Learning, Endogenous Indexation, and Disinflation in the New-Keynesian Model

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Developing a better understanding of the costs of disinflation has long been an important objective for macroeconomic research. Since the 1980s, disinflation episodes and strategies have been studied extensively under the assumption of rational expectations. This assumption implies that central bank announcements regarding future policy plans can help achieve disinflation at little or no cost in terms of lost output in spite of the presence of price level rigidity. Many researchers consider this prediction too optimistic in light of historical experience. Thus, most models used for policy analysis today combine the rational expectations assumption with additional frictions that increase the cost of disinflation, such as exogenous backward-looking indexation of wages and producer prices.

The success of many inflation-targeting countries in lowering inflation in the 1990s provides a new set of case studies that can improve our understanding of inflation-output tradeoffs and serve as a testing ground for macroeconomic modeling. These experiences can serve as the basis for evaluating departures from the benchmark New-Keynesian model with rational expectations and exogenous indexation and investigate the desirability of alternative policy strategies. Chile, which in 1990 became the second country to adopt inflation targeting, constitutes a particularly interesting example as an increasing number of developing economies opt for inflation targeting. The Chilean

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disinflation episode stands out as a very gradual disinflation achieved with temporary annual inflation targets.

In light of the Chilean experience, this paper examines the implications of two departures from the benchmark New-Keynesian model. First, I follow the recent literature on adaptive learning and replace the assumption of rational expectations with recursive least squares learning. Second, I introduce endogenous indexation by allowing firms to choose between backward-looking indexation and the central bank's announced target. At the start of the disinflation episode, indexation is complete and price-setters expect highly persistent inflation. As price-setting firms learn over time, they reassess the likelihood of announced inflation targets and adjust indexation of contracts accordingly.

The findings in this paper indicate that learning and endogenous indexation may reduce the costs of disinflation. A gradual disinflation approach can take advantage of these favorable dynamics to achieve the long-run target at lower output costs. An interesting new result is the finding that announcing and meeting annual targets for inflation results in lower disinflation costs relative to the announcement of a long-run inflation target that will only be met after many years of gradual disinflation. Model simulations of the actual targets announced by the Central Bank of Chile during the disinflation from 1990 to 2001 imply rather favorable learning and indexation dynamics.

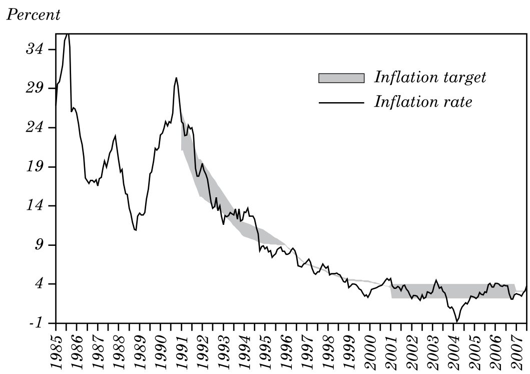
The paper proceeds as follows. Section 1 shortly summarizes several aspects of the Chilean disinflation process and the related literature. Section 2 compares traditional views with the New-Keynesian approach to understanding the costs of disinflation. In section 3, I introduce adaptive learning and endogenous indexation in the New-Keynesian model. Section 4 contrasts immediate and gradual disinflation strategies. In section 5 I formulate different sequences of annual inflation targets and evaluate their performance in implementing disinflation. Section 6 briefly discusses possible approaches to designing dynamically optimal policy, while section 7 concludes.

1. Inflation Targeting and Disinflation: Chile, 1991–2007

Inflation targeting started with public announcements of inflation targets in New Zealand and Chile in 1990. Since then, this monetary policy strategy has been implemented in many economies around the world, including developed countries such as the United Kingdom, Canada, Sweden, Norway, and Australia and an increasing number of developing countries. Many of these developing countries have been able to reduce inflation rates substantially following the adoption of inflation targeting, and they seem to have succeeded in stabilizing inflation at low to moderate rates. Fraga, Goldfajn, and Minella (2003), Corbo and Schmidt-Hebbel (2003), Corbo, Landerretche, and Schmidt-Hebbel (2002), and Mishkin and Schmidt-Hebbel (2001) provide empirical assessments of the performance of inflation targeting in a large number of diverse economies.

Given the increasing popularity of inflation targeting in developing countries, any lessons for policymakers that can be derived from Chile's experience are particularly useful. The Chilean disinflation stands out as a very gradual disinflation. The Central Bank's first official target, which was publicly announced in September 1990, was a range of 15 to 20 percent for annual consumer price index (CPI) inflation between December 1990 and December 1991. From 1991 to 1999, inflation target ranges and point targets were set on an annual basis for the following calendar year. Figure 1 reports the inflation targets (shaded area) along with actual inflation (solid line).

Figure 1. Inflation Targets and Actual Inflation in Chile, 1985-2007



Source: Schmidt-Hebbel and Werner (2002) updated to 2007 by Klaus Schmidt-Hebbel, Gustavo Leyva, and Fabian Gredig from the Central Bank of Chile.

Initially, many observers were skeptical about the importance of the Chilean Central Bank's strategic framework in achieving disinflation. They attributed much of the improvement to good luck in the form of exogenous developments concerning the exchange rate and raw material prices. Calvo and Mendoza (1999), for example, wrote that "factors other than stabilization policies have played an important role in Chilean economic performance, and the dynamics exhibited by key macroeconomic aggregates can be interpreted in part as an endogenous process of adjustment triggered by exogenous shocks." However, the amazing success of the Central Bank of Chile in meeting its annual inflation targets during the disinflation phase from 1990 to 2001 and its continued ability to keep inflation close to or within the target zone of 2 to 4 percent suggest that its strategic framework played an important role.

Aguirre and Schmidt-Hebbel (2007) argue that the short-term annual targets announced during the disinflation phase were observationally equivalent to hard policy targets in full-fledged inflation-targeting regimes. They provide some evidence in favor of this view. In spite of low initial policy credibility and widespread backward-looking price indexation in goods, labor, and financial markets, disinflation was achieved at relatively low costs in terms of associated output losses. Aguirre and Schmidt-Hebbel (2007) suggest that the Central Bank was able to overcome the consequences of backward-looking price indexation and related inflation inertia, and to influence private sector inflation expectations, by pursuing a forward-looking inflation target that served as an explicit nominal anchor. Similarly, Corbo, Landerretche, and Schmidt-Hebbel (2002) draw three main lessons from Chile's experience that should be of interest to other developing countries:

> First, initial progress in reducing inflation toward the target was slow as the public was learning about the Central Bank's true commitment to attaining the target. Second, the gradual phasing in of inflation targeting contributed to declining inflation by lowering inflation expectations and changing wage and price dynamics. Third, with respect to the speed of inflation reduction, a cold-turkey approach would have resulted in a larger sacrifice ratio stemming from higher unemployment during the early years of inflation targeting, when credibility was gradually being built up.

These conclusions suggest that learning by price-setting firms and changes in the degree of backward-looking indexation regarding wages and producer prices played an important role in shaping the costs of disinflation in Chile.[1]

More recently, researchers have developed and estimated sophisticated New-Keynesian dynamic general equilibrium models for policy analysis in Chile.[2] These models share the assumptions of rational expectations and exogenous backward-looking indexation with similar models developed for industrialized economies (see Christiano, Eichenbaum, and Evans, 2005). The New-Keynesian Phillips curve embedded in these models, however, does not seem to be stable. For example, Céspedes, Ochoa, and Soto (2005) report evidence of structural change in the late 1990s. This change is exhibited in a higher weight of expected future inflation—and a correspondingly lower weight of lagged inflation—when producers set their prices. For a sample from 1990 to 2000, they estimate a degree of backward-looking indexation around 0.85, which is essentially indistinguishable from the limiting case of complete indexation. With the sample extended to 2005, however, the degree of indexation declines to around 0.66.

The remainder of this paper explores departures from the standard New-Keynesian model by allowing for adaptive learning and endogenous indexation. I investigate whether the particular choice of inflation-targeting strategy may influence the costs of disinflation by increasing the speed of learning and reducing the degree of backward-looking indexation.

2. Disinflation and the New-Keynesian Phillips Curve

It is conventional wisdom among central bankers that conducting monetary policy so as to keep inflation constant at all times will induce fluctuations in aggregate real output. Historical experience such as the 1980s Volcker disinflation in the United States suggests that a permanent reduction in the rate of inflation cannot be achieved without a temporary decline of output below the economy's potential. Such a cost of disinflation is embedded in the traditional accelerationist Phillips curve:

$$\pi_t = \pi_{t-1} + \lambda x_t. \tag{1}$$

1. See also Herrera (2002) and Lefort and Schmidt-Hebbel (2002).

2. See Caputo, Liendo, and Medina (2007), Caputo, Medina, and Soto (2006), De Gregorio and Parrado (2006), and Céspedes, Ochoa, and Soto (2005).

Here, π_t denotes the rate of inflation and x_t the output gap (that is, the deviation of actual output from the economy's potential).

