# Taylor Projection under Tail Risk

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How to (accurately) approximate the solution to dynamic equilibrium models with tail events, i.e., low probability but large shocks?

Examples include (but are not limited to):

- Rare disasters (e.g., Barro, 2009)
- Climate change (e.g., van den Bremer and van der Ploeg, 2021)
- Natural disasters (e.g., Douenne, 2020)
- Pandemics (e.g., Hong, Wang, and Yang, 2021)
- Wars (e.g., Federle, Meier, Müller, Mutschler, and Schularick, 2024)
- Rare booms (e.g., Bekaert and Engstrom, 2017 and Tsai and Wachter, 2015)

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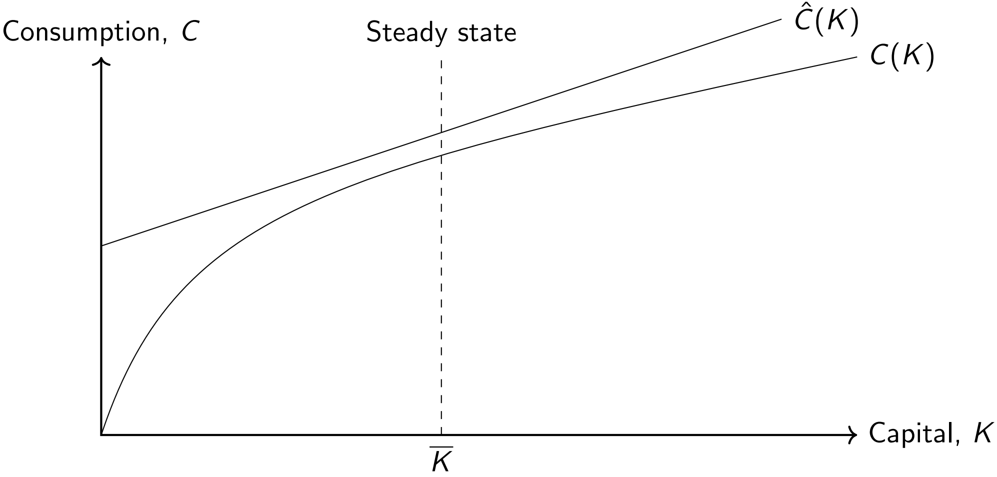
Challenging task! Tail events affect the degree of non-linearities through two channels:



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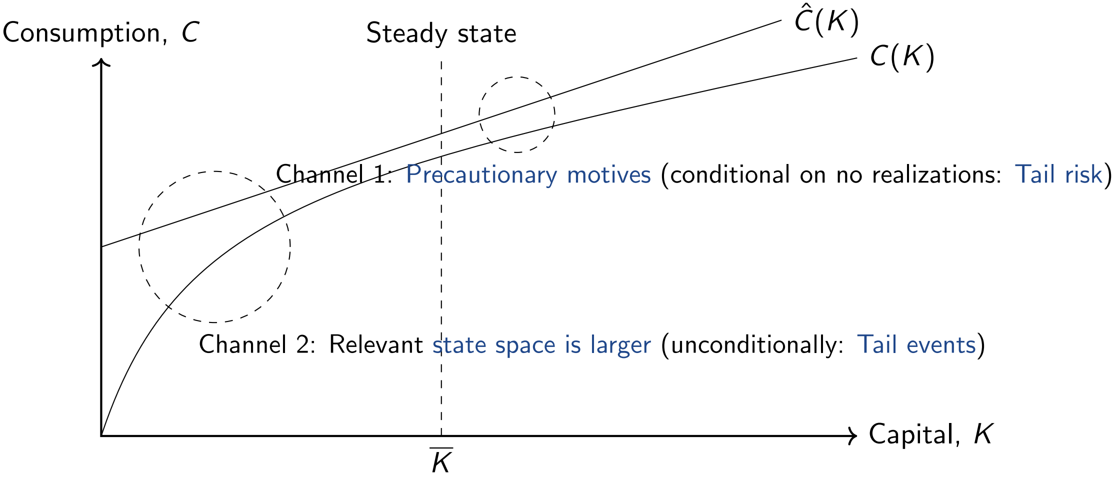

# Contribution

We provide a continuous-time (CT) macroeconomic framework to approximate the solution to dynamic equilibrium models with tail events and risk (jump-diffusion)

- A CT framework for Taylor projection (TP) (Levintal, 2018)
- Proof of convergence as the order of approximation goes to infinity
- Test TP, conditionally and unconditionally, in Gourio (2012) economy:
  - RBC with EZ preferences, time-varying disaster risk, adjustment costs of capital
  - Slow unfolding and recovery from disasters (Nakamura et al., 2013; Gourio, 2008)
- Propose englobed Taylor projection (ETP) that solves issues of
  - Inaccuracies in economies exposed unconditionally to rare disasters
  - Instabilities arising from explosive paths caused by disasters

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# Framework

This system of equations represents a family of continuous-time dynamic stochastic models

$$\mathbf{0} = \mathcal{H}\left(\mathbf{x}, \mathbf{y}, \mathbf{y}_{\mathbf{x}}, \mathbf{y}_{\mathbf{xx}}\right) \tag{1}$$

$$\mathrm{d}\mathbf{x} = \mathbf{b}\left(\mathbf{x}, \mathbf{y}\right)\mathrm{d}t + \boldsymbol{\sigma}\left(\mathbf{x}, \mathbf{y}\right)\mathrm{d}\mathbf{w} + \mathbf{f}\left(\mathbf{x}_{-}, \mathbf{y}_{-}\right)\mathrm{d}\mathbf{N}, \tag{2}$$

$$\mathbf{y} = \mathbf{g}\left(\mathbf{x}\right) \tag{3}$$

- $\mathcal{H}$: a (deterministic) system of functional equations (second-order PDEs) that collects all the equilibrium conditions
- $\mathbf{x}$: vector of state variables (endogenous and exogenous)
- $\mathbf{y}$: vector of control variables with derivatives $\mathbf{y}_{\mathbf{x}}$ and $\mathbf{y}_{\mathbf{xx}}$ wrt the states
- $\mathbf{w}$: vector of independent Brownian motions
- $\mathbf{N}$: vector of stochastically independent Poisson processes with arrival rate vector $\boldsymbol{\lambda}$ and jump size determined by $\mathbf{f}(\cdot)$
- $\mathbf{g}\left(\cdot\right)$: unknown vector of policy functions that solves (1) and (2)

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# Taylor projection

Substituting the unknown solution (3) into the operator (1) yields the new functional operator

$$\mathbf{F}(\mathbf{x}) := \mathcal{H}(\mathbf{x}, \mathbf{g}(\mathbf{x}), \mathbf{g}_{\mathbf{x}}(\mathbf{x}), \mathbf{g}_{\mathbf{xx}}(\mathbf{x})) \tag{4}$$

Taylor projection of order $h$ approximates $\mathbf{g}(\mathbf{x})$ in (4) by polynomial power expansions at an arbitrary point $\bar{\mathbf{x}}$

$$\widehat{\mathbf{g}}(\mathbf{x}; \boldsymbol{\Theta}) = \sum_{i=0}^{h} \widehat{\mathbf{G}}_{\mathbf{x}^i} (\mathbf{x} - \bar{\mathbf{x}})^{\otimes i} \tag{5}$$

where $\boldsymbol{\Theta}$ is a collection of non-repeated coefficients of $\widehat{\mathbf{G}}_{\mathbf{x}^i}$. Replacing $\mathbf{g}(\mathbf{x})$ with (5) in the equilibrium conditions (4) yields

$$\widehat{\mathbf{F}}(\mathbf{x}; \bar{\mathbf{x}}, \boldsymbol{\Theta}) = \mathcal{H}(\mathbf{x}, \widehat{\mathbf{g}}(\mathbf{x}; \boldsymbol{\Theta}), \widehat{\mathbf{g}}_{\mathbf{x}}(\mathbf{x}; \boldsymbol{\Theta}), \widehat{\mathbf{g}}_{\mathbf{xx}}(\mathbf{x}; \boldsymbol{\Theta})) = \mathbf{0} \tag{6}$$

