Online Appendix for: (Un)expected Monetary Policy Shocks and Term Premia

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1. MODEL SOLUTION

1.1 Stationarized Model

Household:

$$V_t = \left[\frac{e^{\varepsilon_{b,t}}}{1-\gamma}\left(\left(c_t - \frac{bc_{t-1}}{e^{z_t^+}}\right)^{1-\gamma} - 1\right) + \frac{e^{\varepsilon_{b,t}}\psi_L\left(1-L_t\right)^{1-\chi}}{1-\chi}\right] + \beta\left(E_t\left[V_{t+1}^{1-\sigma_{EZ}}\right]\right)^{\frac{1}{1-\sigma_{EZ}}} \tag{A-1}$$

$$\lambda_t = e^{\varepsilon_{b,t}}\left(c_t - \frac{bc_{t-1}}{e^{z_t^+}}\right)^{-\gamma} \tag{A-2}$$

$$q_t = \frac{1 - E_t\left[M_{t+1}q_{t+1}\nu\left(\frac{I_{t+1}e^{z_{t+1}^+ + \Psi_{t+1}}}{I_t} - e^{\bar{z}^+ + \bar{\Psi}}\right)e^{\varepsilon_{I,t+1}}\left(\frac{I_{t+1}e^{z_{t+1}^+ + \Psi_{t+1}}}{I_t}\right)^2\right]}{\left(1 - \frac{\nu}{2}\left(\frac{I_t e^{z_t^+ + \Psi_t}}{I_{t-1}} - e^{\bar{z}^+ + \bar{\Psi}}\right)^2 - \nu\left(\frac{I_t e^{z_t^+ + \Psi_t}}{I_{t-1}} - e^{\bar{z}^+ + \bar{\Psi}}\right)\frac{I_t e^{z_t^+ + \Psi_t}}{I_{t-1}}\right)e^{\varepsilon_{I,t}}} \tag{A-3}$$

$$q_t = E_t\left[\frac{M_{t+1}}{e^{\Psi_{t+1}}}\left(r_{t+1}^k + q_{t+1}\left(1-\delta\right)\right)\right] \tag{A-4}$$

$$w_t\lambda_t = e^{\varepsilon_{b,t}}\psi_L\left(1-L_t\right)^{-\chi} \tag{A-5}$$

$$M_t = \beta e^{-z_{t+1}^+}\frac{\lambda_t}{\lambda_{t-1}}\left(V_t\right)^{-\sigma_{EZ}} E_{t-1}\left[V_t^{1-\sigma_{EZ}}\right]^{\frac{\sigma_{EZ}}{1-\sigma_{EZ}}} \tag{A-6}$$

$$1 = M_{t+1}\frac{\exp\left(R_t^f\right)}{\pi_{t+1}} \tag{A-7}$$

Price setting:

$$K_t^p = e^{\varepsilon_{p,t}}y_t\tilde{p}_t^{-\theta_p} + \gamma_p E_t\left[M_{t+1}\left(\frac{\pi_t^{\xi_p}}{\pi_{t+1}}\right)^{1-\theta_p}\left(\frac{\tilde{p}_t}{\tilde{p}_{t-1}}\right)^{-\theta_p}e^{z_t^+ + 1}K_{t+1}^p\right] \tag{A-8}$$

$$\frac{\theta_p - 1}{\theta_p}K_t^p = y_t mc_t\tilde{p}_t^{-\theta_p - 1} + \gamma_p E_t\left[M_{t+1}\left(\frac{\pi_t^{\xi_p}}{\pi_{t+1}}\right)^{-\theta_p}\left(\frac{\tilde{p}_t}{\tilde{p}_{t-1}}\right)^{-\theta_p - 1}\frac{\theta_p - 1}{\theta_p}e^{z_t^+ + 1}K_{t+1}^p\right] \tag{A-9}$$

$$1 = \gamma_p\left(\frac{\pi_{t-1}^{\xi_p}}{\pi_t}\right)^{1-\theta_p} + \left(1-\gamma_p\right)\left(\tilde{p}_t\right)^{1-\theta_p} \tag{A-10}$$

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Intermediate Goods Producer:

$$p_t^+ y_t = e^{a_t}\left(\frac{k_{t-1}}{e^{z_t^+ + \Psi_t}}\right)^{\alpha}(L_t)^{1-\alpha} - \Phi_t \tag{A-11}$$

$$w_t = mc_t e^{a_t}(1-\alpha)\left(\frac{k_{t-1}}{e^{z_t^+ + \Psi_t}}\right)^{\alpha} L_t^{-\alpha} \tag{A-12}$$

$$r_t^k = mc_t e^{a_t}\alpha\left(\frac{k_{t-1}}{e^{z_t^+ + \Psi_t}}\right)^{\alpha-1} L_t^{1-\alpha} \tag{A-13}$$

Aggregation:

$$k_t = (1-\delta)\frac{k_{t-1}}{e^{z_t^+ + \Psi_t}} + e^{\varepsilon_{I,t}}\left(1 - \frac{\nu}{2}\left(\frac{I_t e^{z_t^+ + \Psi_t}}{I_{t-1}} - e^{\bar{z}^+ + \bar{\Psi}}\right)^2\right) I_t \tag{A-14}$$

$$p_t^+ = (1-\gamma_p)(\tilde{p}_t)^{-\theta_p} + \gamma_p\left(\frac{\pi_{t-1}^{\xi_p}}{\pi_t}\right)^{-\theta_p} p_{t-1}^+ \tag{A-15}$$

$$y_t = c_t + I_t + \bar{g}e^{g_t} \tag{A-16}$$

$$z_t^+ = \frac{\alpha}{1-\alpha}\bar{\Psi} + z_t \tag{A-17}$$

Monetary Policy:

$$4R_t^f = 4\rho_R R_{t-1}^f + (1-\rho_R)\left(4\bar{r}^{real} + \eta_y \ln\left(\frac{y_t}{\bar{y}}\right) + 4\ln(\pi_t) + \eta_\pi\left[4\ln(\tilde{\pi}_t) - \ln(\pi_t^*)\right]\right) + \sigma_m\epsilon_{m,t} \tag{A-18}$$

$$\ln\pi_t^* - 4\ln\bar{\pi} = \rho_\pi\left(\ln\pi_{t-1}^* - 4\ln\bar{\pi}\right) + 4\zeta_\pi\left(\ln\pi_{t-1} - \ln\bar{\pi},\right) + \sigma_\pi\epsilon_{\pi,t} \tag{A-19}$$

Shock Processes:

$$g_t = \rho_g g_{t-1} + \sigma_g\epsilon_{g,t} \tag{A-20}$$

$$a_t = \rho_a a_{t-1} + \sigma_a\epsilon_{a,t} \tag{A-21}$$

$$\varepsilon_{I,t} = \rho_I\varepsilon_{I,t-1} + \sigma_I\epsilon_{I,t} \tag{A-22}$$

$$\varepsilon_{b,t} = \rho_b\varepsilon_{b,t-1} + \sigma_b\epsilon_{b,t} \tag{A-23}$$

$$z_t - \bar{z} = \rho_z(z_{t-1} - \bar{z}) + \sigma_z\epsilon_{z,t} \tag{A-24}$$

$$\ln\left(\Omega_t/\bar{\Omega}\right) = \rho_\Omega\ln\left(\Omega_{t-1}/\bar{\Omega}\right) + \sigma_\Omega\epsilon_{\Omega,t} \tag{A-25}$$

1.2 Deterministic Steady State

Given our parameterizations for $\frac{\bar{g}}{\bar{y}}$, $\bar{\pi}$, and $\bar{L}$, the deterministic steady state is

$$\bar{\tilde{p}} = \left[\frac{1-\gamma_p\bar{\pi}^{(\xi_p-1)(1-\theta_p)}}{1-\gamma_p}\right]^{\frac{1}{1-\theta_p}} \tag{A-26}$$

$$\bar{p}^+ = \frac{(1-\gamma_p)\bar{\tilde{p}}^{-\theta_p}}{1-\gamma_p\bar{\pi}^{(1-\xi_p)\theta_p}} \tag{A-27}$$

$$\bar{R}^f = \ln\left(\frac{\bar{\pi}}{\beta}\right) - \left(-\gamma(1-\phi) - \phi\right)\bar{z}^+ \tag{A-28}$$

$$\bar{M} = \beta\exp\left(-\bar{z}^+\right) \tag{A-29}$$

$$\bar{r}^k = \frac{\exp\left(\bar{z}^+ + \bar{\Psi}\right)}{\beta} - (1-\delta) \tag{A-30}$$

$$\overline{mc} = \bar{\tilde{p}}\frac{\theta_p - 1}{\theta_p}\frac{\left(1-\gamma_p\beta\bar{\pi}^{(1-\xi_p)\theta_p}\right)}{\left(1-\gamma_p\beta\bar{\pi}^{(\xi_p-1)\theta_p}\right)} \tag{A-31}$$

$$\bar{k} = \bar{L}\left(\frac{\bar{r}^k}{\overline{mc}\alpha}\frac{1}{\exp\left(\bar{z}^+ + \bar{\Psi}\right)}\right)^{-\frac{1}{1-\alpha}} \tag{A-32}$$

$$\bar{w} = \overline{mc}\left(1-\alpha\right)\left(\exp\left(\bar{z}^+ + \bar{\Psi}\right)\right)^{-\alpha}\left(\frac{\bar{r}^k}{\overline{mc}\alpha}\frac{1}{\exp\left(\bar{z}^+ + \bar{\Psi}\right)}\right)^{\frac{\alpha}{1-\alpha}} \tag{A-33}$$

$$\bar{y} = \bar{r}^k\left(\frac{\bar{k}}{\exp\left(\bar{z}^+ + \bar{\Psi}\right)}\right) + \bar{w}\bar{L} \tag{A-34}$$

