

Data Uncertainty and the Role of Money as an Information Variable for Monetary Policy^{*,†}

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Abstract

In this study, we perform a quantitative assessment of the role of money as an indicator variable for monetary policy in the euro area. We document the magnitude of revisions to euro area-wide data on output, prices, and money, and find that monetary aggregates have a potentially significant role in providing information about current real output. We then proceed to analyze the information content of money in a forward-looking model in which monetary policy is optimally determined subject to incomplete information about the true state of the economy. We show that monetary aggregates may have substantial information content in an environment with high variability of output measurement errors, low variability of money demand shocks, and a strong contemporaneous linkage between money demand and real output. As a practical matter, however, we conclude that money has fairly limited information content as an indicator of contemporaneous aggregate demand in the euro area.

1 Introduction

Many macroeconomic time series are subject to substantial revisions, and hence such data only provide imperfect information about the true state of the economy at a given point in time. In light of these data limitations monetary policymakers and researchers alike have long been interested in identifying indicator variables that provide precise and timely information. At least since the early 1970s, research on the information content of alternative indicators has highlighted the potential usefulness of monetary aggregates; these evaluations have typically been conducted in reduced-form models and models with adaptive expectations.¹ More recently, research on Taylor-style interest rate rules has re-emphasized the importance of “real-time” data uncertainty for the design of monetary policy albeit without considering money’s potential role as an information variable.²

In this study, we perform a quantitative assessment of the role of money as an indicator variable for monetary policy in the euro area. We begin by analyzing the sequence of revisions to euro area-wide data, and find that measures of real output have been subject to substantial revisions over a period of up to nine months, whereas measures of prices and money have generally been subject to relatively minor revisions that occur within a short period of the initial data release. Given this pattern of euro area data revisions, monetary aggregates have a potentially significant role in providing information about the current level of aggregate demand and hence about incipient pressures on the inflation rate.

We then proceed to analyze the information content of money in a forward-looking model in which money has no causal role in influencing output or inflation.³ In particular,

¹Examples of this line of research include Kareken et al. (1973), Friedman (1975, 1990), Tinsley et al. (1980) and Angeloni et al. (1994).

²In particular, a number of studies with U.S. data have found that uncertainty arising from revisions of output gap and inflation measurements may lead to a significant deterioration in the performance of such interest rate rules; see Orphanides (1998), Orphanides et al. (2000) and Rudebusch (2000). For a large-scale analysis of the differences between alternative vintages of U.S. macroeconomic data the reader is referred to Croushore and Stark (1999).

³That is, the short-term nominal interest rate is the instrument of monetary policy, and the money stock does not enter directly into any of the behavioral equations. These assumptions are typical of the current generation of macroeconomic models, including small stylized models (e.g., Rotemberg and Woodford (1997); Fuhrer (1997); Orphanides and Wieland (1998)) as well as large-scale policy models such as the Federal Reserve Board’s FRB/US model (cf. Brayton and Tinsley (1996)), the ECB’s Area-Wide Model (cf. Fagan

our analysis builds on the rational expectations model of the euro area developed by Coenen and Wieland (2000); we augment this model with a calibrated specification for the output revision process, and with the estimated M3 demand equation of Coenen and Vega (2001), which was found to provide a remarkably stable representation of euro area money demand. We assume that the central bank optimally sets the short-term nominal interest rate to minimize a weighted average of inflation volatility and output gap volatility, subject to a small penalty on nominal interest rate movements. We further assume that the central bank and private agents share the same information set and utilize the Kalman filter to make optimal inferences about the true state of the economy (cf. Pearlman et al. (1986), Svensson and Woodford (2000)).⁴

Money can serve as a potentially useful indicator variable in our framework, because we assume that aggregate money demand depends on the *true* level of aggregate output, whereas the central bank and private agents only receive a noisy measure of aggregate output. The rationale behind this assumption is that individual agents' demand for real balances depends on the true level of their individual incomes and the sum of these demands ought to be related to the aggregated actual income. On the other hand, money demand also fluctuates in response to unobserved velocity disturbances that attenuate the strength of its linkage to aggregate demand. Thus, the information content of money depends on the relative variances of output mismeasurements and money demand shocks, and on the strength of the contemporaneous linkage between money demand and real output. We assess the value of this information in terms of the improvement in the policymaker's loss function and in terms of several statistical measures used in the earlier literature, namely,

et al. (2001)), and the multi-country model of Taylor (1993a). An alternative approach, which allows for direct effects of money on inflation, would be the P* model of Hallman et al. (1991) estimated more recently for Germany by Tödter and Reimers (1994) and for the euro area by Gerlach and Svensson (2000). For analysis of this issue in models with explicit microeconomic foundations, see Ireland (2001), McCallum (2001) and Leahy (2001).

⁴In contrast, Dotsey and Hornstein (2000) studied the role of money as an indicator variable in a calibrated model of the U.S. economy with simple measurement error, under the assumption that the true state of the economy is observed by private agents but not by the central bank. It should also be noted that their analysis focused primarily on impulse response functions rather than on measures of the information content of money.

the root mean-squared prediction error (RMSE), the coefficient of determination (R^2), and the entropy of the system. Finally, since money has no causal role in the model, one may view our results as providing a reasonable *lower bound* on the extent to which money provides information about contemporaneous real output.

While the information value of money could also be analyzed in an atheoretical forecasting model,⁵ such a model would not allow us to separate the potential roles of money for monetary policy and consider exclusively the information role motivated by data uncertainty. In the structural model we can exclude the possibility of a direct causal role of money due to real-balance effects or a direct effect on inflation. Furthermore, the structural model distinguishes dynamics due to expectations from those due to policy or adjustment dynamics due to lags of endogenous variables. Finally, it allows us to derive optimal monetary policy and evaluate the information gains in terms of the central bank's loss function.

Our framework demonstrates that monetary aggregates may have substantial information content in an environment with high variability of output mismeasurements, low variability of money demand shocks, and a strong contemporaneous link between money and output. As a practical matter, however, we find that money has fairly limited information content as an indicator of current aggregate demand in the euro area, mainly because the contemporaneous link between M3 and real output is relatively weak. Of course, these results do not rule out other motivations for careful monitoring of monetary aggregates. For example, the prominent role for money in the ECB's monetary policy strategy that is signalled by the announcement of a reference value for the growth rate of M3 is motivated by the usefulness of money as an indicator of potential risks to the medium-term inflation outlook.⁶ Thus, its role under the first pillar of the ECB's strategy is quite different from the potential information value regarding uncertain current output explored in this paper.

Finally, it should be emphasized that our analysis focuses solely on uncertainty regarding

⁵See, for example, the working paper version Coenen, Levin and Wieland (2001) for results based on a VAR model.

⁶See, for instance, Nicoletti-Altimari (2001) who compares the performance of various indicators derived from monetary aggregates to the performance of alternative indicators based on economic and financial data such as output growth, the output gap, the unemployment rate and interest rate spreads.

actual output, and does not address the problem of estimating potential output. While uncertainty about potential output has important consequences for the determination of monetary policy, we neglect this issue here because the money stock is related to actual output via money demand and thus cannot serve as a direct source of information regarding potential output.⁷

The remainder of this paper is organized as follows. Section 2 characterizes the timing and magnitude of revisions to euro area data on aggregate output, prices, and money. Section 3 outlines the behavioral equations of the model, and indicates alternative representations of the output revision process. Section 4 describes our methodology for determining the optimal filtering weights and for evaluating the information content of indicator variables. Section 5 considers the model without money in order to quantify the magnitude of the information problem associated with real-time output mismeasurement. Section 6 adds a highly stylized money demand equation to the model, facilitating a systematic analysis of how the information content of money is affected by the relative variability of output measurement errors and money demand shocks. In light of these findings, Section 7 uses the complete model described above to evaluate the quantitative significance of money as an indicator variable for monetary policy in the euro area. Finally, Section 8 summarizes our conclusions and suggests several directions for future research.

2 Data Uncertainty in the Euro Area

Some macroeconomic data series, such as nominal interest rates, exchange rates, and raw materials prices, are readily available and not subject to revision. In contrast, indicators of aggregate quantities and prices are more difficult to construct, and are frequently subject to substantial revisions as additional information becomes available to the statistical agency.⁸ For the euro area, aggregate data has only become available fairly recently (with

⁷For the implications of uncertainty about output gaps and potential output (or unemployment gaps and the NAIRU) for monetary policy we refer the reader to Ehrmann and Smets (2000), Orphanides (2000) and Wieland (1998) among others.

⁸Further revisions occur on a less frequent basis as the result of definitional changes, such as switching to a different benchmark year for the national income accounts. Such revisions often shift the entire level of

the harmonization of statistical procedures across the individual member countries), and hence the record of initial releases and revisions is necessarily limited. Nevertheless, it is useful to characterize the properties of these revisions in order to shed some light on the degree of data uncertainty in the euro area.

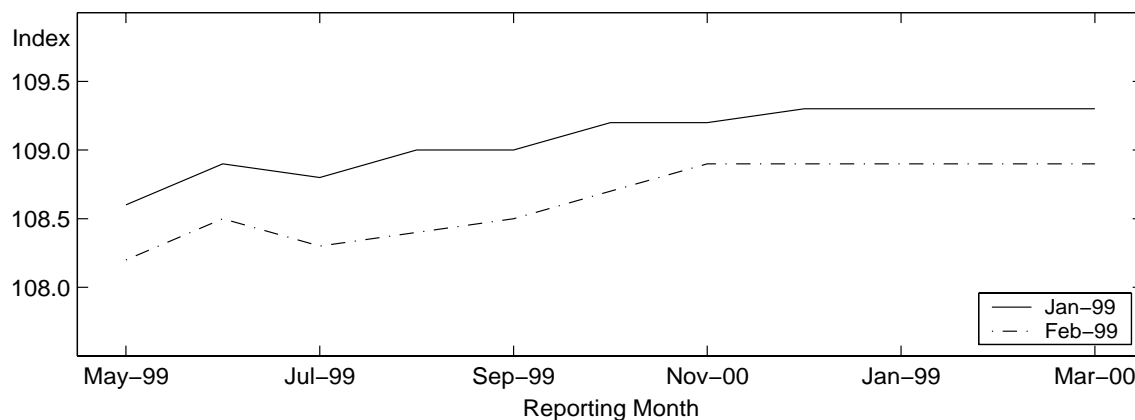
Thus, we proceed to analyze the timing and magnitude of revisions to euro area output, price, and money data, beginning with the advent of European Monetary Union in 1999. As measures of real output, we consider monthly data on industrial production (excluding construction) as well as quarterly data on real GDP. To measure aggregate prices, we consider monthly data on the Harmonized Index of Consumer Prices (HICP) and quarterly data on the GDP price deflator. Finally, we consider monthly data for M3; we focus on this measure of money because Coenen and Vega (2001) found that the demand function for M3 has been remarkably stable. In each case, we utilize real-time data series over the period October 1998 through December 2000, as published in consecutive issues of the European Central Bank's Monthly Bulletin over the period January 1999 through February 2001.⁹

The nature of the revision process is best understood with an example. **Figure 1** shows monthly revisions of industrial production at the start of monetary union. Estimates of euro area industrial production in January and February 1999, for instance, were first published in the May 1999 issue of the ECB Monthly Bulletin. The estimates of the index reported in May were 108.6 and 108.2 for January and February industrial output, respectively. Over the following months the statistical authorities revised these estimates upwards. Revisions only ceased by the end of the year. The magnitude of the revisions over this period was 0.7 in both cases. Clearly, these revisions suggest a significant degree of data uncertainty, which persisted for some time.

Table 1 provides summary statistics regarding the revision process for monthly euro a data series, but may have relatively minor implications for the determination of monetary policy.

⁹The ECB's monthly bulletin is a convenient source for obtaining consistent real-time data. Furthermore, each bulletin represents a reasonably accurate summary of the data available to the ECB Governing Council at its first meeting each month: the cut-off date for inclusion in the bulletin predates each meeting, and the bulletin itself is published a week later. However, in future work it would be interesting to analyze the timing of revisions as published by the statistical agency that actually compiles each data series.

Figure 1: Revisions to Industrial Production



Source: ECB Monthly Bulletin.

area data. The first column is associated with the first revision (one month after the initial publication), the next column reflects the second revision (i.e., the difference between the values published one month and two months following the first publication), and so on until the revision in the tenth month following initial publication. For each series, the first row indicates the largest upward revision at each interval (as a percent of the value published in the previous month), while the second row indicates the largest downward revision, and the third row indicates the mean absolute revision.