The goal of Taylor projection is to find the values of $\boldsymbol{\Theta}$ that satisfy (6) around the neighborhood of $\bar{\mathbf{x}}$

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The goal of Taylor projection is to find the values of $\boldsymbol{\Theta}$ that satisfy (6) around the neighborhood of $\bar{\mathbf{x}}$

This can be achieved by successively computing the first $h$ derivatives of the functional equation (6) with respect to $\mathbf{x}$ and evaluating them at $\bar{\mathbf{x}}$

$$\mathcal{R}(\boldsymbol{\Theta}) = \begin{bmatrix} \widehat{\mathbf{F}}(\bar{\mathbf{x}};\boldsymbol{\Theta}) \\ \text{vec}\left(\widehat{\mathbf{F}}_{\mathbf{x}}(\bar{\mathbf{x}};\boldsymbol{\Theta})\right) \\ \text{vech}\left(\widehat{\mathbf{F}}_{\mathbf{xx}}(\bar{\mathbf{x}};\boldsymbol{\Theta})\right) \\ \vdots \\ \text{vech}\left(\widehat{\mathbf{F}}_{\mathbf{x}^h}(\bar{\mathbf{x}};\boldsymbol{\Theta})\right) \end{bmatrix} = \mathbf{0} \tag{7}$$

This condition is obtained by this Taylor series expansion

$$\underbrace{\mathbf{0} = \mathbf{F}(\mathbf{x})}_{\text{equation (4)}} = \underbrace{\mathbf{F}(\bar{\mathbf{x}})}_{\mathbf{0}} + \underbrace{\mathbf{F}_{\mathbf{x}}(\bar{\mathbf{x}})}_{\mathbf{0}}(\mathbf{x}-\bar{\mathbf{x}}) + \frac{1}{2}\underbrace{\mathbf{F}_{\mathbf{xx}}(\bar{\mathbf{x}})}_{\mathbf{0}}(\mathbf{x}-\bar{\mathbf{x}})^{\otimes 2} + \ldots$$

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## Remarks

1. Theorem: $\widehat{\mathbf{g}}(\mathbf{x};\boldsymbol{\Theta}) \to \mathbf{g}(\mathbf{x})$ as $h \to \infty$ Details
2. Continuous-time versus discrete-time Taylor projection Details
   - Continuous-time Taylor projection has fewer theoretical assumptions
   - Continuous-time Taylor projection has fewer computational costs
3. (CT) Taylor projection vs perturbation (workhorse solution method) Details
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   - Taylor projection adjusts to risk/uncertainty even at first-order ($h = 1$)

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# An illustration: The Ramsey-Cass-Koopmans economy

# Deterministic Ramsey-Cass-Koopmans economy (see e.g. Romer, 2012)

Households maximize lifetime utility by choosing $\{C_t\}_{t=0}^{\infty}$

$$U_0 = \int_{t=0}^{\infty} e^{-\rho t} C_t^{1-\gamma} / [1-\gamma] \, \mathrm{d}t$$

subject to the evolution of capital

$$\mathrm{d}K_t = [f(K_t) - \delta K_t - C_t] \, \mathrm{d}t$$

Optimal $C$ is characterized by the following partial differential equation (PDE)

$$\mathcal{H}(K, C, C_K) = C[f_K(K) - \delta - \rho] - \gamma C_K [f(K) - \delta K - C] = 0$$

Since $C = C(K)$, we may write

$$\mathbf{F}(K) = C(K)[f_K(K) - \delta - \rho] - \gamma C_K(K)[f(K) - \delta K - C(K)] = 0$$

Derivation

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Derivation

As $C$ is an unknown function, consider a first-order polynomial approximant

$$\hat{C}(K) = \theta_0 + \theta_1 \left(K - \bar{K}\right)$$

around an arbitrary point, $\bar{K}$. The coefficients, $\boldsymbol{\Theta} = \left[\begin{array}{cc} \theta_0 & \theta_1 \end{array}\right]^\top$, are unknown

First-order Taylor projection ($h = 1$):

Plug $\hat{C}$ into $\mathbf{F}$ to form $\widehat{\mathbf{F}}$ and then

- Differentiate $\widehat{\mathbf{F}}$ wrt $K$, $\widehat{\mathbf{F}}_K$
- Evaluate $\widehat{\mathbf{F}}$ and $\widehat{\mathbf{F}}_K$ at $\bar{K}$

$$\mathcal{R} = \left[\begin{array}{c} \widehat{\mathbf{F}} \\ \widehat{\mathbf{F}}_K \end{array}\right] = \left[\begin{array}{c} \theta_0 \left(f_K\left(\bar{K}\right) - \delta - \rho\right) - \gamma\theta_1 \left(f\left(\bar{K}\right) - \delta\bar{K} - \theta_0\right) \\ \theta_1 \left(f_K\left(\bar{K}\right) - \delta - \rho\right) + \theta_0 f_{KK}\left(\bar{K}\right) - \gamma\theta_1 \left(f_K\left(\bar{K}\right) - \delta - \theta_1\right) \end{array}\right] = \mathbf{0}$$

That is, two nonlinear equations in two unknowns

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around an arbitrary point, $\bar{K}$. The coefficients, $\mathbf{\Theta} = \left[\begin{array}{ccc} \theta_0 & \theta_1 & \theta_2 \end{array}\right]^\top$, are unknown

Second-order Taylor projection ($h = 2$):

Plug $\hat{C}$ into $\mathbf{F}$ to form $\widehat{\mathbf{F}}$ and then

- Differentiate $\widehat{\mathbf{F}}$ wrt $K$, $\widehat{\mathbf{F}}_K$, and differentiate $\widehat{\mathbf{F}}_K$ wrt $K$, $\widehat{\mathbf{F}}_{KK}$
- Evaluate $\widehat{\mathbf{F}}$, $\widehat{\mathbf{F}}_K$, $\widehat{\mathbf{F}}_{KK}$ at $\bar{K}$

$$\mathcal{R} = \left[\begin{array}{c} \widehat{\mathbf{F}} \\ \widehat{\mathbf{F}}_K \\ \widehat{\mathbf{F}}_{KK} \end{array}\right] = \left[\begin{array}{c} \theta_0 \left(f_K\left(\bar{K}\right) - \delta - \rho\right) - \gamma\theta_1 \left(f\left(\bar{K}\right) - \delta\bar{K} - \theta_0\right) \\ \theta_1 \left(f_K\left(\bar{K}\right) - \delta - \rho\right) + \theta_0 f_{KK}\left(\bar{K}\right) - \gamma\theta_1 \left(f_K\left(\bar{K}\right) - \delta - \theta_1\right) \\ \cdots \end{array}\right] = \mathbf{0}$$

That is, three nonlinear equations in three unknowns

# An RBC model with time-varying disaster risk

The representative household has recursive utility (Epstein-Zin)

$$J_t = \mathbb{E}_t \left[ \int_t^{\infty} f(U_s, J_s) \mathrm{d}s \right]$$

The aggregator (nests CRRA) is

$$f(U, J) = \beta\theta J \left( \left( \frac{U}{((1-\gamma)\beta J)^{\frac{1}{1-\gamma}}} \right)^{1-\frac{1}{\psi}} - 1 \right)$$

with instantaneous utility (consumption and labor supply)

$$U = U(C, L) = C(1-L)^{\nu}$$

A representative firm in the economy is producing a single good using Cobb-Douglas technology

$$Y = K^{\alpha} (zL)^{1-\alpha}, \quad 0 < \alpha < 1$$

The aggregate resource constraint

$$Y = C + I,$$

must hold at all points in time.

