A one-sided spending constraint
- Donations
  - Current-use gifts vs endowment gifts
- Inflation and a nominal spending constraint

A One-Sided Constraint



As μ declines, the constraint binds at a higher level of r_f.

Current-Use and Endowment Gifts

- Consider an endowment that receives donations, proportional to the current level of wealth (for tractability).
- Current-use gifts, arriving at rate g_u, can be spent in the period they are received.
- Endowment gifts, arriving at rate g_e, are added to wealth and spent sustainably later.
- Current-use gifts are equivalent to an increase in the riskfree interest rate.
  - Hence they discourage risktaking (reverse RFY effect).
- Endowment gifts are equivalent to an increase in the rate of time preference.
  - Hence they encourage risktaking because risk has current benefits and deferred costs.
- Intuitively, current-use gifts relax the sustainable spending constraint while endowment gifts tighten it.

Effects of Inflation

- Inflation relaxes the sustainable spending constraint when this constraint is specified in nominal terms.
- Hence low inflation leads to RFY by nominally constrained investors whether we hold the real riskfree rate or the nominal riskfree rate constant.
- These results help to explain why RFY is more of a concern today than in the 1970s.
- Currently, both the real interest rate and inflation are low so our model predicts strong RFY by both real and nominally constrained investors.
- In the 1970s, the real interest rate was low but inflation was even higher, so the nominal interest rate was high and our model does not predict RFY by nominally constrained investors.

Conclusion

- Classical finance theory separates risktaking from intertemporal choice.
- Our model breaks this separation using a sustainable spending constraint: investors take risk as a way to increase current consumption at the cost of more volatile future consumption.
    - The model predicts RFY, stronger when the riskfree rate is low, and more risktaking by impatient investors.
- Rampini and Viswanathan (2010, 2013, 2019) similarly break the separation using models of collateral constraints for firms or borrowing constraints for households.
    - Constrained firms will not put up collateral today, and constrained households will not pay insurance premia today, to manage their future risk exposures.
- The classical result is not as robust as finance theorists have supposed.