A simple experiment serves to illustrate the cost of disinflation. Assume that inflation in period $t = 1$ is equal to 1 percent and that the central bank aims to achieve price stability (that is, an inflation rate of zero percent) in period $t = 2$. Such a reduction in the rate of inflation requires a negative output gap of $-1/\lambda$ percent in period $t = 1$. In the absence of any future shocks that might push inflation up or down, inflation could then be held at zero from period 2 onward by keeping the output gap closed. Thus, the cumulative output loss in absolute terms that is required to achieve a reduction in inflation of 1 percentage point corresponds to $1/\lambda$ percent of total output.

In central bank circles, the cumulative output loss associated with a permanent reduction of the inflation rate by one percent is often referred to as the sacrifice ratio. If equation (1), the accelerationist Phillips curve, is treated as a structural relationship, then the associated sacrifice ratio is constant at $1/\lambda$ and invariant to policy design. In other words, no particular strategy or announcement by the central bank could help in changing the trade-off between output and inflation or in reducing the cumulative output cost of a disinflation. Nevertheless, a central bank that cares about stabilizing output and inflation would always opt for disinflating gradually and spreading the output loss over time.

2.1 The New-Keynesian Perspective on Disinflation

The traditional Phillips curve shown above lacks microeconomic foundations. Fortunately, the New-Keynesian paradigm offers an alternative model of inflation that is consistent with optimizing behavior and rational expectations formation by households and firms. However, the basic version of the New-Keynesian model has a very controversial property. In this model, the macroeconomic policy goals of stabilizing output and inflation do not come into conflict with each other (see Walsh, 2003; Woodford, 2003). This property is often referred to as the divine coincidence. It implies that disinflation can be achieved without any reduction in aggregate output. It is somewhat surprising that a model that incorporates long-lasting nominal rigidities exhibits such a property. To understand its origins, it is helpful to reiterate the elements of the model that drive price-setting and inflation dynamics.

The model is populated by a continuum of monopolistic firms that produce differentiated goods. Importantly, these firms cannot adjust product prices freely in every period. The basic version of the model relies on the mathematically convenient mechanism for modeling price ridigity, as introduced by Calvo (1983). It implies that firms have to wait for a signal to adjust prices. They receive such a signal with probability $1 - \theta$. Every firm that receives a price-setting signal solves a dynamic optimization problem to set its price optimally, taking into account the probabilistic constraint on future price-setting opportunities. A firm j that does not receive a price-setting signal leaves its price unchanged at the zero inflation steady state. Alternatively, if the steady-state rate of inflation, π^S, differs from zero, firm j lets its price grow with that steady-state rate, that is, $P_{j,t} = (1 + \pi^S)P_{j,t-1}$. In other words, firms that are not allowed to reoptimize their price are instead assumed to index to steady-state inflation. In solving their optimization problem, firms are assumed to form rational, model-consistent expectations.

A useful feature of this model is that it can be solved without explicitly tracking the distribution of prices across firms. Aggregation and log-linear approximation deliver a well-known, simple relationship between inflation, expected future inflation, and the output gap—the New-Keynesian Phillips curve:

$$\pi_t - \pi^S = \beta E_t[\pi_{t+1} - \pi^S] + \lambda x_t \tag{2}$$

Here, the output gap, x_t, denotes the difference between actual output and the level of output that would be achieved if prices were flexible. The parameter β refers to the discount factor. The slope parameter, λ, is a function of θ and β.[3]

Again, a simple experiment serves to assess the cost of disinflation. Suppose the central bank enters period $t = 1$ with an inflation target, π^*, equal to 1 percent. Since equation (3) is linear, the steady-state rate of inflation must be equal to the central bank's target, $\pi^S = \pi^*$. In period $t = 2$, the central bank announces a new target rate of zero percent inflation. Market participants would immediately incorporate the new target in their expectations for period $t = 3$. It would imply zero inflation in steady state. As a result, inflation in period $t = 2$

3. To be precise, the baseline version of the model (see Walsh, 2003) implies that λ is determined as follows: $\lambda = (1 - \theta)(1 - \beta\theta)\theta^{-1}(\sigma + \varphi)$. Here, σ^{-1} and φ represent the constant intertemporal elasticity of consumption and labor supply elasticity, respectively.

immediately drops to the new target rate. No reduction in the output gap, x_t, is required to achieve this outcome. Disinflation is costless. It is achieved by influencing market participants' expectations.

The model's implication of costless disinflation stands in contrast to historical experience. For this reason, researchers who have estimated New-Keynesian models using data from leading industrial economies have typically assumed an additional source of price rigidity. One possible approach is to introduce firms that apply a simple rule of thumb in price setting, as in Galí and Gertler (1999). An alternative approach assumes that some firms index prices to past inflation in those periods when they cannot adjust prices optimally (Christiano, Eichenbaum, and Evans, 2005).

Backward-looking indexation has become a popular assumption embedded in many empirically estimated dynamic stochastic general equilibrium (DSGE) models used for monetary policy analysis. Firms that do not receive a Calvo-style signal to adjust prices in the current period are assumed to implement instead a pricing rule based on past inflation, that is, $P_{j,t} = (1 + \pi_{t-1})P_{j,t-1}$. The share of firms that use backward-looking indexation, denoted by κ in the following discussion, is treated as exogenous. Consequently, the log-linear approximation of the New-Keynesian Phillips curve takes the following form:

$$\pi_t - \left(\kappa\pi_{t-1} + (1-\kappa)\pi^S\right) = \beta E_t\left[\pi_{t+1} - \left(\kappa\pi_t + (1-\kappa)\pi^S\right)\right] + \lambda x_t. \tag{3}$$

The current inflation rate then depends on a weighted average of past and expected future inflation. The weight is a function of the share of firms that implement backward-looking indexation:

$$\pi_t = \frac{\kappa}{1+\beta\kappa}\pi_{t-1} + \frac{\beta}{1+\beta\kappa}E_t\left[\pi_{t+1}\right] + \frac{\lambda}{1+\beta\kappa}x_t + \frac{(1-\kappa)(1-\beta)}{1+\beta\kappa}\pi^S. \tag{4}$$

In the limiting case of complete indexation, $\kappa = 1$, the inflation equation simplifies to

$$\pi_t = \frac{1}{1+\beta}\pi_{t-1} + \frac{\beta}{1+\beta}E_t\left[\pi_{t+1}\right] + \frac{\lambda}{1+\beta}x_t. \tag{5}$$

Interestingly, with complete indexation the current inflation rate is independent of steady-state inflation, π^s.

Equation (4) has been estimated for many countries. Estimates for Chile have been obtained by Céspedes, Ochoa, and Soto (2005), Caputo, Medina, and Soto (2006), and Caputo, Liendo, and Medina (2007). Céspedes, Ochoa, and Soto (2005) took care to account for time-variation in the inflation target. In this case, the last term in equation (4) is modified to $(1 - \kappa)(1 + \beta\kappa)^{-1}(\pi_t^* - \beta\pi^*_{t+1})$. As mentioned earlier, they report evidence of structural change. For a sample from 1990 to 2000, they estimate a degree of backward-looking indexation around 0.85, which is essentially indistinguishable from the limiting case of complete indexation. With the sample extended to 2005, however, the degree of indexation declines to around 0.66.

In this paper, I relax two important assumptions of the standard model—namely, the assumption of rational expectations and the assumption of exogenous backward-looking indexation. Relaxing these assumptions is important because of the empirical evidence regarding changes in the degree of inflation persistence during and following the disinflation in Chile. The reduction in inflation persistence may well be due to changes in price setters' beliefs or changes in the degree of backward-looking indexation. I thus depart from the assumption of rational expectations by considering adaptive learning. This follows the lead of Marcet and Sargent (1989), Evans and Honkapohja (2001), Orphanides and Williams (2006a, 2006b), and Gaspar, Smets, and Vestin (2006a, 2006b). A further innovation is rendering the share of firms that implement backward-looking indexation endogenous. In particular, I allow firms to choose between the central bank's inflation target and past inflation as possible indexes. This choice of index is made according to the likelihood that the chosen index better matches the mean of the observed inflation distribution. Firms thus aim to choose the index that seems to provide a better estimate of steady-state inflation.

3. Adaptive Learning and Endogenous Indexation

As shown above, expectations play a key role in determining inflation dynamics. Since the 1980s, research on monetary policy has relied on the assumption of rational expectations and explored its implications for policy design. A drawback of the assumption of rational expectations is that it imputes an unrealistic extent of knowledge to market participants. An interesting alternative approach is adaptive or least-squares learning, which assumes that economic agents behave like econometricians in forming expectations and estimate reduced-form inflation equations. Under certain assumptions, adaptive learning may converge to rational expectations in the long run.