The aggregate capital stock accumulates according to

$$\mathrm{d}K = \left(\Phi\left(I/K\right) - \delta\right) K \mathrm{d}t + \xi_K K \mathrm{d}N, \quad \text{with } K(0) = K_0 \geq 0 \text{ given and } \xi_K \leq 0$$

Disaster dynamics at instant $t$

$$\mathrm{d}N = \begin{cases} 1 & \text{with probability } \lambda \times \mathrm{d}t \\ 0 & \text{otherwise} \end{cases}$$

and a fraction of capital perishes: $-\xi_K$

The arrival rate of disasters follows a square-root process (Wachter, 2013)

$$\mathrm{d}\lambda = \kappa\left(\bar{\lambda} - \lambda\right) \mathrm{d}t + \sigma_\lambda \sqrt{\lambda} \mathrm{d}B_\lambda, \quad \text{with } \lambda(0) = \lambda_0 > 0 \text{ given}$$

The productivity decomposes into a permanent ($z_p$) and transitory ($z_r$) component

$$\log z = \log z_p + \log z_r$$

Each component follows (Hasler and Marfè, 2016)

$$\mathrm{d}\log z_p = \left(\mu - \frac{1}{2}\sigma_{z_p}^2 + \omega q\right)\mathrm{d}t + \sigma_{z_p}\mathrm{d}B_{z_p}$$
$$\mathrm{d}\log z_r = \rho_z\left(q - \log z_r\right)\mathrm{d}t$$

$q$ captures the slow unfolding and subsequent recovery from disasters in productivity

$$\mathrm{d}q = -\phi_q q\mathrm{d}t + \xi_z\mathrm{d}N, \quad q \leq 0, \quad \xi_z < 0$$

## Solving the model with Taylor projection

- $\widehat{\mathbf{F}}$ is a $3 \times 1$ vector of optimality conditions (minimal representation) Details
- We approximate three unknown decision rules

$$\mathbf{g}(\mathbf{x}) \simeq \hat{\mathbf{g}}(\mathbf{x}) = \begin{bmatrix} \hat{v}_k(\mathbf{x}) & \hat{c}(\mathbf{x}) & \hat{L}(\mathbf{x}) \end{bmatrix}^\top$$

where $\mathbf{x} = \begin{bmatrix} k & z_r & q & \lambda \end{bmatrix}^\top$

- A note on how to solve for the value function, $v(\mathbf{x})$ Details
- Standard RBC calibration (reference unit of time is annual) Details

# Economic and numerical implications

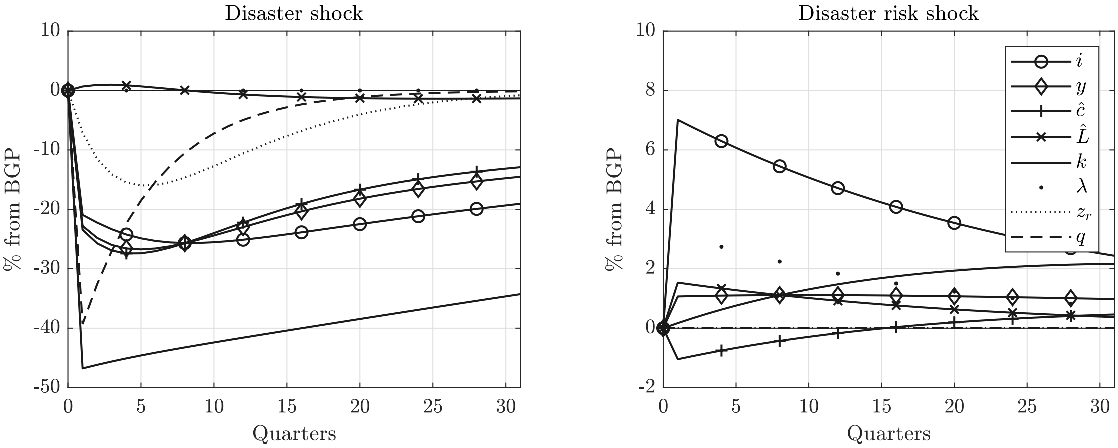


Figure 1: A third-order Taylor projection's impulse response (detrended variables)

We assess the accuracy using two metrics

- Unit free Euler errors, $\mathcal{M}(\mathbf{x})$, that we construct using the equilibrium conditions
- Judd, Maliar, and Maliar, 2017 (JMM) lower bounds
    - Interpretable errors, $\mathbf{d} = \begin{bmatrix} d_c & d_L & d_{v_k} \end{bmatrix}^\top$, in % deviations of the true policy function

$$\begin{bmatrix} \hat{c}(\boldsymbol{x}) \times (1 + d_c/100) \\ \hat{L}(\boldsymbol{x}) \times (1 + d_L/100) \\ \hat{v}_k(\boldsymbol{x}) \times (1 + d_{v_k}/100) \end{bmatrix} = \begin{bmatrix} c(\boldsymbol{x}) \\ L(\boldsymbol{x}) \\ v_k(\boldsymbol{x}) \end{bmatrix}$$

    - "*A simple way to discard numerical approximations that are insufficiently accurate*"

Instead of only looking at mean and max errors, we also consider errors over the distribution of $k$

| | $\log_{10}\mathcal{M}$ | | Average $\log_{10}\mathcal{M}$ sorted by percentile of $k$ | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Mean | Max | $\leq 10$ | (10,25] | (25,50] | (50,75] | (75,90] | $90 <$ |
| TP1 | -1.12 | 4.23 | 0.04 | -0.33 | -1.06 | -2.42 | -1.05 | -0.52 |
| TP2 | -2.03 | 0.513 | -0.22 | -0.72 | -1.86 | -3.99 | -2.01 | -1.31 |
| TP3 | -2.87 | 0.278 | -0.45 | -1.09 | -2.63 | -5.51 | -2.88 | -1.93 |

Table 1: Euler Errors. The table reports Euler errors over the population distribution.

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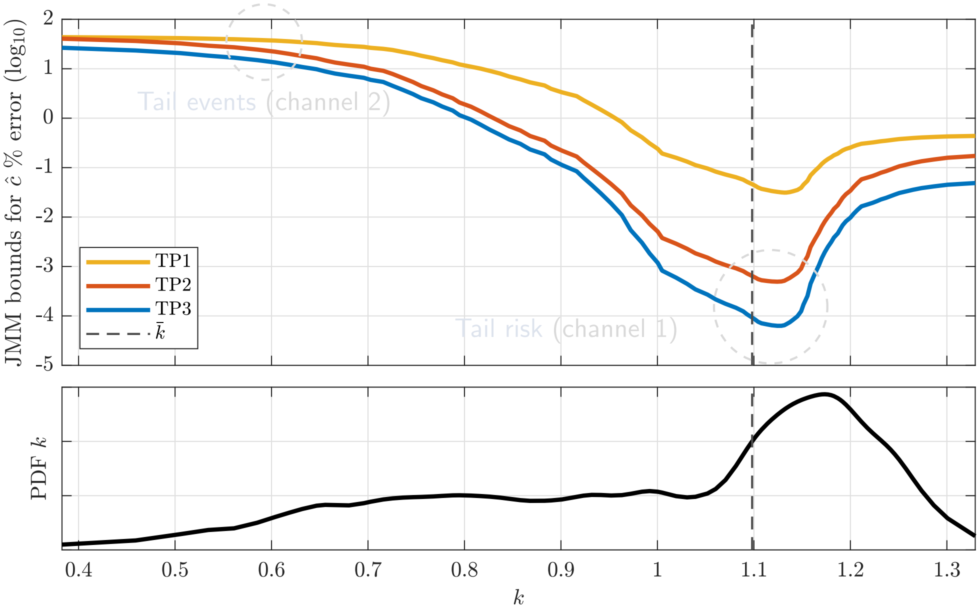
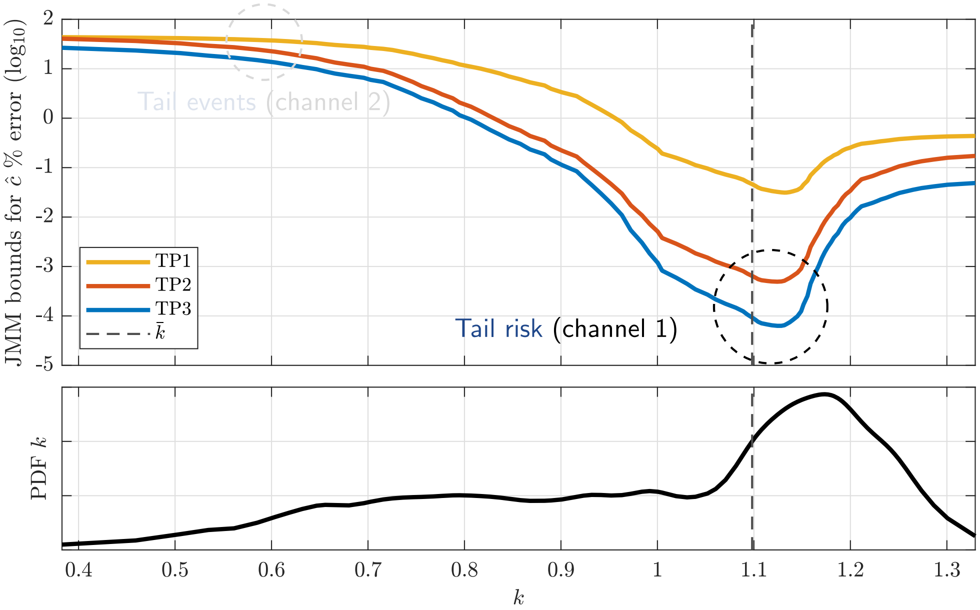