Evidently, the industrial production data are subject to substantial and frequent revisions over the first year after the initial publication. For example, the first monthly revision of this series has a mean absolute value of 0.34 percent, with a maximum upward revision of 0.93 percent and a maximum downward revision of 0.6 percent. While the magnitude of revisions gradually declines as time passes, revisions exceeding 0.1 percent are not unusual during each of the next few months after the initial publication.

In contrast, the consumer price data are typically not revised at all; the only exceptions are apparently due to corrections of reporting errors. Clearly, the lack of revisions does not imply that these data provide an exact measure of aggregate inflation. However, mea-

Table 1: Monthly Euro Area Data Revisions (in Percent)

Month after initial publication	1	2	3	4	5	6	7	8	9
<u>Industrial Production^(a)</u>									
largest upward revision	0.93	0.81	0.54	0.54	0.46	0.27	0.27	0.27	0.18
largest downward revision	-0.60	-0.46	-0.55	-0.27	-0.26	-0.27	-0.27	-0.27	-0.36
mean absolute revision	0.34	0.28	0.24	0.16	0.16	0.11	0.12	0.13	0.13
<u>Consumer Prices^(b)</u>									
largest upward revision	0	0	0	0	0	0	0	0	0
largest downward revision	-0.10	-0.10	0	0	0	0	0	0	0
mean absolute revision	0.03	0.02	0	0	0	0	0	0	0
<u>M3^(c)</u>									
largest upward revision	0.37	0.20	0.14	0.13	0.14	0.06	0.02	0.10	0.03
largest downward revision	-0.18	-0.13	-0.12	-0.03	-0.03	-0.10	-0.11	-0.04	-0.02
mean absolute revision	0.16	0.08	0.06	0.03	0.05	0.04	0.03	0.03	0.02

Source: ECB Monthly Bulletin, January 1999 to February 2001.

Note: ^(a) Index of Industrial Production (excluding construction), seasonally adjusted. ^(b) Harmonized Index of Consumer Prices, Dec. 1998 = 100, not seasonally adjusted. ^(c) M3 Index, Dec. 1998 = 100, seasonally adjusted; calculated from monthly differences in levels adjusted for reclassifications, other revaluations, exchange rate variations etc..

surement biases in the consumer price index have mainly been identified with longer-term factors (such as improving product quality, introduction of new goods and services, and changes in expenditure shares), and hence these biases may not be crucial in evaluating higher-frequency fluctuations in the inflation rate. In any case, as we will see below, the GDP price deflator (which is less susceptible to measurement bias than the HICP) also exhibits relatively small revisions.

Finally, the magnitude of initial data revisions is substantially smaller for M3 than for industrial output: the maximum upward and downward revisions in the first month are less than half as large (in percentage terms). Furthermore, subsequent revisions in M3 are

relatively small and infrequent, so that the mean absolute revision never exceeds 0.1 percent from the second month onwards.

Table 2 reports summary statistics regarding the revision process for real GDP and the GDP price deflator, which are available on a quarterly basis. These statistics indicate that real GDP is subject to fairly large revisions.¹⁰ For example, in the first revision (one quarter after the initial publication), the maximum upward revision exceeds a full percentage point, and the mean absolute revision is about 0.8 percent of the previously published value. Even three quarters after the initial publication, the mean absolute revision of real GDP is about 0.5 percent. In contrast, revisions of the GDP deflator are much smaller: the mean absolute revision is only about 0.1 percent in each of the first two quarters after the initial publication, and subsequent revisions are negligible in magnitude. Evidently, revisions to nominal GDP for the euro area are primarily due to revisions regarding real output rather than prices.

Thus, the monthly and quarterly data yield remarkably similar conclusions regarding real-time data uncertainty in the euro area. Industrial production and real GDP are each subject to relatively large revisions during the first several quarters after the initial publication, indicating that data uncertainty regarding the current level of real output is a non-trivial issue for the euro area.¹¹ By comparison, both measures of aggregate prices (the HICP and the GDP price deflator) and the broad money stock (M3) are subject to relatively small revisions during the first quarter after the initial publication, and to negligible revisions in subsequent quarters.

These results raise the possibility that money can serve as a useful indicator in providing real-time information about fluctuations in real output. An additional advantage of money

¹⁰Some of these revisions have occurred as individual member countries have moved to the ESA95 harmonization of national income accounts and are likely to become smaller as the implementation process is completed in most countries.

¹¹Revisions of the growth rate of real output tend to be smaller. As discussed in the ECB's monthly bulletin of August 2001 (see pages 26-28) the average size of revisions of quarter-on-quarter growth since the first quarter of 1999 was 0.2 percentage points. Throughout this paper, however, we continue to focus on the level rather than the growth rate, because in our view, the level of output relative to the economy's potential is more relevant for determining the appropriate stance of monetary policy and its effect on inflation than the difference between actual and potential growth rates.

Table 2: Quarterly Euro Area Data Revisions (in Percent)

Quarter after initial publication	1	2	3	4	5
<u>Real GDP^(a)</u>					
largest upward revision	1.49	1.21	1.14	0.20	0.19
largest downward revision	-0.91	-0.95	0	-0.02	-0.08
mean absolute revision	0.80	0.69	0.47	0.11	0.14
<u>GDP Price Deflator^(b)</u>					
largest upward revision	0.28	0.38	0.09	0.09	0.09
largest downward revision	-0.10	-0.09	0	-0.09	0
mean absolute revision	0.11	0.14	0.03	0.06	0.05

Source: ECB Monthly Bulletin, January 1999 to February 2001.

Note: ^(a) Seasonally adjusted. ^(b) Seasonally adjusted.

as an indicator variable is that money data typically becomes available earlier than output data.¹² In the following analysis we will primarily focus on the information gain from money in the presence of measurement error, but we will return to the gains arising from the earlier availability of money in the sensitivity analysis at the end of the paper.

3 A Rational-Expectations Model with Data Uncertainty

To quantify the information content of money, we utilize the euro area macroeconomic model of Coenen and Wieland (2000), augmented by the estimated M3 demand equation of Coenen and Vega (2001). Since these equations are specified at a quarterly frequency, it seems reasonable to assume (in light of the results of the previous section) that observations on aggregate output are subject to measurement error, while aggregate prices, money, and

¹²This holds even for monthly data. For example, in June one learns about money growth and inflation in May but about industrial production in April.

nominal interest rates are observed without measurement error.¹³ It also seems reasonable to assume that the money demand of each individual household or firm depends on its own income and expenditures (which are known to that household or firm), while neither private agents nor the central bank observe the true level of aggregate output. Under these assumptions, aggregate money demand will be related to the true level of aggregate income, and hence observations on the money stock can provide useful information about movements in aggregate output.

3.1 The Behavioral Equations

The behavioral equations of the model are indicated in **Table 3**.¹⁴ As shown in equation (M-1), the aggregate price level p_t is determined as a weighted average of overlapping nominal wage contracts signed over the past year. The estimated weighting scheme implies that a weight of 0.32 is placed on the current wage contract w_t , while smaller weights are placed on earlier contracts (e.g., w_{t-3} receives a weight of about 0.18).

The determination of nominal wage contracts is given in equation (M-2). As in Taylor (1980, 1993a), each wage contract is determined by expectations about aggregate prices and perceived output gaps over the duration of the contract.¹⁵ The expectations operator $E_t[\cdot]$ indicates the optimal projection of each variable, conditional on all information available at period t . As noted above, this information set includes the true values of aggregate wages, prices, and interest rates, and noisy observations regarding aggregate output. Since our analysis is focused on the implications of data uncertainty regarding actual output q_t , we assume for simplicity that potential output q_t^* is exogenously determined and known by

¹³In the model considered here, measurement errors of the money stock would have the same effect as money demand shocks in reducing the information content of money as an indicator of aggregate output. Thus, one could always capture the effect of money measurement error by considering a slightly higher variance of the money demand shocks.

¹⁴Wages, prices, output, and money are expressed in logarithmic form, and interest rates are expressed at annualized rates.

¹⁵Coenen and Wieland (2000) also considered relative real wage contract specifications of the type originally proposed by Buiter and Jewitt (1981) and later studied by Fuhrer and Moore (1995). We have performed sensitivity analysis and found that the results reported here concerning the information role of money are not substantially affected by using a relative real wage contract specification instead of equation (M-2).

Table 3: A Rational-Expectations Model with Data Uncertainty

Price Level	$p_t = f_0 w_t + f_1 w_{t-1} + f_2 w_{t-2} + f_3 w_{t-3},$ $\text{where } f_i = 0.25 + (1.5 - i) 0.0456$	(M-1)
Contract Wage	$w_t = E_t \left[\sum_{i=0}^3 f_i p_{t+i} + 0.0115 \sum_{i=0}^3 f_i y_{t+i} \right] + u_t^w,$ $\text{where } y_t = q_t - q_t^* \text{ and } u_t^w \sim \text{i.i.d.}(0, 0.0038^2)$	(M-2)
Aggregate Demand	$y_t = 1.2029 y_{t-1} - 0.2225 y_{t-2} - 0.0942 r_t^l + u_t^d,$ $\text{where } u_t^d \sim \text{i.i.d.}(0, 0.0058^2)$	(M-3)
Money Demand	$\Delta(m - p)_t = 0.071 \Delta^2 q_t + 0.097 (\Delta i_t^s + \Delta i_{t-1}^s)$ $- 0.353 \Delta i_{t-1}^l - 1.052 (\Delta \pi_t + \Delta \pi_{t-1})$ $- 0.132 [(m - p) - 1.125 q$ $+ 0.865 (i^l - i^s) + 6.048 \pi]_{t-2}$ $+ u_t^m,$ $\text{where } \pi_t = p_t - p_{t-1} \text{ and } u_t^m \sim \text{i.i.d.}(0, 0.0023^2)$	(M-4)
Real Interest Rate	$r_t^l = i_t^l - E_t \left[\frac{1}{2} (p_{t+8} - p_t) \right]$	(M-5)
Term Structure	$i_t^l = E_t \left[\frac{1}{8} \sum_{j=0}^7 i_{t+j}^s \right]$	(M-6)

Note: p : aggregate price level; w : nominal contract wage; u^w : contract wage shock; y : output gap; q : output; q^* : potential output; r^l : long-term real interest rate; u^d : aggregate demand shock; m : nominal money balances; i^s short-term nominal interest rate; i^l : long-term nominal interest rate; π : one-quarter inflation rate; u^m : money demand shock.

all private agents and by the central bank.¹⁶ Finally, under these assumptions, it should be noted that the aggregate supply disturbance u_t^w is known to all agents (including the central bank); this implication follows from our assumptions that all agents know the current contract wage and utilize identical information in forming expectations about the variables

¹⁶In fact, equations (M-2) and (M-3) were estimated by Coenen and Wieland (2000) using “final” data on euro area real GDP (that is, data available at the end of 1999), and constructing the output gap by removing a log-linear time trend. In future work, it would be interesting to reestimate these equations using real-time output gap data. However, as the authors note, constructing a real-time output gap data series for the period preceding EMU would be a “courageous undertaking”.

on the right-hand side of equation (M-2).

As shown in equation (M-3), the current output gap depends on the true output gap in each of the previous two quarters and on the *ex ante* long-term real interest rate, r_t^l (which is defined by equation (M-5)). This specification seems reasonable under the assumption that each household or firm makes spending decisions based on its own directly observed income path, and hence aggregate spending depends on the true path of aggregate income.¹⁷ Nevertheless, since the true level of aggregate output is not directly observed, the aggregate demand shock u_t^d is not in the information set of private agents or the central bank.