Following the influential contribution by Evans and Honkapohja (2001), Orphanides and Williams (2006a, 2006b) and Gaspar, Smets, and Vestin (2006a, 2006b) have studied monetary policy design with price-setting firms that form their expectations about future inflation in a least-squares fashion. Motivated by this line of research, I assume that price-setting firms estimate the following regression for inflation:

$$\pi_t = \gamma_t \pi_{t-1} + \varepsilon_t. \tag{6}$$

The parameter γ_t carries a time subscript to allow for episodes with high and low degrees of inflation persistence. I make this assumption because the model will endogenously generate a time-varying degree of inflation persistence. Incorporating this time variation in price setters' perceived inflation equation ensures that expectations formation is consistent with equilibrium outcomes. γ_t is believed to follow a random walk with the variance of innovations denoted by σ^{γ}. Recursive estimation then implies the following updating equations for the price setters' point estimate of the inflation persistence parameter, c_t, and its variance, Σ_t:

$$\begin{aligned} c_t &= c_{t-1} + (\pi_{t-1}^2 \Sigma_{t-1} + \sigma^{\varepsilon})^{-1} \Sigma_{t-1} \pi_{t-1} (\pi_t - c_{t-1}\pi_{t-1}), \\ \Sigma_t &= \Sigma_{t-1} - (\pi_{t-1}^2 \Sigma_{t-1} + \sigma^{\varepsilon})^{-1} \Sigma_{t-1}^2 \pi_{t-1}^2 + \sigma^{\gamma}. \end{aligned} \tag{7}$$

For a derivation of these updating equations using the Kalman filter, see Harvey (1992). The updating equations are also consistent with Bayes' rule under the assumption of normally distributed shocks and beliefs (see Zellner, 1971). In the adaptive learning literature, researchers typically choose from a variety of learning specifications. Branch and Evans (2006) provide a useful exposition of alternative approaches and investigate how well they fit survey expectations.

Given equations (6) and (7), the price setters' expectation of future inflation under least-squares learning, $E_t^{LS}\left[\pi_{t+1}\right]$, corresponds to

$$E_t^{LS}\left[\pi_{t+1}\right] = c_{t-1}\,\pi_t. \tag{8}$$

Here, I follow Gaspar, Smets, and Vestin (2006a, 2006b) in assuming that $E_t^{LS}\left[\pi_{t+1}\right]$ is based on the estimate c_{t-1}, which does not yet incorporate the most recent inflation observation, π_t.[4] Using equation (8) to substitute out expected future inflation in equation (4) yields the following reduced-form inflation equation:

$$\pi_t = \frac{\kappa}{1+\beta(\kappa - c_{t-1})}\pi_{t-1} + \frac{\lambda}{1+\beta(\kappa - c_{t-1})}x_t + \frac{(1-\kappa)(1-\beta)}{1+\beta(\kappa - c_{t-1})}\pi^S. \quad (9)$$

Adaptive learning in the form of the time-varying estimate, c_{t-1}, influences the observed degree of inflation persistence. In addition, the degree of persistence depends on central bank policy.

3.1 Introducing Endogenous Indexation

So far, the degree of backward-looking indexation, κ, has been treated as constant and exogenous. A novel contribution of this paper is to allow for an endogenous determination of a time-varying share of firms that apply backward-looking indexation. I assume that firms would like to pick an index that is a good estimate of steady-state inflation. They have two options. One option is the central bank's announced inflation target, π^*. If the central bank delivers on its promise, then steady-state inflation will be equal to the target. The other option is the most recent observation of inflation, π_{t-1}. If the central bank does not aim to control inflation, the inflation rate will follow a random walk, and past inflation will be the best estimate of future inflation.

Every time firms obtain a new observation on inflation, they investigate whether the target or past inflation better matches the mean of the observed inflation distribution. The probability that the announced inflation target corresponds to the mean of the observed inflation distribution is denoted $s_t = \text{Prob}(\pi^S = \pi^*)$. When a new observation becomes available, s_t is updated as follows:

$$s_{t+1} = \frac{s_t e^{-0.5(\pi_t - \pi^*)^2}}{s_t e^{-0.5(\pi_t - \pi^*)^2} + (1-s_t)e^{-0.5(\pi_t - \pi_{t-1})^2}}. \quad (10)$$

4. Alternatively, one could either use only lagged information, that is $E_t^{LS}[\pi_{t+1}] = c_{t-1}^2 \pi_{t-1}$, or incorporate current inflation in the estimate of the persistence parameter, $E_t^{LS}[\pi_{t+1}] = c_t \pi_t$. The latter specification would require solving a more complicated fixed-point problem.

This updating equation is consistent with Bayes' rule given normal shocks and beliefs.[5]

Firms cannot switch indexes at all times. They are allowed to make a choice regarding the index at the same time as they receive a Calvo-style signal that allows them to adjust their current price optimally. The probability of such a signal is $1 - \theta$. A firm that has received such a signal will then consider whether to switch the index that will apply to its pricing rule in the periods without Calvo signals. One possibility would be to assume that firms switch from backward-looking indexation to the central bank's target as soon as the probability s_t has moved above 0.5 and switch back if this probability falls slightly below 0.5. Such an assumption would seem reasonable in the unlikely case that the index can be switched at zero cost.

Instead, it is assumed that firms only choose to switch the index when there is overwhelming evidence in favor of such a change. Specifically, I introduce a trigger probability, $\bar{S}$. If the firm's current choice of index is π_{t-1}, it will switch to π^* once s_t exceeds $\bar{S}$. Similarly, if the current choice of indexation rate is π^*, the firm will switch back to π_{t-1} if $1 - s_t$ (the probability of π_{t-1}) exceeds the same trigger value. All firms face the same information regarding inflation, so s_t is symmetric across firms. Since the probability of a Calvo signal is $1 - \theta$, a share of $1 - \theta$ firms switches the rate of indexation at any point in time given that there is overwhelming evidence in favor of such a shift.

Finally, the degree of indexation, κ_t, is allowed to vary between complete indexation (that is, $\kappa_t = 1$) and a minimal value of $\underline{\kappa}$ (that is, $\kappa_t \in [\underline{\kappa}, 1]$).[6] Thus, κ_t is governed by the following process:

$$\kappa_t = \begin{cases} \theta\kappa_{t-1} & \text{if } s_t > \bar{S} \text{ and } \kappa_t \geq \underline{\kappa} \\ 1-\theta(1-\kappa_{t-1}) & \text{if } (1-s_t) > \bar{S} \\ \kappa_{t-1} & \text{otherwise} \end{cases} \tag{11}$$

Every period in which s_t exceeds the trigger probability, a share of $1 - \theta$ firms switches from backward-looking indexation to the central bank's target, while a share of θ firms sticks with the past inflation rate.

5. See Wieland (2000a).

6. I maintain a minimal amount of exogenous indexation to ensure that lagged inflation remains a determinant of the equilibrium inflation process under rational expectations. As a result, the learning model uses the correct reduced-form inflation equation under rational expectations.

Since the share of firms using backward-looking indexation varies over time, the reduced-form inflation equation (9) needs to be rewritten as follows:

$$\pi_t = \frac{\kappa_{t-1}}{1+\beta(\kappa_{t-1}-c_{t-1})}\pi_{t-1} + \frac{\lambda}{1+\beta(\kappa_{t-1}-c_{t-1})}x_t + \frac{(1-\kappa_{t-1})(1-\beta)}{1+\beta(\kappa_{t-1}-c_{t-1})}\pi^S. \tag{12}$$

As a short-hand $\delta_{(1,2,3),t}$ denotes the time-varying, reduced-form parameters. Accordingly, the reduced-form inflation equation may be written as

$$\pi_t = \delta_{1,t}\,\pi_{t-1} + \delta_{2,t}\,x_t + \delta_{3,t}. \tag{13}$$

To be able to study disinflation under alternative targeting strategies, it is still necessary to describe the central bank's objectives and the determination of the output gap x_t.

4. Inflation Targeting: Immediate versus Gradual Disinflation

A central bank that has adopted an inflation-targeting strategy is typically assumed to pursue a policy that minimizes the following per-period loss function:

$$l(\pi_t, x_t) = (\pi_t - \pi^*)^2 + \alpha x_t^2. \tag{14}$$

The parameter α refers to the central bank's relative preference for stabilizing output versus inflation.

Two simplifying assumptions keep the technical analysis manageable: the central bank directly controls the output gap, x_t, and it observes the key parameters of the inflation equation as well as the price setters' beliefs regarding inflation persistence, c_{t-1}. Thus, the central bank can take into account the parameters $\delta_{(1,2,3),t}$ of equation (13) in designing its policy. The central bank is not allowed, however, to exploit the dynamic learning process of the price-setters in conducting policy.[7] Under these assumptions, the central bank's dynamic optimization problem corresponds to:

$$\min_{x_t} E_t\left[\sum_{t=1}^{\infty}\beta^{t-1}\left(\pi_t-\pi^*\right)^2+\alpha x_t^2\right], \tag{15}$$

subject to $\pi_t = \delta_{1,t}\, \pi_{t-1} + \delta_{2,t}\, x_t + \delta_{3,t}$.

The extreme cases are strict inflation targeting, $\alpha = 0$, and strict output stabilization, $\alpha \rightarrow \infty$. Strict output stabilization would imply that the central bank always aims to set the output gap, x_t, equal to zero. Consequently, the dynamics of inflation would be governed exclusively by the time-varying parameter, $\delta_{1,t}$, which depends in turn on the degree of backward-looking indexation and the price setters' beliefs regarding inflation persistence. If $\delta_{1,t}$ ever exceeded unity, inflation would spiral out of control. In contrast, strict inflation targeting would ensure that the inflation target is met at all times for any perceived degree of inflation persistence. The resulting output gap policy corresponds to

$$x_t = -\delta_{4,t}\left(\delta_{1,t}\pi_{t-1}+\delta_{3,t}-\pi^*\right). \tag{16}$$

with $\delta_{4,t} = \delta_{2,t}^{-1}$. With a zero inflation target, $\delta_{3,t}$ would also be equal to zero.