JMM bounds for $\hat{c}$ % error ($\log_{10}$)
Tail events (channel 2)
Tail risk (channel 1)
TP1
TP2
TP3
$\bar{k}$
PDF $k$
$k$
2
1
0
-1
-2
-3
-4
-5
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3



JMM bounds for $\hat{c}$ % error ($\log_{10}$)
2
1
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-3
-4
-5
Tail events (channel 2)
Tail risk (channel 1)
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TP2
TP3
$\bar{k}$
PDF $k$
0.4
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1
1.1
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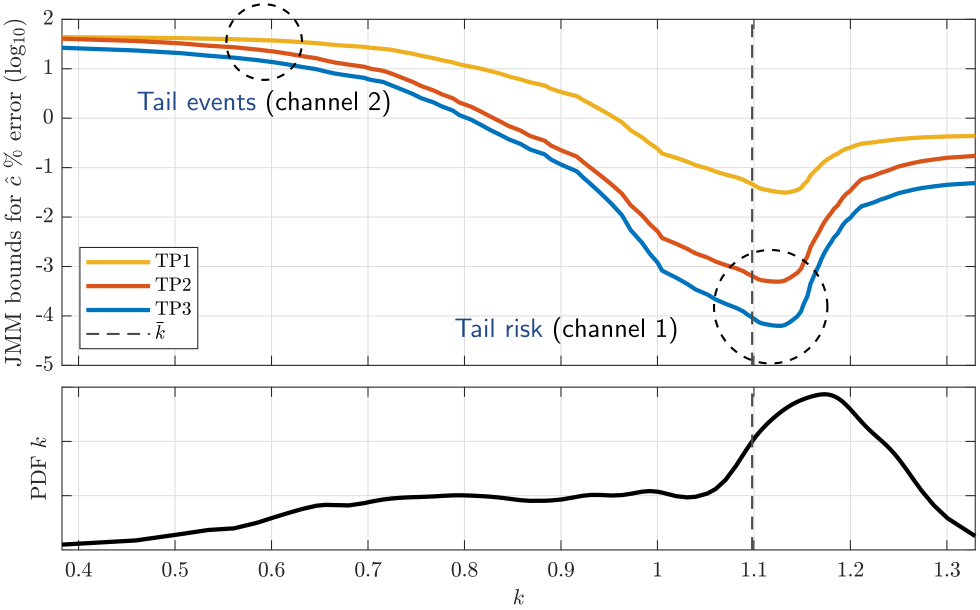
1.3
$k$



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JMM bounds for $\hat{c}$ % error ($\log_{10}$)
TP1
TP2
TP3
$\bar{k}$
PDF $k$
$k$
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0.7
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1.1
1.2
1.3

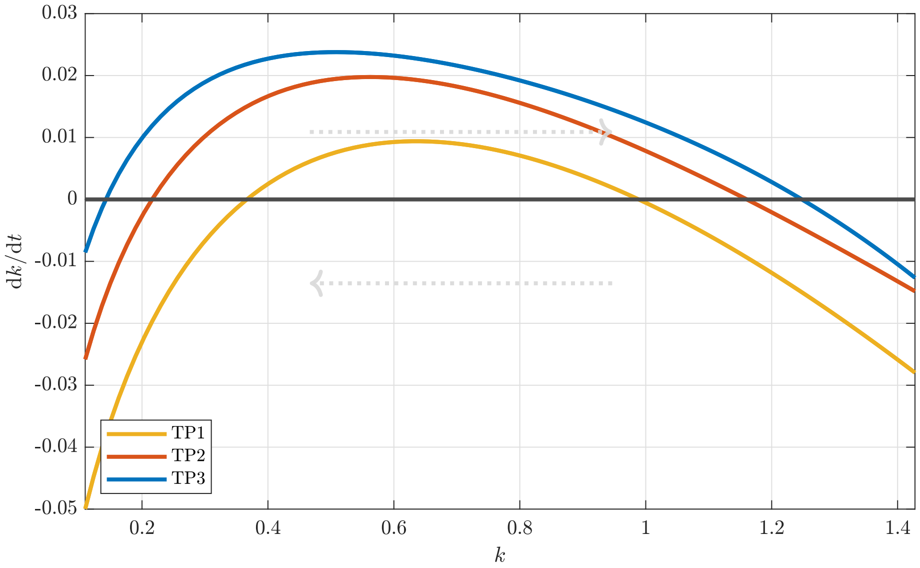
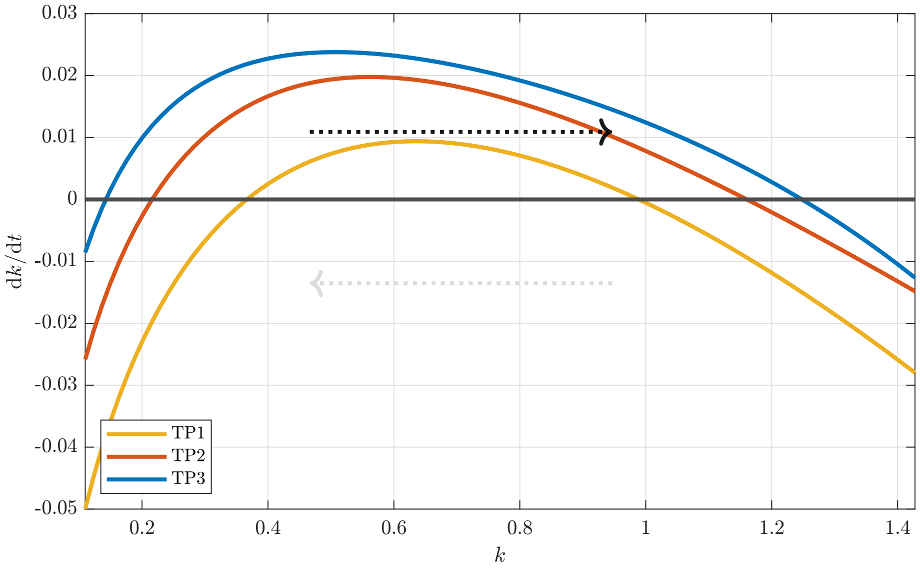



Figure 2: Drift of $k$. All other state variables are at steady state value.

