Discussion of 'Design Limits and Dynamic Policy Analysis ' by Brock, Durlauf, Rondina (BDR)

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Authors' Research Agenda

- The paper is part of a broader agenda:
  - ➔Brock, Durlauf, West (2003), Policy Evaluation in Uncertain Econ. Environments, Brookings Papers.
  - ➔Brock, Durlauf, West (2004), Model Uncertainty and Policy Evaluation: Theory and Empirics.
  - ➔Brock and Durlauf (2004), Elements of a Theory of Design Limits to Optimal Policy, Manchester School.
  - ➔ Brock, Durlauf (2005), Local Robustness Analysis: Theory and Application, JEDC.

Authors' Research Agenda

- Issues on their agenda.
  - ➔Policy under uncertainty.
  - ➔Bayesian model averaging.
  - ➔Robust control.
  - ➔Limits on policy design in the frequency domain.
  - ➔Tradeoffs in the frequency domain.

Frequency Domain Analysis

- Economists typically less familiar with frequency than time domain (not so engineers).
  - ➔Users in economics face somewhat of a hurdle in popularizing applications of their methods.
  - ➔Advanced macro textbooks discuss frequency domain less today than 20 years ago.

Frequency Domain Analysis

- ❐ But, resurgence in the context of robust control (see Hansen and Sargent).
- ❐ Continued useful imports from control engineering into economics.
- ❐ Brock and Durlauf propose design limits as a new import and emphasize usefulness of frequency domain analysis.

Brock, Durlauf and Rondina (2006)

- ❐ BDR 2006 extends JEDC and Manchester School papers.
  - ➔These papers study design limits and their implications for robust policy in single-input / single-output models.
  - ➔SISO:1 policy variable, 1 target variable, scalar ARMA processes + feedback control.
- ❐ This paper focuses on design limits for optimal and simple policy rules in more complicated models.
  - ➔robustness issues not considered here, (possible extension).

Design Limits: The Main Point

1. Model effect of alternative control rules on state variable in frequency domain.
2. Identify how different control rules affect the contributions of fluctuations at each frequency to the overall variance.
3. Characterize the tradeoffs that exist between diminishing the variance contribution of one frequency and another (Bode constraints, not yet exploited in economics).

Design Limits: The Main Point Cont.

4. Design limits imply that policy rules which are effective at reducing low (high) frequency fluctuations inevitably increase high (low) frequency fluctuations.
5. BDR argue that these tradeoffs imply that any ordering of policy rules must carefully account for how policymakers assess frequency-specific components of fluctuations for the variables of interest.

Contributions of BDR (2006)

- Extend derivation of design limits to
  - ➔ MIMO systems: multiple input and multiple output, (more precisely, DIDO)
  - ➔ SIMO: single input and multiple output (more relevant to stabilization policy: one policy variable (i), two targets (y, π).)
- Extend analysis to models with forward-looking, rational expectations.
- Present applications to New-Keynesian model and to Svensson-Rudebusch model with backward-looking expectations.

Design Limits for Beginners

- Back to single input, single output, scalar model without forward-looking expectations, drawing on ‚Elements of ...' paper (example 3.1. in BDR 2006).
- **Law of motion** for *x* with control *u* and zero-mean unobserved random variable ξ, moving-average of innovations, *v:*

$$x_t = A(L)x_{t-1} + bu_{t-1} + \xi_t$$

where $\xi_t = W(L)v_t$

Law of Motion and Feedback Rule

- Feedback rule for control *u*

$$u_{t-1} = -F(L)x_{t-1}$$

Note -F = U in BDR 2006.

- Substitute in law of motion:

$$x_t = A(L)x_{t-1} - bF(L)x_{t-1} + W(L)v_t = \frac{W(L)}{1 - LA(L) + bLF(L)}v_t$$

- u=0 defines free dynamics:

$$x_t^{NC} = A(L)x_{t-1}^{NC} + \xi_t$$

Variance and Frequency Domain

- Fourier transform of lag polynomial C(L)

$$C(\omega) = \sum_{j=-\infty}^{\infty} c_j e^{-ij\omega}$$

- Variance of state $E(x^2)$ expressed as

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{|W(\omega)|^2 \sigma_v^2}{|e^{i\omega} - A(\omega) + bF(\omega)|^2} d\omega = \int_{-\pi}^{\pi} \frac{f_\xi(\omega)}{|e^{i\omega} - A(\omega) + bF(\omega)|^2} d\omega$$