As indicated by equation (M-4), real money balances $(m - p)_t$ are determined by an error correction process involving aggregate output q_t , the short-term nominal interest rate i_t^s , the long-term nominal interest rate i_t^l (defined by equation (M-6)), and the one-quarter inflation rate $\pi_t = p_t - p_{t-1}$.¹⁸ As noted above, this money demand specification indicates that the money stock responds to the true level of aggregate output q_t , and hence money has a potentially useful role as an indicator variable. Of course, this role depends on the stability of the money demand. While estimates of money demand with U.S. data have tended to be unstable, results with German and euro area data have typically been more encouraging.¹⁹

Two additional characteristics of equation (M-4) are crucial in determining the specific information content of money. First, the short-run link between money and output is relatively weak, with an instantaneous income elasticity of only 0.071; evidently, the money stock would provide somewhat more information about current output if the contemporaneous relationship between these two variables were significantly larger. Second, the money demand shock u_t^m has an estimated standard error of 0.23 percent, reflecting the extent to

¹⁷To the extent that individual spending decisions actually reflect agents' perceptions about the aggregate economy, then the output gap equation would need to be augmented by terms such as $E_t[y_{t-1}]$ and $E_t[y_{t-2}]$. We have performed some preliminary analysis of such specifications, but leave further investigation to future research. In this context, a model with more explicit microeconomic foundations that distinguishes between macro- and micro-level uncertainty will be helpful.

¹⁸Because the inflation rate π_t is not annualized, the corresponding coefficients in equation (M-4) appear unusually large.

¹⁹For a recent study regarding U.S. money demand and money's usefulness for U.S. monetary policy see Dotsey et al. (2000).

which money balances move in response to factors other than aggregate output, inflation, and nominal interest rates.

Finally, we assume that the short-term nominal interest rate is the instrument of monetary policy, and that the central bank sets the interest rate each period in order to minimize the following loss function:

$$\mathcal{L}_t = E_t \sum_{j=0}^{\infty} \delta^j \left[(\pi_{t+j} - \pi^*)^2 + \omega_1 y_{t+j}^2 + \omega_2 (i_{t+j}^s - i_{t+j-1}^s)^2 \right],$$

where we choose $\delta = 0.99$, $\omega_1 = 0.5$ and $\omega_2 = 0.1$ as our baseline parameter values.

This loss function may be viewed as a reasonable benchmark representing a monetary policymaker whose primary objective is to stabilize inflation around its target value, π^* , while also seeking to maintain stable output and to avoid large shifts in the policy instrument. In Section 7 below we will also perform some sensitivity analysis regarding the implications of alternative choices for the objective function weights.²⁰

3.2 The Revision Process

In the state-space literature, a typical assumption is that each data point of a given time series is observed just once (possibly subject to some measurement error). In contrast, here we wish to represent a sequence of revisions to the real output data that gradually refines the quality of each individual data point. Statistical agencies construct estimates of variables such as industrial output or GDP based on disaggregated data from individual firms and households.²¹ As more individual observations on disaggregated data are reported to the agency, it publishes revised estimates of the aggregate variable that improve upon the initial data release. Some aggregate series are revised many times in this manner. Since our analysis is focused on the behavior of private agents and the central bank, we do not explicitly model how the statistical agency determines these revisions, using new information on disaggregated variables. Rather, we take the following general representation of a process

²⁰See Coenen et al. (2001) for analysis of the information content of money when the policymaker follows the simple interest rate rule of Taylor (1993b).

²¹In the case of the euro area our understanding is that EUROSTAT constructs euro area estimates based on the national accounts, while national agencies collect disaggregated data.

of successive revisions as the starting point for our analysis:

$$q_t^{(t+j)} = q_t + v_t^{(t+j)}, \quad (1)$$

Here q_t indicates the true level of output at period t , and a parenthetical superscript indicates the “vintage” date of a particular output measurement. Thus, $q_t^{(t)}$ refers to the initial observation of output at time t , while $q_t^{(t+j)}$ is the revised estimate published at time $t + j$, and $v_t^{(t+j)}$ indicates the measurement error of this revised estimate.

In the existing literature, the standard approach has been to assume that each observation is subject to a measurement error that is fixed once and for all at the initial publication date; i.e., $q_t^{(t+j)} = q_t^{(t)}$ and $v_t^{(t+j)} = v_t^{(t)}$ for all $j \geq 0$. In contrast, our analysis explicitly recognizes that in every period the statistical agency may revise previously-published data in light of new information. We do assume, however, that the sequence of data revisions eventually converges to zero; thus, $q_t^{(t+j)}$ converges to a “final” measure $q_t^{(\infty)}$, while the measurement error $v_t^{(t+j)}$ converges to $\bar{v}_t \equiv v_t^{(\infty)}$.²²

We start with the case in which each observation is subject to a single revision; that is, output is measured with error contemporaneously, but its true value is published one quarter later:

$$\begin{aligned} q_t^{(t)} &= q_t + v_t^1 \\ q_t^{(t+1)} &= q_t, \end{aligned} \quad (2)$$

where v_t^1 is serially uncorrelated with mean zero and standard deviation $\sigma(v^1)$, and is uncorrelated with the structural disturbances u_t^d , u_t^w , and u_t^m . We calibrate $\sigma(v^1) = 0.97$ based on the observed revisions described in Section 2. This specification of the revision process implies that the data published by the statistical agency in any given period t includes an error-prone measure of contemporaneous output, $q_t^{(t)} = q_t + v_t^1$, and the true value of the previous period’s output, q_{t-1} . Thus, for this revision process, agents use the Kalman filter in each period to construct an optimal estimate of contemporaneous output, $q_{t|t} \equiv E_t[q_t]$.

²²Thus, our analysis does not consider the implications of occasional redefinitional changes in the entire time series.

As we have seen in Section 2, however, the data on real output are subject to a sequence of substantial revisions for several quarters. Therefore, we also consider the following three-period revision process, in which the “final” measure of period- t output is published after three quarters:

$$\begin{aligned}
q_t^{(t)} &= q_t + \bar{v}_t + v_t^3 + v_t^2 + v_t^1 \\
q_t^{(t+1)} &= q_t + \bar{v}_t + v_t^3 + v_t^2 \\
q_t^{(t+2)} &= q_t + \bar{v}_t + v_t^3 \\
q_t^{(t+j)} &= q_t + \bar{v}_t \qquad \text{for } j \geq 3,
\end{aligned} \tag{3}$$

where $v_t = [v_t^1 \ v_t^2 \ v_t^3]'$ is a vector of serially uncorrelated measurement errors with mean zero and diagonal covariance matrix Σ_{vv} .²³ We calibrate Σ_{vv} in light of the observed revisions from Section 2; in particular, we find that v_t^1 , v_t^2 , and v_t^3 have standard deviations of 0.97 percent, 0.77 percent, and 0.47 percent, respectively.²⁴

Finally, it should be noted that choosing a definitive calibration for the distribution of \bar{v}_t is rather difficult, given the absence of concrete information about the properties of measurement errors that still remain in the “final” published data. Nevertheless, it is clear that assuming a positive variance for \bar{v}_t would imply a higher level of noise in the published output data, and hence would imply larger potential benefits from using money as an indicator variable (given our assumption that money demand is related to the true level of output).

Thus, given our general intention of providing a reasonable *lower bound* for the information content of money, we will proceed henceforth under the assumption that \bar{v}_t has zero variance; that is, agents learn the true value of output after either a single revision or a sequence of three revisions to the published output data. For example, in the case of the three-period revision process, the period- t data vintage includes an initial measure

²³Using the observed revisions from Section 2, we find that the sample correlation between v_t^1 and v_t^2 is negligible, while each has negative sample correlation with v_t^3 ; however, as shown in our earlier work (Coenen et al. (2001)), constructing Σ_{vv} using these sample correlations does not change the results noticeably compared with our baseline assumption of zero contemporaneous correlation between the elements of v_t .

²⁴These standard deviations were computed using the data for which revisions were available for at least three consecutive quarters.

of current output, $q_t^{(t)} = q_t + v_t^3 + v_t^2 + v_t^1$, revised measures of output for the previous two periods, $q_{t-1}^{(t)} = q_{t-1} + v_{t-1}^3 + v_{t-1}^2$ and $q_{t-2}^{(t)} = q_{t-2} + v_{t-2}^3$, and the “final” published measure of output three periods ago that equals its true value, $q_{t-3}^{(t)} = q_{t-3}$. To refine the error-prone measures of current and past output, agents utilize the Kalman filter at each date t to determine the components $q_{t-j|t}$ for $j = 0, 1, 2, 3$; that is, a *preliminary* estimate for the current period, two *revised* estimates for the previous two periods, and the *final* estimate of output three periods ago.

4 Evaluating the Role of Indicator Variables

4.1 The Optimal Filtering Problem

We obtain optimal estimates of output by applying the Kalman filter to our linear rational expectations model of the euro area. Implicitly, we assume that the initial estimate of output, which the statistical agency constructed from disaggregated data, can be improved upon by synthesizing aggregate data on real as well as monetary and financial variables within a fully specified macroeconomic model.²⁵ Given our assumption that private agents and the central bank have the same information concerning aggregate variables, we can follow the methodological approach of Svensson and Woodford (2000), henceforth referred to as SW2000. In particular, the structural model can be expressed in the following form:

$$\begin{bmatrix} X_{t+1} \\ \tilde{E} x_{t+1|t} \end{bmatrix} = A^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + A^2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + B i_t + \begin{bmatrix} u_{t+1} \\ 0 \end{bmatrix}, \quad (4)$$

where X_t is a vector of predetermined variables, x_t is a vector of non-predetermined variables, i_t is the central bank’s policy instrument, u_t is a vector of serially uncorrelated shocks with mean zero and positive semi-definite covariance matrix Σ_{uu} , and the matrices A^1 , A^2 , B and \tilde{E} contain the structural parameters of the model.

²⁵Of course, if the statistical agency were to use optimal filtering to revise its data on aggregate economic variables, the central bank and private agents could utilize the statistical agency’s data without any further refinement (see for instance Sargent (1989)). To our knowledge, EUROSTAT does not use aggregate euro area data and macroeconomic models for optimal filtering in this manner.

In general, the policymaker and market participants do not have complete information about all the elements of X_t nor x_t , but can only directly observe the vector Z_t , which is determined as follows:

$$Z_t = D^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + D^2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + v_t, \quad (5)$$

where v_t is a vector of serially uncorrelated measurement errors with mean zero and positive semi-definite covariance matrix Σ_{vv} . The measurement errors v_t are assumed to be uncorrelated with the shocks u_t at all leads and lags, i.e. $E[u_t v'_\tau] = 0$ for all t and τ . The matrices D^1 and D^2 are selector matrices of appropriate dimension. Here we use $\chi_{\tau|t} = E[\chi_\tau | I_t]$ to denote the rational expectation (that is, the optimal projection) of any variable χ in period τ given information in period t . The information set in period t corresponds to

$$I_t = \left\{ Z_\tau, \tau \leq t; A^1, A^2, B, D^1, D^2, \tilde{E}, \Sigma_{uu}, \Sigma_{vv}, \delta, \omega_1, \omega_2, \pi^* \right\}.$$

In our particular problem, the policymaker and the market participants only have noisy measurements of real output, and learn the true value of output after either a single revision or a sequence of three revisions. To make this setup conformable with the measurement model (5), we augment the generic model (4) in the standard way by embedding current and lagged values of the vector of output measurement errors v_t in the vector of predetermined variables X_t . In the same way, we embed v_{t+1} and Σ_{vv} in u_{t+1} and Σ_{uu} respectively. As a consequence, the vector of measurement errors v_t disappears from the observation equation (5) and, instead, the properly redefined matrix D^1 picks off the current and past values of the output measurement errors.