In the intermediate case, α is positive but not infinite. Such central bank preferences are often called flexible inflation targeting. Under this policy, the output gap falls between the two extremes implied by strict inflation targeting and strict output stabilization, that is, $0 < \delta_{4,t} < \delta_{2,t}^{-1}$. Orphanides and Wieland (2000) provide an analytical formula for the case of $\delta_{1,t} = 1$. Dynamically optimal policies for alternative values of $\delta_{1,t}$ may be computed numerically with the algorithm provided in that paper.[8]

7. I discuss such an ambitious proposal in the last section of the paper. Gaspar, Smets, and Vestin (2006a, 2006b) refer to a central bank with this capability as sophisticated.

8. The matlab code is available from www.volkerwieland.com.

4.1 Model Parameterization and Initial Conditions

Having specified a very stylized but complete macroeconomic model, the next step is to evaluate alternative disinflation strategies. Initial conditions for the disinflation are defined as follows: (i) initial inflation is set at 20 percent, $\pi = 0.2$, similar to the average inflation rate of Chile prior to the start of inflation targeting; (ii) initially all firms implement backward-looking indexation, $\kappa_0 = 1$; and (iii) perceived inflation persistence indicates a unit root in inflation, that is, $c_0 = 1$. Given these initial conditions, the reduced-form inflation equation (13) simplifies to

$$\pi_t = \pi_{t-1} + \lambda x_t, \tag{17}$$

corresponding exactly to equation (1), the accelerationist Phillips curve discussed in section 1. It follows that these initial conditions represent an equilibrium if policy aims exclusively at stabilizing output, that is, if $x_0 = 0$. The parameter values used in the subsequent simulations are summarized in table 1.

Table 1. Parameter Values and Initial Beliefs

Parameter	*Value*	*Economic interpretation*
β	0.99	Discount factor
λ	0.5	Slope of Phillips curve
κ_t	$\kappa_0 = 1$	Degree of indexation to $t-1$ inflation
c_t	$c_0 = 1$	Price setters' initial belief regarding inflation persistence
Σ_t	$\Sigma_0 = 100$	Price setters' initial variance
s_t	$s_0 = 0.1$	Price/index setters' initial belief regarding prob ($\pi^S = \pi^*$)
π_0, π^*	0.2 / 0	Initial inflation: 0.2; long-run inflation target: 0
$\underline{\kappa}$	0.05	Degree of minimal exogenous indexation
θ	0.5	Probability of no price- or index-adjustment signal
$\bar{S}$	0.8	Trigger probability for switching the rate for indexation
σ	2^{-4}	Variance of noise (added later)
σ_γ	10	Belief regarding variability of γ

Source: Author's calculations.

4.2 Immediate versus Gradual Disinflation

The initial conditions summarized above set the stage for the entry of an independent inflation-targeting central bank.[9] This central bank faces very high initial costs of disinflation. The analysis starts by contrasting the immediate disinflation approach that would be implemented under strict inflation targeting with a more gradual approach consistent with a positive weight on output in the central bank's preferences.

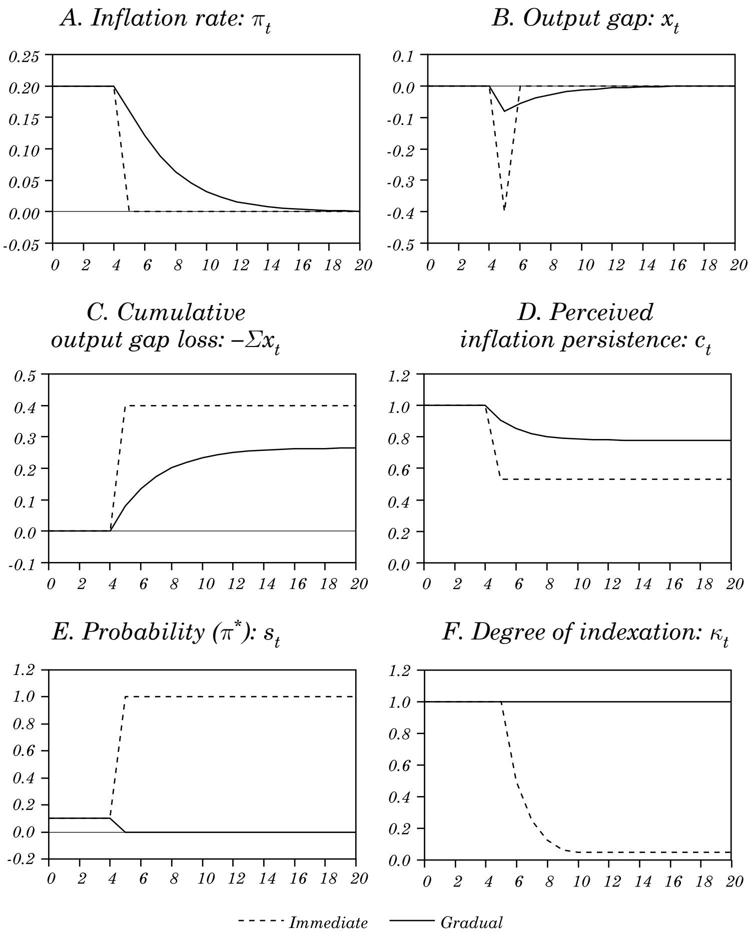
The optimal policy coefficient under strict inflation targeting corresponds to the inverse of the slope of the reduced-form inflation equation and equals $\delta_{4,0} = \delta_{2,0}^{-1} = 2$. In the model, this policy would achieve the inflation target of zero percent within one period but, such an immediate disinflation would result in an output loss of 40 percent in the same period. This outcome is shown by the dotted line in figure 2. In period 5, the central bank introduces a new inflation target of zero percent. The cumulative output loss required to disinflate by 20 percentage points is also realized in period 5. While this approach can be simulated in this simple model, such an immense reduction of total output would not be implementable in practice.

The dramatic experience of immediate disinflation induces price setters to revise their estimates of the inflation persistence parameter, c_t, from 1.0 to about 0.5 (panel D). Furthermore, the probability s_t, which is initially set at 0.1, jumps to 1.0. In other words, the immediate reduction in inflation convinces firms that the central bank's inflation target constitutes a better estimate of the mean of the inflation distribution than the past realization of inflation. Thus, from period 6 onward, the probability s_t exceeds the trigger value $\bar{S}$ (panel E), and firms that receive a Calvo signal will abandon backward-looking indexation and instead choose the central bank's target as their index. Since the probability of such a signal is $1 - \theta$, a share of θ firms continues to implement backward-looking indexation. Thus, κ_t declines over time to the minimum exogenous degree of indexation, $\underline{\kappa}$ (panel F).

A strict inflation-targeting strategy fails to take advantage of the reduction in the cost of disinflation stemming from the decline

9. Sargent, Williams, and Zha (2006) provide a fascinating account of the implications of learning for inflation and stabilization when money growth and inflation are determined by the government's budget constraint rather than by an independent central bank.

Figure 2. Immediate versus Gradual Disinflation



Source: Author's calculations.

in perceived inflation persistence and backward-looking indexation. The reason is simply that the disinflation is completed prior to these favorable developments. Instead, a gradual disinflation strategy might be able to profit from such developments and achieve disinflation at lower output costs. A gradual disinflation strategy is optimal if central bank preferences incorporate output stability—that is, a positive weight α in the loss function (equation 14). In this case, the response parameter, δ_4, in the policy function (equation 16) must be positive but below δ_2^{-1}.

To simulate a gradual disinflation, I set $\delta_{4,t} = \delta_{2,t}/(1+\delta_{2,t}^2)$. Initially, the policy response coefficient, $\delta_{4,t}$, corresponds to 0.4, which is one-fifth of the policy response needed to meet the target immediately. The resulting outcome is depicted in figure 2, with the disinflation again starting in period 5. The initial output decline is much smaller, but it will be sustained for a much longer time than in the case of immediate disinflation. The inflation rate declines gradually. By period 15, inflation is within 0.5 percentage points of the long-run target of zero. If a period in the model is treated as a year, this ten-year disinflation is broadly similar to the Chilean experience between 1991 and 2001.

The cumulative sum of output gap losses is much smaller under the gradual approach than under strict inflation targeting. The cumulative output loss converges to about 26 percent of annual output spread over more than ten years. The reason for the decline in the sacrifice ratio from 2.0 in the case of strict inflation targeting to about 1.3 in the case of gradual disinflation is to be found in adaptive learning. As price-setters observe the fall in the inflation rate, they revise their estimate of inflation persistence downward. This reduction in c_t from 1.0 to about 0.8 adds disinflationary impetus and reduces the costs of disinflation. While the decline in perceived inflation persistence is much smaller under gradual than under immediate disinflation, the gradual approach can take advantage of the resulting reduction in disinflation costs.

With regard to the degree of backward-looking indexation, firms see no reason to switch from backward-looking indexation to the announced inflation target. The announced target is just too far away and progress toward it too slow to change the probability weights on lagged inflation versus the announced target. As a result, endogenous indexation does not come into play in terms of reducing the costs of disinflation under such a gradual disinflation strategy.