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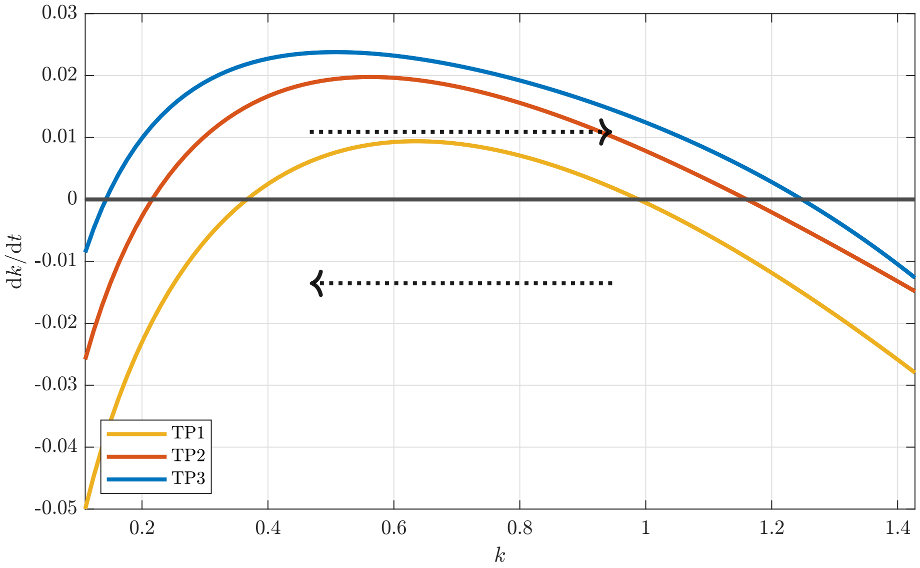


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# Englobed Taylor projection

Inaccuracies and instabilities in tail region: Propose englobed Taylor projection (ETP)

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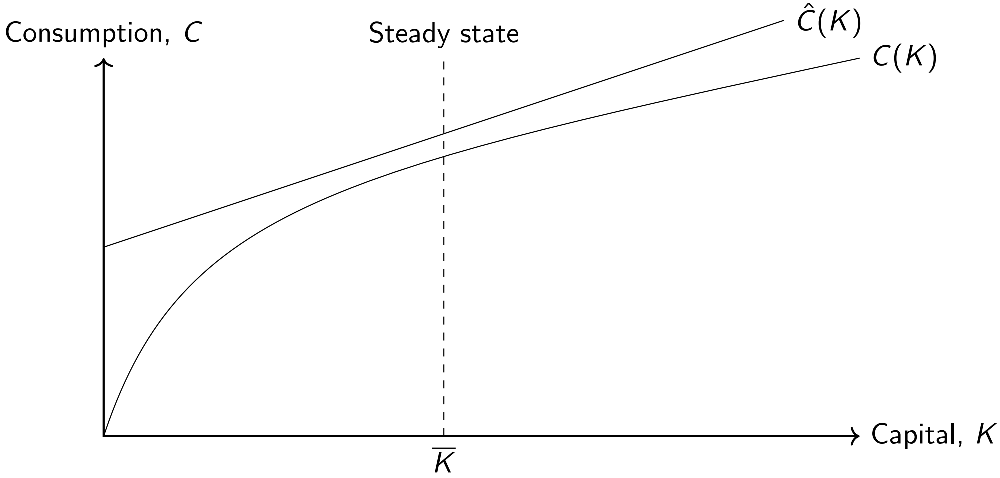

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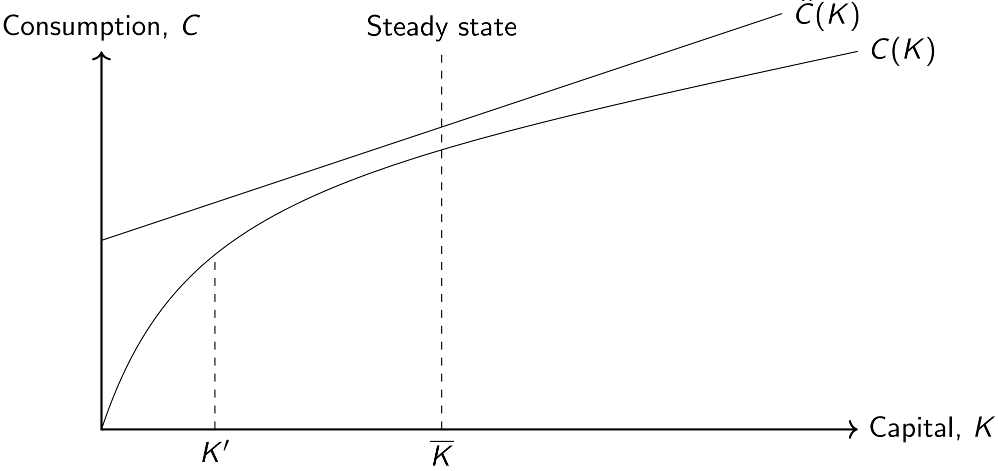

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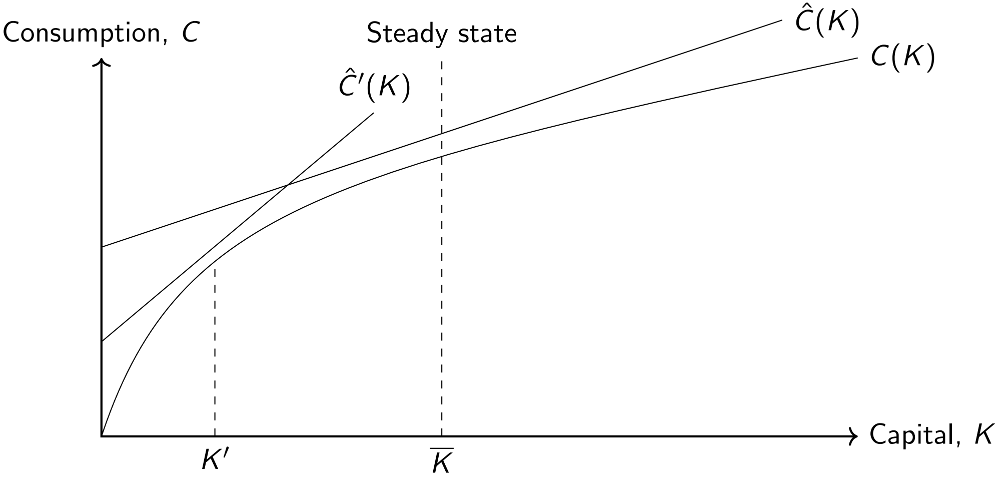

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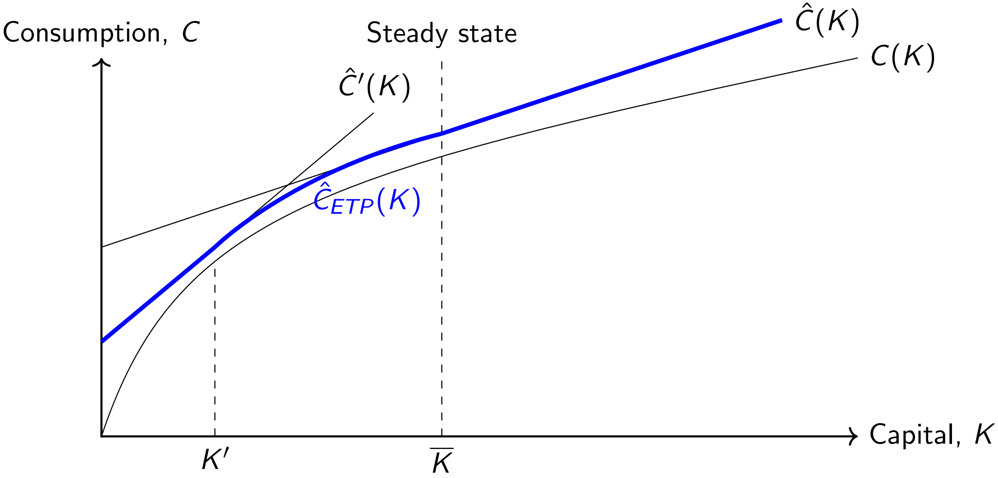


Technically, the "englobed" approximation is given by

$$\widehat{\mathbf{g}}_e(\mathbf{x}; \boldsymbol{\Theta}) = \sum_{s=1}^{n_s} \phi(\mathbf{x}, \bar{\mathbf{x}}_s) \widehat{\mathbf{g}}(\mathbf{x}; \boldsymbol{\Theta}_s, \bar{\mathbf{x}}_s)$$