For the optimal monetary policy under discretion SW2000 show that the policy instrument is a function of the current estimate of the predetermined variables,

$$i_t = F X_{t|t},$$

that the non-predetermined variables fulfill the relationship

$$x_t = G^1 X_t + G^2 X_{t|t}$$

and that the system of equations (4), (5) can be cast into state-space form without instrument and non-predetermined variables,

$$X_{t+1} = H X_t + J X_{t|t} + u_{t+1} \quad (6)$$

$$Z_t = L X_t + M X_{t|t}, \quad (7)$$

where the matrices F , G^1 , G^2 , H , J , L and M are derived in SW2000. This transformation of course simplifies the remaining problem of forming the estimate $X_{t|t}$ considerably.²⁶

Accounting for the contemporaneous effect of the estimate $X_{t|t}$ on Z_t , SW2000 show that the optimal estimate of X_t can be obtained by means of a Kalman filter updating equation. This updating equation is expressed in terms of the innovations in the transformed variables $\bar{Z}_t = Z_t - M X_{t|t}$:

$$X_{t|t} = X_{t|t-1} + K (\bar{Z}_t - \bar{Z}_{t|t-1}) \quad (8)$$

$$= X_{t|t-1} + K L (X_t - X_{t|t-1}). \quad (9)$$

The steady-state Kalman gain matrix K is given by

$$K = PL' (LPL')^{-1},$$

where the matrix P is the steady-state covariance matrix of the innovations $X_t - X_{t|t-1}$ given information in period $t - 1$, i.e., $P = \text{Cov}[X_t - X_{t|t-1} | I_{t-1}]$, and satisfies the relation

$$P = H \left[P - PL' (LPL')^{-1} LP \right] H' + \Sigma_{uu}. \quad (10)$$

We are particularly interested in the weights on the observed indicator vector Z_t under optimal filtering. While the Kalman filter estimate $X_{t|t}$ is obtained in terms of the weighted innovations in the transformed variables \bar{Z}_t , we can recover the optimal weights on the observations of Z_t by substituting $\bar{Z}_t = Z_t - M X_{t|t}$ and $\bar{Z}_{t|t-1} = Z_{t|t-1} - M X_{t|t-1} = L X_{t|t-1}$

²⁶Having eliminated the non-predetermined variables x_t , the estimation of the predetermined variables X_t still requires solving a simultaneity problem. Simultaneity arises because the observable variables Z_t depend on the estimate of the predetermined variables $X_{t|t}$, which in turn depend on the observables used in the estimation.

into (8),

$$X_{t|t} = (I + KM)^{-1}(I - KL) X_{t|t-1} + (I + KM)^{-1}K Z_t.$$

Here we can see that the contemporaneous effect of the estimate $X_{t|t}$ on Z_t merely shows up in the premultiplication of the matrix $(I + KM)^{-1}$. When comparing the weights assigned to different information variables in the subsequent analysis we will refer to the elements of this modified Kalman gain matrix.

The evolution over time of the vector of predetermined variables X_t and its estimate $X_{t|t}$ is simultaneously determined by the transition equation (6) and the Kalman filter updating equation (9) in combination with the prediction formula $X_{t|t-1} = (H + J) X_{t-1|t-1}$ being derived from the former equation. It is convenient to express this system of dynamic equations more compactly as

$$\begin{bmatrix} X_t \\ X_{t|t} \end{bmatrix} = \begin{bmatrix} H & J \\ KLH & (I - KL)H + J \end{bmatrix} \begin{bmatrix} X_{t-1} \\ X_{t-1|t-1} \end{bmatrix} + \begin{bmatrix} I \\ KL \end{bmatrix} u_t.$$

4.2 Measures of Information Content

We evaluate the information content of indicator variables according to the extent that they will reduce the uncertainty surrounding the estimation problem faced by the policymaker and the market participants, and the extent to which the reduction in estimation uncertainty improves the policymaker's loss function.

A convenient measure of estimation uncertainty is the covariance matrix of the projection errors of the vector X_t , given the information set I_t available at period t (that is, information obtained from current and lagged values of the observed vector Z_t). This covariance matrix can be expressed as follows:

$$\text{Cov}[X_t - X_{t|t}|I_t] = P - PL'(LPL')^{-1}LP. \quad (11)$$

For example, one element of X_t is the unobserved aggregate demand shock, u_t^d , and the root mean-squared error (RMSE) of estimating this shock is given by the square root

of the corresponding diagonal element of $\text{Cov}[X_t - X_{t|t}, I_t]$. In the subsequent analysis the RMSE serves as our baseline measure of the estimation uncertainty surrounding the optimal estimate $X_{t|t}$ produced by the application of the Kalman filter.²⁷

In addition to the RMSE, it is useful to consider measuring estimation uncertainty using the concept of entropy (or “expected uncertainty”) taken from the information theory literature. In doing so we follow Tinsley et al. (1980) who employ entropy as a formal measure of the information content of indicator variables. To explain the basic concept and its relationship with the coefficient of determination in linear regression models, R^2 , we restate the relevant general results from Tinsley et al.²⁸

Consider two vectors χ and ξ with joint density $f(\chi, \xi)$. The joint entropy of χ and ξ is given by

$$\mathcal{H}(\chi, \xi) = -\text{E}[\ln(f(\chi, \xi))].$$

The entropy or ‘expected uncertainty’ of χ corresponds to

$$\mathcal{H}(\chi) = -\text{E}[\ln(f(\chi))],$$

where $f(\chi)$ is the marginal density of χ , and the entropy of χ given ξ corresponds to

$$\mathcal{H}(\chi|\xi) = -\text{E}[\ln(f(\chi|\xi))]$$

with $f(\chi|\xi) = f(\chi, \xi)/f(\xi)$ denoting the conditional density of χ given ξ . Since $\mathcal{H}(\chi)$ corresponds to the prior uncertainty associated with χ and the observation ξ may provide additional information with $f(\chi|\xi)$ describing what is known about χ after having observed ξ , $\mathcal{H}(\chi|\xi)$ reflects the posterior uncertainty about χ given ξ . The expected information of the observation ξ with respect to χ is then defined as the difference between the prior uncertainty about χ , $\mathcal{H}(\chi)$, and the posterior uncertainty of χ given ξ , $\mathcal{H}(\chi|\xi)$,

$$\mathcal{I}(\chi|\xi) = \mathcal{H}(\chi) - \mathcal{H}(\chi|\xi).$$

²⁷See Coenen et al. (2001) for additional results regarding the RMSE of the optimal estimate of x_t and the RMSE of multi-step-ahead predictions of X_t and x_t .

²⁸For early uses of the concept of entropy in the economics literature see also Theil (1967).

Using this measure of information content one can derive the expected relative information gain associated with adding a particular indicator variable ζ to the information vector ξ as follows:

$$\mathcal{G}(\chi, \xi, \zeta) = [\mathcal{I}(\chi|\xi, \zeta) - \mathcal{I}(\chi|\xi)]/\mathcal{I}(\chi|\xi).$$

Regarding χ and ξ as jointly distributed normal with covariance matrix Σ , Tinsley et al. show that $\mathcal{I}(\chi|\xi)$ has a particularly simple form. In this case, using the properties of multivariate normal distributions,

$$\mathcal{I}(\chi|\xi) = 0.5 \ln(|\Sigma_{\chi\chi}|/|\Sigma_{\chi\chi} - \Sigma_{\chi\xi} \Sigma_{\xi\xi}^{-1} \Sigma_{\xi\chi}|),$$

where $\Sigma_{\chi\chi}$, $\Sigma_{\xi\xi}$ and $\Sigma_{\chi\xi} = \Sigma'_{\xi\chi}$ are the submatrices of Σ with appropriate dimensions and $\Sigma_{\chi\chi} - \Sigma_{\chi\xi} \Sigma_{\xi\xi}^{-1} \Sigma_{\xi\chi}$ is the conditional covariance matrix of χ given ξ . Thus, under normality, the measure of information content, $\mathcal{I}(\chi|\xi)$, corresponds to the log-distance between the determinants of the covariance matrices of the marginal and the conditional distribution of χ .

The case of univariate χ can then be used to develop an intuitive interpretation of the expected information content $\mathcal{I}(\chi|\xi)$. In this case,

$$\begin{aligned} \mathcal{I}(\chi|\xi) &= 0.5 \ln(\sigma_\chi^2/(\sigma_\chi^2 - \Sigma_{\chi\xi} \Sigma_{\xi\xi}^{-1} \Sigma_{\xi\chi})) \\ &= 0.5 \ln(1/(1 - \beta' \Sigma_{\xi\xi} \beta / \sigma_\chi^2)) \\ &= 0.5 \ln(1/(1 - R_{\chi|\xi}^2)), \end{aligned}$$

where β is the vector of regression coefficients, and $R_{\chi|\xi}^2$ is the population coefficient of determination in the linear regression of χ on ξ . Evidently,

$$R_{\chi|\xi}^2 = 1 - (\exp\{2\mathcal{I}(\chi|\xi)\})^{-1}.$$

These measures of information content can be adapted to our euro area model with rational expectations and data uncertainty by choosing the covariance matrix of the prediction errors $X_t - X_{t|t-1}$ (given by equation (10)) as the covariance matrix of the marginal

distribution $f(\chi)$ and the covariance matrix of the projection errors $X_t - X_{t|t}$ (given by equation (11)) as the covariance matrix of the conditional distribution $f(\chi|\xi)$.²⁹

5 The Information Problem in the Absence of Money

In our framework, the key information problem facing the central bank is to make optimal inferences about the aggregate demand shock, because responding promptly to this shock helps minimize the variability of both output and inflation. To quantify the magnitude of this information problem, we consider the benchmark case in which money is not included in the information set of private agents or the central bank. In this case, the properties of the model are invariant to the specification of money demand, given that money plays no causal role in the model (that is, the short-term nominal interest rate is the instrument of monetary policy, and the money stock does not enter directly into any of the behavioral equations).

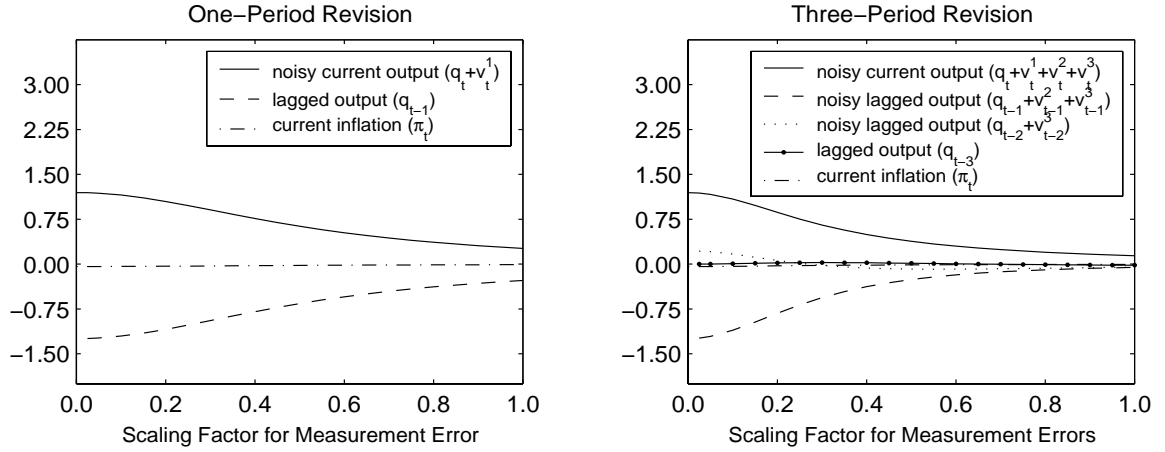
5.1 Optimal Filter Weights

Figure 2 indicates the optimal Kalman filter weights for estimating the aggregate demand shock when money does not enter the information set. The left panel reports these weights for the case in which the output observation is subject to a single revision, while the right panel gives corresponding results for the case of a three-period revision process (as described by equations (2) and (3), respectively). In each panel, we show how the weights vary as a function of the relative magnitude of the revision process, by scaling the standard deviation of each measurement error by the factor λ ; that is, $\lambda = 1$ indicates that each standard deviation has the calibrated value given in Section 3.2.

In the special case of no output mismeasurements ($\lambda = 0$), the aggregate demand shock can be determined exactly using model equation (M-3), that is, $u_t^d = y_t - 1.2029 y_{t-1} + 0.2225 y_{t-2} + 0.0942 r_t^l$. With non-trivial measurement errors ($\lambda > 0$), the optimal weight on each observed variable declines in absolute value. With arbitrarily large output mis-

²⁹See Coenen et al. (2001), Appendix B, for a more detailed discussion.