5. Inflation Targeting: Temporary Inflation Targets

Two important aspects of the Chilean disinflation strategy were its gradual nature and its use of temporary annual inflation targets. Having shown that the gradual approach helps reduce disinflation costs by taking advantage of the reduction in perceived inflation persistence, I now extend the analysis to consider the effect of announcing temporary targets. In the Chilean case, these temporary targets appear to have been pursued quite vigorously. This section thus investigates whether such temporary targets, π_t^*, could have an additional beneficial effect on learning and the degree of indexation and thereby lower the costs of disinflation further.

With temporary targets, the New-Keynesian Phillips curve needs to be slightly modified:

$$\pi_t = \frac{\kappa}{1+\beta\kappa}\pi_{t-1} + \frac{\beta}{1+\beta\kappa}E_t\left[\pi_{t+1}\right] + \frac{\lambda}{1+\beta\kappa}x_t + \frac{(1-\kappa)}{1+\beta\kappa}\left(\pi_t^* - \beta\pi_{t+1}^*\right). \quad (18)$$

Accordingly, the reduced-form inflation equation with adaptive learning and endogenous indexation corresponds to

$$\begin{aligned}\pi_t &= \frac{\kappa_{t-1}}{1+\beta(\kappa_{t-1}-c_{t-1})}\pi_{t-1} + \frac{\lambda}{1+\beta(\kappa_{t-1}-c_{t-1})}x_t \\ &\quad + \frac{(1-\kappa_{t-1})}{1+\beta(\kappa_{t-1}-c_{t-1})}(\pi_t^* - \beta\pi_{t+1}^*) \\ &= \delta_{1,t}\pi_{t-1} + \delta_{2,t}x_t + \delta_{3,t}.\end{aligned} \quad (19)$$

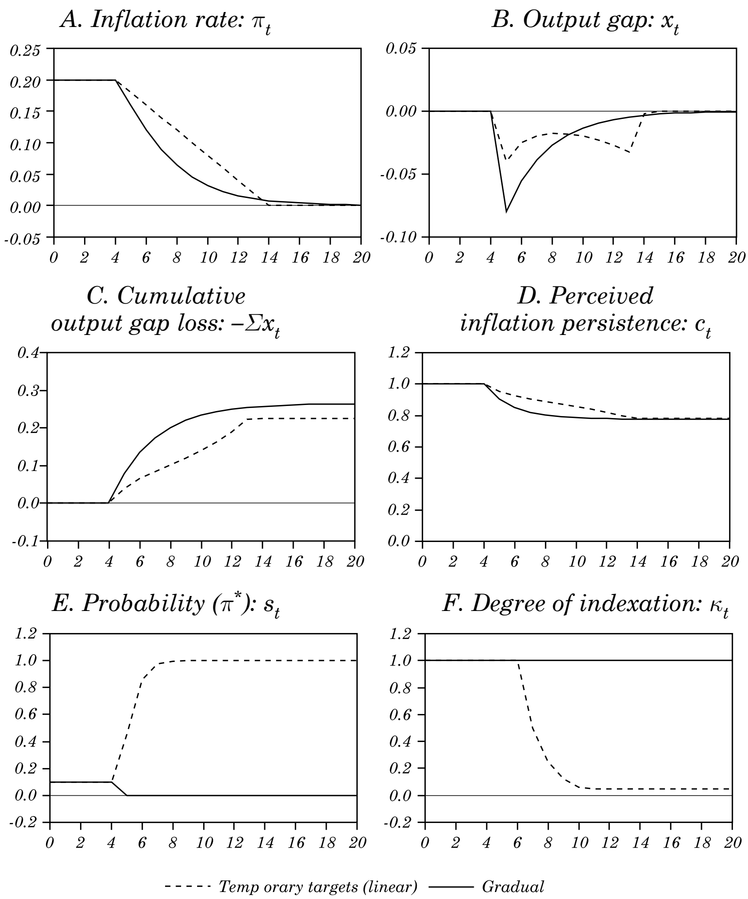
As a first example, consider a gradual, linear reduction in the inflation target by 2 percentage points per year. The long-run target of zero percent inflation is then reached in year 14, ten years after the start of disinflation. I assume that the central bank pursues these annual targets as actively as possible. In other words, the central bank implements strict inflation targeting with respect to temporary targets. After deciding on next year's inflation target, the central bank acts to meet this target. Thus, it pursues the following output gap policy:

$$x_t = -\delta_{4,t}\left(\delta_{1,t}\pi_{t-1} + \delta_{3,t} - \pi_t^*\right), \quad (20)$$

with $\delta_{4,t} = \delta_{2,t}^{-1}$, and $\delta_{(1,2,3)}$ consistent with equation (19).

The disinflation performance with temporary annual targets is shown by the dotted line in figure 3. It compares with the gradual disinflation (that is, the solid line) shown previously in figure 2. In both cases, the parameter governing the perceived degree of inflation persistence, c, declines toward a value of 0.8 (panel D). This decline occurs slightly faster under the gradual disinflation because inflation is initially reduced more quickly than the linear reduction implied by the annual targets.

Figure 3. Temporary Inflation Targets



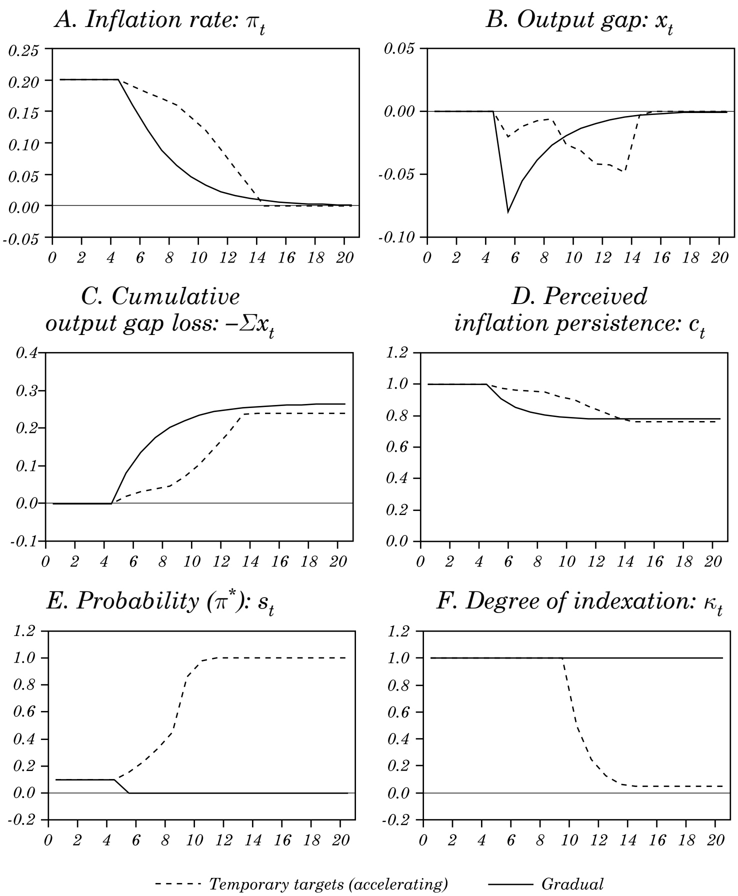
Source: Author's calculations.

An important difference arises with respect to the degree of backward-looking indexation. By announcing and meeting the temporary annual inflation targets, the central bank succeeds in convincing firms that they are better off choosing the central bank's target as an index for the pricing rule applied in those periods without Calvo-style optimal price-adjustment signals. The probability s_t that the central bank's target(s) will represent the mean of the inflation distribution rises quickly (panel E). It exceeds the trigger probability $\overline{S}$ of 0.8 by the second year of the disinflation. Every year from then on, a share of $1 - \theta$ of the firms that previously applied backward-looking indexation switches to using the central bank's targets. As a result, the degree of backward-looking indexation declines fairly rapidly and approaches the minimum level $\underline{\kappa}$ by year 11.

Unlike the gradual disinflation strategy with a long-run target, the strategy with temporary annual targets allows the central bank to take advantage of the endogenous reduction in backward-looking indexation. Firms change their behavior because they can already observe during the first few years of the disinflation that the central bank means to achieve its announced targets. Consequently, the output losses associated with disinflation are lower with annual targets. The cumulative output loss, (panel C) converges to 22 percent of output, that is 4 percent lower than in the case of the gradual disinflation. The sacrifice ratio is reduced to 1.1. Further substantial gains in terms of stabilization performance will accrue in the future. Given the substantial reduction in backward-looking indexation, the central bank will be able to reduce variations in inflation in the event of unexpected shocks at much lower cost in terms of output variability.