$$\Phi = \left(\frac{\bar{k}}{\exp\left(\bar{z}^+ + \bar{\Psi}\right)}\right)^{\alpha}\bar{L}^{1-\alpha} - \bar{y}\bar{p}^+ \tag{A-35}$$

$$\bar{I} = \left(1 - \frac{1-\delta}{\exp\left(\bar{z}^+ + \bar{\Psi}\right)}\right)\bar{k} \tag{A-36}$$

$$\bar{g} = \left(\frac{\bar{g}}{\bar{y}}\right)\bar{y} \tag{A-37}$$

$$\bar{c} = \bar{y} - \bar{g} - \bar{I} \tag{A-38}$$

$$\bar{\lambda} = \left(\bar{c} - \frac{b\bar{c}}{\exp\left(\bar{z}^+\right)}\right)^{-\gamma} \tag{A-39}$$

$$\psi_L = \bar{w}\bar{\lambda}\left(1-\bar{L}\right)^{\chi} \tag{A-40}$$

$$\bar{K}^p = \frac{\bar{y}\bar{\tilde{p}}^{-\theta_p}}{1 - \gamma_p\beta\bar{\pi}^{\left(\xi_p - 1\right)\left(1-\theta_p\right)}} \tag{A-41}$$

$$\bar{V} = \frac{1}{1-\beta}\left(\frac{\left(\bar{c} - \frac{b\bar{c}}{\exp\left(\bar{z}^+\right)}\right)^{1-\gamma} - 1}{1-\gamma} - \frac{\psi_L\left(1-\bar{L}\right)^{1-\chi}}{1-\chi}\right) \tag{A-42}$$

2. RISK-ADJUSTED LINEAR APPROXIMATION

The method of Meyer-Gohde (2016) differs from others in constructing an approximation centered around a risk-adjusted critical point, such as Juillard (2010), Kliem and Uhlig (2016), and Coeurdacier et al. (2011). First, it is direct and noniterative relying entirely on perturbation methods to construct the approximation. Second, it enables us to construct the approximation around (an approximation of) the ergodic mean of the true policy function instead of its stochastic or "risky" steady state, placing the locality of our approximation in a region with a likely high (model-based) data density. The closest methods in the macro-finance term structure literature are Dew-Becker (2014) and Lopez et al. (2015), who both approximate the nonlinear macro side of the model to obtain a linear in states approximation with adjustments for risk and then derive affine approximation of the yield curve taking this macro approximation as given. The exact meaning of these risk adjustments remains unclear, however, whereas the method by Meyer-Gohde (2016) adjusts the coefficients out to the second moments in shocks around the mean of the endogenous variables, itself approximated out to the second moments in shocks.

Thus, instead of either a linear certainty-equivalent or nonlinear non-certainty-equivalent approximation, the method constructs a linear non-certainty-equivalent approximation. By using higher order derivatives of the policy function at the deterministic steady state, it approximates the ergodic mean of endogenous variables and the first derivatives of the policy function around this ergodic mean. Unlike standard higher order polynomial perturbations[1] or affine approximation methods,[2] this linear in states approximation gives us significant computational advantages.

Stacking our n_y endogenous variables into the vector y_t and our n_ε normally distributed exogenous shocks into the vector ε_t, we collect our equations into the following vector of nonlinear rational expectations difference equations

$$0 = E_t[f(y_{t+1}, y_t, y_{t-1}, \varepsilon_t)] = \hat{F}(y_{t-1}, \varepsilon_t) \tag{B-1}$$

where f is an $(n_{eq} \times 1)$ vector valued function, continuously M-times differentiable in all its arguments and with as many equations as endogenous variables $(n_{eq} = n_y)$.

The solution to the functional problem in (B-1) is the policy function

$$y_t = g^0(y_{t-1}, \varepsilon_t) \tag{B-2}$$

Generally, a closed form for (B-2) is not available, so recourse to numerical approximations is necessary. We assume that the related deterministic model

$$0 = f(y_{t+1}, y_t, y_{t-1}, 0) = \overline{F}(y_{t-1}, 0) \tag{B-3}$$

admits the calculation of a fix point, the deterministic steady state, defined as $\overline{y} \in \mathbb{R}^{n_y}$ such that $0 = \overline{F}(\overline{y}, 0)$.

We are, however, interested in the stochastic version of the model and will now proceed to nest the deterministic model, for which we can recover a fix point, and the stochastic model, for which we cannot, within a larger continuum of models, following standard practice in the perturbation DSGE literature.

We introduce an auxiliary variable $\sigma \in [0,\ 1]$ to scale the stochastic elements in the model. The value $\sigma = 1$ corresponds to the "true" stochastic model and $\sigma = 0$ returns the deterministic

[1] Among others, recent third order perturbation approximations for DSGE models of the term structure include Rudebusch and Swanson (2008, 2012), van Binsbergen et al. (2012) Andreasen (2012), and Andreasen et al. (2018). While second order approximations such as Hördahl et al. (2008) provide nonzero but constant premia and De Graeve et al. (2009) is an example of a purely linear model that neglects endogenous premia. Additionally, many recent perturbations, Andreasen and Zabczyk (2015), Andreasen (2012), Andreasen et al. (2018), prune to ensure asymptotic stability.

[2] These approaches separate the macro and financial variables, generally using a (log) linear approximation of the former and an affine approximation for the yield curve following the empirical finance literature. Bonds are priced in an arbitrage free setup using either the endogenous pricing kernel implied by households' stochastic discount factors, as Dew-Becker (2014), Bekaert et al. (2010), and Palomino (2012), or an estimated exogenously specified kernel, as Hördahl et al. (2006), Hördahl and Tristani (2012), Ireland (2015), Rudebusch and Wu (2007), Rudebusch and Wu (2008).

model in (B-3). Accordingly, the stochastic model, (B-1), and the deterministic model, (B-3), can be nested inside the following continuum of models

$$0 = E_t[f(y_{t+1}, y_t, y_{t-1}, \tilde{\varepsilon}_t)] = F(\sigma, y_{t-1}, \tilde{\varepsilon}_t),\ \tilde{\varepsilon}_t \equiv \sigma\varepsilon_t \tag{B-4}$$

with the associated policy function

$$y_t = g(y_{t-1}, \tilde{\varepsilon}_t, \sigma) \tag{B-5}$$

Notice that this reformulation allows us to express the deterministic steady state as the fix point of (B-4) for $\sigma = 0$, i.e., $\overline{y} \in \mathbb{R}^{n_y}$ such that $0 = F(0, \overline{y}, 0) = \overline{F}(\overline{y}, 0)$ and, as a consequence $\overline{y} = g(\overline{y}, 0, 0)$.

We use this deterministic steady state and derivatives of the policy function in (B-5), recovered by the implicit function theorem,[3] evaluated at at $\overline{y}$ (both in the deterministic model, (B-3), and towards our stochastic model, (B-1), to construct our approximation of and around the ergodic mean.

Since y in the policy function (B-5) is a vector valued function, its derivatives form a hypercube.[4] Adopting an abbreviated notation, we write $g_{z^j\sigma^i} \in \mathbb{R}^{n_y \times n_z^j}$ as the partial derivative of the vector function g with respect to the state vector z_t j times and the perturbation parameter σ i times evaluated at the deterministic steady state.

Instead of using the partial derivatives to construct a Taylor series as is the standard procedure,[5] we would like to construct a more accurate linear approximation of the true policy function (B-2), centered at the mean of y_t. Accordingly, we will construct a linear approximation of (B-2) around the ergodic mean, which we formalize in the following.

Proposition 1 *Linear Approximation around the Ergodic Mean*
Nest the means of the stochastic model ($\sigma = 1$) and of the deterministic model ($\sigma = 0$) through

$$\tilde{y}(\sigma) \equiv E\left[g(y_{t-1}, \sigma\varepsilon_t, \sigma)\right] = E\left[y_t\right] \tag{B-8}$$

Then for any $\sigma \in [0,\ 1]$, the linear approximation of the policy function, (B-2), around the mean of y_t defined in (B-8) and that of ε_t is

$$y_t \simeq \tilde{y}(\sigma) + g_y(\tilde{y}(\sigma), 0, \sigma)\left(y_{t-1} - \tilde{y}(\sigma)\right) + g_\varepsilon(\tilde{y}(\sigma), 0, \sigma)\varepsilon_t \tag{B-9}$$

Furthermore, the mean of y_t defined in (B-8) and the two additional unknown functions in this linear approximation

$$\tilde{y_y}(\sigma) \equiv g_y(\tilde{y}(\sigma), 0, \sigma) \tag{B-10}$$
$$\tilde{y_\varepsilon}(\sigma) \equiv g_\varepsilon(\tilde{y}(\sigma), 0, \sigma) \tag{B-11}$$

can be approximated out to second order in σ as

$$\tilde{y}(\sigma) = E\left[y_t\right] \approx \overline{y} + \frac{1}{2}\tilde{y}''(0) \tag{B-12}$$

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[3] See Jin and Judd (2002).