Figure 2: Optimal Indicator Weights for Estimating the Aggregate Demand Shock



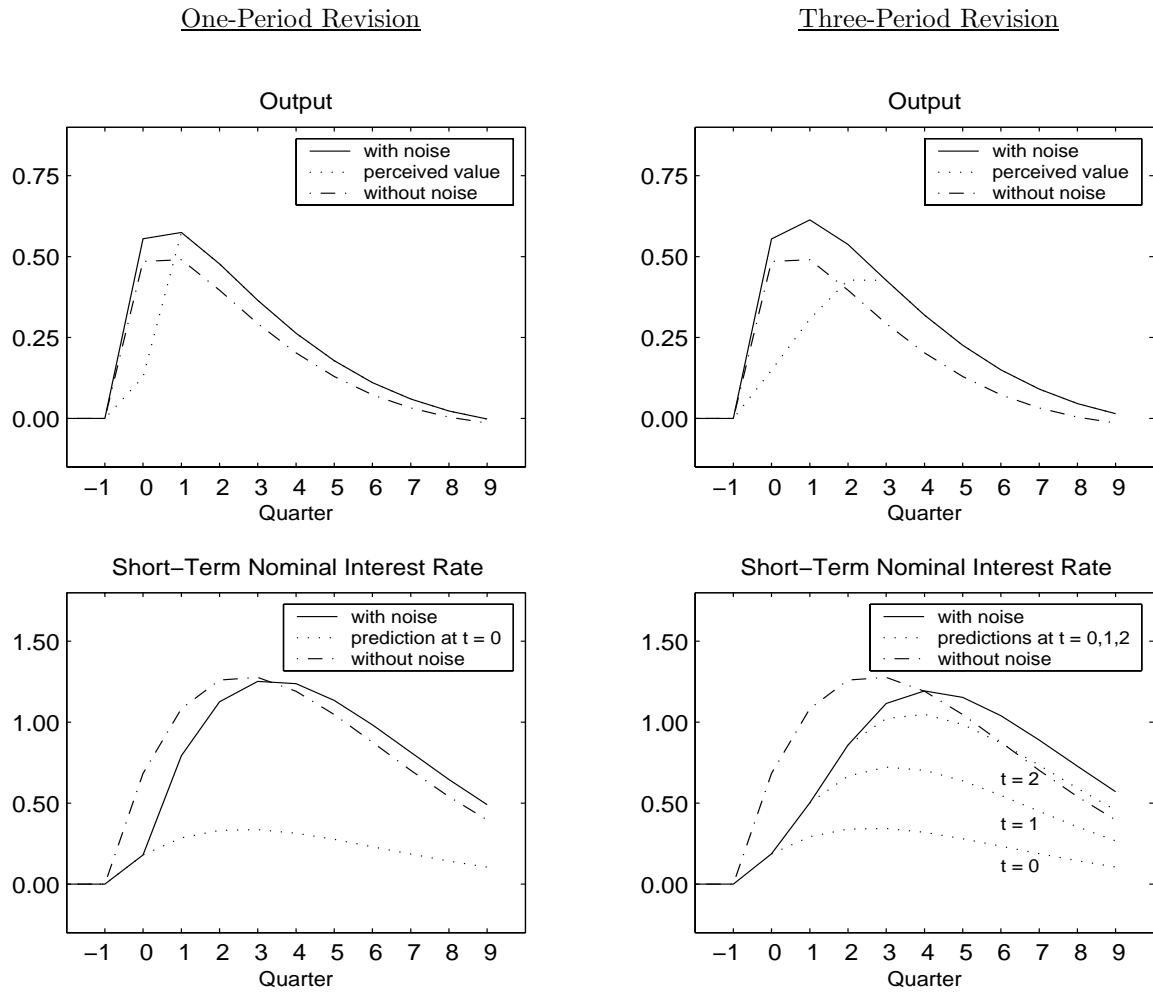
measurements, the observed variables do not provide any information about u_t^d , and the optimal estimate is simply its unconditional mean of zero.

5.2 Impulse Response Functions

To visualize the implications of the optimal filter weights, it is useful to examine the behavior of the economy in response to specific shocks. **Figure 3** depicts the effects of a single aggregate demand shock occurring at time 0 with a magnitude of one standard deviation, with all initial conditions set at their steady-state values. The left column of panels indicates the path of real output and short-term nominal interest rates when the observed output data are subject to a single revision, and the right column of panels gives corresponding results for the case of a three-period revision process.

The aggregate demand shock immediately raises real output about 0.5 percent above steady state (as indicated by the solid line in each upper panel). Since there are no data mismeasurements in this experiment, the level of current output reported at time zero is actually correct. Nevertheless, the perceptions of private agents and the central bank are based on the optimal filter weights that reflect the typical pattern of revisions. Thus, as

Figure 3: Impulse Responses to an Aggregate Demand Shock



indicated by the dotted line in each upper panel, the perceived level of output at time zero is only about 0.1 percent above steady state. This deviation between perceived and actual output disappears after either one or three quarters, depending on which revision process was used to obtain the filter weights.

Given that the rise in reported output at time zero is largely attributed to measurement error, the optimal monetary policy only prescribes a rise of 10-15 basis points in the short-term nominal rate (as indicated by the solid line in each lower panel). As the true magnitude

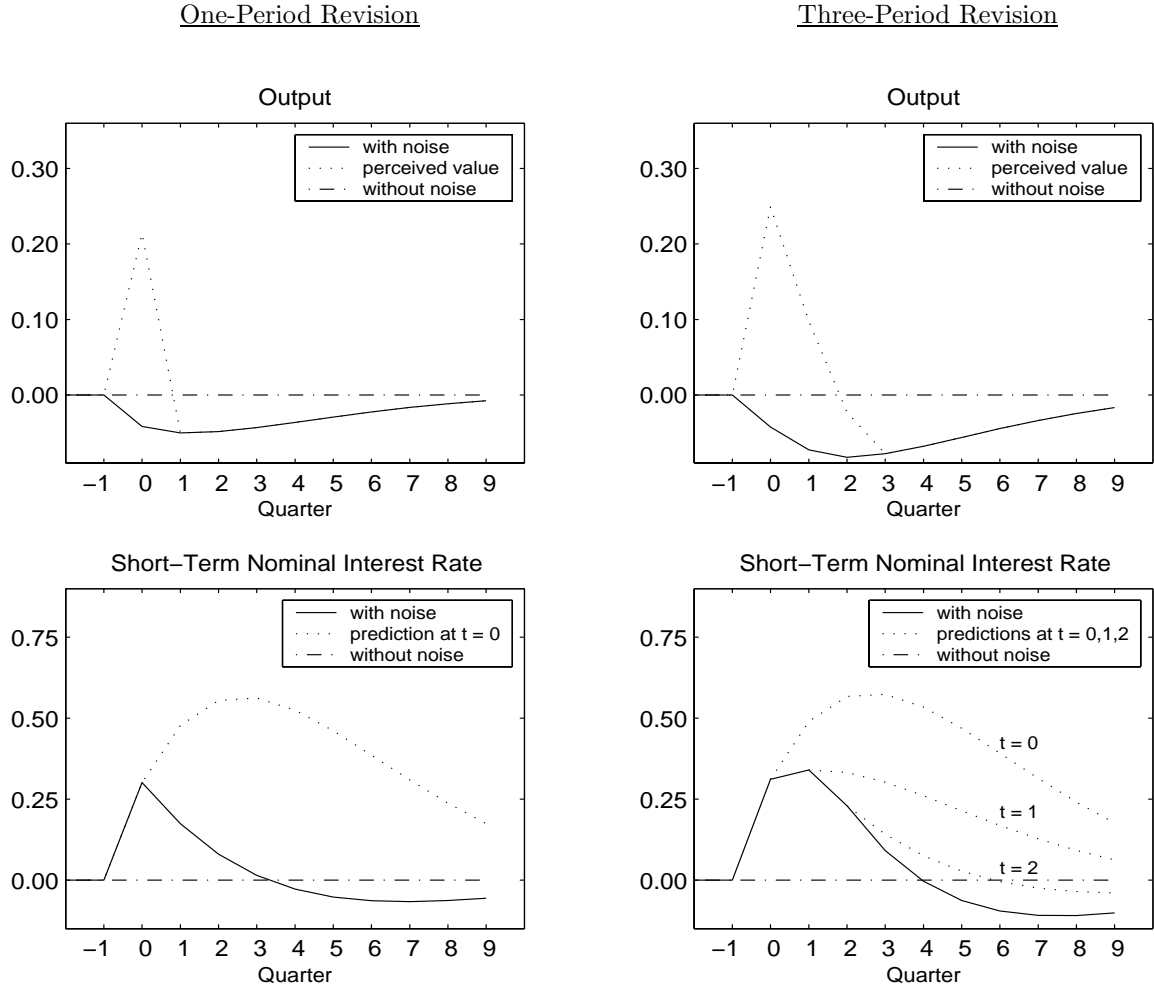
of the aggregate demand shock subsequently becomes clear, short-term rates rise by an additional 100 basis points and then return gradually towards baseline. In the initial period, however, private agents only expect a small amount of additional tightening over the next few quarters (as indicated by the dotted line), and hence the long-term interest rate (not shown) only rises slightly above baseline at time zero.

Evidently, the initial misperception of the aggregate demand shock is associated with non-trivial costs in terms of output stabilization. In particular, if the central bank immediately grasped the true magnitude of the demand shock, the short-term interest rate would jump by 75-100 basis points at time zero (as indicated by the dashed line in each lower panel). With a larger rise in long-term rates at time zero, the impact on real output would be about 0.1 percent less than under either calibrated data revision process (as indicated by the dashed line in each upper panel).

Figure 4 depicts the converse situation in which the reported level of output at time zero is subject to measurement error while the true level of aggregate demand is unchanged. The left column of panels shows the case in which the measurement error is corrected by a single revision one period later, and the right column of panels depicts the case of a three-period revision process.

In this experiment, the reported level of output at time zero contains a measurement error of nearly a full percentage point. Under optimal filtering, however, this observation is largely downweighted, and hence the perceived level of output at time zero is only 0.2 percent above baseline. Nevertheless, the optimal policy prescribes an immediate 25 basis point rise in the short-term nominal interest rate, and private agents expect short rates to rise further before returning gradually to baseline. With the resulting rise in long-term rates at time zero, the true level of output actually *declines* below baseline. Later, when the measurement error is corrected by the revision process, monetary policy reduces rates a bit below baseline to nudge the economy back in the right direction. Thus, while true output never deviates very far from steady state, the output mismeasurement shock does cause a noticeable fluctuation in short-term interest rates.

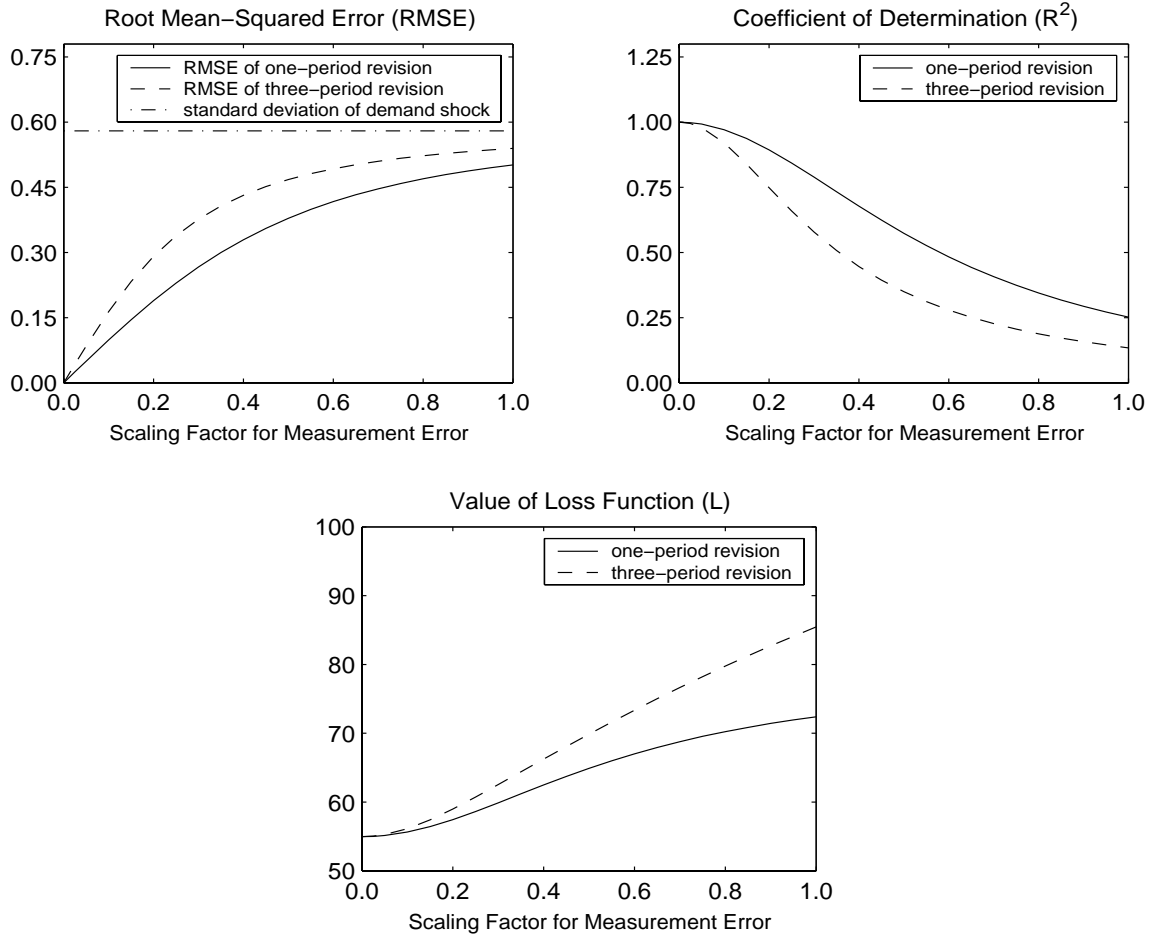
Figure 4: Impulse Responses to an Output Mismeasurement Shock



5.3 Quantifying the Information Problem

Finally, we quantify the information problem in terms of three measures that involve the unconditional second moments of the model variables. The upper panels of **Figure 5** indicate the RMSE and the R^2 associated with estimating the current aggregate demand shock, while the lower panel indicates the value of the policymaker's loss function, \mathcal{L} . In each panel, the solid line corresponds to the single-period revision process, while the dashed line corresponds to the three-period revision process.