Next, I explore three alternative parameterizations of the sequence of annual inflation targets: targets that imply accelerating disinflation; targets that imply decelerating disinflation; and the annual targets set in Chile from 1991 to 2001. In the first case, shown in figure 4 the reduction in the central bank's annual targets accelerates over time (dotted line). The central bank initally lowers the inflation target by one percentage point per year. Starting in year 9, the fifth year of the disinflation, the inflation target is lowered by two percentage points per year. From year 11 onward, the target is lowered by three percentage points per year. The long-run target of zero percent is reached in year 14, after a ten-year disinflation process. Relative to the disinflation with linearly declining targets, accelerating targets initially imply a slower decline in inflation. The output gap incurred during the disinflation increases over time in absolute value. The total cost of disinflation—that is, the cumulative output gap—remains smaller than with the gradual disinflation strategy (solid line) but larger than with linearly declining targets. The cumulative output gap reaches 24 percent, versus 22 percent with linearly-declining targets. Because of the slow pace of disinflation in the first few years, price-setting firms take longer to become convinced that they are better off using the central bank's target as an index for their pricing rules in periods without Calvo-style signals. The probability s_t (panel E) rises slowly and takes five years to exceed the trigger value of 0.8. Only from year 10 onward do those firms that receive Calvo signals start switching from backward-looking indexation to the central bank's targets.

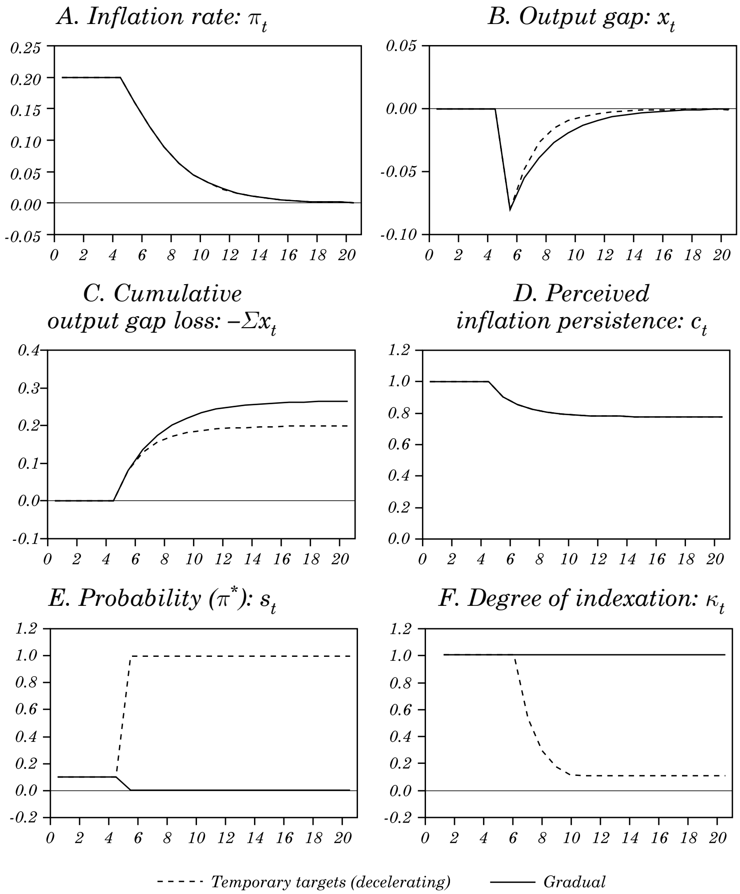
Figure 4. Accelerating Disinflation with Temporary Targets



Source: Author's calculations.

Figure 5 shows the simulation with decelerating targets. In the first year of disinflation, year 5, the central bank aims to lower inflation by 4 percentage points to 16 percent. The speed of disinflation declines in subsequent years. These annual inflation targets (dotted line) are set to be identical to the inflation path that is realized under the gradual disinflation with a long-run target (solid line). Thus, the actual path of inflation (panel A) coincides under these two scenarios. This parameterization is particularly interesting because it provides a ceteris paribus assessment of the reduction in disinflation costs that is achieved by announcing temporary annual

Figure 5. Decelerating Disinflation with Temporary Targets



Source: Author's calculations.

targets. As shown in panel B, the output gap associated with the disinflation with temporary targets (dotted line) is at all times equal to or smaller than (in absolute value) the output gap under gradual disinflation with a long-run target. The total cost of disinflation comes to 20 percent of output—that is, another 2 percent lower than with linearly declining targets. The sacrifice ratio associated with a disinflation from 20 percent to zero inflation is unity. Announcing and achieving the reduction of inflation by 4 percentage points in the first year of the disinflation convinces price-setting firms that the central bank means business. As a result, the probability s_t rises rapidly and firms soon start to abandon the practice of backward-looking indexation.

The annual targets set by the Chilean Central Bank between 1991 and 2001 also implied a decelerating disinflation. In 1990 inflation was substantially above 20 percent. The announced target for 1990 of 15–20 percent thus indicated a significant reduction with the start of the inflation-targeting strategy. Table 2 reports the announced target ranges and point targets, as well as the midpoints of these ranges. From 2001 onward, the Central Bank has aimed to keep inflation within a target range of 2 to 4 percent.

Figure 6 reports a simulation of a disinflation in the New-Keynesian model with adaptive learning and endogenous indexation using the midpoints of the Chilean target ranges from 1991 to 2001. [10] The initial conditions are the same as in the preceding simulations shown in figures 2 to 5. The midpoints of the Chilean target ranges are implemented from year 5 through year 15. To render the cost of disinflation incurred by the pursuit of the Chilean targets in the model comparable to the preceding simulations, I added a further reduction in the inflation target. In period 16, the target is reduced by an additional 3 percentage points so as to reach a long-run target of zero inflation.

The total cost of disinflation in terms of the cumulative output gap loss amounts to 18 percent of gross domestic product (GDP) spread over twelve years (panel C). The sacrifice ratio is 0.9, which is lower than in the simulation with decelerating targets shown in figure 5. This reduction is possible for the following reasons. The initial disinflation steps in years 5, 6, and 7 are vigorous enough to reduce the perceived degree of inflation persistence (panel D) and

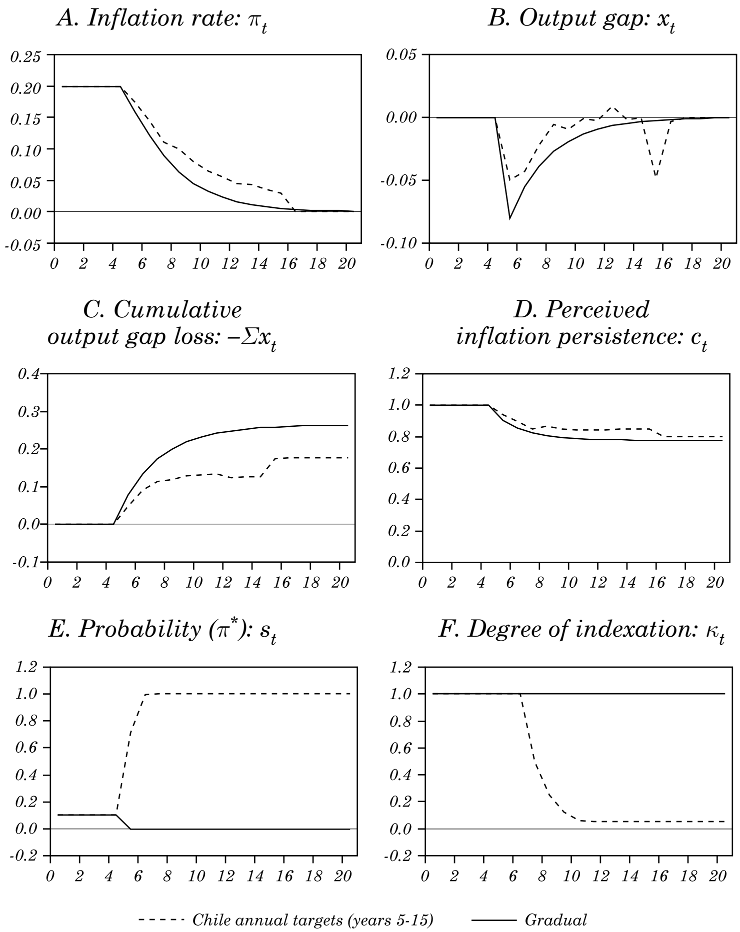
10. I disregard the potential effects of target ranges; see Orphanides and Wieland (2000) for an analysis of such nonlinearities.

Table 2. Chile's Inflation Targets: 1991–2001

Year of target	*1991*	*1992*	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	*2000*	*2001*
Year in model	5	6	7	8	9	10	11	12	13	14	15
Range	15–20	13–16	10–12	9–11	8.0	6.5	5.5	4.5	4.3	3.5	2–4
Midpoint	17.5	14.5	11.0	10.0	8.0	6.5	5.5	4.5	4.3	3.5	3.0

Source: Central Bank of Chile.

Figure 6. Chile's Inflation Targets: 1991–2001 (Years 5 to 15)



Source: Author's calculations.

a. From 2001 onward, the Central Bank of Chile pursued an inflation target zone of 2 to 4 percent with a midpoint of 3 percent. For comparability with the preceding evaluation of disinflation costs, I have added a further 3 percent disinflation step in year 16 to achieve a long-run target of zero inflation.

to raise the probability s_t beyond the trigger level, $\bar{S}$. The degree of backward-looking indexation therefore declines over the course of the disinflation. However, the disinflation stretches out for a longer period than in figure 5 and thereby benefits even more from the reduction in inflation persistence and indexation.