[4] We use the method of Lan and Meyer-Gohde (2014) that differentiates conformably with the Kronecker product, allowing us to maintain standard linear algebraic structures to derive our results as follows:
Let $A(B) : \mathbb{R}^{s\times 1} \to \mathbb{R}^{p\times q}$ be a matrix-valued function that maps an $s \times 1$ vector B into a $p \times q$ matrix $A(B)$, the derivative structure of $A(B)$ with respect to B is defined as

$$A_B \equiv \mathscr{D}_{B^T}\{A\} \equiv \begin{bmatrix} \frac{\partial}{\partial b_1} & \cdots & \frac{\partial}{\partial b_s} \end{bmatrix} \otimes A \tag{B-6}$$

where b_i denotes i'th row of vector B, ^{T} indicates transposition; n'th derivatives are

$$A_{B^n} \equiv \mathscr{D}_{(B^T)^n}\{A\} \equiv \left(\begin{bmatrix} \frac{\partial}{\partial b_1} & \cdots & \frac{\partial}{\partial b_s} \end{bmatrix}^{\otimes[n]}\right) \otimes A \tag{B-7}$$

[5] The Taylor series approximation at a deterministic steady state, assuming (B-5) is C^M with respect to all its arguments, can be written as $y_t = \sum_{j=0}^{M} \frac{1}{j!}\left[\sum_{i=0}^{M-j} \frac{1}{i!} g_{z^j\sigma^i}\sigma^i\right](z_t - \overline{z})^{\otimes[j]}$

$$g_y(\tilde{y}(\sigma), 0, \sigma) \approx g_y + \frac{1}{2}\left(g_{y^2}\left(\tilde{y}''(0) \otimes I_{n_y}\right) + g_{\sigma^2 y}\right) \tag{B-13}$$

$$g_\varepsilon(\tilde{y}(\sigma), 0, \sigma) \approx g_\varepsilon + \frac{1}{2}\left(g_{y\varepsilon}\left(\tilde{y}''(0) \otimes I_{n_\varepsilon}\right) + g_{\sigma^2 \varepsilon}\right) \tag{B-14}$$

where

$$\tilde{y}''(0) = \left(I_{n_y} - g_y\right)^{-1}\left(\left(g_{\varepsilon^2} + \left(I_{n_y} - g_y^{\otimes[2]}\right)^{-1} g_\varepsilon^{\otimes[2]}\right) E\left[\varepsilon_t^{\otimes[2]}\right] + g_{\sigma^2}\right) \tag{B-15}$$

PROOF:
See Meyer-Gohde (2016).

3. DATA

Real GDP: *BEA NIPA table 1.1.6 line 1 (A191RX1).*

Nominal GDP: *BEA NIPA table 1.1.5 line 1 (A191RC1).*

Implicit GDP Deflator: the ratio of **Nominal GDP** to **Real GDP**.

Private Consumption: Real consumption expenditures for non-durables and services is the sum of *BEA NIPA table 1.1.5 line 5 (DNDGRC1)* and *BEA NIPA table 1.1.5 line 6 (DNDGRC1)* deflated by the implicit GDP deflator.

Private Investment: Total real private investment is the sum of Gross Private Investment *BEA NIPA table 1.1.5 line 7 (A006RC1)* and Personal Consumption Expenditures: Durable Goods *BEA NIPA table 1.1.5 line 4 (DDURRC1)* deflated by the implicit GDP deflator.

Civilian Population: This series is calculated from monthly data of *civilian noninstitutional population over 16 years (CNP16OV)* from the U.S. Department of Labor: Bureau of Labor Statistics.

Policy Rate: 3-Month Treasury Bill: Secondary Market Rate *TB3MS* provided by Board of Governors of the Federal Reserve System. The quarterly aggregation is end of period.

Treasury Bond Yields: 1-year, 2-year, 3-year, 5-year, and 10-year zero-coupon bond yields measured end of quarter. The original series are daily figures based on the updated series by Adrian et al. (2013).
Source: `https://www.newyorkfed.org/research/data_indicators/term_premia.html`

Nominal Interest Rate Forecasts: 1-quarter (TBILL3) and 4-quarter (TBILL6) ahead forecasts of the 3-Month Treasury Bill. The time series are the median responses by the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia.
Source: `https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/data-files`

4. ENDOGENOUS PRIOR

Following Del Negro and Schorfheide (2008), we assume $\hat{F}$ to be a vector that collects the first moments of interest from our pre-sample and $F_M(\theta)$ be a vector-valued function which relates model parameters and ergodic means

$$\hat{F} = F_M(\theta) + \eta \tag{D-1}$$

where η is a vector of measurement errors. In our application, we assume that the error terms η are independently and normally distributed. Hence, we express eq. (D-1) as a quasi-likelihood function which can be interpreted as the conditional density

$$\begin{aligned}\mathcal{L}\left(F_M(\theta)\,|\hat{F},T^*\right) &= \exp\left\{-\frac{T^*}{2}\left(\hat{F}-F_M(\theta)\right)'\Sigma_\eta^{-1}\left(\hat{F}-F_M(\theta)\right)\right\} \\ &= p\left(\hat{F}|F_M(\theta),T^*\right)\end{aligned} \tag{D-2}$$

This quasi-likelihood is small for values of θ that lead the DSGE model to predict first moments that strongly differ from the measures of the pre-sample. The parameter T^* captures, along with the standard deviation of η, the precision of our beliefs about the first moments. In practice we set T^* to the length of the pre-sample.

For the application in this paper, we assume that the vector $\hat{F}$ contains the mean of inflation and the means of proxies for the level, slope, and curvature factors of the yield curve. We include the mean of inflation because the non-linearities in our model impose strong precautionary motives that push the predicted ergodic mean of inflation away from its deterministic steady state, $\bar{\pi}$, as is also discussed by Tallarini (2000) and Andreasen (2011). Regarding $\mathcal{L}\left(F_M(\theta)\,|\hat{F}\right)$, we assume that $E_t\left[400\pi|\theta\right]$ is normally distributed with mean 2.5 and variance 0.1.

We follow, e.g., Diebold et al. (2006) and specify common proxies for the level, slope, and curvature factors of the yield curve. Specifically, the proxy for the level factor is $\left(R^{\$}_{1,t}+R^{\$}_{8,t}+R^{\$}_{40,t}\right)/3$, with all yields expressed in annualized terms and the nominal yield of the 1-quarter Treasury Bond equal to the policy rate in the model. Additionally, the proxies for the slope and curvature factors are defined as $R^{\$}_{1,t}-R^{\$}_{40,t}$ and $2R^{\$}_{8,t}-R^{\$}_{1,t}-R^{\$}_{40,t}$, respectively. Regarding $\mathcal{L}\left(F_M(\theta)\,|\hat{F}\right)$, we assume that the ergodic mean of each factor is normally distributed, with the mean equal to its empirical counterpart of the pre-sample. Moreover, we assume that the means of level, slope, and curvature have a variance of 22, 12, and 9 basis points respectively. Thus, the means and variances can be interpreted as $\hat{F}$ value and the variance of the measurement error η in eq. (D-1).

Additionally, we use the second moments of macroeconomic variables, about which we have a priori knowledge, to inform our prior distribution and apply the approach of Christiano et al. (2011). This approach uses classical large sample theory to form a large sample approximation to the likelihood of the pre-sample statistics. The approach is conceptually similar to the one proposed by Del Negro and Schorfheide (2008), but differs in some important respects. Specifically, Del Negro and Schorfheide (2008) focus on the model-implied p-th order vector autoregression, which implies that the likelihood of the second moments is known exactly conditional on the DSGE model parameters and requires no large-sample approximation in contrast to the approach by Christiano et al. (2011). Yet, the latter approach is more flexible insofar as the statistics to target are concerned. Accordingly, let S be a column vector containing the second moments of interest, then, as shown by Christiano et al. (2011) under the assumption of large sample, the estimator of S is

$$\hat{S}\sim N\left(S^0,\frac{\hat{\Sigma}_S}{T}\right) \tag{D-3}$$

with S^0 the true value of S, T the sample length, and $\hat{\Sigma}_S$ the estimate of the zero-frequency spectral density. Now, let $S_M(\theta)$ be a function which maps our DSGE model parameters θ into S. Then, for n targeted second moments and sufficiently large T, the density of $\hat{S}$ is given by

$$p\left(\hat{S}|\theta\right)=\left(\frac{T}{2\pi}\right)^{\frac{n}{2}}\left\|\hat{\Sigma}_S\right\|^{-\frac{1}{2}}\exp\left\{-\frac{T}{2}\left(\hat{S}-S_M(\theta)\right)'\hat{\Sigma}_S^{-1}\left(\hat{S}-S_M(\theta)\right)\right\} \tag{D-4}$$

In our application, S is a set of variances of macroeconomic variables (GDP growth, consumption growth, investment growth, inflation, and the policy rate). In sum, the overall endogenous prior distribution takes the following form

$$p\left(\theta|\hat{F},\hat{S},T^*\right)=C^{-1}p(\theta)\,p\left(\hat{F}|F_M(\theta),T^*\right)p\left(\hat{S}|\theta\right) \tag{D-5}$$

where $p(\theta)$ is the initial prior distribution and C a normalization constant. Two points are noteworthy. First, while the initial priors are independent across parameters, as is typical in Bayesian analysis, the endogenous prior is not independent across parameters. Second, the normalization constant C is necessary for, e.g., posterior odds calculation but not for estimating the model. Accordingly, we do not calculate this constant, which has otherwise to be approximated (see, for example, Del Negro and Schorfheide, 2008, Kliem and Uhlig, 2016). So, the posterior distribution is given by

$$p\left(\theta|X, \hat{F}, \hat{S}, T^*\right) \propto p\left(\theta|\hat{F}, \hat{S}, T^*\right) p\left(X|\theta\right) \tag{D-6}$$

with $p(X|\theta)$ the likelihood of the data conditional on DSGE model parameters θ.