where $|C(\omega)|^2 = C(\omega)C(-\omega)$

Sensitivity Function

- Identify how the control rule affects each of the frequency specific components of Ex^2
- Multiply and divide with $\left|e^{i\omega} - A(\omega)\right|^2$

$$Ex_t^2 = \int_{-\pi}^{\pi} \frac{f_\xi(\omega)}{\left|e^{i\omega} - A(\omega)\right|^2 \left|e^{i\omega} + b\left(e^{i\omega} - A(\omega)\right)^{-1} F(\omega)\right|^2} d\omega =$$

$$\int_{-\pi}^{\pi} \frac{f_{x^{NC}}(\omega)}{\left|e^{i\omega} + b\left(e^{i\omega} - A(\omega)\right)^{-1} F(\omega)\right|^2} d\omega$$

Sensitivity Function cont.

$$Ex_t^2 = \int_{-\pi}^{\pi} \left|S(\omega)\right|^2 f_{x^{NC}}(\omega) d\omega$$

where $$S(\omega) = \frac{1}{e^{i\omega} + b\left(e^{i\omega} - A(\omega)\right)^{-1} F(\omega)}$$

and $$f_{x^{NC}}(\omega) = \frac{f_\xi(\omega)}{\left|e^{i\omega} - A(\omega)\right|^2}$$

- **The effects of a policy on a state are summarized by the sensitivity function S(ω).**

Sensitivity Function cont.

Since $E\left(x_t^{NC}\right)^2 = \int_{-\pi}^{\pi} f_{x^{NC}}(\omega) d\omega$ and $Ex_t^2 = \int_{-\pi}^{\pi} f_x(\omega) d\omega$,

S(ω) indicates how each frequency-specific component of the uncontrolled process is translated into a frequency-specific component of the controlled process.

What sorts of constraints exist on the choice of S() that may be achieved by the choice of feedback law F(L) ?

Discrete-Time Bode Constraint

- K_{bode} greater or equal 0.

$$\int_{-\pi}^{\pi} \ln\left(\left|S(\omega)\right|^2\right) d\omega = K_{Bode}$$

- Depends on model. K=0 if uncontrolled process is stable, K>0 if uncontrolled process explosive.

Results in SISO

- Impossible for $\left|S(\omega)\right| < 1 \ \forall \omega \in [-\pi,\pi]$;

  otherwise $\int_{-\pi}^{\pi} \ln\left(\left|S(\omega)\right|^2\right) d\omega$ <0.

- Impossible to design policy such that

  $$f_x(\omega) \le f_{x^{NC}}(\omega) \ \forall \omega \in [-\pi,\pi]$$

- Fundamental tradeoff!

Some Comments and Questions

- Backward-looking models: excluding explosiveness implies important frequency tradeoffs. Ok, but didn't we know that.
- Recall basic accelerationist Phillips curve:

  $$\pi_t = \pi_{t-1} + \beta y_{t-1} + \varepsilon_t$$

- To stabilize inflation need output to respond to inflation shocks. Policy tradeoff. Business cycle fluctuations in output in order to stabilize inflation.

Some Comments and Questions

- Forward-looking models: interesting that Bode constraint can go negative and improvement over uncontrolled process is feasible over all frequencies. Does that rely on commitment? What about discretionary case?

- Forward-looking models: what are the frequency-trade-offs related to excluding multiplicity of equilibria?

Some Comments and Questions

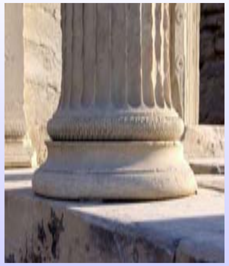
- Would be interesting to explore policy rules that focus on specific frequencies.
- For example when policy objective is defined over subset of frequencies.

  $$\left|S(\omega)\right| \le M < 1 \ \forall \omega \in [-\bar{\omega},\bar{\omega}]$$

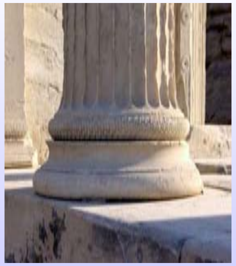
- Positive analysis in case of some central banks (see next application).
- Normative reasons would require analysis of frequency specific welfare?

And an Application!
ECB‘s Pillars and Frequency Domain

Money

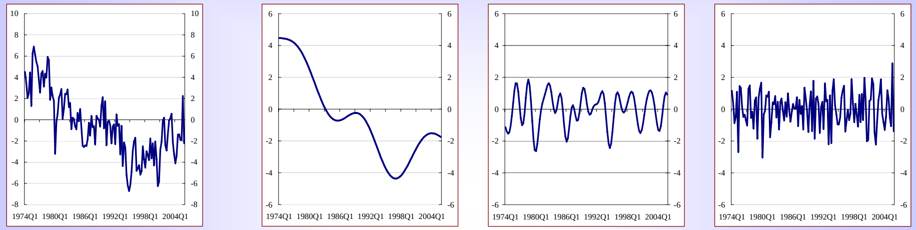


Output (gap?)