Figure 5: Measures of the Information Problem



Of course, with no output mismeasurement, the estimated value of u_t^d has zero RMSE, and the R^2 equals unity. As the degree of output mismeasurement increases, the RMSE rises towards the asymptote of 0.58 percent (that is, the unconditional standard deviation of u_t^d), and the R^2 falls towards zero. With the calibrated data revision process ($\lambda = 1$), **Figure 5** indicates that the policymaker has relatively little information about the current aggregate demand shock; that is, the RMSE is close to 0.5 percent, while the R^2 is only 25 percent.

As shown in the lower panel of **Figure 5**, output mismeasurement generates non-

negligible costs in terms of higher output and inflation volatility, even under the optimal monetary policy. In particular, consider the policymaker’s loss function \mathcal{L} for the calibrated magnitude of mismeasurements ($\lambda = 1$) compared with the case of no measurement errors ($\lambda = 0$): the policymaker’s loss function is nearly 30 percent higher for the calibrated one-period revision process, and more than 50 percent higher for the calibrated three-period revision process. These results provide a clear rationale for investigating the extent to which additional indicator variables such as money can serve a useful role when reported output is subject to substantial measurement error.

6 The Potential Information Content of Money

In this section, we investigate the potential information content of money in a somewhat stylized setting in which reported output is subject to a single revision (as specified in equation (2)), while money demand is determined by the following quantity equation (instead of the more complicated dynamic specification in the baseline model of Section 3):

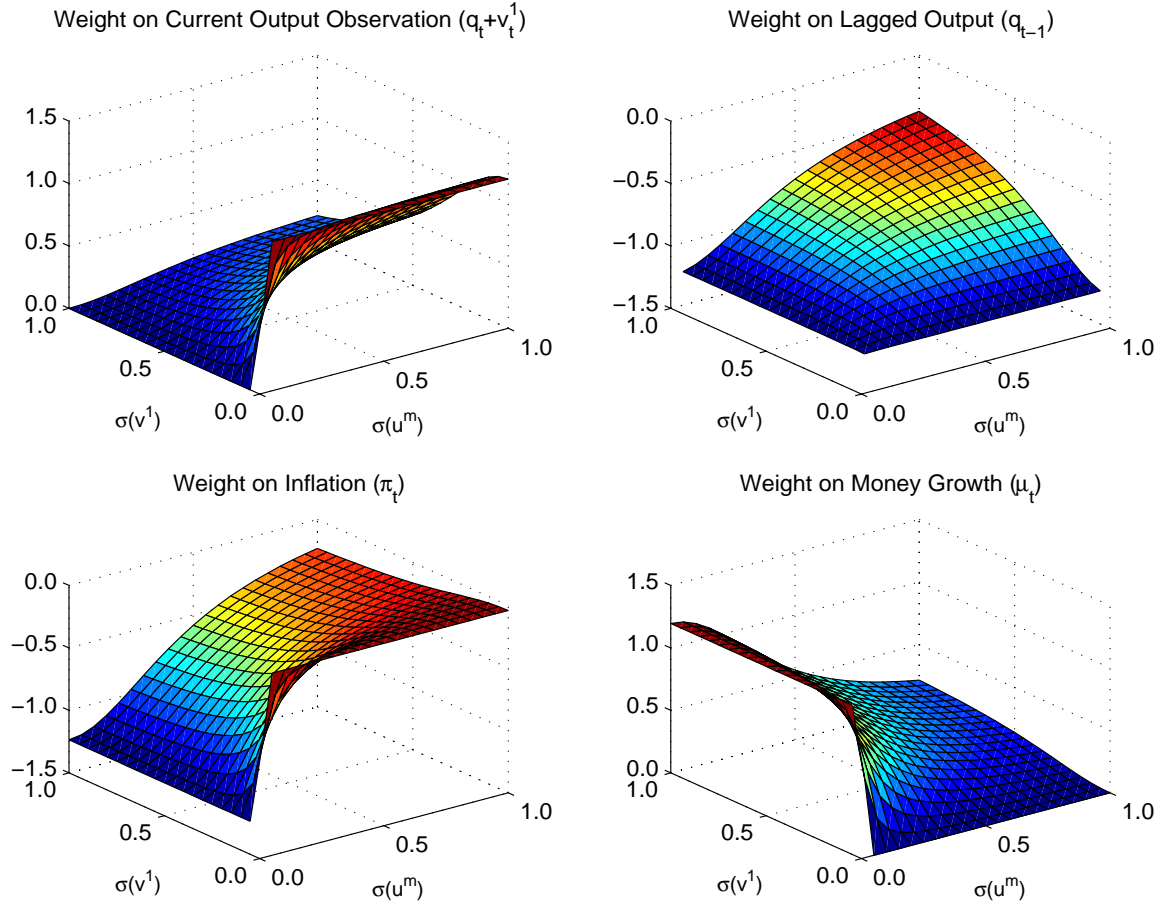
$$m_t - p_t = q_t + u_t^m, \tag{12}$$

where the exogenous disturbance u_t^m is assumed to be serially uncorrelated with mean zero and standard deviation $\sigma(u^m)$.

As discussed in Section 3, we assume that money demand evolves in response to the true level of output, and hence can serve as a useful indicator variable. In fact, under our assumption that aggregate prices are known by all agents, money would be a perfect indicator of true output in the limiting case with no money demand shocks ($\sigma(u^m) = 0$). On the other hand, of course, money would provide no useful additional information if output were observed without measurement error ($\sigma(v^1) = 0$). More generally, the role of money as an indicator variable will depend on the relative volatility of money demand disturbances and output mismeasurements.

Figure 6 indicates the optimal filtering weight on each of the four variables that play a role in estimating the current aggregate demand shock, namely, the noisy current output

Figure 6: Optimal Filtering Weights with Money as an Indicator Variable



observation $q_t^{(t)}$, the true value of lagged output q_{t-1} (which becomes known at time t), the inflation rate π_t , and the money growth rate $\mu_t = m_t - m_{t-1}$. In each panel, the filtering weight on the specified variable is measured on the vertical axis, while the two axes in the horizontal plane denote the standard deviations of the output measurement error $\sigma(v^1)$ and the money demand shock $\sigma(u^m)$, respectively.

With regard to the weights on current and lagged output and inflation as a function of the output measurement error, we confirm the findings for the case without money. As the measurement error regarding output increases the weights assigned to noisy current output,

lagged output and inflation decrease (in absolute terms). This is the case for any level of the standard deviation of the money demand shock as can be seen by moving from right to left along the dimension which corresponds to the output measurement error.

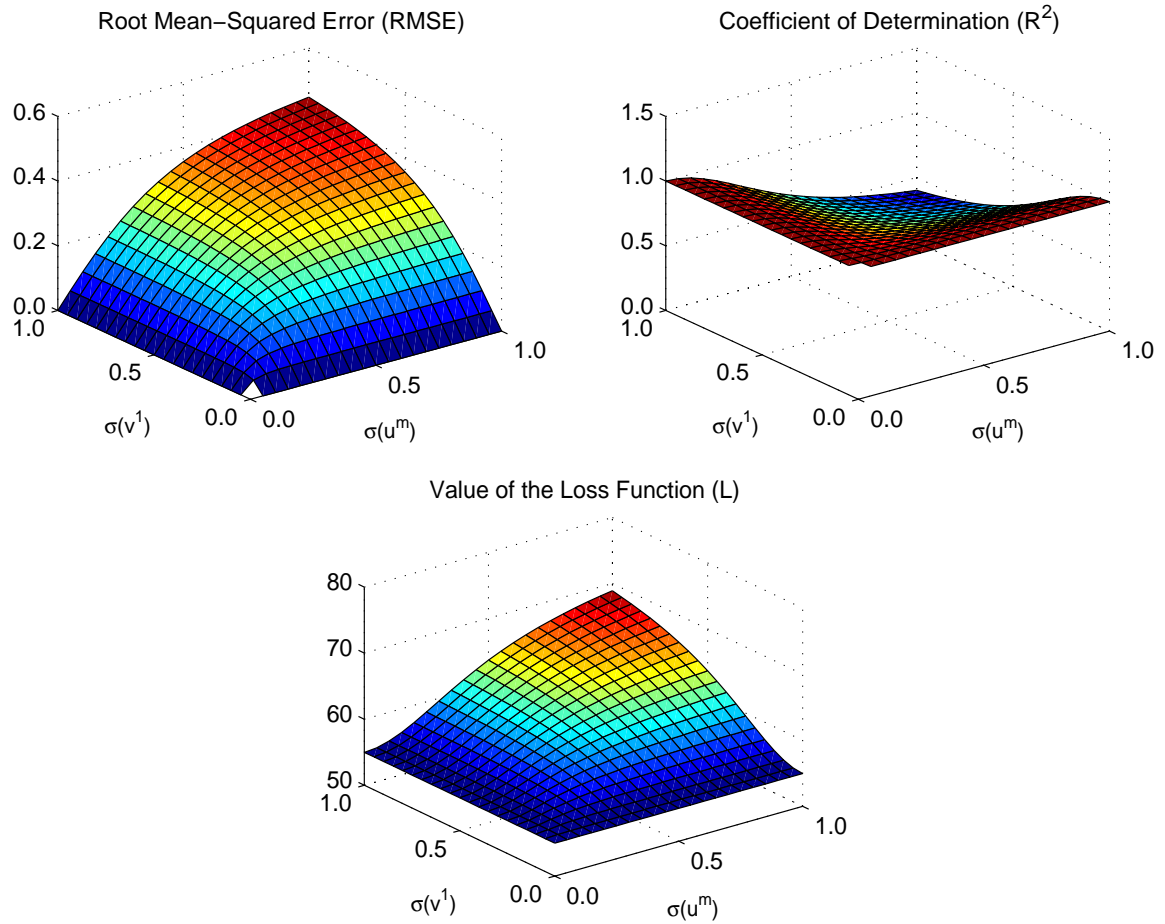
We find that money plays an significant role in estimating the current aggregate demand shock if the relative magnitude of the money demand shock is not too large. The weight assigned to money is largest in the absence of money demand shocks (that is, $\sigma(u^m) = 0$). In this case, the decision maker can infer the true value of output (and consequently, the aggregate demand shock) directly from the money growth rate, since lagged output and inflation are observed exactly.

As $\sigma(u^m)$ increases, however, the weight on money growth in the optimal estimate of output declines. With the possibility of money demand shocks, the decision maker cannot be sure whether a money growth observation that seems inconsistent with observed output is an indication of a mismeasurement of actual output or of a money demand shock. While the weight on money declines, one can see that it declines more slowly the greater the standard deviation of the output measurement error. Finally, it is of interest to note that a higher standard deviation of the money demand shock also reduces the absolute value of the weights on inflation and lagged output.

Figure 7 quantifies the information problem using the same measures as in the previous section. In particular, the top two panels show RMSE and R^2 associated with the contemporaneous estimate of the aggregate demand shock, while the lower panel indicates the value of the policymaker's loss function \mathcal{L} . The RMSE is zero and R^2 equals unity along either axis in the horizontal plane, corresponding to the special cases of no output mismeasurement ($\sigma(v^1) = 0$) or no money demand shocks ($\sigma(u^m) = 0$). In all three panels, the surface of the graph varies monotonically with the values of $\sigma(v^1)$ and $\sigma(u^m)$, as long as both standard deviations are strictly positive.