The baseline version of the New-Keynesian model does not include structural shocks in the inflation equation. Such shocks are often added either to capture the presence of measurement error or to reflect missing variables or other sources of rigidity. I now proceed to introduce random shocks in the New-Keynesian Phillips curve:

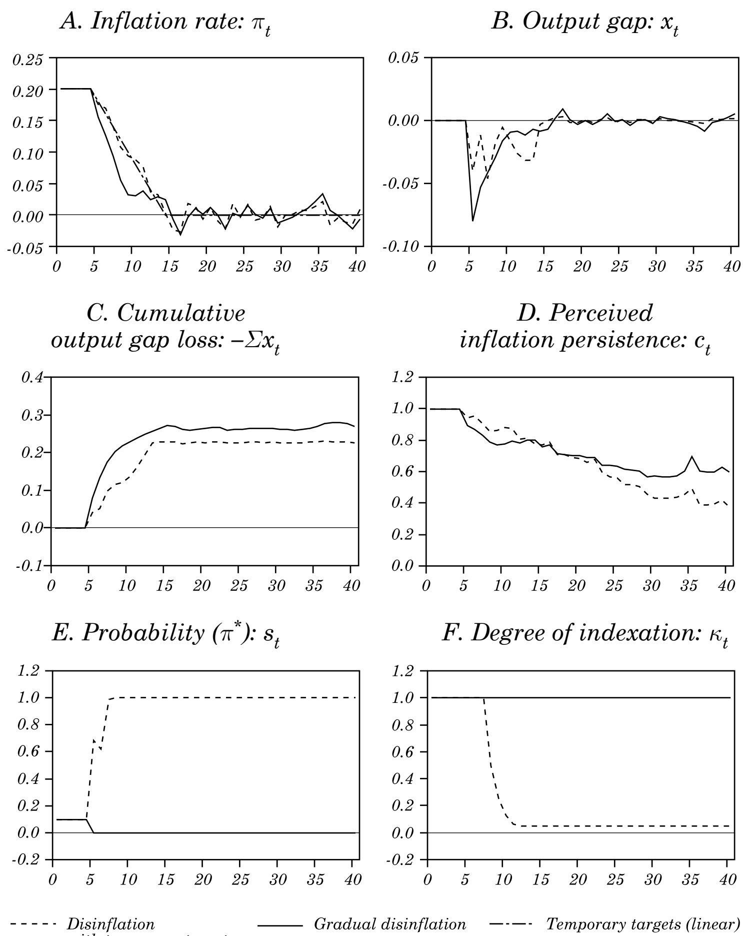
$$\pi_t = \frac{\kappa}{1+\beta\kappa}\pi_{t-1} + \frac{\beta}{1+\beta\kappa}E_t\left[\pi_{t+1}\right] + \frac{\lambda}{1+\beta\kappa}x_t + \frac{(1-\kappa)}{1+\beta\kappa}\left(\pi_t^* - \beta\pi_{t+1}^*\right) + \eta_t. \tag{21}$$

The shocks are denoted by η_t and are normally distributed with zero mean and variance $\sigma_\eta = 2^{-4}$. The timing of expectations formation, policy actions, and shocks is such that the shocks are realized after time t expectations have been formed and policy has been set. The shocks thus introduce noise in inflation that cannot be avoided by contemporaneous policy actions. However, in the period following the shock, the central bank will act to minimize further consequences from these variations that would occur as a result of the intrinsic persistence of inflation. To this end, the central bank induces offsetting variations in the output gap.

The fluctuations of inflation and output that result from random shocks and subsequent policy responses have an important influence on the dynamics of learning and endogenous indexation. On the one hand, such shocks imply that the central bank never meets its target exactly. Firms may therefore find it more difficult to assess whether it is better to use past inflation or the central bank's target as an index for their pricing rules in periods without Calvo signals. On the other hand, the fact that the central bank will set policy to counter the consequences of unforeseen shocks to inflation will generate information regarding the degree of inflation persistence and induce adaptive learning. Fluctuations may thus increase the speed of learning and reduce inflation persistence, and the costs of disinflation may decline further.

Figure 7 shows dynamic simulations with a particular draw of random shocks, η. The length of time covered is forty years, rather than twenty as in the preceding figures. The figure compares the outcome under a gradual disinflation with a long-run target (solid line) with a disinflation based on linearly declining annual targets (dash-dotted line). Panel A reports the actual inflation rates, which exhibit some random fluctuations, together with the annual targets (dotted line).

Figure 7. Shocks Accelerate Learning and Perceived Persistence Declines

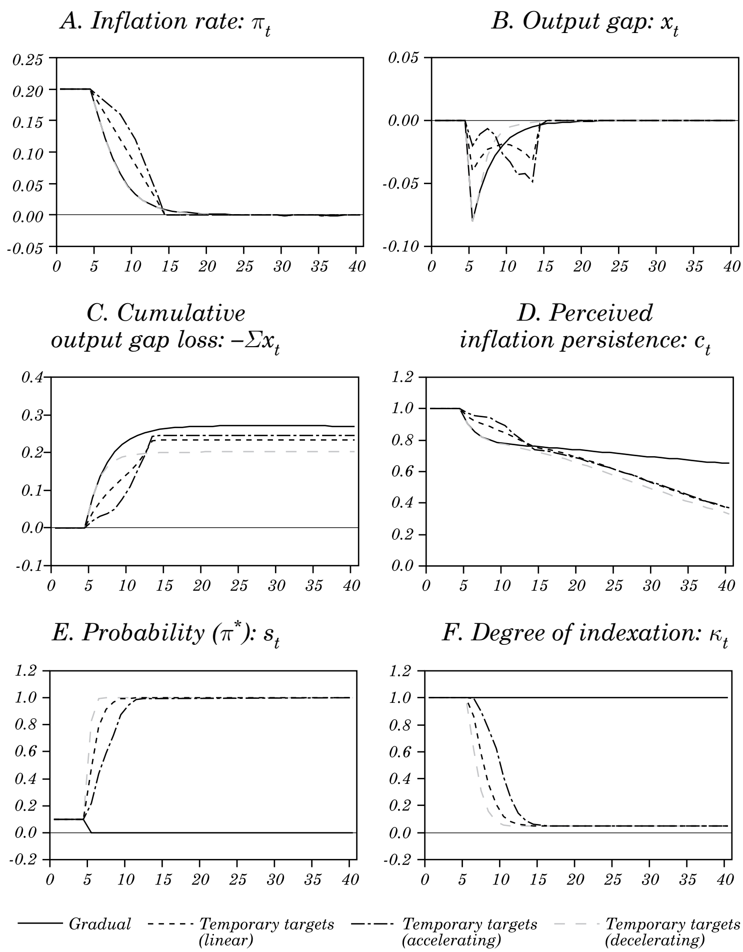


Source: Author's calculations.

Two aspects of these stochastic simulations are of particular interest. Panel D shows that the perceived degree of inflation persistence continues to decline even after the disinflation process has been completed. It is the policy response to the consequences of unforeseen shocks that stabilizes inflation fluctuations and drives down price setters' estimates of the persistence parameter, c_t. This decline is much more pronounced in the simulation with annual targets. By year 40, it reaches 0.4, while it is still at 0.6 in the gradual disinflation with long-run target. The reason is that the structural persistence from indexation is ultimately much smaller in the simulation with annual targets. The central bank's announcement and achievement of these targets has convinced firms to switch from backward-looking indexation to using the target rates. The probability s_t measuring the usefulness of central bank targets for indexation does not increase as smoothly as in the absence of unforeseeable random shocks. In figure 3, panel E, the probability s_t rises rapidly and smoothly above the trigger level in the simulation with linearly decline targets. In figure 7, panel E, it moves up and down a little bit before rising further above the trigger level. This finding shows that the switch from backward-looking indexation to the central bank targets is influenced by the particular series of shocks.

Figure 7 only reports the outcomes for a single draw of shocks. The strategy with temporary inflation targets need not always outperform the gradual disinflation strategy in terms of output losses. To shed further light on the likely outcomes, I simulated a thousand series of shocks drawn from a normal distribution and compute averages across these thousand simulations. The averages are reported in figure 8, which shows averages for the gradual disinflation with a long-run target (solid line), with linearly-declining annual targets (dotted line), with decelerating targets (dash-gray line) and with accelerating targets (dash-dotted line). The results are quite similar to the simulation without shocks, although they are not the same because of the nonlinearity resulting from adaptive learning and indexation. The ranking of speeds of disinflation (panel A) and cumulative output losses (panel C) remains unchanged. The perceived degree of inflation persistence reaches 0.4 for all three types of temporary targets by year 40. After many more years, it converges to a small but positive value consistent with the persistence implied by the minimum degree of backward-looking indexation under rational expectations. The increase in the probability s_t, (panel E) is fastest with decelerating targets and slowest with accelerating targets. As a result, the degree of backward-looking indexation declines most quickly with decelerating targets and most slowly with accelerating targets. In the case of a gradual disinflation with long-run targets, backward-looking indexation remains complete.

Figure 8. Averages over a Thousand Simulations



Source: Author's calculations.