Frequency decomposition of M3 (Source ECB)

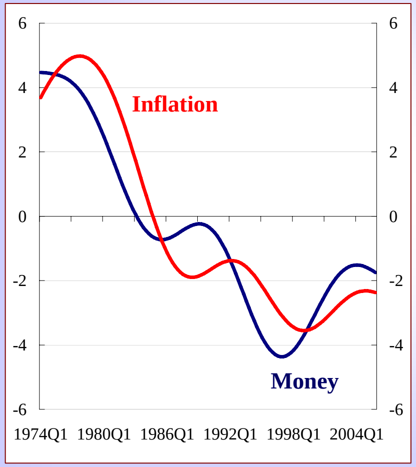
annualised quarter-on-quarter changes, deviations from mean



M3 = Low frequency component + Business cycle frequency component + High frequency component

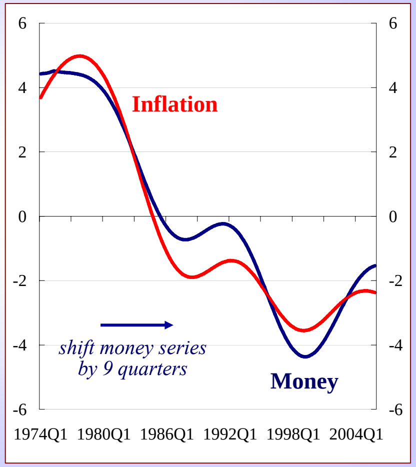
Low Frequency Money Growth leads Inflation (Source ECB)

annualised quarter-on-quarter changes, deviations from mean, periodicity > 8 years



Source: ECB calculations, based on Bruggeman, et al (2005).

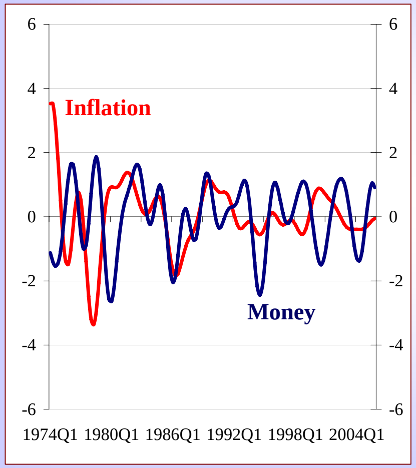
annualised quarter-on-quarter changes, deviations from mean, periodicity > 8 years



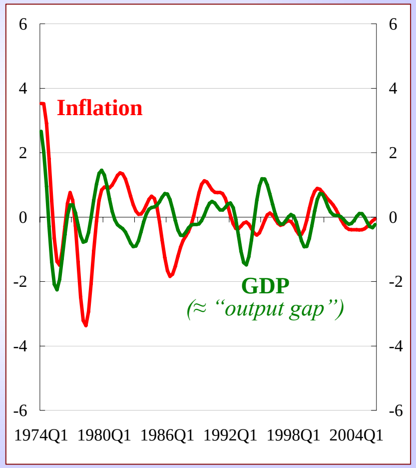
Source: ECB calculations, based on Bruggeman, et al (2005).

Output and Inflation at Business Cycle Frequency (Source ECB)

annualised quarter-on-quarter changes, deviations from mean, 2 years < periodicity < 8 years



Source: ECB calculations, based on Bruggeman, et al (2005).



Source: ECB calculations.

Frequencies and Feedback Law

- Consider a monetarist confident about causality but very sceptical about output gap measures:
  - ➔Leans towards pinning down low-frequency fluctuations as signaled by money growth and leaves output gap alone.

Frequencies and Feedback Law Cont.

- Consider a Keynesian confident on causality and sceptical about information in money:
  - ➔Accelerationist Phillips curve requires a feedback law that induces output fluctuations to offset inflation shocks (pin down inflation).
  - ➔If you also care about output volatility (flexible inflation targeting) then only partially offset inflation shocks and observe correlation at business cycle frequency.

Field of Study for Frequency Domain Analysis

- What are good rules to target low frequencies?
- How to adjust these rules for uncertainties for example
  - ➔about output gap measures,
  - ➔ or about causality,
  - ➔or about link between money and inflation?