Name	Symbol	Domain	Density	Para(1)	Para(2)
Relative risk aversion	$RRA/100$	$\mathbb{R}^+$	Uniform	0	20
Calvo parameter	γ_p	$[0,1)$	Beta	0.5	0.1
Investment adjustment	ν	$\mathbb{R}^+$	Gamma	4.0	0.75
Habit formation	b	$[0,1)$	Beta	0.5	0.1
Intertemporal elas. substitution	IES	$[0,1)$	Beta	0.25	0.1
Steady state inflation	$100\left(\bar{\pi}-1\right)$	$\mathbb{R}^+$	Uniform	0	6
Interest rate AR coefficient	ρ_R	$[0,1)$	Beta	0.8	0.1
Interest rate inflation coefficient	η_π	$\mathbb{R}^+$	Gamma	1	0.15
Interest rate output coefficient	η_y	$\mathbb{R}^+$	Gamma	0.5	0.1
Inflation target coefficient	$100\zeta_\pi$	$[0,1)$	Beta	0.3	0.1
AR coefficient technology	ρ_a	$[0,1)$	Beta	0.75	0.1
AR coefficient preference	ρ_b	$[0,1)$	Beta	0.75	0.1
AR coefficient investment	ρ_i	$[0,1)$	Beta	0.75	0.1
AR coefficient gov. spending	ρ_g	$[0,1)$	Beta	0.75	0.1
AR coefficient inflation target	ρ_π	$[0,1)$	Beta	0.95	0.025
AR coefficient long-run growth	ρ_z	$[0,1)$	Beta	0.75	0.1
AR coefficient fixed costs	ρ_Ω	$[0,1)$	Beta	0.75	0.1
S.d. technology	$100\sigma_a$	$\mathbb{R}^+$	InvGam	0.5	2
S.d. preference	$100\sigma_b$	$\mathbb{R}^+$	InvGam	0.5	2
S.d. investment	$100\sigma_i$	$\mathbb{R}^+$	InvGam	0.5	2
S.d. monetary policy shock	$100\sigma_m$	$\mathbb{R}^+$	InvGam	0.5	2
S.d. government spending	$100\sigma_g$	$\mathbb{R}^+$	InvGam	0.5	2
S.d. inflation target	$100\sigma_\pi$	$\mathbb{R}^+$	InvGam	0.06	0.03
S.d. long-run growth	$100\sigma_z$	$\mathbb{R}^+$	InvGam	0.5	2
S.d. fixed costs	$100\sigma_\Omega$	$\mathbb{R}^+$	InvGam	0.5	2
ME 1-year T-Bill	$4R^{\$}_{4,t}$	$\mathbb{R}^+$	InvGam	0.005	∞
ME 2-year T-Bill	$4R^{\$}_{8,t}$	$\mathbb{R}^+$	InvGam	0.005	∞
ME 3-year T-Bill	$4R^{\$}_{12,t}$	$\mathbb{R}^+$	InvGam	0.005	∞
ME 5-year T-Bill	$4R^{\$}_{20,t}$	$\mathbb{R}^+$	InvGam	0.005	∞
ME 10-year T-Bill	$4R^{\$}_{40,t}$	$\mathbb{R}^+$	InvGam	0.005	∞
ME 1Q-expected policy rate	$4E_t\left[R^f_{t,t+1}\right]$	$\mathbb{R}^+$	InvGam	0.005	∞
ME 4Q-expected policy rate	$4E_t\left[R^f_{t,t+4}\right]$	$\mathbb{R}^+$	InvGam	0.005	∞

Table I: Initial prior distribution. Para(1) and Para(2) correspond to means and standard deviations for the Beta, Gamma, Inverted Gamma, and Normal distributions and to the lower and upper bounds for the Uniform distribution.

Table I summarizes the initial prior distributions of the remaining parameters. While the prior distributions for most of the parameters are chosen following the literature, it is noteworthy to highlight some deviations. First, we do not use a prior for the preference parameters, γ and α_{EZ}, directly, but rather impose priors for the intertemporal elasticity of substitution, IES, and the coefficient relative risk aversion, RRA, and solve for the underlying parameters. The intertemporal elasticity of substitution, IES, in our model with external habit formation is

$$IES = \frac{1}{\gamma}\left[1 - \frac{b}{\exp\left(\bar{z}^{+}\right)}\right] \tag{D-7}$$

To maintain the macroeconomic fit of the model, we have to ensure that the IES is below one, with the prior being a beta distribution. We follow Swanson (2012) by using his closed-form expressions for risk aversion, RRA, which takes into account that households can vary their labor supply. Hence, our model implies

$$RRA = \frac{\gamma}{1 - \frac{b}{\exp(\bar{z}^{+})} + \frac{\gamma}{\chi}\left(1-\bar{l}\right)\frac{\bar{w}}{\bar{c}}} + \alpha_{EZ}\frac{1-\gamma}{1 - \frac{b}{\exp(\bar{z}^{+})} - \left(1 - \frac{b}{\exp(\bar{z}^{+})}\right)^{\gamma}\bar{c}^{\gamma-1} + \frac{\bar{w}\left(1-\bar{l}\right)}{\bar{c}}\frac{1-\gamma}{1-\chi}} \tag{D-8}$$

where $\bar{l}$ is the steady state labor supply, while $\bar{c}$ and $\bar{w}$ are consumption and the real wage in the deterministic steady state, respectively. Given the wide range of different estimates for relative risk aversion in the macro- and finance literatures, we initially assume a uniform prior with support over the interval 0 to 2000; our endogenous prior approach, however, does impose an informative prior. We proceed analogously for the deterministic steady state of inflation and choose an uninformative initial prior distribution. Finally, we add measurement errors to the 1-year, 2-year, 3-year, 5-year, and 10-year Treasury bond yields as well as to the expected policy rate expected 1 and 4-quarters ahead. By adding measurement errors along the yield curve, we are following the empirical term structure literature (see, for example, Diebold et al., 2006) and the measurement errors on the expectations of the short rate align the imperfect fit of the data with the model's rational expectation assumption.

5. SUPPLEMENTARY RESULTS

5.1 Initial Prior vs Posterior Plots

5.2 Predicted Moments

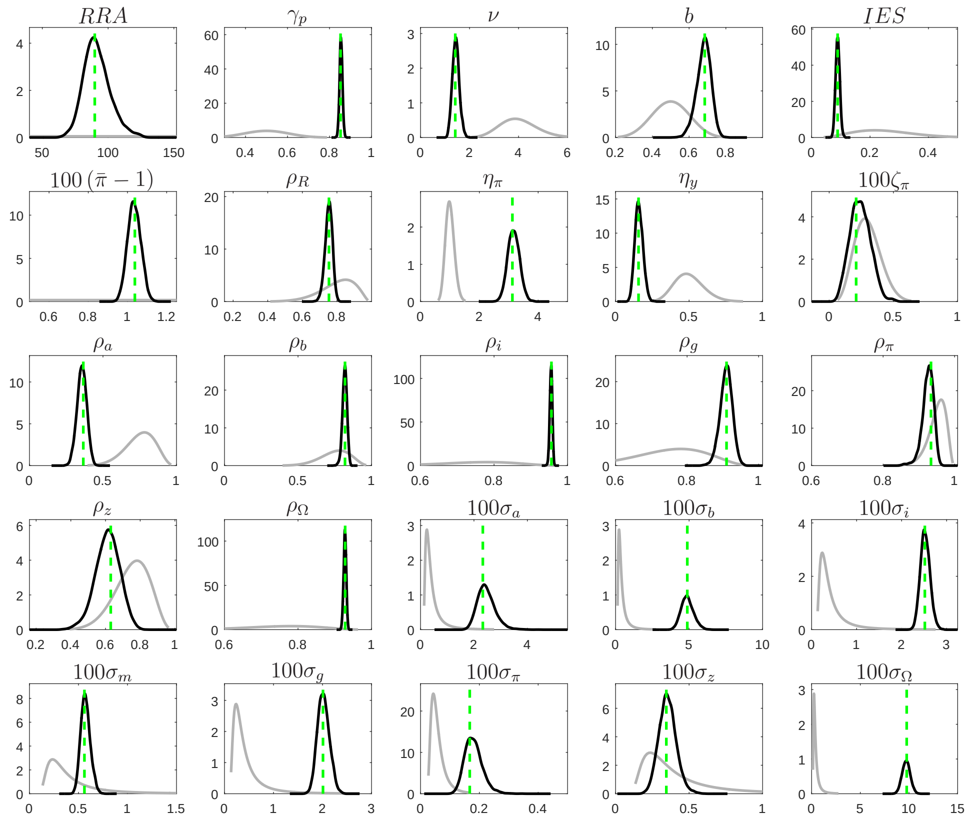


Figure 1: Prior (gray) and posterior (black) distribution of the model parameters, the green dashed line indicates the posterior mode.

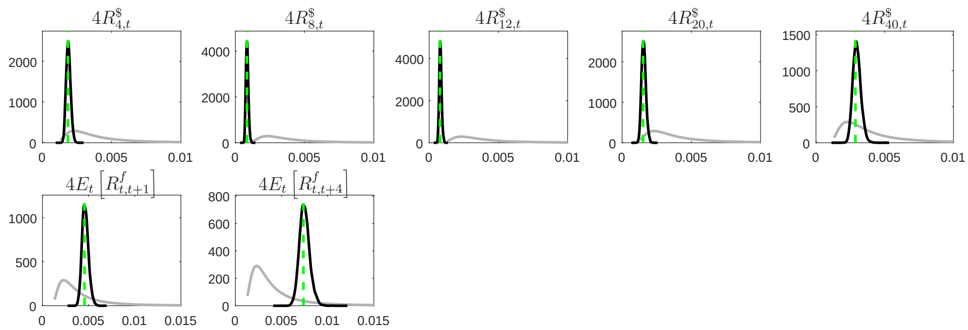


Figure 2: Prior (gray) and posterior (black) distribution of measurement errors, the green dashed line indicates the posterior mode.

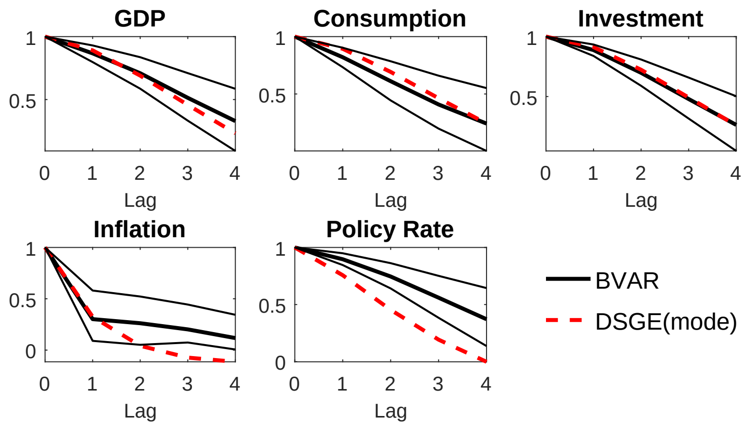


Figure 3: Predicted autocorrelation of selected HP-filtered macro variables at the posterior mode and the corresponding population moments of the data calculated by using a Bayesian vector autoregression model with two lags. The thin black lines represent the 90% probability bands.