- $\phi(\mathbf{x}, \bar{\mathbf{x}}_s) \in [0, 1]$ are scalar weight functions centered at $\bar{\mathbf{x}}_1, \bar{\mathbf{x}}_2, \ldots, \bar{\mathbf{x}}_s, \ldots, \bar{\mathbf{x}}_{n_s}$ and $\sum_{s=1}^{n_s} \phi(\mathbf{x}, \bar{\mathbf{x}}_s) = 1 \quad \forall \mathbf{x}$
- $\widehat{\mathbf{g}}(\mathbf{x}; \boldsymbol{\Theta}_s, \bar{\mathbf{x}}_s)$ is a Taylor projection approximation at $\bar{\mathbf{x}}_s$

We prove convergence of the englobed Taylor projection algorithm for any choice of grid, $\bar{\mathbf{x}}_1, \bar{\mathbf{x}}_2, \ldots, \bar{\mathbf{x}}_s, \ldots, \bar{\mathbf{x}}_{n_s}$,

$$\widehat{\mathbf{g}}_e(\mathbf{x}; \boldsymbol{\Theta}) \to \mathbf{g}(\mathbf{x}) \quad \text{as } h \to \infty$$

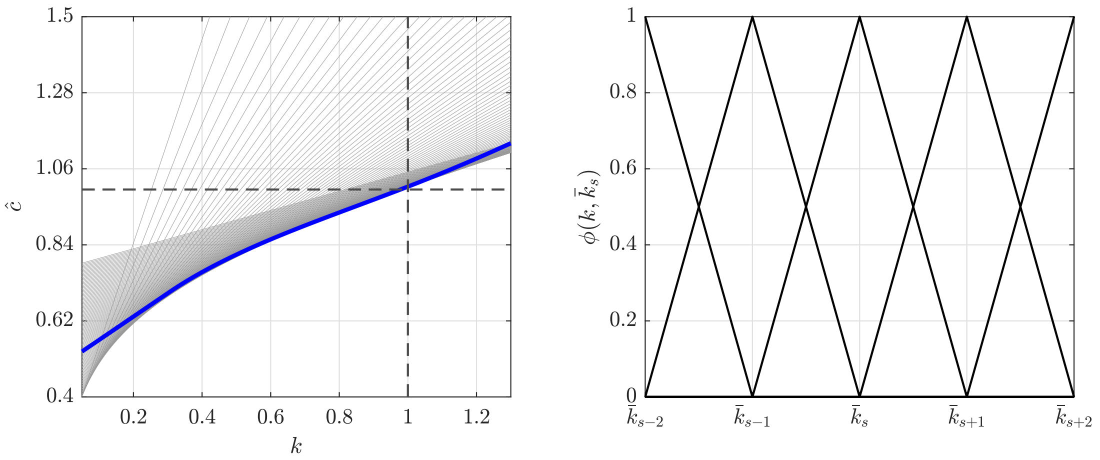
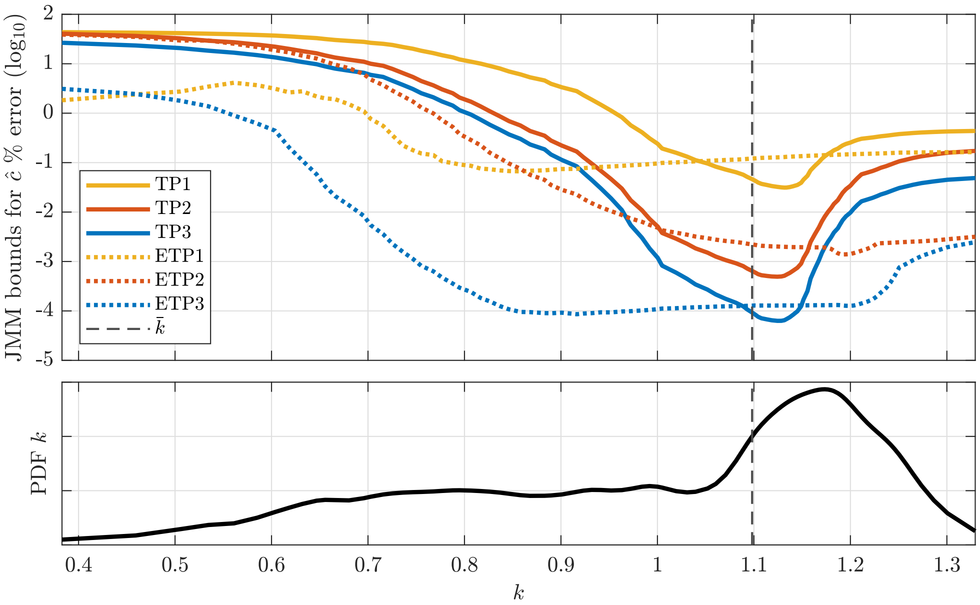



Figure 3: A first-order englobed Taylor projection



JMM bounds for $\hat{c}$ % error ($\log_{10}$)
2
1
0
-1
-2
-3
-4
-5
TP1
TP2
TP3
ETP1
ETP2
ETP3
$\bar{k}$
PDF $k$
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
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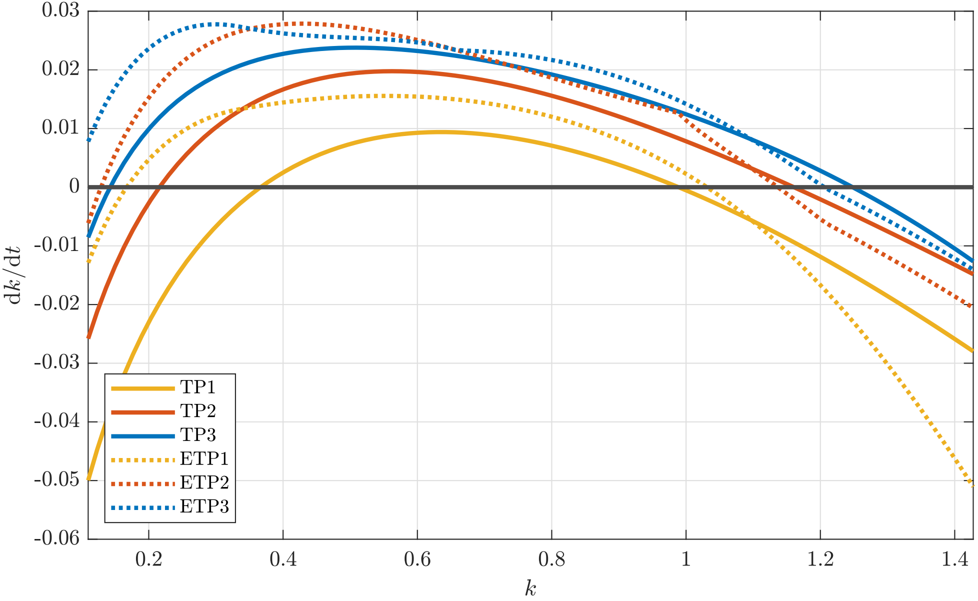
1.3
$k$



0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
dk/dt
0.2
0.4
0.6
0.8
1
1.2
1.4
k
TP1
TP2
TP3
ETP1
ETP2
ETP3

# Conclusion

We extend Taylor projection to approximate the solution of continuous-time dynamic stochastic models driven by jump-diffusion processes

1. We provide proof of convergence
2. Demonstrate applicability in an economy hit by infrequent disasters by gauging errors and stability
3. To reduce *i*) approximation errors and *ii*) instability induced by large jumps, we propose englobed Taylor projection to ensure stable and precise simulations of economies exposed to rare disasters

Our appendix provides additional applications: a simple New Keynesian economy, an endogenous growth (AK) economy with disaster risk, and a finite resource extraction problem

Thank you for your time!