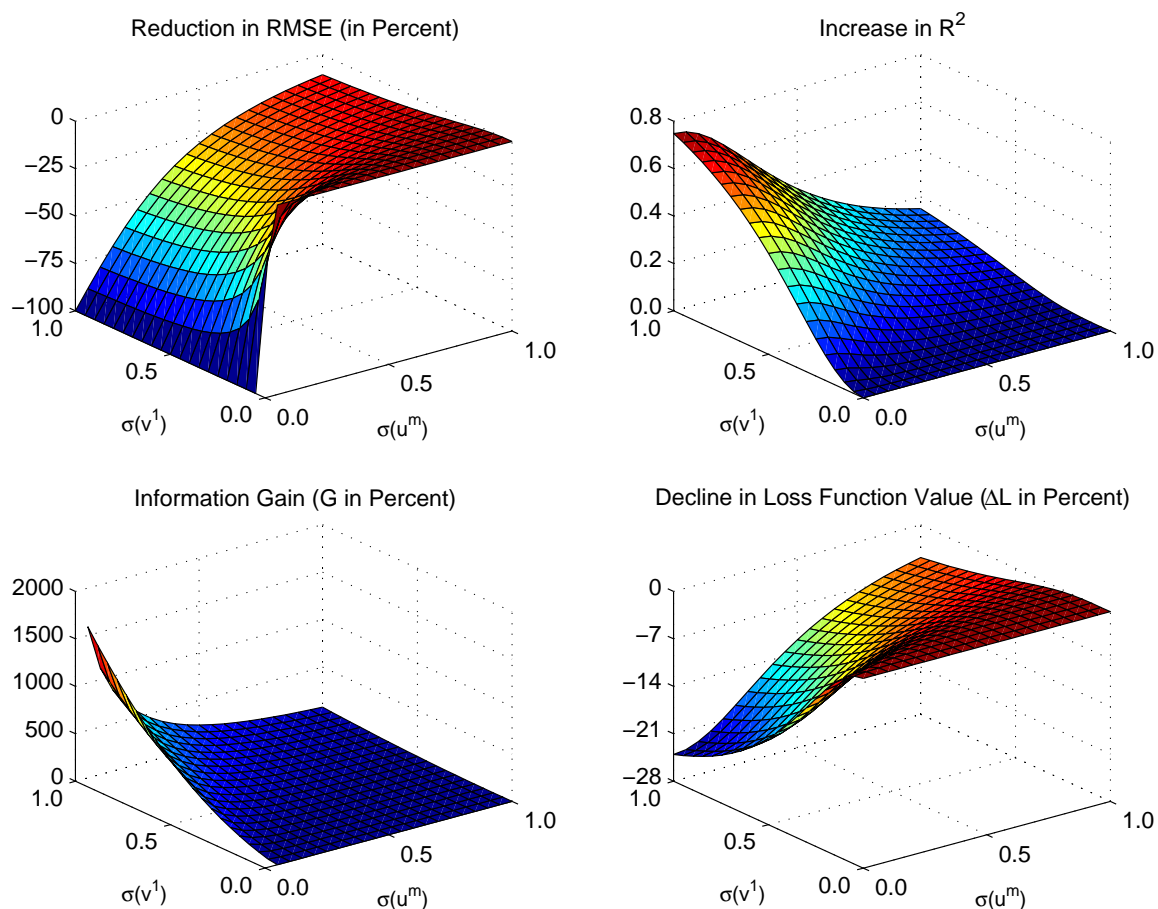
Finally, **Figure 8** depicts four measures of the information content of money in estimating the aggregate demand shock: the reduction in RMSE (in percent), the improvement in R^2 , the expected relative gain in information \mathcal{G} (in percent), and the reduction in the

Figure 7: The Information Problem with Money as an Indicator Variable



policymaker's loss function \mathcal{L} (in percent). When output measurement errors are relatively large compared with money demand shocks (that is, $\sigma(v^1) > 0.5$ percent and $\sigma(u^m) < 0.5$ percent), the improvement in the quality of the estimate can be substantial: in such cases, money reduces the RMSE by 20 percent or more and raises the R^2 by at least 0.18. The information gain from utilizing money as an information variable is also very high under these conditions, while the policymaker's loss function can be reduced by as much as 25 percent.

Figure 8: The Information Content of Money



7 The Quantitative Significance of Money as an Indicator Variable

Having illustrated the role of money in a somewhat simplified model, we now proceed to quantify the information content of money using the full model given in Section 3. In particular, we utilize the empirical money demand equation given in model equation (M-4) with the estimated standard deviation of the money demand shock, and we consider the 3-quarter output revision process given in equation (3) as well as the simpler 1-quarter revision process (given by equation (2)) that was used in the previous section. We also

consider the implications of the increased magnitude of money demand disturbances during the period following the implementation of EMU in early 1999.

7.1 The Optimal Indicator Weights

Table 4 indicates the optimal weight on each indicator variable used in estimating the current aggregate demand shock, u_t^d , for each specification of the output revision process. The upper panel shows these weights when money is not included in the information set, while the lower panel indicates the weights when current money growth is utilized in constructing the optimal estimate. In the latter case, we consider two alternative values for the standard deviation of money demand shocks: the estimate obtained by Coenen and Vega (2001) for the pre-EMU period (1980:4 to 1998:4), which is equal to 0.23 percent, and the standard deviation of one-step-ahead forecast errors associated with this money demand equation over the more recent period (1999:1 to 2000:4), which amounts to 0.43 percent. While the latter value is markedly higher, this may be viewed as a temporary phenomenon associated with the initial transition to EMU.

When the revision process is completed in a single period, the previous period's output is known with certainty (that is, $q_{t-1}^{(t)} = q_{t-1}$). In this case, longer lags of output do not contain any additional information regarding the period t aggregate demand shock. Thus, as shown by the first row of the upper panel, the optimal filter places non-zero weight on the noisy current output observation $q_t^{(t)}$, the previous period's output level q_{t-1} , and the current inflation rate π_t .

In contrast, when the revision process takes three periods, the current and previous two output observations contain measurement error, while the true value of q_{t-3} is revealed in the latest data vintage. In this case, as indicated in the second row of the upper panel, the optimal filter places non-zero weight on all four output observations (that is, $q_t^{(t)}$, $q_{t-1}^{(t)}$, $q_{t-2}^{(t)}$, and $q_{t-3}^{(t)}$) as well as the current inflation rate.

When current money growth is included in the information set, we see that this indicator variable receives substantial weight in estimating the current aggregate demand shock. The

Table 4: Optimal Indicator Weights for Estimating the Aggregate Demand Shock

Revision Process	$q_t^{(t)}$	$q_{t-1}^{(t)}$	$q_{t-2}^{(t)}$	$q_{t-3}^{(t)}$	π_t	μ_t
<u>Filtering without Money</u>						
One-Period Revision	0.275	-0.287	0	0	-0.009	—
Three-Period Revision	0.140	-0.055	-0.051	-0.020	-0.005	—
<u>Filtering with Money</u>						
A. Pre-EMU Standard Deviation of Money Demand Shock						
One-Period Revision	0.281	-0.229	0	0	0.012	0.356
Three-Period Revision	0.150	-0.042	-0.070	-0.047	0.018	0.384
B. Initial-EMU Standard Deviation of Money Demand Shock						
One-Period Revision	0.277	-0.271	0	0	-0.003	0.101
Three-Period Revision	0.143	-0.051	-0.057	-0.027	0.002	0.114

precise weight depends on the specification of the output revision process and the standard deviation of money demand shocks, but the notable point is that the magnitude of this weight is roughly similar to that placed on the noisy current output observation. Of course, interpreting the specific pattern of filtering weights is rather difficult, and hence we now proceed to quantify the information content of money using the measures described above.

7.2 Measures of Information Content

Table 5 characterizes the information role of money in estimating the current aggregate demand shock under each of the two assumptions about the revision process. As a benchmark for comparison, the first three columns of the table provide results for the case when money growth is included in the information set, namely, the RMSE of the demand shock estimate, the associated R^2 , and the value of the policymaker's loss function \mathcal{L} . Evidently, the precision of the demand shock estimates is not very high, regardless of how the output

Table 5: The Information Role of Money in the Estimated Model

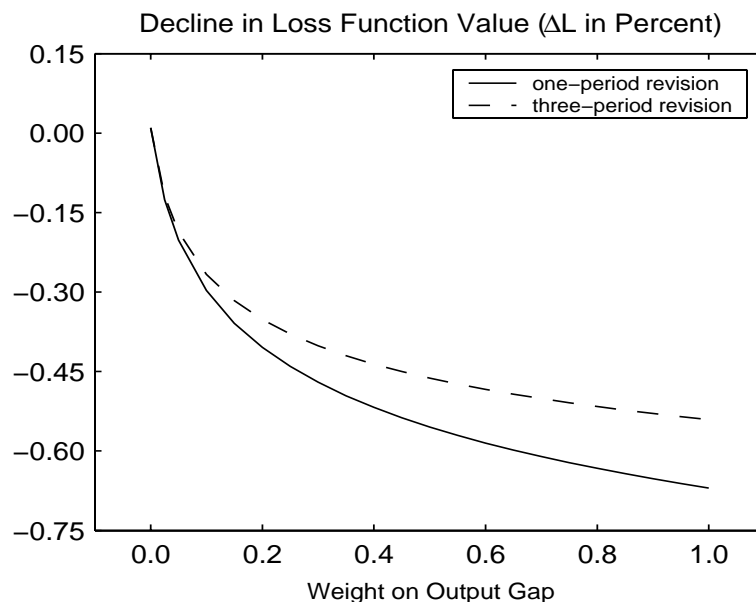
Revision Process	RMSE	R^2	\mathcal{L}	Information Content			
				$\% \Delta \text{RMSE}$	ΔR^2	$\% \mathcal{G}$	$\% \Delta \mathcal{L}$
A. Pre-EMU Standard Deviation of Money Demand Shock							
One-Period Revision	0.49	0.28	71.73	-1.17	0.02	7.72	-0.55
Three-Period Revision	0.53	0.16	85.05	-1.33	0.02	18.63	-0.46
B. Initial-EMU Standard Deviation of Money Demand Shock							
One-Period Revision	0.50	0.27	72.01	-0.34	0.01	2.23	-0.16
Three-Period Revision	0.54	0.14	85.33	-0.41	0.01	5.64	-0.14

revision process is specified.

The remainder of **Table 5** reveals the extent to which current money growth increases the precision of the estimated demand shock and thereby reduces the policymaker's loss function compared to the situation when money is not included in the information set. In particular, the RMSE is reduced by about 1.5 percent, while the R^2 increases by about 0.02. Measured in terms of lower entropy, the information gain \mathcal{G} is somewhat more impressive: about 10 to 20 percent, depending on the specification of the output revision process. Finally, as indicated in the last column, using money as an indicator variable reduces the policymaker's loss function \mathcal{L} by about 0.5 percent.

The lower panel of **Table 5** shows that these measures of the information content of money are markedly lower when we calibrate the standard deviation of money demand shocks to the results for the early-EMU period. These results are completely consistent with the findings of the previous section: for a given degree of output mismeasurement, the information content of money shrinks with the magnitude of innovations to money demand.

Figure 9: Sensitivity to the Policymaker's Stabilization Objectives



7.3 Further Sensitivity Analysis

Now we briefly summarize some additional sensitivity analysis regarding the results presented above.

First, it is useful to consider the extent to which the information content of money depends on the stabilization objectives of monetary policy. In our framework, money is potentially useful in making inferences about the contemporaneous level of real output, but provides no information about current inflation (for which the reported data is subject to little or no revision) and negligible information regarding future inflation.³⁰ Thus, in this context money will have negligible information content in a strict inflation targeting regime, but may serve as an important indicator variable for a central bank that places substantial weight on stabilizing both output and inflation.

Figure 9 shows how the information content of money (as measured by $\% \Delta \mathcal{L}$) varies

³⁰The latter point is documented in our earlier working paper; cf. Coenen et al. (2002).