Name	Symbol	Mean 50%	Mean 5%	Mean 95%	S.d. 50%	S.d. 5%	S.d. 95%
1-year real T-Bill	$R_{4,t}$	2.68	1.09	4.26	1.81	1.28	2.62
2-year real T-Bill	$R_{8,t}$	3.00	1.58	4.42	1.53	1.04	2.29
3-year real T-Bill	$R_{12,t}$	3.17	1.90	4.43	1.35	0.91	2.04
5-year real T-Bill	$R_{20,t}$	3.33	2.31	4.34	1.10	0.74	1.66
10-year real T-Bill	$R_{40,t}$	3.73	3.05	4.43	0.72	0.48	1.08
1-year nominal term premium	$TP^{\$}_{4,t}$	37.36	27.93	46.96	8.83	5.92	13.39
2-year nominal term premium	$TP^{\$}_{8,t}$	77.14	55.92	98.46	19.55	13.16	29.58
3-year nominal term premium	$TP^{\$}_{12,t}$	99.69	71.02	128.40	25.64	17.25	38.89
5-year nominal term premium	$TP^{\$}_{20,t}$	129.06	91.12	167.03	31.37	20.90	48.30
10-year nominal term premium	$TP^{\$}_{40,t}$	202.69	148.52	256.89	37.85	24.37	60.81
1-year real term premium	$TP_{4,t}$	23.95	18.76	29.57	4.82	3.21	7.36
2-year real term premium	$TP_{8,t}$	56.96	42.78	71.59	13.22	8.86	20.04
3-year real term premium	$TP_{12,t}$	74.54	54.60	94.85	18.10	12.17	27.40
5-year real term premium	$TP_{20,t}$	93.09	66.69	119.70	22.09	14.80	33.75
10-year real term premium	$TP_{40,t}$	138.88	101.06	176.17	26.26	17.08	41.68
1-year inflation risk premium	$TP^{\pi}_{4,t}$	13.34	8.77	18.03	4.14	2.78	6.30
2-year inflation risk premium	$TP^{\pi}_{8,t}$	20.03	12.52	27.98	6.47	4.30	9.93
3-year inflation risk premium	$TP^{\pi}_{12,t}$	24.93	15.50	35.19	7.67	5.04	11.97
5-year inflation risk premium	$TP^{\pi}_{20,t}$	35.68	22.99	49.76	9.37	6.01	15.02
10-year inflation risk premium	$TP^{\pi}_{40,t}$	63.46	44.89	84.47	11.61	7.17	19.44

Table II: Simulated moments of further financial variables.

Note: The simulated moments are based on 1200 parameter vector draws from the posterior. For each draw, we simulate 1000 time series for each variable of interest. After removing a burn-in of 5000 periods for each simulation the final simulated time series have the same length (T=100) as the vector of observables. The number in brackets indicate 5% and 95% probabilities. All returns are measured in annualized percentage points and all risk premia are measured in annualized basis points.

5.3 Risk-Adjusted Impulse Responses versus Generalized Impulse Responses

Here, we compare our impulse responses using the solution method of Meyer-Gohde (2016) with generalized impulse responses from a standard nonlinear solution method (see Koop et al. (1996), Andreasen et al. (2018)). We use our posterior mean parameters and compute a standard third order perturbation of our model. The generalized impulse response of a variable y^j_{t+s} to a shock ε^i_t is given by

$$GIRF(s, \omega, y_{t-1}) = E\left[y^j_{t+s}|y_{t-1}, \varepsilon^i_t = \omega\right] - E\left[y^j_{t+s}|y_{t-1}\right] \tag{E-1}$$

To calculate the impulse responses, we run 10,000 simulations of 5,040 periods each for the third order perturbation, where an impulse ω occurs at period 5,001.[6] We start the simulations from the deterministic steady state and then discard the first 5,000 periods so that the simulated values will likely have converged to the ergodic distribution. The average value over all the simulations, as well as the 90% and 68% coverage of the simulations can be found in figure 4.

The figure also contains impulse responses from standard linear approximations around the deterministic steady state. Whereas both the generalized impulse response and the impulses calculated from the risk adjusted linear approximation are in deviations from the ergodic mean, the standard linear approximation returns impulses in deviations from the deterministic steady state. While it is tempting to look for the term premia to span the distance between our risk-adjusted and a standard linear approximation for bond yields, the different points of approximation that encompass covariance terms and the like preclude this.

As can clearly be seen in the figure, our risk adjusted linear approximated model is very successful in capturing the effects of monetary policy changes that a fully nonlinear approximation would predict. In contrast to the standard linear approximation, the nonlinearity in risk captured by the method we use captures the effects on term premia. Conspicuously, the forward guidance experiment from the main paper is missing here. Both this and the estimation of our model would be nontrivial tasks for a standard nonlinear approximation. Thus, we conclude that the gains from maintaining linearity in states by using the risk adjusted linear approximation outweigh the costs of apparently small accuracy loses.

Our approximation is noncertainty equivalent despite its linearity in states; i.e., the underlying risk in the economy affects the predicted response to any shock. To illustrate this, figure 5 contains the impact responses of the yield curve and components to monetary policy shocks (1) at our posterior mean estimates and (2) at our posterior mean estimates with the variance of all other shocks set to zero. Additionally, the impact responses of the standard deterministic linear approximation are also plotted.

Under the standard linear approximation at the deterministic steady state, the impulse response functions are invariant to the volatility of shocks. Under the risk adjusted solution, they differ significantly due to the risk dependence of the solution. This also underlines why having a rich, estimated stochastic environment is essential even to analyses focusing on a single aspect of the macroeconomy (say, monetary policy) in the absence of certainty equivalence.

[6]To maximize comparability with the main text, we ensure that the average impulse leads to a 50 basis point drop in the policy rate on impact. Due to the nonlinearity in states of the third order perturbation, we cannot simply scale the impulse responses, but must solve a fixed point problem to recover the ω that leads to this 50 basis point drop.

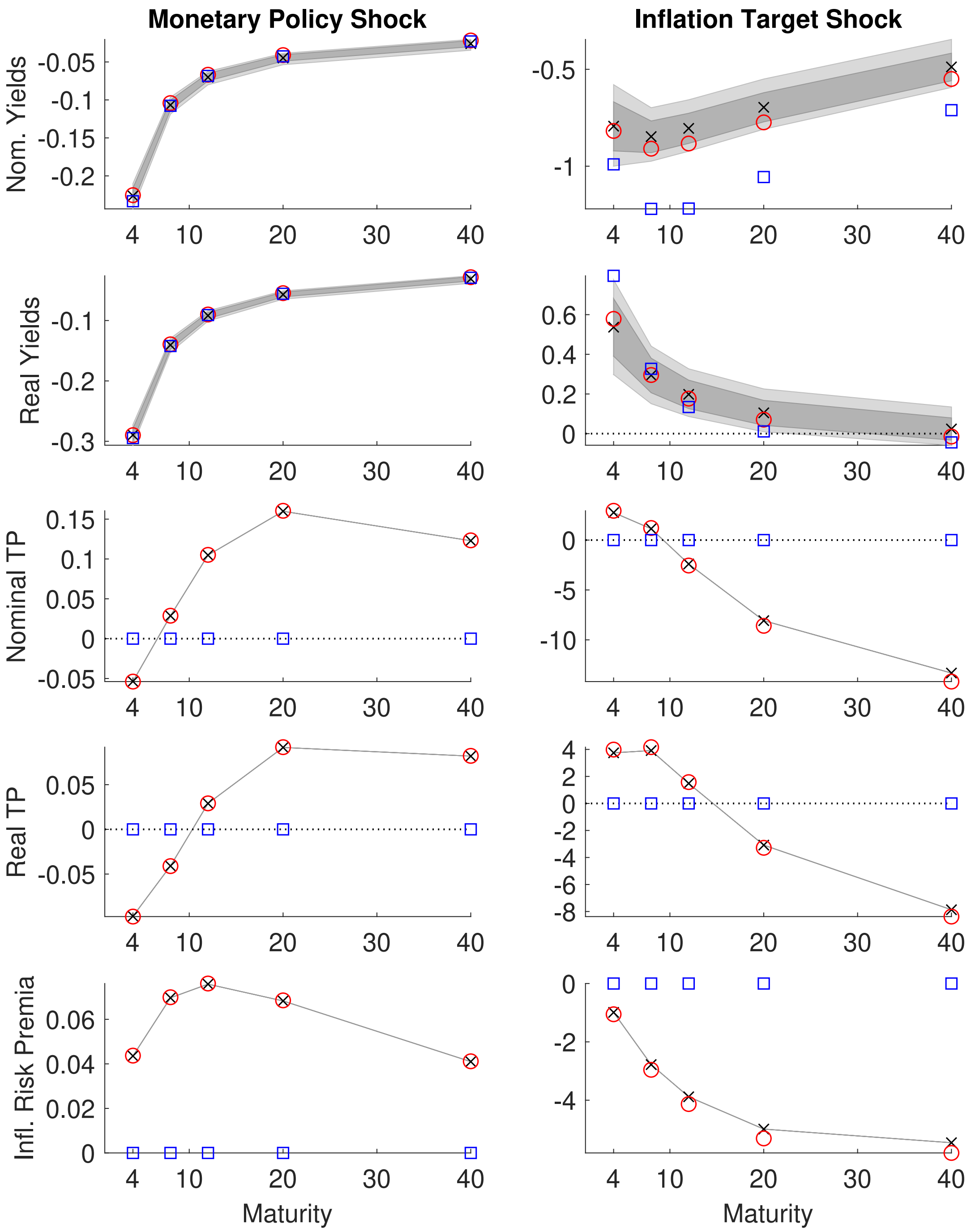


Figure 4: Solution method and impact responses of nominal and real term structures.