# Appendix

## Derivations, Ramsey-Cass-Koopmans economy

HJB reads

$$\rho V = C^{1-\gamma}/[1-\gamma] + V_K\left(f\left(K\right) - \delta K - C\right)$$

FOC

$$C^{-\gamma} - V_K = 0 \Rightarrow C(K)$$

Costate

$$V_K\left(f_K\left(K\right) - \delta - \rho\right) + V_{KK}\left(f\left(K\right) - \delta K - C\right) = 0$$

Using the FOC, substitute $V_K$ and $V_{KK}$ with $C$ and $C_K$ into costate to get **F**

Return

## Social planner problem (Gourio, 2012)

The social planner chooses sequences $\{C_t, L_t\}_{t=0}^{\infty}$ such that that

$$V\left(K_0, z_{r,0}, z_{p,0}, q_0, \lambda_0\right) = \max_{\{C_t, L_t\}_{t=0}^{\infty}} J_0,$$

subject to

$$\begin{aligned}
\mathrm{d}K_t =& \left(\Phi\left(\frac{K_t^{\alpha}\left(z_{r,t} z_{p,t} L_t\right)^{1-\alpha} - C_t}{K_t}\right) - \delta\right) K_t \mathrm{d}t + \xi_K K_{-t} \mathrm{d}N_t, \\
\mathrm{d}z_{p,t} =& \left(\mu + \omega q_t\right) z_{p,t} \mathrm{d}t + \sigma_{z_p} z_{p,t} \mathrm{d}B_{z_p,t}, \\
\mathrm{d}z_{r,t} =& \rho_z \left(q_t - \log z_{r,t}\right) z_{r,t} \mathrm{d}t, \\
\mathrm{d}q_t =& -\phi_q q_t \mathrm{d}t + \xi_z \mathrm{d}N_t, \\
\mathrm{d}\lambda_t =& \kappa\left(\overline{\lambda} - \lambda_t\right) \mathrm{d}t + \sigma_\lambda \sqrt{\lambda_t} \mathrm{d}B_{\lambda,t},
\end{aligned}$$

We characterize the competitive equilibrium as the evolution of the costate of capital

$$
\begin{aligned}
0 = & \left[\Phi_{\iota}\iota_k k + \Phi(\iota) - \delta - \mu - \omega q + \gamma\sigma_{z_p}^2 + \left(\mu + \omega q - \frac{1}{2}\gamma\sigma_{z_p}^2\right)(1-\gamma) + f_v\right] v_k \\
& + \left[\Phi(\iota) - \delta - \mu - \omega q + \gamma\sigma_{z_p}^2\right] k v_{kk} + \sigma_{z_p}^2 k v_{kk} + \frac{1}{2}\sigma_{z_p}^2 k^2 v_{kkk} \\
& + \rho_z (q - \log(z_r)) z_r v_{z_r k} - \phi_q q v_{qk} \\
& + \kappa\left(\bar{\lambda} - \lambda\right) v_{\lambda k} + \frac{1}{2}\sigma_{\lambda}^2 \lambda v_{\lambda\lambda k} + \lambda\left[\tilde{v}_k (1+\xi) - v_k\right], \qquad (8)
\end{aligned}
$$

along with first-order conditions for any interior solution

$$
\begin{aligned}
f_u u_c + \Phi_{\iota}\iota_c k v_k &= 0, \qquad (9) \\
f_u u_L + \Phi_{\iota}\iota_L k v_k &= 0, \qquad (10)
\end{aligned}
$$

Together, they form the equilibrium vector, **F**

Return

## Assumptions

1. $\mathcal{H}$ and $\mathbf{g}$ are analytic functions on an open set $\mathbb{X} \ni \bar{\mathbf{x}}$
2. As the order of approximation approaches infinity, $h \to \infty$, the derivatives of the policy function at the approximation point, $\mathbf{g}_{\mathbf{x}^h}(\bar{\mathbf{x}})$, go sufficiently fast to zero

## Theorem

- Let Assumptions 1 and 2 hold. Then, $\widehat{\mathbf{g}}(\mathbf{x}; \boldsymbol{\Theta})$ converges to $\mathbf{g}(\mathbf{x})$ as $h \to \infty$

Return

- Continuous-time framework implies (using Itô calculus) the absence of any conditional expectation operator in the equilibrium conditions

$$\mathcal{H}\left(\mathbf{x}, \mathbf{y}, \mathbf{y}_{\mathbf{x}}, \mathbf{y}_{\mathbf{xx}}\right) = \mathbf{0}.$$

But on the flip side, the derivatives of policy functions are needed

- Discrete-time framework (Levintal, 2018)

$$\mathbb{E}_t\left[\mathbf{F}\left(\mathbf{x}_t, \mathbf{x}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t+1}\right)\right] = 0.$$

The framework requires approximations of $\mathbb{E}_t[\cdot]$

- Implications
  - Theoretical: proof of convergence requires fewer assumptions
  - Computational: cost of $\mathbf{y}_{\mathbf{x}^h}$ is small (exploit $\mathbf{y}$'s polynomial structure)

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## Computational differences

- A perturbation approximation of order $h$ needs information from derivatives of equilibrium equations of order $h+2$ (Gaspar and Judd, 1997)
- An $h$-th order Taylor projection only needs derivatives of equilibrium equations of order $h$
- Example: Five equilibrium equations and three state variables. A 2nd order perturbation approximation needs 125 derivatives, while Taylor projection needs 45

## Properties (first-order)

- Perturbation adjusts for risk in levels but slopes are certainty equivalent
- Taylor projection adjusts for risk in both level and slope coefficients

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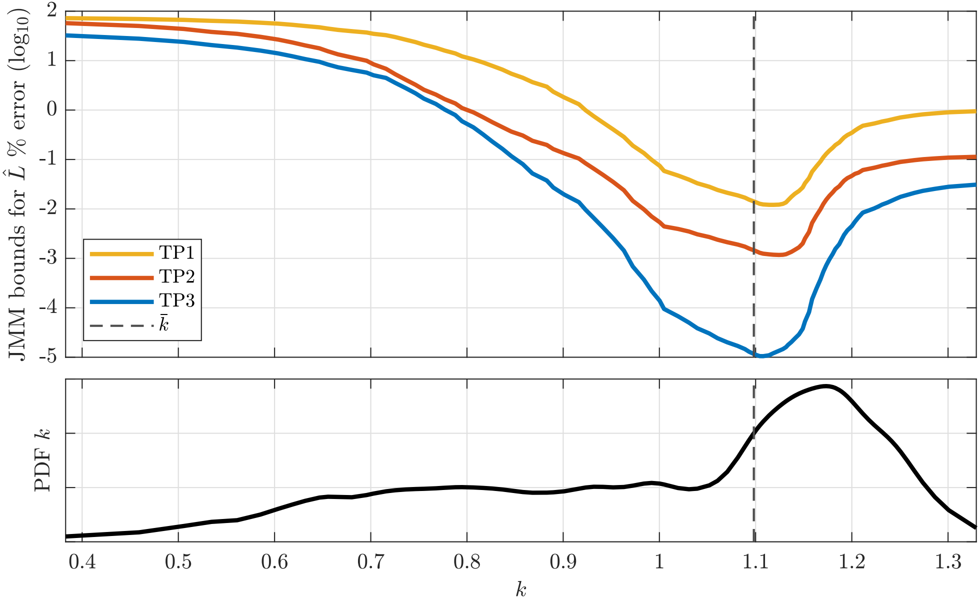
## A note on the value function

- Stochastic differential utility $\Rightarrow$ policy functions also depend on the (unknown) value function $v(\mathbf{x})$
- We do not approximate the value function directly. Instead

$$\hat{v}(\mathbf{x}) = \int \hat{v}_k(\mathbf{x}) \mathrm{d}k$$

  using the approximant of the costate variable for capital
- The yet unknown constant of integration is pinned down by the maximized HJB