Table 6: Sensitivity to the Income Elasticity of Money Demand

Information Measure	Revision Process	Scaling Factor for the Income Elasticity of Money Demand		
		1	2	4
% Δ RMSE	One-Period Revision	-1.17	-4.46	-14.94
	Three-Period Revision	-1.33	-5.25	-16.46
ΔR^2	One-Period Revision	0.02	0.06	0.20
	Three-Period Revision	0.02	0.09	0.26
% \mathcal{G}	One-Period Revision	7.72	29.85	105.86
	Three-Period Revision	18.63	74.85	249.86
% $\Delta \mathcal{L}$	One-Period Revision	-0.55	-2.07	-6.57
	Three-Period Revision	-0.46	-1.23	-3.54

with the relative weight on output volatility in the objective function. As noted in Section 3, our baseline analysis has assumed that the policymaker only places half as much weight on output volatility compared with inflation volatility; in this case, the use of money as an indicator variable reduces the loss function by about 0.5 percent. The reduction in the loss function is somewhat greater for a policymaker who places equal weight on stabilizing both output and inflation, and is negligible for a policymaker focused solely on stabilizing the inflation rate.

Second, it is worthwhile to consider the implications of alternative money demand specifications with a stronger contemporaneous relationship between money and output. For the preceding analysis, we have used the demand function for M3 because this money aggregate has exhibited reasonable stability in the euro area over the past two decades (cf. Coenen and Vega (2001)). However, the estimated coefficient that determines the instantaneous income elasticity of M3 is rather small. Narrower money aggregates (such as M1) typi-

Table 7: Optimal Indicator Weights when Current Output is Unobserved

Revision Process	$q_t^{(t)}$	$q_{t-1}^{(t)}$	$q_{t-2}^{(t)}$	$q_{t-3}^{(t)}$	π_t	μ_t
One-Period Revision	—	0.082	0	0	0.028	0.465
Three-Period Revision	—	0.026	-0.020	-0.033	0.026	0.436

cally have a much tighter relationship with current output. As shown in **Table 6**, we find that a higher instantaneous income elasticity substantially raises the information content of money. Thus, to the extent that monetary analysis can identify structural changes and special factors that generate shifts in the demand function for a narrow aggregate (such as M1), the central bank would be able to utilize such an aggregate in reducing the data uncertainty associated with current output.

Finally, we have assumed for simplicity that private agents and the central bank are able to utilize a noisy estimate of contemporaneous output at each point in time. Given the actual time delays in releasing GDP data, however, it may be more realistic to assume that no output estimate is available until the subsequent period (especially since our model is specified at a quarterly frequency). As shown in **Tables 7 and 8**, the importance of using money as an indicator variable increases in this case.

Table 8: The Information Role of Money when Current Output is Unobserved

Revision Process	RMSE	R^2	\mathcal{L}	Information Content			
				$\% \Delta \text{RMSE}$	ΔR^2	$\% \mathcal{G}$	$\% \Delta \mathcal{L}$
One-Period Revision	0.57	0.03	77.53	-1.58	0.03	—	-0.93
Three-Period Revision	0.57	0.03	92.54	-1.49	0.03	—	-0.69

Note: The measure of information content \mathcal{G} is not well-defined for the particular estimation problem when current output is unobserved.

8 Conclusion

To explore the information role of money in the presence of data uncertainty we have extended the euro area macroeconomic model of Coenen and Wieland (2000) by incorporating the euro area-wide money demand model of Coenen and Vega (2001) and an empirically calibrated model of the revision process of aggregate euro area output. Using this framework we have found that money can play an important role as an information variable and may result in major improvements in current output estimates. However, the specific nature of this role depends on the magnitude of the output measurement error relative to the money demand shock.

In particular, we have found noticeable but small improvements in output estimates due to the inclusion of money growth in the information set when the standard deviation of money demand shocks equals the estimated value from Coenen and Vega (2001). Sensitivity analyses indicate that money plays a quantitatively more important role with regard to output estimation if the central bank assigns a high weight to output stabilization, if current output is completely unobserved, or if the direct contemporaneous link between output and money growth strengthens. Of course, as the construction of euro area aggregate output data is improved over time, the magnitude of the revisions discussed in Section 2 is likely to decline. Nevertheless, evidence concerning U.S. data vintages collected by Croushore and Stark (1999) indicates that data uncertainty will remain an important issue even once the data collection technology has matured.

Throughout the paper we consider a relatively limited role of money by focusing exclusively on the information content of money with respect to output measurement and by excluding the possibility of a direct role of money in output and inflation determination. In this sense, our quantitative results only indicate a lower bound on the usefulness of money. An alternative model that allows for significant direct effects of money on inflation and could be used in future research is the so-called P^* model.

Also, as noted earlier, we have focused attention on a framework with symmetric in-

formation regarding aggregate output data as far as private market participants and the central bank are concerned. We have also conducted some exploratory analysis under the assumption of asymmetric information regarding aggregate data that is used by Dotsey and Hornstein (2000) and Svensson and Woodford (2001). However, in our view this assumption is undesirable if it implies that a representative agent by knowing his individual income can also infer aggregate income and demand while the policymaker only observes a noisy estimate of aggregate demand. We plan to study the asymmetric case in more detail in the future in a model that would allow us to differentiate more carefully between individual and aggregate uncertainty.

Finally, another interesting avenue for future research would be to compare optimal filtering to simple filtering rules in keeping with the recent debate on optimal versus simple monetary policy rules. For example, one could investigate the performance of simple rules that respond only to observed output growth, inflation and money growth instead of optimal estimates of the output gap. A recent study that considers an example of a simple filtering rule in the context of NAIRU uncertainty is Meyer, Swanson and Wieland (2001).

References

- Angeloni, I., C. Cottarelli and A. Levy, 1994, Cross-border deposits, aggregation, and money demand in the transition to EMU, *Journal of Policy Modelling*, 16, 27-54.
- Brayton, F., and P. Tinsley, 1996, A guide to FRB/US: A macroeconomic model of the United States, Finance and Economics Discussion Series, 1996-42, Federal Reserve Board.
- Buiter, W. H. and I. Jewitt, 1981, Staggered wage setting with real wage relativities: Variations on a theme of Taylor, *The Manchester School*, 49, 211-228, reprinted in W. H. Buiter (ed.), 1989, *Macroeconomic theory and stabilization policy*, Manchester: Manchester University Press.
- Coenen, G. and J. L. Vega, 2001, The demand for M3 in the euro area, *Journal of Applied Econometrics*, 16, 727-748.
- Coenen, G. and V. Wieland, 2000, A small estimated euro area model with rational expectations and nominal rigidities, ECB Working Paper No. 30, European Central Bank, September.
- Coenen, G., A. Levin and V. Wieland, 2001, Data uncertainty and the role of money as an information variable for monetary policy, ECB Working Paper No. 84, European Central Bank, November.
- Croushore, D. and T. Stark, 1999, A real-time data set for macroeconomists, Federal Reserve Bank of Philadelphia Working Paper, 99-4, June.
- Dotsey, M. and A. Hornstein, 2000, Is the behavior of money useful to a discretionary policy maker?, mimeo, Federal Reserve Bank of Richmond.
- Dotsey, M., C. Lantz, and L. Santucci, 2000, Is money useful in the conduct of monetary policy?, Federal Reserve Bank of Richmond *Economic Quarterly*, 86, 4, 23-48.
- European Central Bank, 2001, Framework and tools of monetary analysis, *ECB Monthly Bulletin*, May, 41-58.
- Ehrmann, M. and F. Smets, 2000, Uncertain potential output: Implications for monetary policy, ECB Working Paper No. 59, European Central Bank, April.
- Fagan, G., J. Henry and R. Mestre, 2001, An area-wide model (AWM) for the euro area, ECB Working Paper, No. 42, European Central Bank, January.
- Friedman, B. M., 1975, Targets, instruments and indicators of monetary policy, *Journal of Monetary Economics*, 1, 443-473.
- Friedman, B. M., 1990, Targets and instruments of monetary policy, in B. M. Friedman and F. H. Hahn (eds.), *Handbook of monetary economics*, Amsterdam: North Holland,

1185-1230.

Fuhrer, J. C., 1997, Inflation/Output Variability Trade-Offs and Optimal Monetary Policy, *Journal of Money, Credit and Banking*, 29, 214-234.

Fuhrer, J. C. and G. R. Moore, 1995, Inflation persistence, *Quarterly Journal of Economics*, 110, 127-159.

Gerlach, S. and L. E. O. Svensson, 2000, Money and inflation in the euro area: A case for monetary indicators?, *BIS Working Paper No. 98*.

Hallman, J. J., R. D. Porter and D. H. Small, 1991, Is the price level tied to the M2 aggregate in the long run?, *American Economic Review*, 81, 841-858.

Ireland, P. N., 2001, Money's role in the monetary business cycle, paper presented at the ECB workshop on "Stochastic dynamic general-equilibrium (SDGE) models and their uses in monetary policy" held at the ECB in Frankfurt on 5-6 June 2001.

Kareken, J. H., T. Muench and N. Wallace, 1973, Optimal open market strategies: The use of information variables, *American Economic Review*, 63, 156-172.

Leahy, J., 2001, Comment on McCallum: Monetary policy analysis in models without money, *Federal Reserve Bank of St. Louis Review*, 83, 4, 161-163.

McCallum, B. T., 2001, Monetary policy analysis in models without money, *Federal Reserve Bank of St. Louis Review*, 83, 4, 144-160.

Meyer, L. H., E. T. Swanson and V. Wieland, 2001, NAIRU Uncertainty and Nonlinear Policy Rules, *American Economic Review*, 91, 226-231.

Nicoletti-Altimari, S., 2001, Does money lead inflation in the euro area?, *ECB Working Paper No. 63*, European Central Bank, May.

Orphanides, A., 1998, Monetary Policy evaluation with noisy information, *Finance and Economics Discussion Series, 98-50*, Board of Governors of the Federal Reserve System.

Orphanides, A., 2000, The quest for prosperity without inflation, *ECB Working Paper No. 15*.

Orphanides, A., R. D. Porter, D. Reifschneider, R. Tetlow and F. Finan, 2000, Errors in the measurement of the output gap and the design of monetary policy, *Journal of Economics and Business, 52*, 117-143. *Finance and Economics Discussion Series, 99-45*, Board of Governors of the Federal Reserve System.

Orphanides, A. and V. Wieland, 1998, Price stability and monetary policy effectiveness when nominal interest rates are bounded at zero, *Finance and Economics Discussion Series, 98-35*, Board of Governors of the Federal Reserve System.

- Pearlman, J. G., D. Currie and P. Levine, 1986, Rational expectations models with partial information, *Economic Modelling*, 3, 90-105.
- Rotemberg, J. and M. Woodford, 1997, An optimization-based econometric framework for the evaluation of monetary policy, in *NBER Macroeconomics Annual 1997*, Cambridge, MA: The MIT Press, 297-346.
- Rudebusch, G. D., 2000, Is the Fed too timid? Monetary policy in an uncertain world, Federal Reserve Bank of San Francisco.
- Sargent, T. J., 1989, Two models of measurement and the investment accelerator, *Journal of Political Economy*, 97, 251-287.
- Svensson, L. E. O. and M. Woodford, 2000, Indicator variables for optimal policy, ECB Working Paper No. 12.
- Svensson, L. E. O. and M. Woodford, 2001, Indicators for optimal policy under asymmetric information, NBER Working Paper No. 8255.
- Taylor, J. B., 1980, Aggregate dynamics and staggered contracts, *Journal of Political Economy*, 88, 1-24.
- Taylor, J. B., 1993a, *Macroeconomic policy in the world economy: From econometric design to practical operation*, New York: W.W. Norton.
- Taylor, J. B., 1993b, Discretion versus policy rules in practice, *Carnegie-Rochester Conference Series on Public Policy*, 39, 195-214.
- Theil, H., 1967, *Economics and information theory*, Amsterdam: North-Holland.
- Tinsley, P. A., P. A. Spindt and M. E. Friar, 1980, Indicator and filter attributes of monetary aggregates: a nit-picking case for disaggregation, *Journal of Econometrics*, 14, 61-91.
- Tödter, K. H. and H. E. Reimers, 1994, P-Star as a link between money and prices in Germany, *Weltwirtschaftliches Archiv*, 130, 273-289.
- Wieland, V., 1998, Monetary policy under uncertainty about the natural unemployment rate, *Finance and Economics Discussion Series*, 98-22, Board of Governors of the Federal Reserve System.