Note: The figure shows the impact response across all maturities to a surprise 50 basis point policy rate cut and a surprise cut in the inflation target leading to a 50 basis point policy rate cut. The deviations of yields are in percentage points while the deviations of risk premia are presented in basis points. The black crosses (median) and shaded areas (90% and 68% coverage) give generalized impulse responses calculated with a full third order perturbation at our posterior mode. The red circles give the responses from the risk-adjusted linear approximation. The blue squares give the responses from a standard linear approximation.

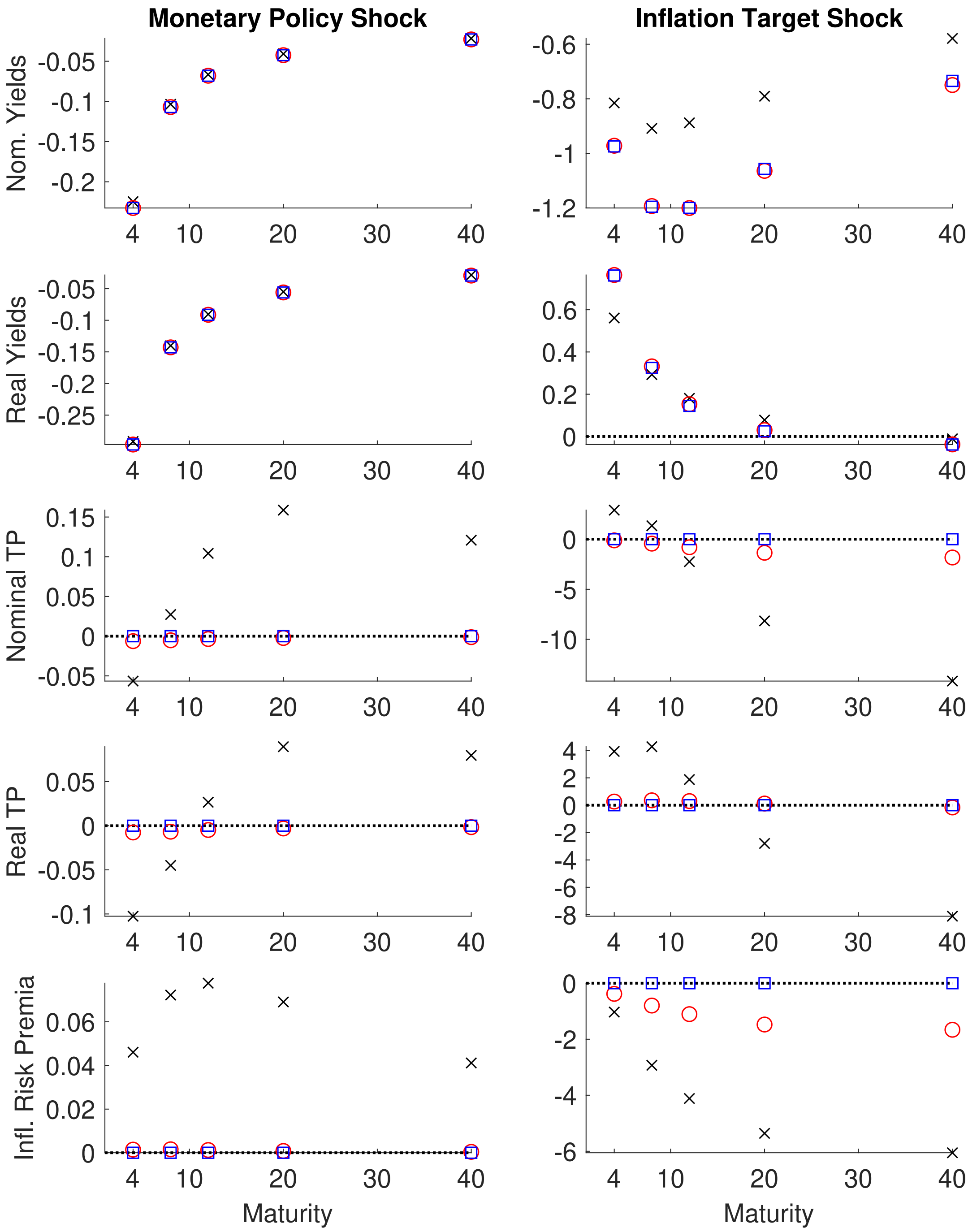


Figure 5: Risk dependence of impact responses of nominal and real term structures.

Note: The figure shows the impact response across all maturities to a surprise 50 basis point policy rate cut and a surprise cut in the inflation target leading to a 50 basis point policy rate cut. The deviations of yields are in percentage points while the deviations of risk premia are presented in basis points. The black crosses give the responses at our posterior mode from the risk-adjusted linear approximation. The red circles give the responses from the risk-adjusted linear approximation with the variances of all other shocks set to zero. The blue squares give the responses from a standard linear approximation.

5.4 Extending the Sample until 2019Q4

In this subsection, we present the results, and particularly the model implied term premium, that we obtain by the sample out to 2019:Q4. In this extended sample, we ignore the lower zero bound, both from a modelling perspective as well as from a data perspective. We decided against using the shadow short rate (see, for example, Wu and Xia, 2016) to proxy the central bank's unconventional monetary policy as the shadow rate would be not in line with, for example, the 1 and 4-quarter ahead expectations of the 3-month T-Bill from the Survey of Professional Forecasters. Accordingly, we fix the monetary policy rule parameters to the benchmark estimates, as these parameters otherwise would be either difficult to identify and/or most likely biased. Similarly, we have decided to calibrate the price stickiness and as well as the deterministic steady state of inflation, both of which seem to be difficult to identify in this extended sample. Moreover, we change the calibration of the technology and investment trends to fit the means of GDP and investment growth accordingly. While we use the same initial prior as in our benchmark estimation and follow the implementation identically, the resulting endogenous priors themselves are different due to the extended sample.

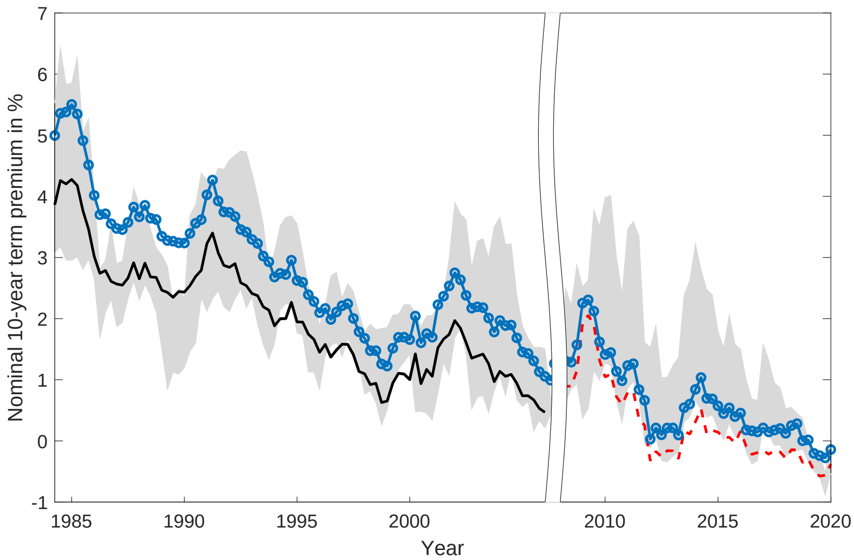


Figure 6: Model implied 10-year nominal term premia.

Note: The blue-circle line shows the model implied 10-year nominal term premium based on the estimation with the extended sample until 2019Q4. In comparison, the black solid line and the red-dashed line show implied 10-year nominal term premia based on the benchmark estimation using data until 2007Q4. Moreover, the figure displays the range of corresponding estimates in the literature (gray area) as described in more detail in the caption of figure 1 in the main text.

Figure 6 shows again like figure 1 in the main text the model implied 10-year nominal term premium out to 2019Q4 with the parameter estimates from the benchmark sample until 2007Q4. Additionally, figure 6 shows the same term premium with the model parameters estimated using the extended sample out to 2019Q4. Moreover, we compare the model implied term premium with those of the empirical literature (Kim and Wright, 2005, Adrian et al., 2013, among others). Both estimates show the same pattern and are highly comparable with the empirical estimates. This gives us confidence regarding our estimations of the term premium and, in particular, those based on our benchmark sample until 2007q4. Despite the fact that the model implied term premia are very similar, our model misses different channels for unconventional monetary policy (e.g. portfolio balance effect, signaling effects) as well as the ZLB, all of which can have significant effects on term premia as shown by the recent empirical literature (Swanson and Williams, 2014, Swanson, 2020). While these channels are obviously missing in our model, the remaining channels may have captured their effects, potentially resulting in biased parameter estimates. Table III shows the parameter estimates for the extended sample. There are some differences to the corresponding table in the main text, which are worth discussing in more detail. First, the relative risk aversion and the persistence of the inflation target shock become smaller, both have been highlighted as key features in the literature to achieve high and volatile nominal term premia in DSGE models. So for the extended sample, other channels must play a more amplified role. Overall, the standard deviations of the shocks have increased, which points to a bigger overall level of and role for risk and uncertainty in the model. Particularly, the increase in the standard deviation of shocks to and decrease in the persistence of the inflation target could be interpreted as a monetary policy that is expected to be less predictable and more uncertain. A finding, which in our view is strongly related to the inadequate structural covering of monetary policy during the last third of the sample. As a consequence, we consider our model as being best suited for the sample period between 1984 and 2008. To cover the recent period with ZLB and unconventional monetary appropriately, we would have to incorporate several model features which is beyond the scope of the present paper. Certainly, the investigation of the effects of different unconventional monetary policy tools within a structural model is an interesting avenue for further research.