Return



JMM bounds for $\hat{L}$ % error ($\log_{10}$)
2
1
0
-1
-2
-3
-4
-5
TP1
TP2
TP3
$\bar{k}$
PDF $k$
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
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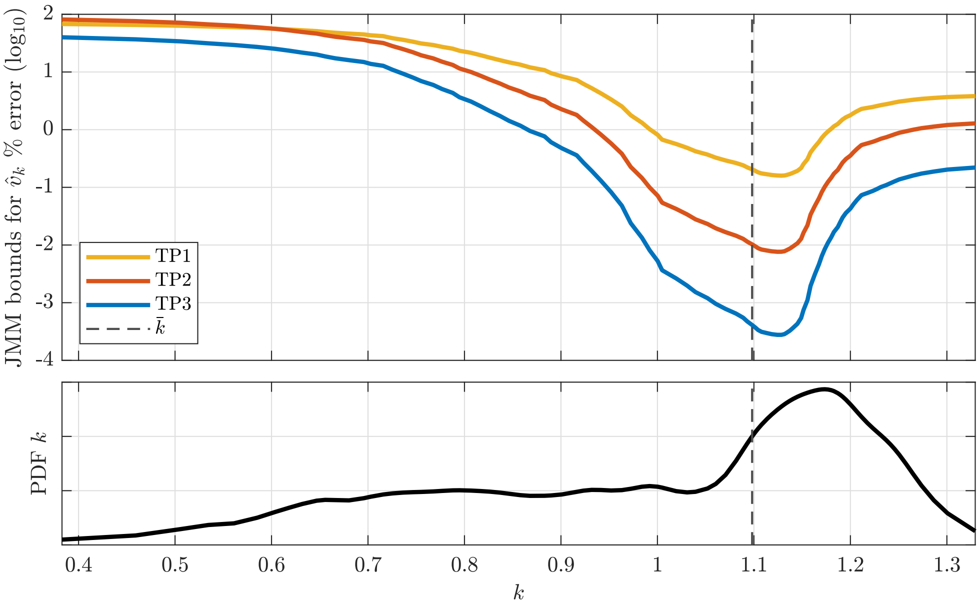
JMM bounds for $\hat{v}_k$ % error ($\log_{10}$)
2
1
0
-1
-2
-3
-4
TP1
TP2
TP3
$\bar{k}$
PDF $k$
0.4
0.5
0.6
0.7
0.8
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1
1.1
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1.3
$k$

Return

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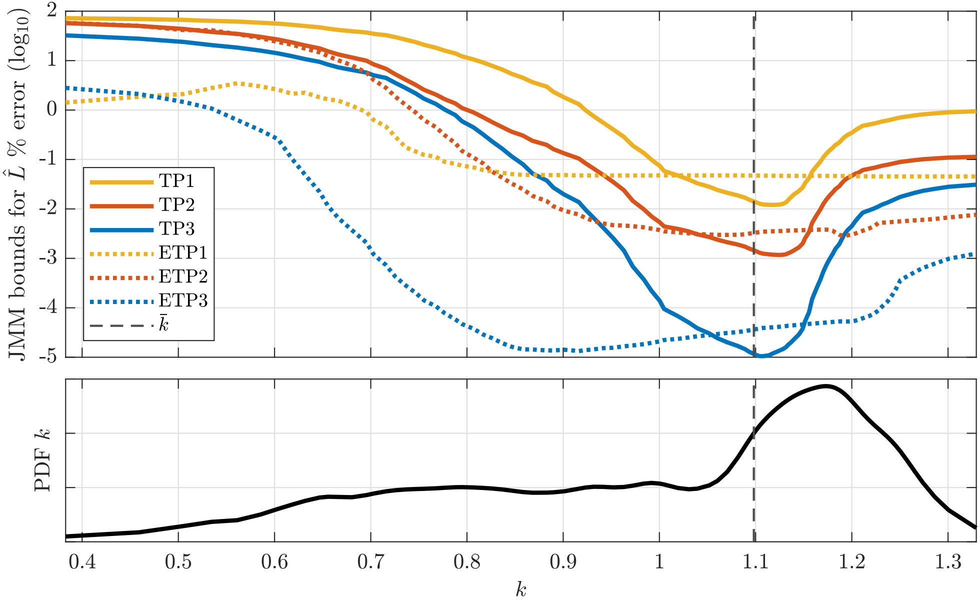
| TP3 | -2.87 | 0.28 | -0.45 | -1.09 | -2.63 | -5.51 | -2.88 | -1.93 |
| ETP1 | -1.78 | -0.02 | -1.22 | -1.74 | -1.99 | -1.86 | -1.77 | -1.65 |
| ETP2 | -2.45 | 0.38 | -0.32 | -1.07 | -2.69 | -3.39 | -3.33 | -2.38 |
| ETP3 | -4.54 | 0.10 | -2.10 | -4.49 | -5.14 | -5.04 | -4.97 | -3.63 |

Table 2: Error values. The table reports JMM error measures over the population distribution implied by ETP3.

Return



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2
1
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TP1
TP2
TP3
ETP1
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PDF $k$
0.4
0.5
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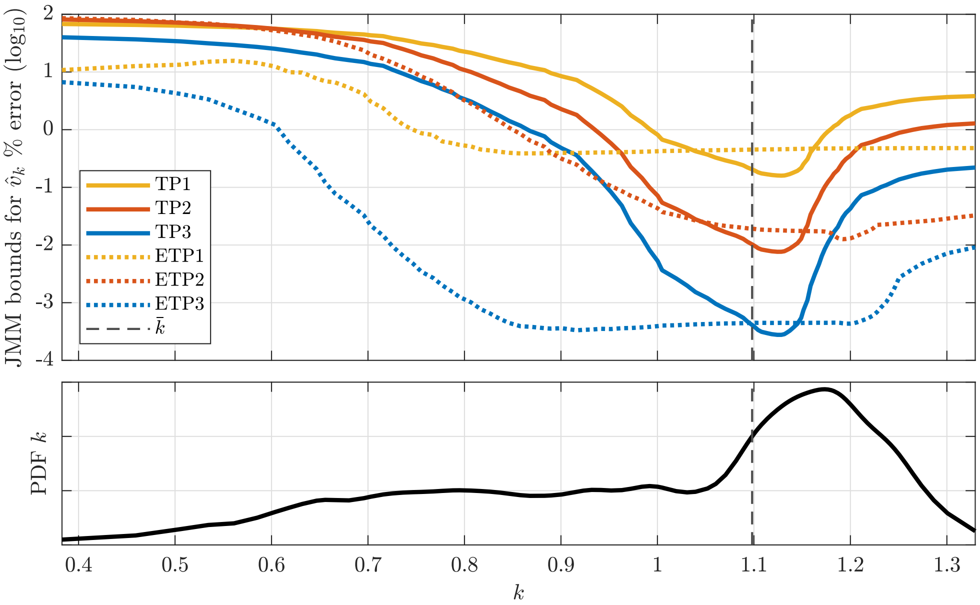
1.1
1.2
1.3
$k$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2
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-3
-4
TP1
TP2
TP3
ETP1
ETP2
ETP3
$\bar{k}$
PDF $k$
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
$k$

Return

## The calibration follows an annual unit of time

| Parameters | Value |
|---|---|
| Household's preferences | |
| Coefficient of relative risk aversion, $\gamma$ | 3.8000 |
| Intertemporal elasticity of substitution, $\hat{\psi}$ | 2.0000 |
| Leisure preference, $\nu$ | 2.3300 |
| Subjective discount rate, $\beta$ | 0.0410 |
| Production | |
| Capital share in output, $\alpha$ | 0.3400 |
| Depreciation rate of capital, $\delta$ | 0.0776 |
| Elasticity, investment-to-capital ratio, wrt Tobin's $q$, $\eta$ | 10.000 |
| Det. steady-state value, investment-to-capital ratio, $\bar{\iota}$ | $\delta + \mu$ |

| Parameters | Value |
|---|---|
| **Exogenous processes** | |
| Trend growth of TFP, $\mu$ | 0.0200 |
| Standard deviation of TFP shock, $\sigma_{z_p}$ | 0.0200 |
| Mean arrival rate of Poisson process, $\overline{\lambda}$ | 0.0280 |
| Persistence of disaster risk, $\kappa$ | 0.2000 |
| Standard deviation of disaster risk, $\sigma_{\lambda}$ | 0.1000 |
| Permanent destruction in productivity after a jump, $\omega$ | 0.1900 |
| Destruction rate of capital/productivity, $\xi_K = \xi_z = \xi$ | -0.4742 |
| Speed of recovery, $\phi_q$ | 0.7526 |
| Speed of disaster realization, $\rho_z$ | 0.7527 |

Return

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