6. A "Sophisticated" Central Bank versus One That Learns

These findings suggest that the performance of monetary policy could be improved further by allowing the central bank to observe and exploit the nonlinear dynamics stemming from adaptive learning and endogenous indexation—that is, equations (7), (10), and (11), in the design of dynamically optimal policy. Gaspar, Smets, and Vestin (2006a) study such an optimal policy problem with adaptive learning, but without endogenous indexation. They introduce the label "sophisticated" for a central bank that is capable of exploiting learning dynamics. In my model, such a sophisticated central bank would solve the following dynamic optimization problem:

$$\min_{x_t} E_t\left[\sum_{t=1}^{\infty}\beta^{t-1}\left(\pi_t - \pi^*\right)^2 + \alpha x_t^2\right], \tag{22}$$

subject to $\pi_t = \delta_{1,t}\pi_{t-1} + \delta_{2,t}x_{\mathrm{t}} + \delta_{3,t} + \eta_t$ and equations (7), (8), (10), and (11).

The optimal policy is nonlinear because it takes into account the nonlinearities arising from recursive estimation of the degree of inflation persistence—that is, equations (7) and (8)—and endogenous indexation—that is, equations (10) and (11).

Following Gaspar, Smets, and Vestin (2006a, 2006b), the central bank's choice variable is assumed to be the output gap and the central bank is assumed to aim at a long-run inflation target. An alternative approach, inspired by the present paper, would be to use annual inflation targets as the central bank's choice variable. A particular choice of temporary target would then automatically imply a given output gap according to the strict inflation-targeting policy shown by equation (20).

The optimization problem defined by (22) corresponds to a nonlinear dynamic programming problem with four state variables: (π_{t-1}, c_{t-1}, Σ_{t-1}, s_{t-1}). Numerical approximation of such a problem is complicated but within the reach of current methodology. However, optimal policy design here relies on rather courageous assumptions regarding the central bank's knowledge of private sector expectations formation. The central bank is assumed not only to observe the private sector's beliefs, but also to know the exact learning dynamics. The policy that could be implemented by such an extremely knowledgeable central bank might provide a useful benchmark for model-based comparison, but it does not represent a strategy that could be implemented in practice. I propose instead an alternative approach to policy design under uncertainty that can be pursued under more realistic informational assumptions.

Optimal policy design that could be implemented with the information available to central banks in practice takes recourse to learning. In this case, the central bank would learn about inflation dynamics by recursively estimating the relevant parameters of the reduced-form inflation equations (13) or (19). Contrary to the price-setting firms in the model, which were assumed to simply estimate a regression of inflation on its own lag, the central bank can spend more resources on learning. Certainly, central bank econometricians regularly estimate Phillips curves that include the effect of policy on inflation via the output gap, x_t.

In the model studied in this paper, central bank learning could be applied to the reduced-form inflation equation consistent with adaptive learning and endogenous indexation—that is,

$$\pi_t = \delta_{1,t}\, \pi_{t-t} + \delta_{2,t}\, x_t + \delta_{3,t} + \eta_t. \tag{23}$$

Central bank beliefs regarding the three time-varying parameters may be summarized by the vector $\mathbf{d}_t = (d_{1,t}, d_{2,t}, d_{3,t})$ and associated covariance matrix $\mathbf{\Sigma}_{d,t}$.[11]

$$\text{var}\left[\begin{pmatrix} d_{1,t} \\ d_{2,t} \\ d_{3,t} \end{pmatrix}\right] = \mathbf{\Sigma}_d = \begin{pmatrix} v_t^1 & v_t^{12} & v_t^{13} \\ v_t^{12} & v_t^2 & v_t^{23} \\ v_t^{13} & v_t^{23} & v_t^3 \end{pmatrix}. \tag{24}$$

The vector of state variables that characterize central bank beliefs contains nine variables, the three means, three variances, and three covariances. The associated updating equations for recursive least squares with time-varying parameters correspond to the following:[12]

$$\begin{pmatrix} d_{1,t} \\ d_{2,t} \\ d_{3,t} \end{pmatrix} = \begin{pmatrix} d_{1,t-1} \\ d_{2,t-1} \\ d_{3,t-1} \end{pmatrix} + \mathbf{\Sigma}_{t-1}\mathbf{X}_t F^{-1}\,(\pi_t - d_{1,t-1}\pi_{t-1} - d_{2,t-1}x_t - d_{3,t-1}), \tag{25}$$

11. See Wieland (2006). Related work on central bank learning in this context includes Cogley, Colacito, and Sargent (2007), Ellison (2006), Svensson and Williams (2007), and Wieland (2000a,2000b).

12. For a derivation of the updating equations using Bayes' rule or the Kalman filter, see Zellner (1971) and Harvey (1992), respectively.

$$\Sigma_{d,t} = \Sigma_{d,t-1} - \Sigma_{d,t-1}\mathbf{X}_t F^{-1}\mathbf{X}_t' \Sigma_{d,t-1} + \boldsymbol{\sigma}_d,$$

where $\mathbf{X}_t' = (\pi_{t-1}\, x_t\ 1)$. F refers to the conditional variance of inflation and , $F = \mathbf{X}_t \Sigma_{d,t-1} \mathbf{X}_t' + \sigma^{\eta}$.

The information requirements for such a learning central bank are much less stringent than for the sophisticated central bank discussed above. Only inflation and output observations are needed. Potential output could be subsumed in the time-varying intercept. A fruitful area for future research would be to reassess the disinflation policies in the preceding section under the assumption that the central bank learns about the time-varying parameters governing the inflation process in this manner. Wieland (2000a, 2000b, 2006) and Beck and Wieland (2002) compute optimal learning policies for such problems with up to two unknown parameters and compare their performance to passive learning policies that do not take into account the central bank's own updating equations in optimization. At the least, policy design under passive learning could be applied to the policy problem in this paper.

7. Conclusions and Extensions

This paper has shown that inflation-targeting strategies can lower the costs of disinflation and future inflation stabilization. I have explored two channels through which such a reduction may take place: adaptive learning and endogenous indexation. Arguably, both channels may have played an important role in Chile's disinflation experience.

If market participants learn adaptively rather than form rational expectations, then history matters. As the central bank acts to bring inflation under control, market participants will observe the consequences of these actions and revise their beliefs regarding the degree of inflation persistence. Over time, adaptive learning lowers the cost of disinflation. A gradual approach to disinflation can take advantage of this beneficial effect.

Endogenous indexation implies that price-setting firms are allowed to choose between past inflation and the central bank's target as an index for their pricing rule in periods without Calvo-style signals to set prices optimally. Firms assess the likelihood that announced inflation targets determine steady-state inflation and adjust the indexation of contracts accordingly. A strategy of announcing and achieving short-term targets for inflation is able to influence the degree of backward-looking indexation. It implies that firms are able to observe fairly quickly whether the central bank acts to meet the targets it proclaims. When the central bank follows through on its commitments, the likelihood that firms switch from backward-looking indexation to the central bank's announced targets rises. Short-term annual targets that are pursued aggressively help reduce the degree of indexation more effectively than a strategy with a long-run target that is achieved only gradually.

This analysis suggests that dynamic general equilibrium models estimated under the assumptions of rational expectations and an exogenous, constant degree of backward-looking indexation may misjudge the costs of disinflation in two ways. First, the assumption of rational expectations may overstate the central bank's power to influence the costs of disinflation through words alone, whether they be announcements or verbal commitments. Learning implies that announcements need to be followed by action to convince market participants. The resulting reduction in inflation persistence is influenced by policy actions, as well as economic shocks. Second, the assumption of exogenous indexation may lead to model estimates that overstate the cost of disinflation and inflation-output trade-offs. Endogenous reductions in the degree of backward-looking indexation as inflation rates decline to a low level consistent with announced targets would present the central bank with more favorable trade-offs.

This research presents a number of interesting and potentially important possible extensions. These extensions concern the optimal design of monetary policy, the formation of expectations, the role of the interest rate, the role of the exchange rate, and the degree of openness of the economy. With regard to dynamically optimal policy design, two possible approaches were proposed in section 6 of the paper, including the derivation of the dynamically optimal policy that takes into account the nonlinear learning dynamics present in the model. Although such a policy relies on unrealistic informational assumptions, it would form a useful benchmark for comparison with practically implementable policies, such as the policy with central bank learning proposed in section 6.

As to the formation of expectations, it would be useful to evaluate the implications of alternative adaptive learning specifications (see Branch and Evans, 2006; Milani, 2007) for the cost of disinflation. It would also be interesting to study endogenous indexation under rational expectations. The quantitative effects of endogenous indexation could thus be studied separately from those stemming from adaptive learning.

The model consider here is very stylized. The central bank has been assumed to control the output gap directly. Instead, the transmission from the central bank's primary policy instrument (namely, the nominal short-term interest rate) and the output gap could be modeled explicitly. In other words, the model can be extended to include the log-linearized Euler equation of households—that is, the New-Keynesian IS curve. This extension would support exploration of a host of new questions regarding the design of interest rate rules and the conditions for stability under learning (see also Llosa and Tuesta, 2007).

Finally, Chile, like many inflation-targeting countries, is a small open economy. During the disinflation in Chile, favorable shocks to the exchange rate and the terms of trade may have played an important role in cushioning the economy. These effects could be examined by extending the analysis of learning and endogenous indexation conducted in this paper to a small open economy. In an open economy, further practical questions arise such as whether to target domestic inflation or CPI inflation and how to account for the exchange rate in interest rate policy.

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