Name	Symbol	Mode	Mean	5%	95%
Relative risk aversion	RRA	38.325	39.580	30.057	48.934
Investment adjustment	ν	1.390	1.408	1.222	1.594
Habit formation	b	0.561	0.545	0.466	0.628
Intertemporal elas. substitution	IES	0.104	0.104	0.094	0.114
Inflation target coefficient	$100\zeta_\pi$	0.248	0.267	0.115	0.410
AR coefficient technology	ρ_a	0.517	0.508	0.448	0.565
AR coefficient preference	ρ_b	0.899	0.899	0.886	0.913
AR coefficient investment	ρ_i	0.954	0.952	0.947	0.958
AR coefficient gov. spending	ρ_g	0.943	0.942	0.927	0.958
AR coefficient inflation target	ρ_π	0.698	0.699	0.661	0.731
AR coefficient long-run growth	ρ_z	0.644	0.618	0.503	0.726
AR coefficient fixed cost	ρ_Ω	0.957	0.957	0.954	0.960
S.d. technology	$100\sigma_a$	1.519	1.541	1.349	1.733
S.d. preference	$100\sigma_b$	5.999	5.912	5.236	6.565
S.d. investment	$100\sigma_i$	2.767	2.746	2.575	2.926
S.d. monetary policy shock	$100\sigma_m$	0.486	0.493	0.426	0.559
S.d. government spending	$100\sigma_g$	2.365	2.368	2.211	2.539
S.d. inflation target	$100\sigma_\pi$	0.646	0.643	0.564	0.718
S.d. long-run growth	$100\sigma_z$	0.334	0.349	0.266	0.432
S.d. fixed cost	$100\sigma_\Omega$	10.103	10.038	9.338	10.685
ME 1-year T-Bill	$400R^{\$}_{4,t}$	0.165	0.166	0.146	0.186
ME 2-year T-Bill	$400R^{\$}_{8,t}$	0.081	0.082	0.069	0.095
ME 3-year T-Bill	$400R^{\$}_{12,t}$	0.075	0.077	0.065	0.088
ME 5-year T-Bill	$400R^{\$}_{20,t}$	0.154	0.157	0.134	0.180
ME 10-year T-Bill	$400R^{\$}_{40,t}$	0.309	0.314	0.272	0.355
ME 1Q-expected policy rate	$400E_t\left[R^f_{t,t+1}\right]$	0.416	0.418	0.375	0.461
ME 4Q-expected policy rate	$400E_t\left[R^f_{t,t+4}\right]$	0.680	0.687	0.621	0.757

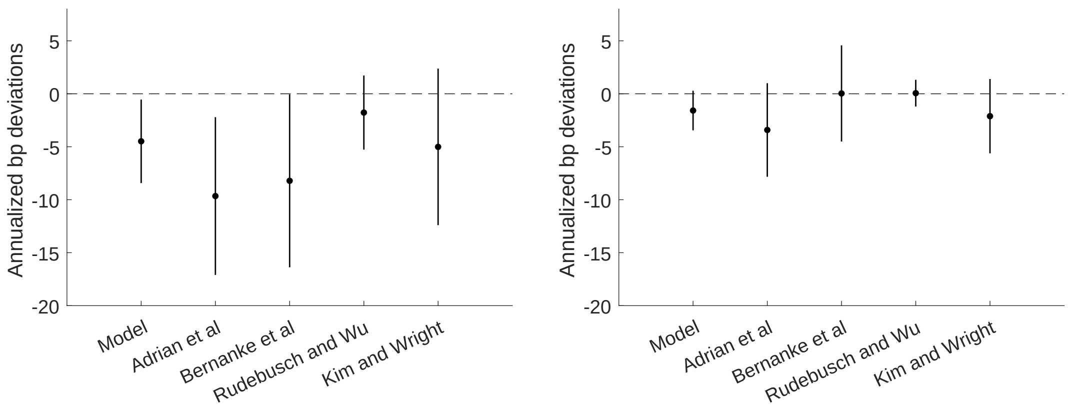
Table III: Posterior statistics. Posterior means and parameter distributions are based on a standard MCMC algorithm with two chains of 50,000 parameter vector draws each, 50% of the draws used for burn-in, and a draw acceptance rates about 1/3.

5.5 Empirical evidence

In this subsection, we compare the impulse responses from our structural model with those from the empirical literature in greater detail. In particular, we apply a linear local projection following Jordà (2005). Our model setup is very flexible and encompasses the commonly used linear projections in the empirical literature (e.g. Hanson and Stein, 2015, Nakamura and Steinsson, 2018, Crump et al., 2016). The linear model is given as follows

$$x_{t+h} = \alpha_h + \psi_h(L)\, z_{t-1} + \beta_h shock_t + \varepsilon_{t+h} \quad \text{for} \quad h = 0, 1, 2 \ldots, \tag{F-1}$$

where x is the variable of interest, z a vector of control variables, $\psi_h(L)$ a polynomial in the lag operator, and *shock* the identified monetary policy shock. In our applications, $\psi_h(L)$ is a polynomial of order 2, the vector of controls z comprise GDP growth and inflation along with the variable of interest and the identified shock (see, for example, Stock and Watson, 2018). Finally, the variables of interest x are nominal yields and nominal term premia with a maturity between 4 and 40 quarters. Throughout the paper, we use Newey-West standard errors to account for autocorrelation and heteroscedasticity. We use a lag truncation parameter of 2 which is larger than the local projection horizon $h = 0$. Accordingly, figure 6 in the main text presents the results for $h = 0$ of this local linear projection. For comparison, we scaled all results so that the median response of the 2-year bond is equal to 0.1 annualized percentage points. The results are similar for the model implied historical term premia as well as for the estimates from Adrian et al. (2013). The left panel of figure 7 extends this result using alternative measures of the 10-year nominal term premium from the literature. As the available sample differs in length among the estimates, figure 7 shows the results for 1984:Q1-2005:Q4, while figure 6 in the main text is based on our full sample 1983:Q1-2007:Q4.



(a) monetary policy measure: Model

(b) monetary policy measure: Romer and Romer

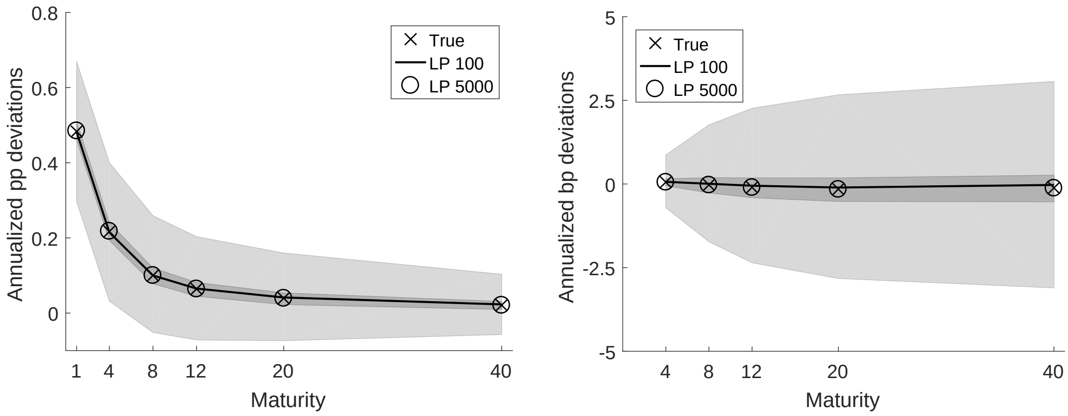
Figure 7: Impact effect of monetary policy shock on 10-year nominal term premia.

Note: The dots and vertical lines show median response and 95% confidence bands from the local projection for different historical 10-year nominal term premia as dependent variable, respectively. We use the Newey-West correction for the standard errors.

In the following, we perform a Monte-Carlo exercise to evaluate the small sample properties of the linear projection estimator. At the posterior mean, we simulate 1,0000 time series with a length of 10,000 for all variables of interest, control variables, and monetary policy shocks from the model. After discarding the first 5,000 observations, we run two sets of local linear projections with a sample length of 100 and 5,000 respectively. Figure 8 presents the results. On average, both linear projections deliver estimates close to the true, theoretical response and, therefore, show no systematic small sample bias (Jordà, 2005). However, the Monte-Carlo exercise shows a high estimation uncertainty in small samples, consistent with the wide range of quantitatively and qualitatively different estimates in the empirical literature.

5.6 Impulse response functions

The three columns in figure 9 contain the IRFs of macroeconomic variables to a surprise shock to the policy rate (left column), to a surprise inflation target shock (middle column), and to a



(a) Nominal yield curve (b) Nominal term premium

Figure 8: Monte-carlo experiment

Note: Impact effect of monetary policy shock on nominal yields and nominal term premia for different maturities. The dark gray area and light gray area represent the 95% confidence interval from the local projection with sample length 5000 and 100, respectively. The circles and solid line are the corresponding median responses while the crosses present theoretical response. All results are based on 1000 simulations at the posterior mean.

four-quarter ahead forward guidance shock (right column). All shocks are normalized to yield a median lowering of the policy rate by 50 basis points on impact (or in four quarters for the forward guidance shock).

The responses of the macroeconomy to the surprise policy rate shock are contained in first column of figure 9. As is standard in the literature, the expansionary policy due to surprise policy rate cut (left column of figure 9) leads to an increase in aggregate demand and its components as well as inflation. As the policy rate begins to return to its mean level with inflation still elevated, the resulting increase in expected real rates reverses the expansion, depressing aggregate demand and its components, before the macroeconomy then settles back to its mean position after around 10 quarters.

The middle column of figure 9 shows the impulse responses to a surprise inflation target shock. The reduction in the inflation target is accompanied with a nearly two annualized percentage point reduction in inflation, roughly the same magnitude as the reduction of the target, which corresponds to a substantial change in the systematic behavior of monetary policy. The lowering of the policy rate is hump shaped with the maximal decrease of about 110 annualized basis points occurring about a year after the lowering of the inflation target. This lowering of the policy rate is not sufficient to overcome the initial contractionary effects of the lowered inflation target and associated disinflation as can be seen by the negative responses on aggregate demand. Moreover, our results illustrate that a shock to the inflation target is much more long lasting and therefore has stronger effects on business cycle and lower frequencies, in contrast to a simple innovation to the Taylor-rule which quickly dissipates. This confirms the interpretation of Rudebusch and Swanson (2012) that a change in the inflation target, or more generally a change in the systematic behavior of monetary policy, introduces long-run nominal risk into the economy.

The right column in figure 9 shows the evolution of macroeconomic variables following the forward guidance experiment. Similarly to most studies, we find that forward guidance increases macroeconomic activity and substantially increases inflation. Output and inflation both increase on impact with output reaching its peak after 3 quarters and falling slightly below its mean value after 12 quarters. The response to the announcement is driven by expectations of lower nominal short term interest rates and of future inflation. Expected higher inflation leads to a rise in current inflation through forward looking price setting, with a consequential fall in current and expected

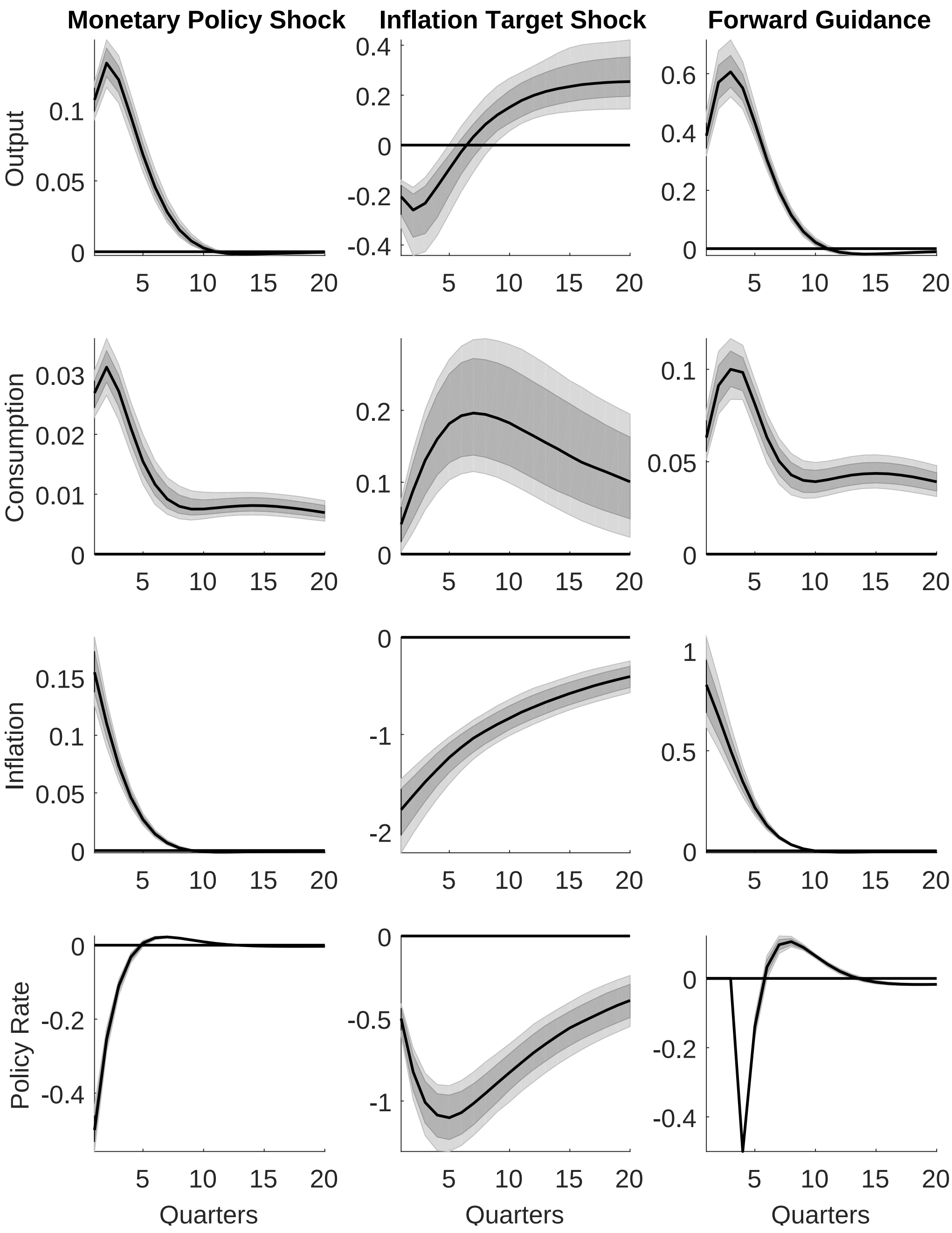


Figure 9: Posterior impulse responses of macro variables

Note: The figure shows a surprise 50 basis point policy rate cut, a surprise cut in the inflation target leading to a 50 basis point policy rate cut, and forward guidance of a 50 basis point policy rate cut in 4 quarters. The deviations of yields are in percentage points while the deviations of risk premia are presented in basis points. Shaded areas represent the 90% and 68% posterior credible sets.

real interest rates and associated increase in economic activity on impact. Therefore, comparable to a change in the inflation target, forward guidance communicates the central bank's commitment to allow higher inflation in the future, which has more stronger and more long lasting effects on households' expectation and so on their precautionary savings motives.

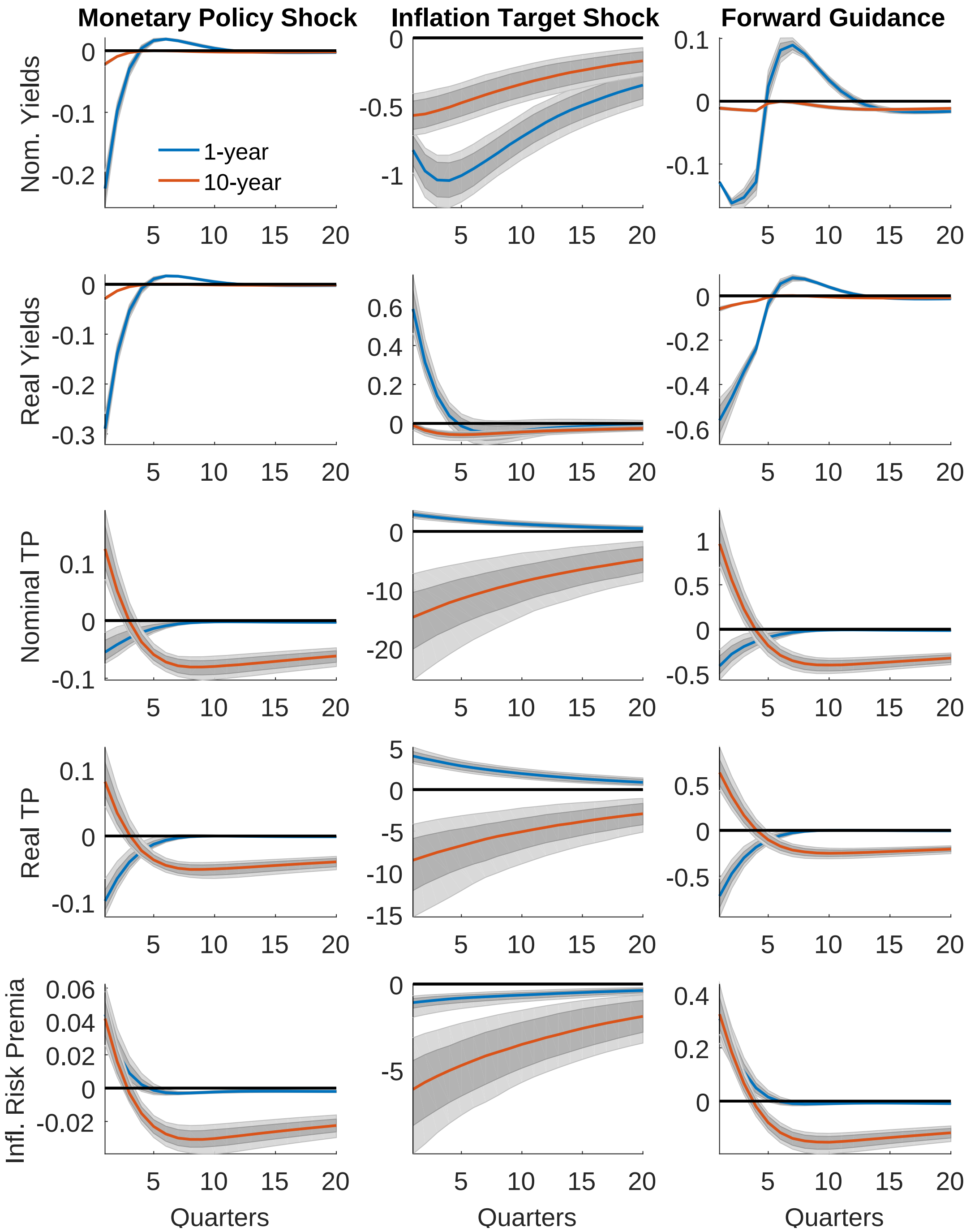


Figure 10: Posterior impulse responses of nominal and real term structure at the short and long end.

Note: The figure shows a surprise 50 basis point policy rate cut, a surprise cut in the inflation target leading to a 50 basis point policy rate cut, and forward guidance of a 50 basis point policy rate cut in 4 quarters. The deviations of yields are in percentage points while the deviations of risk premia are presented in basis points. Shaded areas represent the 90% and 68% posterior credible sets.

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